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Mechanics of Continuum Manipulators, A Comparative Study of Five Methods with Experiments

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Abstract.: Investigations on control and optimization of continuum manipulators have resulted in a number of kinematic and dynamic modeling approaches each having their own advantages and limitations in various applications. In this paper, a comparative study of five main methods in the literature for kinematic, static and dynamic modeling of continuum manipulators is presented in a unified mathematical framework. The five widely used methods of Lumped system dynamic model, Constant curvature, two-step modified constant curvature, variable curvature Cosserat rod and beam theory approach, and series solution identification are re-viewed here with derivation details in order to clarify their methodological differences. A comparison between computer simulations and experimental results using a STIFF-FLOP continuum manipulator is presented to study the advantages of each modeling method.

Keywords: continuum manipulator, dynamic, lumped system, constant curvature, variable curvature, Cosserat, beam theory, series solution, experiments.

1 INTRODUCTION

Traditional limitations posed by conventional rigid linked robots, such as vast occupied space, rigidity, and relatively low dexterity, has resulted in an emerging trend during these recent years for scientists to show increasing interest in the concept of continuum robots. Taking inspiration from biological examples such as octopus tentacles, chameleon tongue and elephant trunk, researchers are looking into the possibility of replicating similar navigational and grasping characteristics by harnessing the hyper-redundant

dancy demonstrated in nature [1]. This class of robots promises considerable performance improvements in different areas which currently witness the presence of traditional robots, such as surgical applications, underwater manipulation, rehabilitation, repair, etc. [2], [3], [4]. As a natural by-product of this trend, kinematic and dynamic modeling and analysis of these types of robots have gained similar attention within the research community. However, the inherent nature of continuum robots being highly deformable has put forward new challenges in this regard.

The most popular kinematic model for a bent continuum manipulator is provided by the Constant Curvature (CC) model. The CC model simplifies the kinematics of a continuum manipulator by assuming that the backbone kinematics in a planar deformed state can be expressed by a CC profile. In [3], Webster and Jones deliver a thorough discussion on this subject and reviewed several methods for kinematic modelling of continuum robots using two separate sub-mappings: a robot-specific map relating the mechanics of the loads with the system material strains, and a general map to relate these strains to the spatial kinematic configuration of the manipulator. The general robot-independent mapping might suffer from singularity, as discussed in [5], where a new shape function approach suggested by Godage can handle this limitation. Although being commonly used as a simplifying assumption, the CC assumptions are usually not accurate in the presence of external forces. The recent dynamic models to address this limitation can be categorized into four groups: 1) Lumped model elements using Lagrangian demonstration, which approximates the continuum manipulator with a number of rigid-link pieces combined with springs and dampers in-between [6]. The total kinetic energy of the system as the degrees of freedom goes to infinity is similar to that of a continuum manipulator. 2) The Euler-Bernoulli beam model [7,8] and/or the Principle of Virtual Work (PVW) [9,10] using CC kinematic. 3) Variable Curvature (VC) kinematics [11] with Cosserat rod model (or equivalent beam theory method), as explained in [12,13] which leads to an optimization boundary value problem (BVP). Tunay presented an approximate solution for the weak-form integral equations resulting from this BVP in a finite element discretized form [14]. 4) Approximate solutions based on the identification of the system with a polynomial [15], or shape function based series solutions [5] which construct a setup-specific model. These identified models are similar to single direct shooting methods in solving BVPs in mechanics of material community. The approximate solutions, appropriate for control purposes, are more precise and faster in terms of computation but do not account for the structural characteristics, while the lumped model elements and Cosserat rod model suffer from heavy calculations despite being suitable for design and optimization. In most of these methods, numerical inaccuracy and singularities in deriving the inverse kinematics are inevitable. We have recently introduced a new analytical method to model compound continuum manipulators in presence of external forces utilizing the principle of virtual work and experimental observation of the deformed system in which the CC is an initial but not essential assumption for the kinematic map. This can be categorized as an approximate solution which incorporates the compound structural characteristics of a continuum manipulator and provides a deformation map for the manipulator cross-section.

Fig. 1. Experiment with body loads: a) base, b) magnetic trackers, c) STIFF-FLOP module, d) force sensor, e) Aurora tracker (I), experiments with extensive external force (II), a pneumatic braided extensile chamber (III).

In this paper, a comparative study between five modeling approaches for kinematic, static and dynamic of a continuum manipulator is presented and their accuracy, advantages and limitations are discussed in comparison to experimental results using a STIFF-FLOP continuum actuator module [2,16] (Fig. 1.I,II). In the process; simple mathematic derivation of the manipulator deformation energy for the body and braided actuator chambers and mechanical maps based on Neo-Hookean assumptions are discussed; a comparison between different models for a braided extensile pneumatic actuator is presented and a novel tow-step modified solution is presented based on CC kinematics and Castigliano's method for beam deflections to enable using of CC assumption in the presence of high body and external loading.

In section II a review of derivation and implementation of the five modelling methods for continuum manipulators (Lumped system dynamic model, CC, two-step modified CC, VC Cosserat rod and beam theory approach, and series solution identification) are presented. The models' applications are discussed based on our simulation result performance and accuracy in section III followed by conclusion and discussion.

2 Modelling, Simulation and Experimental Framework

The spatial orientation of a continuum manipulator as the system control outputs can be derived based on CC or VC kinematics. This general map can be used as the system control model by relating the actuator chambers' length and manipulator tip position and orientation; however, a more precise model should consider the mechanical properties of the manipulator too [12]. Slenderness and softness of most continuum manipulators make external and body forces important in the modeling of their behavior. Body specific static and dynamic map models are used to relate these loads with the kinematic parameters of a continuum manipulator as are discussed in this section.

The simulation results are based on experimental setup dimensions (Table 1) and compared with the results from three set of experiments; 1) random pressurization of a STIFF-FLOP module in a two-module manipulator to study 3D deformation under extensive body weight (Fig. 1.I); 2) planar deformation of a module with extensive external load at the tip (Fig. 1.II); and 3) actuation of a pneumatic braided extensile chamber (Fig. 1.III). The error is defined as the difference in the length of the tip position vector in simulation and experiments and the error percentage is the ratio of the error to the experimental value of the tip position vector length.

Table. 1.
Experiment pa-
rameters.

Par.	Value
r_{p1} [mm]	2.5
r_{p2} [mm]	3
r_0 [mm]	8.5
γ_0 [deg]	89
r_1 [mm]	4.5
r_2 [mm]	12.5
l [mm]	44.0
E [KPa]	100
m_b [Kg]	0.0245
m_e [Kg]	0.011
g [m/s ²]	9.81
μ_t [Ns/m]	0.1
μ_r [Ns/m]	5e-4

Algorithm. 1. Sample “AutoTMTDyn” input code for lumped model of planar motion

```

body(i).type = 'rigid' ;   body type of disk i
body(i).m = m_b/n ;      disk i mass, n: number of disks
body(i).l = l_b/n ;      disk i length
body(i).l_com = [ 0 0 l/(2n) ] ;   disk i COM position
joint(i).first = i-1 ;    joint i 1st body
joint(i).second = i ;     joint i 2nd body
joint(i).tr(1).trans = [ 0 0 l_b/n ] ;   joint i 1st translation
joint(i).tr(1).rot = [ 0 0 ] ;   joint i 1st rotation
joint(i).tr(2).trans = [ 0 0 inf ] ;   joint i 2nd translation, inf: for DOF
joint(i).tr(2).rot = [ 2 inf ] ;   joint i 2nd translation
joint(i).dof(1).init = 0 ;   1st DOF initial condition
joint(i).dof(2).init = 1e-5 ;   2nd DOF initial condition
joint(i).dof(1).spring = [ k_i, 0 ] ;   1st DOF spring coeff. & initial value
joint(i).dof(1).damp = [  $\mu$ , 0 ] ;   1st DOF viscous & Coulomb damping
joint(i).dof(2).spring = [ k_i, 0 ] ;   2nd DOF spring coeff. & initial value
joint(i).dof(2).damp = [  $\mu$ , 0 ] ;   2nd DOF viscous & Coulomb damping
exload(1).body = n ;   external load reference body
exload(1).tr(1).trans = [ 0 0 l/n ] ;   tip external load relative position
exload(1).tr(1).rot = [ 0 0 ] ;   tip external load relative orientation

```

Equivalent Lumped System Model

- *Kinematics*: A continuum manipulator can be assumed as a highly articulated rigid link system with an infinite number of disks connected through spring-damper supported spherical joints [6], [17] (Fig. 2.II). We used an axial transformation (ρ) followed by a 3-2-1 Euler angle set (η, ζ, ξ),

$$R_{ri} = R_z(\eta_i) \cdot R_y(\zeta_i) \cdot R_x(\xi_i) \rightarrow \Gamma_{ri} = \begin{bmatrix} R_{ri} & \rho_i \\ 0 & 1 \end{bmatrix}, \quad R_i = \prod_{j=1}^i R_{rj}, \quad \Gamma_i = \prod_{j=1}^i \Gamma_{rj}, \quad (1)$$

where R_a is the rotation matrix around a axis in the local frame, Γ is the transformation matrix, ρ is the axial transformation vector, subscript r denotes the relative rotation matrix between each two disks, i denotes the disk number and j is a general numerator. We use C_x for $\cos(x)$ and S_x for $\sin(x)$. Centre of masses (COM) position vector ($\rho_i = \Gamma_i \rho_{ri}$), linear velocity ($\dot{\rho}_i$) and angular velocity ($[\omega_{ci}]_\times = R_{i,t} \cdot R_i^T$) in Cartesian coordinates and w.r.t the spatial frame can be found afterward. $[\omega]_\times$ is the skew-symmetric matrix of the vector ω and subscript comma (.) means derivative w.r.t. the following parameter. The axial linear transformation and Euler angle rotations of all joints can be assumed to be equal to model a uniformly deforming link [18] while assuming separate free DOFs results in a VC kinematic model. Joints' stiffness and damping values can be identified from experimental results or can be derived using deformation energy of elastic material [6]. We left these two values to be constant. Cross-sectional deformation can be considered as a change in the inertia and diameter of the rigid links.

- *Dynamic Model*: Having the stiffness values and the gravitational potential energy function based on the initial shape of the manipulator, principle of virtual work (PVW) in static case and Lagrange method in the dynamic case can be used to model the system mechanics [6]. A high DOF BVP is formed in the static case and can be solved by a numerical optimization method. The dynamic equations are more efficient to be solved

Fig. 2. Lumped system dynamic model results for a model with three rigid links and independent joints compared to the experimental results with body loads (I), Lumped system model diagram (II).

using a forward Runge-Kutta numerical integration method. A matrix form of equation of motion (EOM) using TMT method is presented to derive the dynamic model [19]. TMT method is a simple and clear method which eliminates the highest order derivatives in each step and results in a simplified matrix form of unconstrained Lagrange EOM, ideal for numerical simulation of complex large dynamic systems as

$$T_{(q)} = [\rho_i \quad \dots]^T_{[3n_i \times 1]}, \quad \omega_{ci} = \bar{\omega}_{ci} q_{,t}, \quad M = \text{diag}[m_1 m_1 m_1 I_{1[3 \times 3]} \quad \dots], \quad \bar{T} = \begin{bmatrix} T_{(q),q} \\ \bar{\omega}_c \\ \vdots \end{bmatrix},$$

$$d = (\bar{T} q_{,t})_{,q,t}, \quad \bar{M} = \bar{T}^T M \bar{T}, \quad d_{\text{EOM}} = \bar{T}^T [\sum f_c - M d] + f_q, \quad \bar{M} q_{,tt} = d_{\text{EOM}}, \quad (2)$$

where q is the generalized state vector, T is the transformation matrix for multi body links' COM position and rotation in terms of generalized coordinates, M is the system's mass and inertia matrix, m_i and I_i are the i^{th} link mass and inertia matrices, f_c and f_q are the external conservative and non-conservative forces in Cartesian (i.e. gravitational forced) and joint space (i.e. joint inputs) respectively, n_i is the number of bodies and n_q is the number of states. $\bar{\omega}_{ci}$ is a $3 \times n_q$ coefficient matrix derived by collecting $q_{,t}$ elements in ω_{ci} . The actuator forces are considered as concentrated force and torque acting uniformly on all DOFs. The translational and rotational stiffness are found from Euler-Bernoulli relation for each segment and by considering a symmetric deformation for the cross-section assuming an incompressible beam, where $\Pi \lambda_i = 1$ and λ_i is the Cauchy-Green principle stretch in the i direction [12] as explained later, $k_t = nEa/(\lambda_i^2 l)$ and $k_r = nEJ_d/(\lambda_i^3 l)$, where n is the number of segments, E is the modules of elasticity, a is the segment initial area, J_d is the initial second moment of area around the d axis and λ is the axial elongation of each link as $\lambda_i = 1 + n\varepsilon/l$.

We used "AutoTMTDyn", a software package in Matlab programming language that drives the TMT vector form of the Lagrange EOM using simple inputs about the kinematics of the system [19], to simulate a three-segment lumped model of a STIFF-FLOP manipulator module (Fig. 2.II). A sample input code for planar motion is presented in Algorithm 1. The model can capture transient behavior of the manipulator

Fig. 3. CC and axial deformation parameters (I), deformed cross-section predicted by the general geometry deformation method (II), braided pneumatic chamber models (For $E=100$ [Kpa]) compared to experimental results (III).

with high accuracy and good performance [6]. Despite the static models, the role of proper identification or modeling of the damping coefficient is important to capture an accurate dynamic model for the system. The model may become unstable and hard to simulate for higher number of links without the uniform deformation assumption, however shows to be accurate even for a small number of DOFs. The forward dynamics of the system is easier to solve and this method is suitable for dynamic control design purposes using traditional control theories for rigid body systems. Although some structural characteristics of the system is considered in the modeling procedure, this method cannot provide insight in the material property for structural design and optimization purposes. Simulation results are presented in Fig. 2.I showing 22% mean error compared to the dynamic experimental results with body loads.

Constant Curvature Kinematics and Mechanics

Kinematic Model: As the most common used model for the kinematic representation, the manipulator is considered as a continuous CC curve as in Fig. 3.I [3]. Here, a geometric map (f_G) and a structural specific map (f_S) describe the system mechanics. f_G maps the curvature parameters (l_m : central axis length, κ : curvature, ϕ : polar orientation angle) to curve tip position (ρ_{tip}) which can be found from CC assumptions using a set of transformations given by $R_y(\phi)-R_z(\kappa l)-\rho_{xy}-R_y(-\phi)$ [3], where $\rho_{xy} = [(1 - C_{\kappa l})/\kappa \quad S_{\kappa l}/\kappa \quad 0]^T$ and,

$$f_G(l, \kappa, \phi) = \begin{bmatrix} R_y(\phi) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_z(\theta) & \rho_{xy} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_y(-\phi) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}, 0 = [0 \quad 0 \quad 0]^T, \quad (3)$$

This map suffers from inherent singularity for $\kappa = 0$. An effective method to overcome this singularity is using the actuator lengths as the system states in the differential inversion method to find the system model [3,5].

f_S maps the input parameters space (actuator lengths (l_c) for hydraulic and tendon driven models) to curvature parameters space. For a module with three actuators we have [3],

$$f_S = \begin{cases} \phi = \tan^{-1}(\sqrt{3}(l_{c2} - l_{c3})/(l_{c2} + l_{c3} - 2l_{c1})) \\ \kappa = 2\sqrt{l_{c1}^2 + l_{c2}^2 + l_{c3}^2 - l_{c1}l_{c2} - l_{c1}l_{c3} - l_{c2}l_{c3}}/(d_c(l_{c1} + l_{c2} + l_{c3})), \\ l_m = (l_{c1} + l_{c2} + l_{c3})/3 \end{cases}, \quad (4)$$

- *Euler-Bernoulli Beam Model:* For pneumatic actuators, a second step is needed to map the pressure inputs to actuator length. In the case of the well-known "OCT-Arm"

series of continuum robots [20] where the modules are created from actuators without a supporting shell, a map between the actuator pressure and elongation is needed. A simple model can be derived by assuming the manipulator deformation as superposition of the elongation and bending of an Euler-Bernoulli beam in the ϕ direction due to the static balance between the axial force (f_{EB}) and bending moment (τ_{EB}) of the actuator at the tip, external loads and the body loads. The axial elongation is $\lambda_l = 1 + f_{EB}/(Ea)$ or more precisely, by considering the cross-section symmetric deformation of an incompressible material as explained later and solving $\lambda_l = 1 + f_{EB}\lambda_l^2/(Ea)$ for λ_l and the curvature is $\kappa = \tau_{EB}/EJ_\phi$ or more precisely, $\kappa = \tau_{EB}\lambda_l^2/EJ_\phi$ by considering the cross-section deformation after elongation. The simulation results assuming fixed cross-section for pneumatic chambers in comparison to experiments with body loads (Fig. 1.I) is presented in Fig. 4.I,II.

- *Geometry Deformation Model for Compound Structure*: A more accurate model can be derived by considering the compound structure consisting of the body shell and braided actuators. A supporting body shell is usually used to prevent the actuator chambers from bulking, i.e. STIFF-FLOP [2,16], f_s need to be derived by considering the body deformation energy (w_s). A simple solution for w_s can be found by only considering the body axial deformation and neglecting the bending and twisting as in the simple model for braided actuators. A more accurate but complex solution can be derived by extending our previous work [10] where the geometry deformation method is used as follows for the bending and elongation case,

$$I_1 = (\lambda(2 + \lambda^3) - 2\kappa x \sqrt{\lambda(1 - \lambda^3)}) / (1 - 2\kappa x \sqrt{\lambda}), w_s = E/6 \oint I_1 dx. \quad (5)$$

This method predicts the cross-sectional deformation as in Fig. 3.II. Then the principle of virtual energy becomes $w_{C,q} + w_{G,q} + w_{S,q} + w_{B,q} = 0$, where $q = [l_m, \kappa, \phi]$ is the curvature parameters vector and w_B is for external load and body forces. This can be solved for $p = f_s^{-1}(q)$. Simulation results in comparison to experiments for a STIFF-FLOP manipulator section are presented in Fig. 4.I,II. The model has an average of 28% error mainly because of CC assumption errors; however, it increases the model accuracy by 3% compared to a simple Euler-Bernoulli beam method.

- *Braided Pneumatic Actuator Exact Mechanics*: For pneumatic braided extensible actuators two analytical solutions are presented based on principle of virtual work ($\sum w_{i,q} = 0$), incompressibility assumption and Neo-Hookean deformation energy relation ($u_d = E/6 (\sum_{i=1}^3 \lambda_i - 3)$) [9,10,12], where w is the virtual work or energy and q is the vector of system states. The principle of virtual energy for a chamber is $w_{C,\lambda} + w_{G,\lambda} = 0$, where w_C is the chamber body deformation energy, w_G is the air pressure work, $\lambda = l/l_0$ (considered as the only system state) is the axial stretch, l_1 and l_d are the module axis initial and deformed length. This can be solved for $p = f_s^{-1}(q)$. The most famous model to incorporate the braiding effect, used by Trivedi for the first time for continuum manipulators, assumes the outer surface of a constant volume cylindrical chamber is constrained to the helical braids deformation as $\lambda_3^2 S_{\gamma_1}^2 + \lambda^2 C_{\gamma_1}^2 = 1$, where λ_3 is the radial stretch [12,21]. His presentation of this formula in his paper, [12], has some errors which are corrected here,

$$p = E(r_{c2}^2 - r_{c1}^2)(\lambda_{\gamma}^2 - 1) / 6\lambda_{\gamma}^4 r_{c1}^2 (1 - \lambda_{\gamma}^2 C_{\gamma}^2)^3 \times (\lambda_{\gamma}^6 (-2S_{\gamma}^6 + 5S_{\gamma}^4 - 4S_{\gamma}^2 + 1) + \lambda_{\gamma}^4 (7S_{\gamma}^6 - 16S_{\gamma}^4 + 11S_{\gamma}^2 - 2) + \lambda_{\gamma}^2 (7S_{\gamma}^6 - 4S_{\gamma}^4 - 3S_{\gamma}^2 + 1) + 3S_{\gamma}^4) \quad (6)$$

where γ is the initial thread helix angle, r_{c1} and r_{c2} are the chamber inner and outer initial radius and E is the module of elasticity.

- *Braided Pneumatic Actuator Simplified Mechanics*: For a dense threaded chamber, as in most actuators of this type, where $\gamma \approx \pi/2$ and the chamber deformation does not change γ significantly, the thread helix radius change becomes negligible ($\lambda_3 \approx 1$, $r_c \approx r_{c0}$) and the relation simplifies to $p = E(r_{c2}^2 - r_{c1}^2)(\lambda_{\gamma}^2 - 1)(\lambda_{\gamma}^2 + 3) / 6\lambda_{\gamma}^4 r_{c1}^2$. We have recently used a new so-called geometry deformation method inspired by Rivlin's work on the "Problem of Flexure" [22] with the same assumption as Trivedi [12] resulting in a new more complex way to reach the same results [9]. In both derivations, the braiding is on the outer surface of the chamber. If the manipulator has no shell as in OctArm [12], this gives the map f_S between the pressure and chamber length.

A third simpler model is possible to drive by assuming a dense thread ($\gamma \approx \pi/2 \rightarrow r_c \approx r_{c0}$) constraint on the inner body of the chamber. For the gas work, we have $w_G = pa_{p0}l_{c0}(\lambda - 1)$, where a_{p0} is the initial internal area of the chamber. The outer radius of the chamber shrinks as the actuator extends. A homogeneous incompressible cylinder undergoing axial elongation with axial deformation ration λ , experiences radial and circumferential deformation with stretch of $1/\sqrt{\lambda}$ to satisfy incompressibility criteria [23]. The Neo-Hookean model suggests that the deformation energy of body becomes $w_S = Ea_{t0}l_0(\lambda^2 + 2/\lambda - 3)/6$ as in [23], where a_{t0} is the initial chamber thickness area. From the principle of virtual work, the pressure can be found as, $p = Ea_{t0}(\lambda - 1/\lambda^2)/(3a_{p0})$. A comparison between the results of these methods for one actuator from STIFF-FLOP manipulator is presented in Fig. 3.III. The figure shows the good accuracy of the inner dense thread model in predicting the values and overall behavior. Trivedi's exact and dense thread models have almost the same result with high error despite the results reported in [12]. We believe that this is due to the very small size and thickness of STIFF-FLOP actuators compared to OCTArm ones. The inner dense thread method is used to model the STIFF-FLOP manipulator using CC approach in the next sections.

- *Simple Compound Model*: Neglecting the bending in the body shell and the actuator chamber deformation energy (w_C), we can assume the braids are on the inner surface of the chambers. Then we simply have $w_G = \sum p\pi r_{i0}^2(l_{Ci} - l_{Ci0})$ and $w_{G,q} + w_{S,q} + w_{B,q} = 0$ as the system static model where l_{Ci} is the i^{th} chamber axial length as in [3],

$$l_{c1} = l_m(1 - \kappa d_C C_{(-\phi)}/2), \quad l_{c2|3} = l_m(1 - \kappa d_C C_{(-\phi \pm 2\pi/3)}/2). \quad (7)$$

A careful choice of the combination of models for the actuator and body shell is important to achieve the best accuracy as well as comprehensiveness. The simulation results for the forward model compared to the experimental results with body loads

Fig. 4. Simulation results for CC and two-step modified CC (MD) models compared to the experiments with body loads for a STIFF-FLOP module (I,II), two-step modified CC (MD) model diagram (III).

(Fig. 1.I) show 28% error for the general geometry deformation method (GD) and fix cross-section actuator (fix r), 31% for the Euler-Bernoulli model (EB) with fix cross-section and 28% for the Neo-Hookean symmetric elongation (NH sym.) model and extensible braided actuator (elong. helix) model, showing 3% increase in the accuracy of compound models as well as their comprehensive modeling ability (Fig. 4.I,II).

Two-Step Modified Constant Curvature Model

To compensate high modeling error related to CC assumption, Mahvash et. al. used a two-step approximate model where the CC map is being used for non-contact map of an eccentric tube catheter and a modifying map based on beam theory is used to model the deflections from the CC geometry due external and body loadings [24]. Here we investigate a similar solution for continuum manipulators. Any change in the input pressures or actuator lengths results in a CC geometry. Then the resulted curved beam undergoes small deformations due to body and external loads assuming a linear stress-strain relation (Euler-Bernoulli beam model) (Fig. 4.III). Castigliano's method for a linear-elastic structures is used [25] to find an analytical solution for the modifying deformation at the tip and the middle of the manipulator,

$$\begin{aligned}
 f_{y(\theta)} &= (\rho f_{B_Y}(\theta_t - \theta) + f_{e_Y})C_{(\theta)} + (\rho f_{B_X}(\theta_t - \theta) + f_{e_X})S_{(\theta)}, \\
 \tau_{x(\theta)} &= -\rho^2 f_{B_Z}(C_{(\theta_t - \theta)} - 1) + \rho f_{e_Z}S_{(\theta)}(\theta_t - \theta) - \tau_{e_Y}S_{(\theta)} + \tau_{e_X}C_{(\theta)}, \\
 \tau_{y(\theta)} &= \rho^2 f_{B_Z}(\theta - \theta_t + S_{(\theta_t - \theta)}) - \rho f_{e_Z}(1 - C_{(\theta_t - \theta)}) + \tau_{e_Y}C_{(\theta)} + \tau_{e_X}S_{(\theta)}, \\
 \tau_{z(\theta)} &= \rho^2 c f_{B_Y}(C_{(\theta_t)} - C_{(\theta)}) + \tau_{e_Z} + \rho f_{e_X}(S_{(\theta)} - S_{(\theta_t)}) + \rho^2 c((mg - f_{B_Y}l_d)(S_{(\theta_t)} - S_{(\theta)} + \\
 &\quad (\theta - \theta_t)C_{(\theta)}))/l_d + \rho^2 f_{B_X}(C_{(\theta_t)} - C_{(\theta)} - (\theta - \theta_t)S_{(\theta)}), \\
 w_E &= \int_0^{\theta_t} (\tau_x^2/(2EI_{xx}) + \tau_z^2/(2EI_{zz}) + \tau_y^2/(2GJ_{yy}) + f_y^2/(2EA))d\theta, \tag{8}
 \end{aligned}$$

where the shear strains are neglected, $\rho = 1/\kappa$, $\theta_t = l_d/\kappa$, $f_{B_X/Y/Z}$ is the external uniform load, $f_{e_X/Y/Z}$ and $\tau_{e_X/Y/Z}$ are the external load and torque vector at the tip, $I_{xx} =$

$I_{zz} = \pi r_{s0}^4 / (4\lambda^2)$ and $J_{yy} = \pi r_{s0}^4 / (2\lambda^2)$ are the cross-section first and second moment of area, G is the shear module, subscript x,y,z is the local curvilinear frame attached to COM and tangent to backbone curve and subscript X,Y,Z are for the main inertial frame. A correction factor ($c=0.01$) is used for the forces in y direction in calculating $\tau_z(\theta)$ to adjust for the change in the moment arm in the static equilibrium compared to the initial CC configuration which should be identified for each loading condition from experiments. While this is a limiting factor, without this correction factor the formula results in a large error if the loading condition in the CC initial state is considerably different with the final equilibrium state. The final tip deflection from the constant curve geometry we have, $\epsilon_x = w_{E,fe_x}$, $\epsilon_y = w_{E,fe_y}$, $\epsilon_z = w_{E,fe_z}$ which can be derived analytically, not presented here due to the limitation of space. The actuator forces are not mentioned as they have been considered to find the initial CC geometry. The modified model improves the accuracy by %1 for all the CC models to preserves the accuracy of CC model in the presence of small external and body forces (Fig. 4.II); however, it is not enough for precise control design. CC based models suffer from inaccuracy in presence of external and body forces and singularity at the straight orientations. However, they provide a simple solution to the continuous behavior of the manipulators being used for real-time or simple path planning and control applications [3]. Use of principle of virtual work or the introduced two-step modified model enables this method to preserve its accuracy in the presence of small body and external loads and to account for the structural characteristics of the manipulator which helps with the design optimization problems [9,10]. Lagrange dynamic method can be used for dynamic modeling of a continuum manipulator with CC assumption which mostly suffers from singularities and low accuracy.

Variable Curvature Kinematics and Mechanics

The VC model assumes the backbone consists of a series of infinitesimal CC curves [11,12,25,26] to improve the backbone deformation model as the most important factor in increasing the manipulator modelling accuracy.

- *Variable Curvature Kinematics*: VC kinematics presents the relation between the local curvilinear frames with unit vectors d_i along the manipulator backbone curve tangential ($\hat{t}$), normal ($\hat{n}$) along a fix point in the cross-section, e.g. location of actuator 1, and binormal ($\hat{b}$) axes (Fig. 5.III). The curve spatial configuration usually expressed in inertial Cartesian coordinates. The local frame is the cross-section physical frame and $[\hat{n}, \hat{t}, \hat{b}]$ are not Frenet–Serret curvilinear frame directors here. A set of rotation matrices (R_s) gives the relation between the local transnational strains ($\rho_{(s),s}$) and the derivative of the manipulator spatial geometry ($\rho_{(s),s}$) as $\rho_{(s),s} = R_{(s)} \cdot (v + [0, 1, 0]^T)$, where v is the linear strain vector in the local curvilinear frame which the rotation matrix R_s rotates to be expressed in the inertial reference frame. This rotation representation is equal to the variation of the position vector from a local frame to the next one ($\rho_{(s),s}$) along the backbone unit length (s). The variation of $R(s)$ along the curve length is found from the local curvatures and torsion (u) as $R_{(s),s} = R_{(s)} [u]_\times$. Cosserat rod or beam theory methods are used to find v and u based on the system loads. Euler-Bernoulli equations for a rod is a special case of these methods with infinite shear modulus (G). Integrating to find ρ_s and $R(s)$ results in the system VC kinematics [12].

Fig. 5. Cosserat/beam theory model results vs. experiments with extensive external loads (I), Variable curvature kinematics (II), free body diagram for one element (III).

- *Cosserat Rod Method*: Cosserat rod model derive the equilibrium between the forces on each infinitesimal element of a continuum media using free body diagram of each element [26,27] (Fig. 5.III) as $n_{(s)} + f_{(s)} = b_{ab}\rho_{(s),tt}$ and $m_{(s)} + \rho_{(s),s} \times n + \tau_{(s)} = J\omega_{(s),t}$, where the time dependent terms on the right side are dropped for static solutions. Hooke's law is been used to relate the variation of the boundary forces to the strains as $n_{(s)} = K_l v_{(s)}$ and $m_{(s)} = K_b u_{(s)}$, where $K_l = \text{diag}(Ga_b, Ea_b, Ga_b)$ and $K_b = \text{diag}(EJ_1, GJ_2, EJ_3)$ are the stiffness diagonal matrices. A simple version of this relation in the planar case is presented in [12]. Rearranging the derivations to find $v_{(s)}$ and $u_{(s)}$ based on the loads, results in the static mechanic map .

- *BVP Solution*: The static and kinematic map form a BVP for the strain and position vector spatial derivatives [26] that needs to be integrated over the volume and time. One can use finite element approaches [28]. Alternatively, for a forward integration method, the system four states (u, v, ρ, R) should be known at the start point. An optimization based method is needed to guess and update the base point states where the initial position and orientation are known. The problem of finding proper initial guesses is simplified by finding the base states based on beam theory.

- *Beam Theory Approach*: Beam theory method can be used instead of the Cosserat rod method to find the static map where a combination of a holistic and an element-wise view of the system is considered. Both methods result in the same set of equations and identical in the simulation results. The loads at each cross-section are derived from writing a static equilibrium between the element internal stress vector and all the forces and moments acting on the cross-section that is located between the element section and the beam free end. This is different from Cosserat model where the equilibrium is considered between the element boundary forces. Here we need to know the geometry of the manipulator at the equilibrium state and finding the acting loads on the cross-sections usually needs an integration over the manipulator length between the element position and the free end. These limit the application of this method especially for finite element implementation, however, it simplifies the BVP by eliminating the spatial derivatives of v and u (we work with the strains in this method) and provides the oppor-

tunity to solve the forward integration method using approximate series solutions (concatenation, direct single and multiple shooting). The beam theory method results in the following static map for a continuum manipulator in the static case

$$v = K_l^{-1} \cdot (R^T \cdot (f_L + f_b) + f_{p_d}), \quad u = K_b^{-1} \cdot (R^T \cdot (\tau_{b_d} + \tau_{L_d}) + \tau_{p_d}), \quad (9)$$

where $f_{b(s)} = ba(l-s)g$ is the body weight force, $f_{p_d} = \Sigma_{i=1}^3 p_i a_c \cdot [0, 1, 0]^T$ is the pneumatic pressure force assuming fix cross-section (a dense thread), $\tau_{L_d} = (\rho_{L_d} - \rho_{d(s)}) \times f_L$ is the external load moment, $\tau_{b_d} = ba \int_s^l (\rho_{d(s)} - \rho_{d(\epsilon)}) d\epsilon \times g$ is the body weight moment, $\tau_{p_d} = a_p r_{o_d} p \cdot [r_2, 0, r_3]$ is the pneumatic pressure moment exerting at the manipulator tip, where $r_{2|3} = r_{o_d} [S|C_{(0)}, S|C_{(2\pi/3)}, S|C_{(-2\pi/3)}]^T$, and $r_{o_d} = r_o/\sqrt{\lambda}$ is the deformed placement radius for the pneumatic chambers, all in deformed state. The deformed values in each element are found by using the local curvature parameters $\lambda_l = v_t + 1$ and $\kappa = \sqrt{u_a^2 + u_b^2}/(v_t + 1)$, assuming a uniform cross-section.

- *Single Shooting Optimization Based Solution:* We can approximate the solution to the BVP with a series of finite terms, i.e. a polynomial of order n as a function of s , and investigating the equilibrium condition. This is considered as a special case of separation of variable method for Partial Differential Equation (PDE) systems that provides a weak form solution. The terms are consisting of separate space dependent shape functions and time dependent coefficients where the time dependent part is constant in static cases. The constants are being optimized so the guessed series solution represents the static equilibrium of the system. We fit a curve to seven points using Matlab "interp1" function and then use Matlab "fsolve" function for optimization of these points' positions in the plane starting from a straight configuration as the initial guess. The algorithm usually needs only two trials to find the equilibrium configuration for a single-curve formation and three trials for a double-curve formation, i.e. in presence of high external force (Fig. 5.I) showing the possibility of real-time implementation of this method. A similar result can be achieved by having a polynomial of order at least three to model the "S" shape configuration of a single module under high external loads with good accuracy. As observed in previous research [13], the overall accuracy of the Cosserat rod model is about 16% better than the CC models with a 12% mean error in comparison to the experimental results with extensive external loads. It is relatively hard to model the dynamics of a system using Cosserat and beam theory methods; however, they are the best in terms of accuracy to find the manipulator configuration in static equilibrium points.

Identification Based Series Solutions

Coefficients of a series solution can be identified based on the experimental results [5,15]. Godage presented an identification based Taylor series solution for the kinematic map of a continuum manipulator (the relation between actuator chamber length and manipulator geometry) and later implemented the resulted identified solution in the Lagrange EOM for dynamic modeling of a continuum manipulator [5]. The spatial geometry is described using a time-dependent coefficient vector ($\gamma(t)$) and a shape function matrix ($\Psi(\epsilon)$) as $r(\epsilon, t) = [x(\epsilon, t) \quad y(\epsilon, t) \quad z(\epsilon, t)]^T = \Psi(\epsilon)\gamma(t)$, where $\epsilon \in$

Fig. 6. Identification based series model results compared to experiments with body loads.

$[0,1]$ is a scalar variable defining the position of each point on the manipulator axis with 0 for the base and 1 for the tip. The final model can be as simple as a polynomial of order five [5]. Godage used three equations each with 56 shape functions of the form $A_1 l_1^{A_2} \epsilon^{A_3+1} R^{-A_3}$ where A_1, A_2 and A_3 are integers to be identified based on optimization, $A_1 \& A_2 \in [0,1]$ and $A_3 \in [0,5]$ [5]. Choice of the shape function and time-dependent coefficient can be based on observation, guessed solution or completely arbitrary. Similar to other weak approximation solutions, they should satisfy the boundary and initial conditions and the convergence should be investigated for increasing the number of terms. Here we introduce a simpler series with eight time dependent coefficients (C_{ij}) to be identified using Least Square Value optimization as,

$$\Psi = C_{3 \times 8} \cdot \text{diag}[\epsilon \quad \epsilon \quad \epsilon \quad \epsilon \quad \epsilon^2 \quad \epsilon^2 \quad \epsilon^2 \quad \epsilon^3], \quad (10)$$

where $\gamma(t) = [1 \quad l_{c1} \quad l_{c2} \quad l_{c3} \quad l_{c1}l_{c2} \quad l_{c1}l_{c3} \quad l_{c2}l_{c3} \quad l_{c1}l_{c2}l_{c3}]^T$ is for a system with kinematic inputs and $\gamma(t) = [1 \quad p_1 \quad p_2 \quad p_3 \quad p_1p_2 \quad p_1p_3 \quad p_2p_3 \quad p_1p_2p_3]^T$ for a system with pressure inputs. To satisfy boundary conditions we have $[C_{11} \quad C_{21} \quad C_{31}] = [x_0 \quad y_0 \quad z_0]_{\text{tip}}$. The model trains and performs very fast, suitable for real-time and precise control purposes. The simulation results show 11% mean error which is the most accurate results in predicting the static behavior of the manipulator with body loads after being trained with 30% of the experimental points (Fig. 6). However, the structural information which is necessary for design optimization is not incorporated in the model. Besides the complexity of the series solution make any further analytical analysis such as dynamic modeling, controller design and stability analysis a hard task. This method incorporates the same level of structural parameters as the Cosserat method and can be assumed as a basis for both optimization and control frameworks.

3 Discussion and Comparison

In table 3, a comparison between the different modeling methods is presented based on the derivation and implementation complexity, the level of structural characteristics in the model, sensitivity to model parameters, computation cost, overall accuracy and suggested applications. It worth mentioning that the information in table 3 is provided based on the feasibility, ease and popularity of the model usage for a specific purpose; however, all the methods can be used for different purposes after some modification.

Table. 3. Comparisons of different modelling approaches.

	<i>Lumped Sys.</i>	<i>CC & Euler-Bernoulli</i>	<i>CC & PVW</i>	<i>Modified CC</i>	<i>Cosserat/Beam Theory</i>	<i>Series Id.</i>
Spec.s	Dynamic, Forward / Inverse	Static, Forward / Inverse	Static, Inverse	Static, Forward	Static, Forward	Static, Forward / Inverse
Complexity	Hard	Easy	Medium	Medium	Hard	Medium
Struct. Details	Medium	Low	High	Low/Medium	Medium	Low
Param. Sensitivity	Medium / High	Low	High	Medium	Medium	Low
Comp. Cost	High	Low	Medium	Low	High	Low/Medium
Accuracy (Error,%)	Medium (22%)	Low – Med. (31%)	Low – Med. (28%)	Low – Med. (27%)	High (12%)	High (11%)
Application	Simulation, Control	Non-Accurate Control	Design	Non-Accurate Control	Simulation	Control

4 Conclusion

In this paper, a review on derivation and implementation of five different approaches for modeling of continuum manipulators is presented. Lumped system dynamic model, Constant curvature, two-step modified constant curvature, variable curvature Cosserat rod and beam theory approach and series solution identification are derived and the modeling results are compared with experimental data using a STIFF-FLOP continuum manipulator section. Our study shows the competency of the identification based model in comparison to the other methods in accuracy and performance. From a different perspective, we conclude that dynamic behavior prediction and control method implementation are streamlined in lumped system model. Different CC models based on the principle of virtual work show less accuracy but the better incorporation of structural characteristics which makes them suitable for design optimization purposes. Furthermore, through the sections of this paper, simple mathematical derivations of the manipulator deformation energy are discussed; a comparison between different models for a braided extensile pneumatic actuator is presented and a novel two-step modified CC solution using Castigliano's method is suggested to simplify the design optimization, control and path planning problems of continuum manipulators.

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References

1. S. Hirose and M. Mori, "Biologically inspired snake-like robots," in Proc. IEEE Int. Conf. Robot. Biomimetics, 2004, pp. 1–7.

2. M. Cianchetti, T. Ranzani, G. Gerboni, T. Nanayakkara, K. Althoefer, P. Dasgupta, and A. Menciassi, "Soft robotics technologies to address shortcomings in today's Minimally Invasive Surgery: The STIFFFLOP approach," *Soft Robotics*, vol. 1(2), pp. 122–131, 2014.
3. R. J. Webster and B. a. Jones, "Design and Kinematic Modeling of CC Continuum Robots: A Review," *Int. J. Rob. Res.*, vol. 29, no. 13, pp. 1661–1683, Jun. 2010.
4. J. S. Mehling, M. Diftler, M. Chu, and others, "A minimally invasive tendril robot for in-space inspection," in *Biomedical Robotics and Biomechanics, 2006. BioRob 2006. The First IEEE/RAS-EMBS International Conference on, 2006*, pp. 690–695.
5. I. S. Godage, D. T. Branson, E. Guglielmino, and others, "Shape function-based kinematics and dynamics for variable length continuum robotic arms," in *Robotics and Automation (ICRA), 2011 IEEE International Conference on, 2011*, pp. 452–457.
6. I. S. Godage, R. Wirz, I. D. Walker, and R. J. Webster III, "Accurate and Efficient Dynamics for Variable-Length Continuum Arms: A Center of Gravity Approach," *Soft Robotics*, vol. 2, no. 3, pp. 96–106, 2015.
7. Y. Shapiro, A. Wolf, and K. Gabor, "Bi-bellows: Pneumatic bending actuator," *Sensors and Actuators A: Physical*, vol. 167, no. 2, pp. 484–494, Jun. 2011.
8. J. Fraś, J. Czarnowski, M. Maciaś, and J. Głowka, "Static modeling of multisection soft continuum manipulator for stiff-flop project," in *Recent Advances in Automation, Robotics and Measuring Techniques, 2014*, vol. 267, pp. 365–375.
9. S. M. H. Sadati, A. Shiva, A. Ataka, et. al., "A Geometry Deformation Model for Compound Continuum Manipulators with External Loading," in *In Robotics and Automation (ICRA), 2016 IEEE International Conference on., 2016*.
10. S. M. H. Sadati et al., "A Geometry Deformation Model for Braided Continuum Manipulators," *Frontiers in Robotics and AI*, vol. 4, Jun. 2017.
11. T. Mahl, A. E. Mayer, A. Hildebrandt, and O. Sawodny, "A variable curvature modeling approach for kinematic control of continuum manipulators," in *2013 American Control Conference, 2013*, pp. 4945–4950.
12. D. Trivedi, A. Lotfi, and C. Rahn, "Geometrically Exact Models for Soft Robotic Manipulators," *IEEE Transactions on Robotics*, vol. 4, no. 24, pp. 773–780, 2008.
13. I. D. Walker, "Continuous Backbone 'Continuum' Robot Manipulators," *International Scholarly Research Notices (ISRN Robotics)*, vol. 2013, p. e726506, Jul. 2013.
14. I. Tunay, "Spatial continuum models of rods undergoing large deformation and inflation," *IEEE Trans. Robot.*, vol. 29, no. 2, pp. 297–307, 2013.
15. G. Chen, M. T. Pham, and T. Redarce, "Sensor-based guidance control of a continuum robot for a semi-autonomous colonoscopy," *Rob. Auton. Syst.*, vol. 57, no. 6–7, pp. 712–722, 2009.
16. J. Fraś, J. Czarnowski, M. Maciaś, J. Głowka, M. Cianchetti, and A. Menciassi, "New STIFF-FLOP module construction idea for improved actuation and sensing," in *2015 IEEE International Conference on Robotics and Automation (ICRA), 2015*, pp. 2901–2906.
17. B. Conrad and M. Zinn, "Closed loop task space control of an interleaved continuum-rigid manipulator," in *Robotics and Auto. (ICRA), 2015 IEEE Int. Conf. on, 2015*, pp. 1743–1750.
18. J. Jung, R. S. Penning, and M. R. Zinn, "A modeling approach for robotic catheters: effects of nonlinear internal device friction," *Advanced Robotics*, vol. 28, no. 8, pp. 557–572, 2014.
19. S. Sadati, S. Naghibi, and M. Naraghi, "An Automatic Algorithm to Derive Linear Vector Form of Lagrangian Equation of Motion with Collision and Constraint," *Procedia Computer Science*, vol. 76, pp. 217–222, 2015.
20. D. Trivedi and C. D. Rahn, "Model-based shape estimation for soft robotic manipulators: The planar case," *Journal of Mechanisms and Robotics*, vol. 6, no. 2, p. 021005, 2014.
21. W. Liu and C. Rahn, "Fiber-reinforced membrane models of McKibben actuators," *Journal of Applied Mechanics*, vol. 70, no. 6, pp. 853–859, 2003.

22. R. S. Rivlin, "Large Elastic Deformations of Isotropic Materials. V. The Problem of Flexure," *Proc. of the Roy. Soc. A: Math., Phy. & Eng. Sci.*, vol.195, no.1043, pp.463–473, 1949.
23. A. N. Gent, *Engineering with rubber: how to design rubber components*. Carl Hanser Verlag GmbH Co KG, 2012.
24. M. Mahvash and P. E. Dupont, "Stiffness control of a continuum manipulator in contact with a soft environment," in *Intelligent Robots and Systems (IROS)*, 2010 IEEE/RSJ International Conference on, 2010, pp. 863–870.
25. R. C. Juvinall and K. M. Marshek, *Fundamentals of machine component design*, vol. 83. John Wiley & Sons New York, 2006.
26. M. Neumann and J. Burgner-Kahrs, "Considerations for Follow-the-Leader Motion of Extensible Tendon-driven Continuum Robots," 2016.
27. J. Burgner-Kahrs, D. C. Rucker, and H. Choset, "Continuum Robots for Medical Applications: A Survey," *IEEE Transactions on Robotics*, vol. 31, no. 6, pp. 1261–1280, Dec. 2015.
28. C. Duriez and T. Bieze, "Soft Robot Modeling, Simulation and Control in Real-Time," in *Soft Robotics: Trends, Applications and Challenges*, C. Laschi, J. Rossiter, F. Iida, M. Cianchetti, and L. Margheri, Eds. Springer International Publishing, 2017, pp. 103–109.