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A New Design of Membership-Function-Dependent Controller for T-S Fuzzy Systems Under Imperfect Premise Matching

Changzhu Zhang, *Member, IEEE*, Hak-Keung Lam, *Senior Member, IEEE*, Jianbin Qiu, *Senior Member, IEEE*, Chengju Liu, and Qijun Chen

Abstract—This paper is concerned with the problem of membership-function-dependent controller design for a class of discrete-time T-S fuzzy systems. Based on the partition method of premise variable space, the original T-S fuzzy model is equivalently converted into a kind of piecewise-fuzzy system. Then by employing some staircase functions, the continuous membership functions are approximated by a series of discrete values, via which the information of membership functions is brought into the stability analysis to reduce the design conservatism. With piecewise Lyapunov functions, the approaches to the piecewise-fuzzy state feedback and observer-based output feedback controller design are proposed, respectively, in terms of linear matrix inequalities such that the closed-loop system is asymptotically stable with a prescribed $\mathcal{H}_\infty$ performance level. It is shown that the membership functions of the fuzzy model and fuzzy controllers are not necessarily the same, which allows more design flexibility. Finally, two illustrative examples are provided to show the effectiveness of the developed methods.

Index Terms—Takagi-Sugeno fuzzy systems, membership-function-dependent, Lyapunov stability, imperfect premise matching, controller design

I. INTRODUCTION

It is known that nearly all the physical plants and industrial processes in practice are nonlinear, and these different kinds of nonlinearities impose great difficulties on stability analysis and controller synthesis. During the past few decades, as an effective way to tackle these complex nonlinear systems, fuzzy logic control has received much attention in control community. The fuzzy logic control provides a means of converting a linguistic control strategy based on expert knowledge into an automatic control strategy, and many significant results

reported in the literature have shown that it offers an effective method for the control problem of complex nonlinear systems or even nonanalytic systems (please refer to [1]–[7] and references therein). To mention a few, in [5] a fuzzy logic controller was designed and implemented on a test vehicle for the lateral motion control, which is modularized as a feedback, preview, and gain scheduling rule bases. The results on stability analysis of continuous-time fuzzy-model-based control systems were reviewed in [2], where some fundamental and essential aspects related to membership-function-dependent analysis methods are summarized. In [6], the problem of robust adaptive fuzzy cooperative tracking control for a class of uncertain nonlinear multi-agent systems was studied, which are subject to time delays and dead-zone nonlinearities. For a class of chaotic systems with unknown functions and disturbances, the problem of asymptotic stabilization for such systems were investigated in [7] based on the adaptive fuzzy logic control theory.

In general, fuzzy control can be classified into model-free approach and model-based approach [2], [8]–[12]. Among various types of model-based fuzzy control approaches, Takagi-Sugeno (T-S) fuzzy models have attracted plenty of research interests and a great deal of research effort has been devoted to the problem of stability analysis and controller synthesis based on T-S fuzzy models in recent years [13]–[17]. With T-S fuzzy models, a nonlinear plant can be approximated in a general form by a series of linear subsystems via fuzzy membership functions, which locally describe the dynamics of the nonlinear plant [18]–[22]. In this case, one can resort to the fruitful linear multivariable system theory for the stability analysis and controller synthesis for these nonlinear systems represented by T-S fuzzy models in a unified framework. In [23], a sufficient condition is firstly provided for the quadratic stability of T-S fuzzy systems by considering a Lyapunov function for all subsystems involved. Then, the problem of stability analysis and stabilization for T-S fuzzy systems are converted into the task of searching for a common positive definite Lyapunov matrix, and plenty of significant results on relaxed quadratic stability conditions can be found in [3], [24]–[26] and references therein. However, it should be noted that the stability analysis and design results must be valid for all subsystems with a common quadratic Lyapunov matrix, which are very conservative. In some cases, it is very difficult to find such a common matrix, especially for some highly nonlinear complex fuzzy systems. Moreover, it is often the case that a common quadratic Lyapunov matrix does not exist for some

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fuzzy systems even if they are asymptotically stable [27], [28]. In order to release the conservatism of the conditions based on common Lyapunov functions (CLFs), a great number of results are reported in the literature on the problem of stability analysis and controller synthesis for fuzzy systems in the framework of piecewise- or fuzzy-Lyapunov-function-based approaches [28]–[33]. It has been demonstrated that piecewise or fuzzy Lyapunov functions (PLFs or FLFs) could offer much richer classes of Lyapunov function candidates than common quadratic Lyapunov functions. Moreover, further relaxations can be expected by introducing some slack variables in stability analysis and controller synthesis in different ways [34], [35].

On the other hand, most control design techniques via linear matrix inequalities are based on determining some matrix variables to guarantee the positiveness of a double fuzzy summation, and nearly all the stability conditions aforementioned are independent of the “shape” of the membership functions of fuzzy systems. However, these results tend to be conservative as some important properties of membership functions are not used in the stability analysis. Sometimes it might be the case that the analysis strategies on the original nonlinear model could provide better solutions than those on its equivalent fuzzy T-S one [36]. Therefore, the information of membership functions plays an important role for the relaxation of the results on stability analysis and controller synthesis, and some related membership-function-dependent analysis strategies were revealed in the existing literature [2], [36]–[40]. To mention a few, with the known minimum and maximum grades of membership, new relaxed stability conditions were derived by employing the information of the uncertain membership functions [39]. In [36], a series of relaxed linear matrix inequality (LMI) conditions were presented for fuzzy control that incorporates the membership function shape information in the form of polynomial constraints. By employing the staircase membership functions and some slack matrices, the LMI-based stability conditions were obtained, which include the membership functions of both fuzzy model and fuzzy controller, and then an approach to the state feedback controller design was proposed in [40]. For the most-recent advances on this topic, please refer to the survey paper [2] and references therein.

It is also worthy to mention that almost all the aforementioned results are about stability analysis or state feedback controller design for T-S fuzzy systems under consideration of the information of membership functions. However, in some cases it is difficult or even impossible to obtain all the state information of the physical plant to be controlled for subsequent controller design. Thus, some other control strategies deserve further investigation with membership-function-dependent analysis approaches, such as, observer-based output feedback control. In the existing membership-function-dependent stability analysis conditions were derived with the common quadratic Lyapunov functions, which is a source of conservatism as mentioned above. To the best of our knowledge, few research effort has been devoted to the membership-function-dependent stability analysis and controller synthesis for T-S fuzzy systems subject to external disturbances within

the piecewise- or fuzzy-Lyapunov-function-based framework. All these motivate our present work.

In this paper, the problem of membership-function-dependent controller design is considered for a class of discrete-time T-S fuzzy systems within the piecewise Lyapunov-function-based framework. To overcome the difficulty in using the continuous information of membership functions, staircase membership functions are employed to approximate the original membership functions of fuzzy plant and fuzzy controllers. With the piecewise Lyapunov functions, the approaches to the piecewise-fuzzy state feedback controller and the observer-based output feedback controller designs are studied, respectively, which guarantee the closed-loop fuzzy system to be asymptotically stable with a predefined $\mathcal{H}_\infty$ performance level. It is shown that all the synthesis conditions are formulated in terms of linear matrix inequalities, and two illustrative examples are provided to demonstrate the effectiveness and advantages of the proposed methods.

The remainder of this paper is organized as follows. Section II is devoted to problem formulation. In Section III, the main results of stability analysis and controller design of piecewise-fuzzy state feedback controller and observer-based output feedback controller are presented, respectively. The simulation results are given in Section IV. Finally, we conclude the paper in Section V.

Notations: The notations used throughout the paper are fairly standard. The superscript “ T ” stands for matrix transpose; $\mathbb{R}^n$ denotes the n -dimensional Euclidean space; $\mathbb{R}^{m \times n}$ is the set of all real m by n matrices and $\mathbb{S}^n$ denotes the set of $n \times n$ real symmetric matrices; $\mathbf{I}_n$ and $\mathbf{0}_{m \times n}$ represent n -dimensional identity matrix and m by n zero matrix, respectively. We use an asterisk (*) to represent a term that is induced by symmetry in symmetric matrices. $\text{diag}\{\dots\}$ stands for a block-diagonal matrix. The notation $P > 0$ means that P is real symmetric and positive definite. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

II. MODEL DESCRIPTION AND PROBLEM FORMULATION

In this section, we study the control problem for a class of discrete-time T-S fuzzy dynamic systems, which are used to describe a complex nonlinear physical plant by the smooth combination of local analytic linear models with fuzzy membership functions. To facilitate the controller design, we first re-formulate the T-S fuzzy model into a piecewise form. And then, two piecewise-fuzzy control laws, i.e., state feedback control and observer-based output feedback control, are proposed.

A. Discrete-time T-S Fuzzy Dynamic Models

The discrete-time T-S fuzzy system with external disturbance is given as follows:

Plant Rule $\mathfrak{R}_p^m$: IF $f_1(x(t))$ is $\mathcal{M}_1^m$, AND $\dots$, AND $f_\Psi(x(t))$ is $\mathcal{M}_\Psi^m$, THEN

$$\begin{cases} x(t+1) = A_m x(t) + B_m u(t) + D_m w(t) \\ z(t) = L_m x(t) + N_m u(t) \\ y(t) = C x(t), m \in \mathfrak{R}_p \triangleq \{1, 2, \dots, p\} \end{cases} \quad (1)$$

where $\mathcal{M}_\alpha^m$ is a fuzzy term of rule m corresponding to the known function $f_\alpha(x(t))$ ($\alpha = 1, 2, \dots, \Psi$) and $m \in \mathfrak{R}_P$. $f(x(t)) = [f_1(x(t)), \dots, f_\Psi(x(t))]^T$ is the premise variable vector. $x(t) \in \mathbb{R}^{n_x}$ stands for the state vector and $u(t) \in \mathbb{R}^{n_u}$ is the control input; $y(t) \in \mathbb{R}^{n_y}$ is the measured output. Without loss of generality, assuming the system output matrix C to be of full-row rank, $z(t) \in \mathbb{R}^{n_z}$ is the regulated output, $w(t) \in \mathbb{R}^{n_w}$ is the external disturbance belonging to $l_2[0, \infty)$. A_m, B_m, D_m, L_m , and N_m are known constant matrices.

Let $\mu_m(x(t))$ be the normalized fuzzy membership function given as

$$\mu_m(x(t)) = \frac{\prod_{j=1}^{\Psi} \mu_j^m[f_j(x(t))]}{\sum_{i=1}^p \prod_{j=1}^{\Psi} \mu_j^i[f_j(x(t))]} \geq 0, \sum_{m=1}^p \mu_m(x(t)) = 1 \quad (2)$$

where $\mu_j^m[f_j(x(t))]$ represents the grade of membership of $f_j(x(t))$ in M_j^m . For brevity, $\mu_m(x(t))$ is denoted as μ_m without ambiguity. Following a standard fuzzy inference method, the compact form of T-S fuzzy system in (1) can be re-expressed as

$$\begin{cases} x(t+1) = A(\mu)x(t) + B(\mu)u(t) + D(\mu)w(t) \\ z(t) = L(\mu)x(t) + N(\mu)u(t) \\ y(t) = Cx(t) \end{cases} \quad (3)$$

where

$$\begin{cases} A(\mu) := \sum_{m=1}^p \mu_m A_m, B(\mu) := \sum_{m=1}^p \mu_m B_m \\ D(\mu) := \sum_{m=1}^p \mu_m D_m, L(\mu) := \sum_{m=1}^p \mu_m L_m \\ N(\mu) := \sum_{m=1}^p \mu_m N_m. \end{cases}$$

In this paper, an approach to the robust stability analysis and controller synthesis in the piecewise-Lyapunov-function-based framework is developed, and to facilitate the subsequent controller design, the original T-S fuzzy dynamic model is firstly converted into the equivalent piecewise-fuzzy system. To this end, we partition the premise variable space Ω^P into two kinds of polyhedral regions, i.e., crisp regions and fuzzy regions [28]. Denote those regions as Ω_l^P ($l \in \Lambda_P$) with Λ_P being the set of polyhedral region indices.

Define $\mathcal{I}_P(l)$ as the index set of the fired rules in each region Ω_l^P , that is,

$$\mathcal{I}_P(l) := \{m | \mu_m(x(t)) > 0, f(x(t)) \in \Omega_l^P, m \in \mathfrak{R}_P, l \in \Lambda_P\}, \text{ as:}$$

and then the crisp and fuzzy regions can be given as

$$\begin{cases} \Omega_l^P := \{f(x(t)) | \mu_k(x(t)) = 1, \text{ for all } k \in \mathcal{I}_P(l)\} \\ \Omega_l^P := \{f(x(t)) | 0 \leq \mu_k(x(t)) < 1, \text{ for all } k \in \mathcal{I}_P(l)\}. \end{cases}$$

With the above partition method, in the l th region Ω_l^P , one has $\sum_{m \in \mathcal{I}_P(l)} \mu_m = 1$, and the fuzzy system can be further expressed as

$$\begin{cases} x(t+1) = \mathcal{A}_l x(t) + \mathcal{B}_l u(t) + \mathcal{D}_l w(t) \\ z(t) = \mathcal{L}_l x(t) + \mathcal{N}_l u(t) \\ y(t) = Cx(t) \end{cases} \quad (4)$$

where

$$\begin{cases} \mathcal{A}_l = \sum_{m \in \mathcal{I}_P(l)} \mu_m A_m, \mathcal{B}_l = \sum_{m \in \mathcal{I}_P(l)} \mu_m B_m \\ \mathcal{D}_l = \sum_{m \in \mathcal{I}_P(l)} \mu_m D_m, \mathcal{L}_l = \sum_{m \in \mathcal{I}_P(l)} \mu_m L_m \\ \mathcal{N}_l = \sum_{m \in \mathcal{I}_P(l)} \mu_m N_m. \end{cases}$$

For subsequent use, let a set $\mathcal{T}_P$ denote all possible region transitions during the evolution of the system dynamics

$$\mathcal{T}_P := \{(l, i) | f(x(t)) \in \Omega_l^P, f(x(t+1)) \in \Omega_i^P, l, i \in \Lambda_P\}.$$

B. Piecewise-fuzzy State Feedback Controller

A fuzzy controller with q fuzzy rules is to be designed for the T-S fuzzy system in (1). Following the similar partition method, we assume that the premise variable space of the controller is divided into several regions Ω_s^C ($s \in \Lambda_C$) and let Λ_C and $\mathcal{I}_C(s)$ denote the set of region indices and the index of fired rules in the s th region Ω_s^C , respectively. The piecewise-fuzzy state feedback controller is constructed as follows:

Region Rule s : IF $g(x(t)) \in \Omega_s^C$, $s \in \Lambda_C$,

Controller Rule $\mathfrak{R}_C^n$: IF $g_1(x(t))$ is $\mathcal{N}_1^n$, AND $\dots$, AND $g_\Phi(x(t))$ is $\mathcal{N}_\Phi^n$, THEN

$$u(t) = K_{sn}x(t), n \in \{1, 2, \dots, q\}, \quad (5)$$

where $g(x(t)) = [g_1(x(t)), \dots, g_\Phi(x(t))]^T$ is the premise variable vector and K_{sn} , ($n \in \mathcal{I}_C(s)$, $s \in \Lambda_C$) are the state feedback controller gains of each local model in each region to be determined later. Thus, in the s th region, the controller can be given by

$$u(t) = \mathcal{K}_s x(t), s \in \Lambda_C, \quad (6)$$

where $\mathcal{K}_s = \sum_{n \in \mathcal{I}_C(s)} \nu_n K_{sn}$ and ν_n or $\nu_n(x(t))$ is the normalized fuzzy membership function for the controller given by

$$\nu_n = \frac{\prod_{j=1}^{\Phi} \nu_j^n[g_j(x(t))]}{\sum_{i=1}^q \prod_{j=1}^{\Phi} \nu_j^i[g_j(x(t))]} \geq 0, \sum_{n=1}^q \nu_n = 1.$$

Similarly, we define a set $\mathcal{T}_C$ to denote all possible region transitions from Ω_s^C to Ω_j^C , $s, j \in \Lambda_C$

$$\mathcal{T}_C := \{(s, j) | g(x(t)) \in \Omega_s^C, g(x(t+1)) \in \Omega_j^C, s, j \in \Lambda_C\}.$$

Based on the mathematical expressions of physical plant and state feedback controller in piecewise-fuzzy form in (4) and (6), respectively, the closed-loop system can be obtained

$$\begin{cases} x(t+1) = (\mathcal{A}_l + \mathcal{B}_l \mathcal{K}_s)x(t) + \mathcal{D}_l w(t) \\ z(t) = (\mathcal{L}_l + \mathcal{N}_l \mathcal{K}_s)x(t) \\ y(t) = Cx(t), (l \in \Lambda_P, s \in \Lambda_C). \end{cases} \quad (7)$$

Remark 2.1: From the expression of state feedback controller in (5), it is worth noting that: 1) a fuzzy-weighted state feedback controller gain K_{sn} is proposed in each region, that is, there are several controller gains in fuzzy regions instead of one single controller gain in one region. In this case, it offers a greater design flexibility for the state feedback controller. 2) Moreover, in the controller formulation, it is not required that the T-S fuzzy physical plant and fuzzy state feedback controller share the same membership functions, which also offers the design flexibility for the membership functions of fuzzy controller as the membership functions for fuzzy controller can be selected to be less complex than those of the physical plant.

C. Piecewise-fuzzy Observer-based Output Feedback Controller

In the previous subsection, we consider the piecewise-fuzzy state feedback controller for T-S fuzzy systems by taking the unmatched premise variables into account. However, it might not be the case in practice that all the state information is available at the controller side as usually some states cannot be directly measured. Thus, in this section, we consider the problem of piecewise-fuzzy observer-based output feedback control for the T-S fuzzy systems under the piecewise-Lyapunov-function-based framework.

Similarly, we define Ω_s^C ($s \in \Lambda_C$), Λ_C , and $\mathcal{I}_C(s)$ as the partitioned regions, the set of region indices, and the index of fired rules in the s th region Ω_s^C , respectively. For the discrete-time T-S fuzzy system in (1) and its piecewise form in (4), we consider the following piecewise-fuzzy observer-based output feedback controller with unmatched premise variables:

Region Rule s : IF $g(\hat{x}(t)) \in \Omega_s^C$, $s \in \Lambda_C$,

Observer-based Controller Rule $\mathfrak{R}_C^n$: IF $g_1(\hat{x}(t))$ is $\mathcal{N}_1^n$ AND $\dots$, AND $g_\Phi(\hat{x}(t))$ is $\mathcal{N}_\Phi^n$, **THEN**

$$\begin{cases} \hat{x}(t+1) = A_n \hat{x}(t) + B_n u(t) + R_{sn}(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C \hat{x}(t) \\ u(t) = K_{sn} \hat{x}(t), n \in \{1, 2, \dots, q\} \end{cases} \quad (8)$$

where $g(\hat{x}(t)) = [g_1(\hat{x}(t)), \dots, g_\Phi(\hat{x}(t))]^T$ is the premise variable depending on the estimated states $\hat{x}(t) \in \mathbb{R}^{n_x}$, and R_{sn} and K_{sn} ($n \in \mathcal{I}_C(s)$, $s \in \Lambda_C$) are the observer and controller gains of each local model in each region.

With the partition approach, the fuzzy observer-based controller in piecewise-fuzzy form can be given as

$$\begin{cases} \hat{x}(t+1) = \hat{A}_s \hat{x}(t) + \hat{B}_s u(t) + \hat{R}_s(y(t) - \hat{y}(t)) \\ \hat{y}(t) = C \hat{x}(t) \\ u(t) = \hat{K}_s \hat{x}(t) \end{cases} \quad (9)$$

where

$$\begin{cases} \hat{A}_s = \sum_{n \in \mathcal{I}_C(s)} \hat{\nu}_n A_n, \hat{B}_s = \sum_{n \in \mathcal{I}_C(s)} \hat{\nu}_n B_n, \\ \hat{R}_s = \sum_{n \in \mathcal{I}_C(s)} \hat{\nu}_n R_{sn}, \hat{K}_s = \sum_{n \in \mathcal{I}_C(s)} \hat{\nu}_n K_{sn} \end{cases}$$

and $\hat{\nu}_n$ or $(\nu_n(\hat{x}(t)))$ is the normalized fuzzy membership function given as

$$\hat{\nu}_n = \frac{\prod_{j=1}^{\Phi} \nu_j^n[g_j(\hat{x}(t))]}{\sum_{i=1}^q \prod_{j=1}^{\Phi} \nu_j^i[g_j(\hat{x}(t))]} \geq 0, \sum_{n=1}^q \hat{\nu}_n = 1.$$

For subsequent use, we also define a set $\mathcal{T}_C$ to denote all possible region transitions of the observer dynamics from Ω_s^C to Ω_j^C , $s, j \in \Lambda_C$

$$\mathcal{T}_C := \{(s, j) | g(\hat{x}(t)) \in \Omega_s^C, g(\hat{x}(t+1)) \in \Omega_j^C, s, j \in \Lambda_C\}.$$

Defining the estimation error as $e(t) = x(t) - \hat{x}(t)$, and by combining (4) and (9), the augmented closed-loop system can be rewritten as

$$\begin{cases} \xi(t+1) = \bar{A}_{ls} \xi(t) + \bar{D}_l w(t) \\ z(t) = \bar{L}_s \xi(t) \end{cases} \quad (10)$$

where

$$\begin{cases} \xi(t) = [x^T(t) \ e^T(t)]^T, \bar{D}_l = \begin{bmatrix} \mathcal{D}_l \\ \mathcal{D}_l \end{bmatrix} \\ \bar{A}_{ls} = \begin{bmatrix} \mathcal{A}_l + \mathcal{B}_l \hat{K}_s \\ \mathcal{A}_l - \hat{A}_s + (\mathcal{B}_l - \hat{B}_s) \hat{K}_s - \mathcal{B}_l \hat{K}_s \\ \hat{A}_s - (\mathcal{B}_l - \hat{B}_s) \hat{K}_s - \hat{R}_s C \end{bmatrix} \\ \bar{L}_s = [\mathcal{L}_l + \mathcal{N}_l \hat{K}_s \ -\mathcal{N}_l \hat{K}_s]. \end{cases}$$

For the given discrete-time T-S fuzzy dynamic system in (1) and piecewise form in (4), the objective of this paper is to design the admissible piecewise-fuzzy state feedback controller and observer-based output feedback controller in the form of (5) and (8), respectively, such that the closed-loop systems in (7) and (10) are asymptotically stable with guaranteed $\mathcal{H}_\infty$ performance, that is, the l_2 -norm from the disturbance $w(t)$ to the regulated output $z(t)$ is less than γ under zero initial conditions

$$\sum_{t=0}^{\infty} z^T(t) z(t) \leq \gamma^2 \sum_{t=0}^{\infty} w^T(t) w(t)$$

for all $w(t) \in l_2[0, \infty)$.

III. MAIN RESULTS

In this section, the results on piecewise-fuzzy controller design are presented based on piecewise Lyapunov functions for the discrete-time T-S fuzzy systems. By considering the shape of membership functions of the physical plant and controllers, the membership-function-dependent approaches to the controller design are proposed.

A. State Feedback Controller Design

Theorem 3.1: Considering the discrete-time T-S fuzzy model given in (1), the piecewise-fuzzy state feedback controller in (5) with unmatched premise variables, and a series of predefined scalars $\bar{\mu}_m$ and $\bar{\nu}_n$ utilized to respectively approximate the membership functions μ_m and ν_n , and θ_{mn} such that $\mu_m \nu_n - \bar{\mu}_m \bar{\nu}_n - \theta_{mn} \geq 0$, the closed-loop system in (7) is asymptotically stable with a guaranteed $\mathcal{H}_\infty$ performance level γ if there exist $M_{lsij} \in \mathbb{S}^{2n_x+n_w+n_z}$, $P_{ls} \in \mathbb{S}^{n_x}$, $G_s \in \mathbb{R}^{n_x \times n_x}$, and $\tilde{K}_{sn} \in \mathbb{R}^{n_u \times n_x}$, such that the following conditions hold:

$$P_{ls} > 0, W_{lsij} \geq 0 \quad (11)$$

$$\sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} [\theta_{mn} M_{lsij} + (\bar{\mu}_m \bar{\nu}_n + \theta_{mn})(\Xi_{lsij}^{mn} + W_{lsij})] < 0, \quad (12)$$

$$\begin{aligned} & \Xi_{lsij}^{mn} + M_{lsij} + W_{lsij} < 0, \\ & m \in \mathcal{I}_P(l), n \in \mathcal{I}_C(s), l, i \in \Lambda_P, \\ & s, j \in \Lambda_C, (l, i) \in \mathcal{T}_P, (s, j) \in \mathcal{T}_C, \end{aligned} \quad (13)$$

where

$$\Xi_{lsij}^{mn} = \begin{bmatrix} P_{ls} - G_s - G_s^T & * & * & * \\ \mathbf{0}_{n_w \times n_x} & -\gamma^2 \mathbf{I}_{n_w} & * & * \\ A_m G_s + B_m \tilde{K}_{sn} & D_m & -P_{ij} & * \\ L_m G_s + N_m \tilde{K}_{sn} & \mathbf{0}_{n_z \times n_w} & \mathbf{0}_{n_z \times n_x} & -\mathbf{I}_{n_z} \end{bmatrix}.$$

Moreover, the admissible piecewise-fuzzy state feedback controller gains in the form of (5) are given by

$$K_{sn} = \tilde{K}_{sn} G_s^{-1}. \quad (14)$$

Proof: Consider the following piecewise Lyapunov function:

$$V(t) = x^T(t) P_{ls}^{-1} x(t), f(x(t)) \in \Omega_l^P, \\ g(x(t)) \in \Omega_s^C, l \in \Lambda_P, s \in \Lambda_C, \quad (15)$$

where $0 < P_{ls}$ are Lyapunov matrices to be determined.

Define $\eta(t) = [x^T(t) w^T(t)]^T$ and $\Delta V(t) = V(t + 1, x(t + 1)) - V(t, x(t))$. With the given piecewise Lyapunov function, it is sufficient to show the system (7) satisfying the aforementioned requirements in last section if the following inequality holds:

$$\mathcal{J}(t) \triangleq V(t + 1, x(t + 1)) - V(t, x(t)) \\ + z^T(t) z(t) - \gamma^2 w^T(t) w(t) < 0. \quad (16)$$

Without loss of generality, we assume that $f(x(t + 1)) \in \Omega_i^P, g(x(t + 1)) \in \Omega_j^C$ ($i \in \Lambda_P, j \in \Lambda_C$), and $(l, i) \in \mathcal{T}_P, (s, j) \in \mathcal{T}_C$. Along the trajectory of (7), the left-hand-side of equation (16) is reformulated as:

$$\mathcal{J}(t) = [(A_l + B_l K_s) x(t) + D_l w(t)]^T \\ \times P_{ij}^{-1} [(A_l + B_l K_s) x(t) + D_l w(t)] \\ - x^T(t) P_{ls}^{-1} x(t) + z^T(t) z(t) - \gamma^2 w^T(t) w(t) \\ = \eta^T(t) \tilde{\Xi}_{lsij}(\mu, \nu) \eta(t), \quad (17)$$

where

$$\tilde{\Xi}_{lsij}(\mu, \nu) = \begin{bmatrix} -P_{ls}^{-1} & * & * & * \\ \mathbf{0}_{n_w \times n_x} & -\gamma^2 \mathbf{I}_{n_w} & * & * \\ A_l + B_l K_s & D_l & -P_{ij} & * \\ \mathcal{L}_l + N_l K_s & \mathbf{0}_{n_z \times n_w} & \mathbf{0}_{n_z \times n_x} & -\mathbf{I}_{n_z} \end{bmatrix}.$$

Obviously, if $\tilde{\Xi}_{lsij}(\mu, \nu) < 0$, one can conclude that the closed-loop system in (7) is asymptotically stable with a guaranteed $\mathcal{H}_\infty$ performance level γ .

On the other hand, considering the definitions of system matrices and controller gain in piecewise-fuzzy form, we have

$$\tilde{\Xi}_{lsij}(\mu, \nu) = \sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} \mu_m \nu_n \hat{\Xi}_{lsij}^{mn}, \quad (18)$$

where

$$\hat{\Xi}_{lsij}^{mn} = \begin{bmatrix} -P_{ls}^{-1} & * & * & * \\ \mathbf{0}_{n_w \times n_x} & -\gamma^2 \mathbf{I}_{n_w} & * & * \\ A_m + B_m K_{sn} & D_m & -P_{ij} & * \\ L_m + N_m K_{sn} & \mathbf{0}_{n_z \times n_w} & \mathbf{0}_{n_z \times n_x} & -\mathbf{I}_{n_z} \end{bmatrix}.$$

Therefore, a sufficient condition for the closed-loop T-S fuzzy system in (7) to be asymptotically stable with a prescribed $\mathcal{H}_\infty$ performance level γ can be given by

$$\tilde{\Xi}_{lsij}(\mu, \nu) = \sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} \mu_m \nu_n \hat{\Xi}_{lsij}^{mn} < 0, \quad (19)$$

from which we can see that the information of membership functions $\mu_m(x(t))$ and $\nu_n(x(t))$ is involved.

To further reduce the design conservatism, we introduce a series of discrete values $\bar{\mu}_m$ and $\bar{\nu}_n$ to approximate the

membership functions μ_m and ν_n , which can be determined according to the membership functions with respect to some points on the abscissa axis. In this case, it can be easily known that $\sum_{m \in \mathcal{I}_P(l)} \bar{\mu}_m = 1$ and $\sum_{n \in \mathcal{I}_C(s)} \bar{\nu}_n = 1$. To facilitate the subsequent controller design, some slack matrices $0 \leq \hat{W}_{lsij} \in \mathbb{S}^{2n_x + n_w + n_z}$ are introduced, and thus we have

$$\tilde{\Xi}_{lsij}(\mu, \nu) \leq \sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} \left[\bar{\mu}_m \bar{\nu}_n \hat{\Xi}_{lsij}^{mn} \right. \\ \left. + (\mu_m \nu_n - \bar{\mu}_m \bar{\nu}_n) (\hat{\Xi}_{lsij}^{mn} + \hat{M}_{lsij}) + \mu_m \nu_n \hat{W}_{lsij} \right. \\ \left. + (\theta_{mn} - \theta_{mn}) (\hat{\Xi}_{lsij}^{mn} + \hat{M}_{lsij} + \hat{W}_{lsij}) \right] \\ = \sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} \left[\theta_{mn} \hat{M}_{lsij} \right. \\ \left. + (\bar{\mu}_m \bar{\nu}_n + \theta_{mn}) (\hat{\Xi}_{lsij}^{mn} + \hat{W}_{lsij}) \right] \\ + \sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} (\mu_m \nu_n - \bar{\mu}_m \bar{\nu}_n - \theta_{mn}) \\ \times (\hat{\Xi}_{lsij}^{mn} + \hat{M}_{lsij} + \hat{W}_{lsij}) \quad (20)$$

where $\hat{M}_{lsij}$ are arbitrary symmetric matrices with appropriate dimensions, θ_{mn} are known scalars such that $\mu_m \nu_n - \bar{\mu}_m \bar{\nu}_n - \theta_{mn} \geq 0$.

From the terms on the right-hand-side of the equation in (20), we can conclude that inequality (16) holds if the following conditions are satisfied:

$$\sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} \left[\theta_{mn} \hat{M}_{lsij} \right. \\ \left. + (\bar{\mu}_m \bar{\nu}_n + \theta_{mn}) (\hat{\Xi}_{lsij}^{mn} + \hat{W}_{lsij}) \right] < 0, \quad (21)$$

$$\hat{\Xi}_{lsij}^{mn} + \hat{M}_{lsij} + \hat{W}_{lsij} < 0, \quad (22)$$

$$l, i \in \Lambda_P, s, j \in \Lambda_C, (l, i) \in \mathcal{T}_P, (s, j) \in \mathcal{T}_C.$$

Now, we are in the position to derive the controller gains based on the stability conditions given above. It should be noted that the matrix variables P_{ls}^{-1} and P_{ij} co-exist in (21) and (22), which leads to some difficulties in the solution of optimization problem. Thus, we adopt the widely used inequality as follows by introducing some nonsingular slack matrices $\mathcal{G}_s \in \mathbb{R}^{n_x \times n_x}$ ($s \in \Lambda_C$):

$$P_{ls} - G_s - G_s^T + G_s P_{ls}^{-1} G_s^T \\ = (P_{ls} - G_s) P_{ls}^{-1} (P_{ls} - G_s)^T \geq 0, \quad (23)$$

which yields

$$-G_s P_{ls}^{-1} G_s^T \leq P_{ls} - G_s - G_s^T. \quad (24)$$

Now, performing a congruence transformation to (21) and (22) with $\mathcal{G}_s = \text{diag}\{G_s, I_{n_w}, I_{n_x}, I_{n_z}\}$ and considering the inequality in (24), conditions (21) and (22) hold if

$$\sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} \left[\theta_{mn} \mathcal{G}_s^T \hat{M}_{lsij} \mathcal{G}_s \right. \\ \left. + (\bar{\mu}_m \bar{\nu}_n + \theta_{mn}) (\hat{\Xi}_{lsij}^{mn} + \mathcal{G}_s^T \hat{W}_{lsij} \mathcal{G}_s) \right] < 0, \quad (25)$$

$$\hat{\Xi}_{lsij}^{mn} + \mathcal{G}_s^T \hat{M}_{lsij} \mathcal{G}_s + \mathcal{G}_s^T \hat{W}_{lsij} \mathcal{G}_s < 0,$$

$$l, i \in \Lambda_{\mathcal{P}}, s, j \in \Lambda_{\mathcal{C}}, (l, i) \in \mathcal{T}_{\mathcal{P}}, (s, j) \in \mathcal{T}_{\mathcal{C}}, \quad (26)$$

with

$$\tilde{\Xi}_{lsij}^{mn} = \begin{bmatrix} P_{ls} - G_s - G_s^T & * & * & * \\ \mathbf{0}_{n_w \times n_x} & -\gamma^2 \mathbf{I}_{n_w} & * & * \\ A_m G_s + B_m K_{sn} G_s & D_m & -P_{ij} & * \\ L_m + N_m K_{sn} G_s & \mathbf{0}_{n_z \times n_w} & \mathbf{0}_{n_z \times n_x} & -\mathbf{I}_{n_z} \end{bmatrix},$$

which further implies the stability condition with a guaranteed $\mathcal{H}_{\infty}$ performance level γ in (16).

By defining $\tilde{K}_{sn} = K_{sn} G_s$, $W_{lsij} = G_s^T \tilde{W}_{lsij} G_s$, and $M_{lsij} = G_s^T M_{lsij} G_s$, we readily have (25) and (26) from (11)-(13) in *Theorem 3.1*. The proof is thus completed.

Remark 3.1: In *Theorem 3.1*, the stability analysis and controller synthesis are investigated for discrete-time T-S fuzzy systems under piecewise-Lyapunov-function-based framework. It should be noted that, to further reduce the design conservatism, some information of the membership functions is incorporated by choosing finite discrete values of these membership functions. To one extreme, if μ_m and ν_n can be well approximated by a great number of $\bar{\mu}_m$ and $\bar{\nu}_n$ (that is, $\bar{\mu}_m \bar{\nu}_n \cong \mu_m \nu_n$), respectively, the stability condition with a guaranteed $\mathcal{H}_{\infty}$ performance level γ in (11)-(13) will reduce to $\sum_{m \in \mathcal{I}_{\mathcal{P}}(l)} \sum_{n \in \mathcal{I}_{\mathcal{C}}(s)} \bar{\mu}_m \bar{\nu}_n \tilde{\Xi}_{lsij}^{mn} < 0$ by letting $M_{lsij} = 0$, $W_{lsij} = 0$, and $\theta_{mn} = 0$. On the other hand, with the introduction of slack matrices M_{lsij} , W_{lsij} , and the increasing number of partitioned regions, the computational load will become higher.

In *Theorem 3.1*, a membership-function-dependent approach to the state feedback controller design is proposed under piecewise-Lyapunov-function-based framework. However, if the information of membership functions is not considered in the stability analysis as in some existing literatures, such as [31], we will directly derive the following conditions on state feedback controller design.

Corollary 3.1: For the discrete-time T-S fuzzy model given in (1) and the fuzzy-piecewise state feedback controller in (5) with unmatched premise variables, the closed-loop system in (7) is asymptotically stable with a guaranteed $\mathcal{H}_{\infty}$ performance level γ if there exist $P_{ls} \in \mathbb{S}^{n_x}$, $G_s \in \mathbb{R}^{n_x \times n_x}$, and $\tilde{K}_{sn} \in \mathbb{R}^{n_u \times n_x}$, such that the following conditions hold:

$$\Xi_{lsij}^{mn} = \begin{bmatrix} P_{ls} - G_s - G_s^T & * & * & * \\ \mathbf{0}_{n_w \times n_x} & -\gamma^2 \mathbf{I}_{n_w} & * & * \\ A_m G_s + B_m \tilde{K}_{sn} & D_m & -P_{ij} & * \\ L_m + N_m \tilde{K}_{sn} & \mathbf{0}_{n_z \times n_w} & \mathbf{0}_{n_z \times n_x} & -\mathbf{I}_{n_z} \end{bmatrix} < 0$$

$$m \in \mathcal{I}_{\mathcal{P}}(l), n \in \mathcal{I}_{\mathcal{C}}(s), l, i \in \Lambda_{\mathcal{P}},$$

$$s, j \in \Lambda_{\mathcal{C}}, (l, i) \in \mathcal{T}_{\mathcal{P}}, (s, j) \in \mathcal{T}_{\mathcal{C}}, \quad (27)$$

and moreover, the admissible piecewise-fuzzy state feedback controller in the form of (5) can be derived by (14).

The proof of *Corollary 3.1* can be easily completed based on (19) by extracting the membership functions.

Remark 3.2: In the aforementioned results on controller design, it is not required that the piecewise-fuzzy state feedback controller shares the same membership functions with physical plant. However, if the $\mathcal{H}_{\infty}$ performance analysis for the closed-loop system in (7) and the corresponding

state feedback controller design are conducted with parallel distributed compensation (PDC) method, i.e., $p = c$ and $\mu_m(x(t)) = \nu_m(x(t))$ for all $m \in \{1, 2, \dots, p\}$, the conservatism of the controller design results in *Theorem 3.1* can be further relaxed under the membership-function-dependent analysis strategy with some approaches reported in the existing literatures [15], [25], [26], [40].

B. Observer-based Output Feedback Controller Design

In this subsection, we present the main result on the piecewise-fuzzy observer-based output feedback controller design.

Theorem 3.2: Consider the T-S fuzzy discrete-time system in (4) and the observer-based output feedback controller in the form of (8) with unmatched premise variables. For the given scalars $\bar{\mu}_m$ and $\bar{\nu}_n$ respectively approximating the membership functions μ_m and ν_n , and θ_{mn} satisfying $\mu_m \bar{\nu}_n - \bar{\mu}_m \bar{\nu}_n - \theta_{mn} \geq 0$, the closed-loop system (9) is asymptotically stable with a guaranteed $\mathcal{H}_{\infty}$ performance γ if there exists a set of matrices $P_{ls} \in \mathbb{S}^{2n_x}$, $S_{1s(1)} \in \mathbb{R}^{n_y \times n_y}$, $S_{1s(2)} \in \mathbb{R}^{(n_x - n_y) \times n_y}$, $S_{1s(3)} \in \mathbb{R}^{(n_x - n_y) \times (n_x - n_y)}$, $\tilde{K}_{sn} \in \mathbb{R}^{n_u \times n_x}$, $\tilde{R}_{sn} \in \mathbb{R}^{n_x \times n_y}$, M_{lsij} , and $W_{lsij} \in \mathbb{S}^{4n_x + n_z + n_w}$ such that the following linear matrix inequality conditions are satisfied

$$P_{ls} > 0, W_{lsij} \geq 0, \quad (28)$$

$$\sum_{m \in \mathcal{I}_{\mathcal{P}}(l)} \sum_{n \in \mathcal{I}_{\mathcal{C}}(s)} [\theta_{mn} M_{lsij} + (\bar{\mu}_m \bar{\nu}_n + \theta_{mn})(\Pi_{lsij}^{mn} + W_{lsij})] < 0, \quad (29)$$

$$(\Pi_{lsij}^{mn} + M_{lsij} + W_{lsij}) < 0,$$

$$m \in \mathcal{I}_{\mathcal{P}}(l), n \in \mathcal{I}_{\mathcal{C}}(s), l, i \in \Lambda_{\mathcal{P}},$$

$$s, j \in \Lambda_{\mathcal{C}}, (l, i) \in \mathcal{T}_{\mathcal{P}}, (s, j) \in \mathcal{T}_{\mathcal{C}}, \quad (30)$$

where

$$\left\{ \begin{array}{l} \Pi_{lsij}^{mn} = \begin{bmatrix} P_{ls} - S_s^T - S_s & * & * & * \\ \mathbf{0}_{n_w \times 2n_x} & -\gamma^2 \mathbf{I}_{n_w} & * & * \\ \tilde{\mathcal{A}}_s^{mn} & D_m & -P_{ij} & * \\ \tilde{\mathcal{L}}_s^{mn} & \mathbf{0}_{n_z \times n_w} & \mathbf{0}_{n_z \times 2n_x} & -\mathbf{I}_{n_z} \end{bmatrix} \\ \tilde{\mathcal{A}}_s^{mn} = \begin{bmatrix} A_m S_{1s} + B_m \tilde{K}_{sn} \\ (A_m - A_n) S_{1s} + (B_m - B_n) \tilde{K}_{sn} \\ -B_m \tilde{K}_{sn} \\ A_n S_{1s} - (B_m - B_n) \tilde{K}_{sn} - \tilde{R}_{sn} J_3^T \end{bmatrix} \\ \tilde{\mathcal{L}}_s^{mn} = [L_m S_{1s} + N_m \tilde{K}_{sn} - N_m \tilde{K}_{sn}] \\ \tilde{R}_{sn} = [\tilde{R}_{sn} \mathbf{0}_{n_x \times (n_x - n_y)}], S_s = \text{diag}\{S_{1s}, S_{1s}\} \\ S_{1s} = J_3 \begin{bmatrix} J_2^{-1} J_1^T S_{1s(1)} & \mathbf{0}_{n_y \times (n_x - n_y)} \\ S_{1s(2)} & S_{1s(3)} \end{bmatrix} J_3^T, \end{array} \right.$$

and J_1, J_2 , and J_3 are obtained via the singular value decomposition (SVD) of the output matrix C , i.e., $C = J_1 [J_2 \mathbf{0}_{n_y \times (n_x - n_y)}] J_3^T$. Moreover, the observer and controller gains can be calculated via

$$R_{sn} = \tilde{R}_{sn} S_{1s(1)}^{-1}, K_{sn} = \tilde{K}_{sn} S_{1s}^{-1}, s \in \Lambda_{\mathcal{C}}, n \in \mathcal{I}_{\mathcal{C}}(s). \quad (31)$$

Proof: We consider the following piecewise quadratic Lyapunov function as in state feedback case:

$$V(t) = \xi^T(t) P_{ls}^{-1} \xi(t), f(x(t)) \in \Omega_l^{\mathcal{P}},$$

$$g(\hat{x}(t)) \in \Omega_s^c, l \in \Lambda_P, s \in \Lambda_C, \quad (32)$$

where $0 < P_{ls} \in \mathbb{S}^{2n_x}$ are Lyapunov matrices to be determined, and the following inequality will be used to guarantee the closed-loop system in (10) to be asymptotically stable with a prescribed $\mathcal{H}_\infty$ performance level γ

$$V(t+1) - V(t) < -z^T(t)z(t) + \gamma^2 w^T(t)w(t). \quad (33)$$

Following the similar line in *Theorem 3.1*, it can be easily shown that the aforementioned inequality holds if along the trajectories of system (10) the following inequality is satisfied:

$$\sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} \mu_m \hat{v}_n \hat{\Pi}_{lsij}^{mn} < 0, \quad (34)$$

where

$$\begin{cases} \hat{\Pi}_{lsij}^{mn} = \begin{bmatrix} -P_{ls}^{-1} & * & * & * \\ \mathbf{0}_{n_w \times 2n_x} & -\gamma^2 \mathbf{I}_{n_w} & * & * \\ \bar{\mathcal{A}}_s^{mn} & \bar{\mathcal{D}}^m & -P_{ij} & * \\ \bar{\mathcal{L}}_s^{mn} & \mathbf{0}_{n_z \times n_w} & \mathbf{0}_{n_z \times 2n_x} & -\mathbf{I}_{n_z} \end{bmatrix} \\ \bar{\mathcal{A}}_s^{mn} = \begin{bmatrix} A_m + B_m K_{sn} \\ A_m - A_n + (B_m - B_n) K_{sn} \\ -B_m K_{sn} \\ A_n - (B_m - B_n) K_{sn} - R_{sn} C \end{bmatrix} \\ \bar{\mathcal{D}}^m = \begin{bmatrix} D_m \\ D_m \end{bmatrix} \\ \bar{\mathcal{L}}_s^{mn} = \begin{bmatrix} L_m + N_m K_{sn} & -N_m K_{sn} \end{bmatrix} \end{cases}$$

By introducing the slack matrices $M_{lsij} \in \mathbb{S}^{4n_x + n_z + n_w}$, and $0 \leq W_{lsij} \in \mathbb{S}^{4n_x + n_z + n_w}$, the inequality (34) holds if

$$\begin{aligned} & \sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} \left[\theta_{mn} \hat{M}_{lsij} \right. \\ & \quad \left. + (\bar{\mu}_m \bar{v}_n + \theta_{mn})(\hat{\Pi}_{lsij}^{mn} + \hat{W}_{lsij}) \right] \\ & \quad + \sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} (\mu_m \hat{v}_n - \bar{\mu}_m \bar{v}_n - \theta_{mn}) \\ & \quad \times (\hat{\Pi}_{lsij}^{mn} + \hat{M}_{lsij} + \hat{W}_{lsij}) < 0, \quad (35) \end{aligned}$$

where $\bar{\mu}_m$ and $\bar{v}_n$ are a series of discrete values to approximate the membership functions μ_m and $\hat{v}_n$, respectively, and θ_{mn} are known scalars satisfying $\mu_m \hat{v}_n - \bar{\mu}_m \bar{v}_n - \theta_{mn} \geq 0$ ($m \in \mathcal{I}_P(l), n \in \mathcal{I}_C(s)$).

By considering the matrix inequality in (23), and performing a congruence transformation to (35) with $\mathcal{S}_s = \text{diag}\{S_s, \mathbf{I}_{n_w}, \mathbf{I}_{2n_x}, \mathbf{I}_{n_z}\}$ and its transpose, we have the following two conditions such that the closed-loop is asymptotically stable with $\mathcal{H}_\infty$ performance level γ :

$$\sum_{m \in \mathcal{I}_P(l)} \sum_{n \in \mathcal{I}_C(s)} [\theta_{mn} M_{lsij} + (\bar{\mu}_m \bar{v}_n + \theta_{mn})(\tilde{\Pi}_{lsij}^{mn} + W_{lsij})] < 0, \quad (36)$$

$$(\tilde{\Pi}_{lsij}^{mn} + M_{lsij} + W_{lsij}) < 0, \quad (37)$$

for $m \in \mathcal{I}_P(l), n \in \mathcal{I}_C(s), l, i \in \Lambda_P, s, j \in \Lambda_C, (l, i) \in \mathcal{T}_P, (s, j) \in \mathcal{T}_C$, where $W_{lsij} = S_s^T \tilde{W}_{lsij} S_s$, $M_{lsij} = S_s^T \tilde{M}_{lsij} S_s$, and

$$\tilde{\Pi}_{lsij}^{mn} = \begin{bmatrix} P_{ls} - S_s^T - S_s & * & * & * \\ \mathbf{0}_{n_w \times 2n_x} & -\gamma^2 \mathbf{I}_{n_w} & * & * \\ \bar{\mathcal{A}}_s^{mn} S_s & \bar{\mathcal{D}}^m S_s & -P_{ij} & * \\ \bar{\mathcal{L}}_s^{mn} S_s & \mathbf{0}_{n_z \times n_w} & \mathbf{0}_{n_z \times 2n_x} & -\mathbf{I}_{n_z} \end{bmatrix}.$$

Without loss of generality, we assume the output matrix C to be with full row rank, and then its SVD can be given as follows:

$$\begin{aligned} C &= J_1 [J_2 \mathbf{0}_{n_y \times (n_x - n_y)}] J_3^T, \\ J_1 J_1^T &= \mathbf{I}_{n_y}, J_3 J_3^T = \mathbf{I}_{n_x}, \end{aligned} \quad (38)$$

where $J_1 \in \mathbb{R}^{n_y \times n_y}$, $J_2 \in \mathbb{R}^{n_y \times n_y}$, and $J_3 \in \mathbb{R}^{n_x \times n_x}$ are all nonsingular matrices.

To facilitate the calculations of observer and controller gains, we specify the matrix S_s to be with the structure as follows:

$$\begin{aligned} S_s &= \text{diag}\{S_{1s}, S_{1s}\}, \\ S_{1s} &= J_3 \begin{bmatrix} J_2^{-1} J_1^T S_{1s(1)} & \mathbf{0}_{n_y \times (n_x - n_y)} \\ S_{1s(2)} & S_{1s(3)} \end{bmatrix} J_3^T, \end{aligned}$$

where $S_{1s(1)} \in \mathbb{R}^{n_y \times n_y}$, $S_{1s(2)} \in \mathbb{R}^{(n_x - n_y) \times n_y}$, $S_{1s(3)} \in \mathbb{R}^{(n_x - n_y) \times (n_x - n_y)}$, and $S_{1s(1)}$ and $S_{1s(3)}$ are nonsingular.

Then, with (38) and (39) one can obtain

$$\begin{aligned} R_{sn} C S_{1s} &= R_{sn} J_1 [J_2 \mathbf{0}_{n_y \times (n_x - n_y)}] J_3^T \\ &\quad \times J_3 \begin{bmatrix} J_2^{-1} J_1^T S_{1s(1)} & \mathbf{0}_{n_y \times (n_x - n_y)} \\ S_{1s(2)} & S_{1s(3)} \end{bmatrix} J_3^T \\ &= [\tilde{R}_{sn} \mathbf{0}_{n_y \times (n_x - n_y)}] J_3^T \triangleq \tilde{R}_{sn} J_3^T. \end{aligned} \quad (39)$$

By defining $\tilde{K}_{sn} = K_{sn} S_{1s}$, and $\tilde{R}_n = R_{sn} S_{1s(1)}$, we readily have (36)-(37), and (29)-(30), which further implies (34) and (33). Thus, the proof is completed.

Remark 3.3: From the inequalities given in *Theorems 3.1* and *3.2*, it can be observed that the information of membership functions is incorporated in the stability and $\mathcal{H}_\infty$ performance analysis within the piecewise-Lyapunov-function-based framework, by which the design conservatism is further reduced (please see Table II in Example 4.1). However, it should be noted that two different types of space separations are introduced, that is, the partition on premise variable space for system conversion from T-S fuzzy model to equivalent piecewise-fuzzy form and the division on x -axis of membership functions for approximation of these functions with a series of discrete values. In this case, with increase of the partitions, the number of stability conditions in terms of matrix inequalities will increase accordingly, and thus, a computational problem might arise for some systems with more complex membership functions.

Remark 3.4: With the introduction of slack matrices S_s in *Theorem 3.2*, a sufficient condition in terms of linear matrix inequality for the observer and controller design is derived, which can be directly solved via some commercially available softwares, such as, MatLab. However, it is noted that, to facilitate the linearization procedure of inequality constraints in (36) and (37), S_s is specified with a structural constraints

in (38), which inevitably introduced some degree of design conservatism as expected. To further reduce the conservatism, the cone complementarity linearization (CCL) design strategy can be adopted [41]. For more details about CCL, please refer to [42].

IV. SIMULATION RESULTS

In this section, two illustrative examples are given to show the effectiveness of the proposed approaches to the design of piecewise-fuzzy state feedback controller and observer-based output feedback controller.

Example 4.1: Consider a discrete-time T-S fuzzy system in the form of (1) with three local models, and the system parameters are given as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.9000 & -1.1800 \\ -0.0900 & -0.6300 \end{bmatrix}, A_2 = \begin{bmatrix} 1.0500 & -0.800 \\ 0.0500 & -0.2000 \end{bmatrix} \\ A_3 &= \begin{bmatrix} 1.3440 & -1.4280 \\ 0.3360 & -1.1200 \end{bmatrix}, B_1 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, D_1 = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix} \\ B_2 &= B_3 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, D_2 = D_3 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} \\ L_m &= [0 \ 0.2], N_m = 0.1, C = [1 \ 1] \end{aligned}$$

for $m \in \{1, 2, 3\}$.

Fig. 1. Membership functions for the physical plant in Example 4.1

The membership functions of the physical plant are depicted in Figure 1, from which it can be seen that we have three regions Ω_1^P , Ω_2^P , and Ω_3^P according to the space partition method mentioned in Section II. Moreover, with the partition, one has $\mathcal{I}_P(1) = \{1, 2\}$, $\mathcal{I}_P(2) = \{2\}$, and $\mathcal{I}_P(3) = \{2, 3\}$.

Now, a piecewise-fuzzy state feedback controller in the form of (5) with three fuzzy rules is employed to stabilize the fuzzy system and guarantee a certain level of $\mathcal{H}_\infty$ performance of the closed-loop system. To facilitate the design of the piecewise-fuzzy controller, we assume that the membership functions of the state feedback controller are given by trapezoid as shown in Figure 2, which are not same as those of physical plant.

Now, we consider using the staircase functions to approximate the original membership functions of both fuzzy model and piecewise-fuzzy controller. To this end, the x -axis is uniformly divided into different intervals in each region

Fig. 2. Membership functions for the controller in Example 4.1

Ω_l^P ($\Omega_s^C, l, s \in 1, 2, 3$). Please see Figure 1 in Ω_1^P . We choose the membership function values at the middle point of each interval to approximate the membership functions. It can be easily observed that, with the increasing number of the partitioned intervals, approximation errors among the discrete values $\bar{\mu}_m$, $\bar{\nu}_n$ and the corresponding μ_m and ν_n become small. To guarantee $\mu_m \nu_n - \bar{\mu}_m \bar{\nu}_n - \theta_{mn} \geq 0$, let $\theta_{mn} = \mu_m \nu_n - \bar{\mu}_m \bar{\nu}_n - 10^{-4}$. To demonstrate the effectiveness of the proposed method, we select the step length as 1.0, and the obtained values of θ_{mn} are listed in Table I. Now, by *Theorem 3.1*, we obtain an admissible piecewise-fuzzy state feedback controller for the system in *Example 4.1*, which are given as follows

$$\begin{cases} K_{11} = \begin{bmatrix} -0.0155 & 0.0324 \end{bmatrix}, K_{12} = \begin{bmatrix} -0.0297 & 0.0198 \end{bmatrix} \\ K_{32} = \begin{bmatrix} -0.9502 & 0.4583 \end{bmatrix}, K_{33} = \begin{bmatrix} -1.0103 & 0.4416 \end{bmatrix} \\ K_{22} = \begin{bmatrix} -0.9052 & 0.3700 \end{bmatrix} \end{cases}$$

and the optimal $\mathcal{H}_\infty$ performance index is $\gamma_{\min} = 2.0475$.

To further illustrate the advantage of the proposed method over some existing fuzzy controller design methods, such as, piecewise-Lyapunov-function-based framework and common-Lyapunov-based framework, we consider different cases with step length 1.0, 0.4, and 0.2, that is, the definition range of membership functions $[-10, 10]$ is uniformly divided into 20, 50, 100 intervals, respectively. A detailed comparison of the obtained minimum $\mathcal{H}_\infty$ performance indices $\gamma_{\min}$ under different scenarios and design methods is summarized in Table II. The results clearly show that better performance can be obtained via the developed method in this paper over the existing ones under piecewise-Lyapunov-function-based and common-Lyapunov-based framework, and moreover, with the increasing number of partitioned intervals, the $\mathcal{H}_\infty$ performance can be further improved.

Example 4.2: In this example, the truck-trailer model formulated in [43] is used to demonstrate the effectiveness of the piecewise-fuzzy observer-based output feedback controller design in this paper. As the control purpose of this system is to back up a truck-trailer along a desired trajectory, a simplified

TABLE I
THE CORRESPONDING VALUES OF θ_{mn} WITH STEP LENGTH 1.0 IN EXAMPLE 4.1

	Ω_1^P, Ω_1^C							
θ_{11}	-0.0629	-0.0661	-0.0942	-0.1873	-0.0691	-0.0056	-3.9217×10^{-4}	-1.0990×10^{-4}
θ_{12}	-0.0623	-0.0623	-0.0571	-0.0902	0.0780	-0.0109	-0.0012	-2.4853×10^{-4}
θ_{21}	-2.0292×10^{-4}	-0.0012	-0.0109	-0.0780	-0.0902	-0.0571	-0.0623	-0.0626
θ_{22}	-1.0989×10^{-4}	-3.9217×10^{-4}	-0.0056	-0.0691	-0.1873	-0.0942	-0.0661	-0.0629
	Ω_3^P, Ω_3^C							
θ_{22}	-0.0629	-0.0661	-0.0942	-0.1873	-0.0691	-0.0056	-3.9217×10^{-4}	-1.0990×10^{-4}
θ_{23}	-0.0626	-0.0623	-0.0571	-0.0902	-0.0780	-0.0109	-0.0012	-2.0314×10^{-4}
θ_{23}	-2.0314×10^{-4}	-0.0012	-0.0109	-0.0780	-0.0902	-0.0571	-0.0623	-0.0626
θ_{33}	-1.0990×10^{-4}	-3.9217×10^{-4}	-0.0056	-0.0691	-0.01873	-0.0942	-0.0661	-0.0629

TABLE II
COMPARISON OF MINIMUM $\mathcal{H}_\infty$ PERFORMANCE FOR DIFFERENT CASES IN EXAMPLE 4.1

	<i>Theorem 3.1</i> (step length=0.2)	<i>Theorem 3.1</i> (step length=0.4)	<i>Theorem 3.1</i> (step length=1.0)	PLF-based method	CLF-based method
$\gamma_{\min}$	1.1338	1.5021	2.0475	2.8350	5.3660

model is given as follows:

$$\begin{cases} x_1(t+1) = (1 - \frac{v_0 t_0}{L_0})x_1(t) + \frac{v_0 t_0}{L_0}u(t) + 0.1w(t) \\ x_2(t+1) = x_2(t) + \frac{v_0 t_0}{L_0}x_1(t) + 0.1w(t) \\ x_3(t+1) = x_3(t) + v_0 t_0 \sin(x_2(t) + \frac{v_0 t_0}{L_0}x_1(t)) + 0.1w(t) \end{cases}$$

where $x_1(t)$ is the angle difference between truck and trailer, and $x_2(t)$ is the angle of the trailer; $x_3(t)$ is the vertical position of the rear end of the trailer; $u(t)$ is the control input of steering angle; $w(t)$ is the external disturbance and L_0 is the length of the trailer. The parameters in this model are given as $L_0 = 5.5\text{m}$, $v_0 = -1.0\text{m/s}$, $t_0 = 2.0\text{s}$, and the measurement and regulated output are, respectively, given by

$$\begin{aligned} y(t) &= \begin{bmatrix} 7 & -2 & 0.03 \\ 0.8 & 2.1 & 0 \end{bmatrix} x(t), \\ z(t) &= [0.5 \ 0 \ 0] x(t) + 0.2u(t). \end{aligned}$$

By the fuzzy modeling approach in [43], one can obtain the corresponding T-S fuzzy approximation of the nonlinear system, and the Gaussian-type membership functions are employed as the fuzzy basis functions [44], which are given in Figure 3.

Then the T-S fuzzy dynamics of the truck-trailer model is given as follows:

Plant Rule $\mathfrak{R}_{\mathcal{P}}^m$: IF $x_2(t) + \frac{v_0 t_0}{L_0}x_1(t)$ is M_1^m , THEN

$$\begin{cases} x(t+1) = A_m x(t) + B_m u(t) + D_m w(t) \\ z(t) = L_m x(t) + N_m u(t) \\ y(t) = Cx(t), m \in \mathfrak{R}_{\mathcal{P}} \triangleq \{1, 2\}, \end{cases}$$

and the system matrices are

$$\begin{aligned} A_1 &= \begin{bmatrix} 1 - \frac{v_0 t_0}{L_0} & 0 & 0 \\ \frac{v_0 t_0}{L_0} & 1 & 0 \\ \frac{v_0^2 t_0^2}{2L_0} & v_0 t_0 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 - \frac{v_0 t_0}{L_0} & 0 & 0 \\ \frac{v_0 t_0}{L_0} & 1 & 0 \\ \frac{dv_0^2 t_0^2}{2L_0} & dv_0 t_0 & 1 \end{bmatrix}, \\ B_1 &= B_2 = \begin{bmatrix} \frac{v_0 t_0}{L_0} & 0 & 0 \end{bmatrix}^T, D_1 = D_2 = [0.1 \ 0.1 \ 0.1]^T, \end{aligned}$$

where d equals to $\frac{0.01}{\pi}$ to guarantee the system to be theoretically controllable.

Fig. 3. The membership functions in Example 4.2

Consider the piecewise-fuzzy output feedback controller in the form of (8) with the same membership functions in Figure 3. It is ready for the design of observer-based output feedback controller according to *Theorem 3.2* with the step length $\pi/24$. A feasible solution is obtained with the optimal $\mathcal{H}_\infty$ performance index $\gamma_{\min} = 0.2747$, and the corresponding observer and controller gains are given as follows:

$$\begin{aligned} \begin{cases} K_{11} = [1.9409 \ -1.1070 \ 0.0047] \\ K_{21} = [1.9409 \ -1.1070 \ 0.0047] \\ K_{22} = [1.9314 \ -0.9790 \ 0.0050] \end{cases} \\ \begin{cases} R_{11} = \begin{bmatrix} 0.2235 & 0.1941 \\ -0.0822 & 0.4348 \\ -1.3578 & 2.1514 \end{bmatrix} \\ R_{21} = \begin{bmatrix} 0.2208 & 0.1659 \\ -0.0808 & 0.4022 \\ -1.4246 & 2.1988 \end{bmatrix} \\ R_{22} = \begin{bmatrix} 0.2175 & 0.1678 \\ -0.0827 & 0.4033 \\ -1.4205 & 2.3368 \end{bmatrix} \end{cases} \end{aligned}$$

Fig. 4. The closed-loop state responses without disturbance in Example 4.2

Fig. 5. The closed-loop state responses with $w(t) = e^{-0.05}\sin(0.4\pi t)$ in Example 4.2

and (K_{11}, R_{11}) and $(K_{21}, K_{22}, R_{21}, R_{22})$ are the controller and observer gains when the dynamics of the piecewise-fuzzy observer evolves in region Ω_1^C and Ω_2^C , respectively.

To illustrate the asymptotical stability of the closed-loop system with the obtained observer-based output feedback controller, we show some simulation results under different conditions with $w(t) = 0$. In Figure 4, the state responses under $x(0) = [\frac{2}{5}\pi \ \frac{3}{4}\pi \ 0]^T$, $\hat{x}(0) = [0 \ 0 \ 0]^T$, and $w(t) = 0$ are shown. Now, we consider the $\mathcal{H}_\infty$ performance of the closed-loop system with the external disturbance given by $w(t) = e^{-0.05}\sin(0.4\pi t)$ under zero initial condition. In Figure 5 and Figure 6, the curves of state response of the closed-loop system and the ratio $\sqrt{\sum_{i=0}^{\kappa} z^T(i)z(i)} / \sqrt{\sum_{i=0}^{\kappa} w^T(i)w(i)}$ are plotted, respectively, from which we can see that the ratio is obviously less than $\gamma_{\min} = 0.2747$, further implying the effectiveness of the proposed method.

V. CONCLUSION

In this paper, the problem of membership-function-dependent controller design for T-S fuzzy system is considered under piecewise-Lyapunov-function-based framework. By employing the staircase functions, the continuous information of original membership functions is approximated and

Fig. 6. The response of ratio $\frac{\sqrt{\sum_{i=0}^{\kappa} z^T(i)z(i)}}{\sqrt{\sum_{i=0}^{\kappa} w^T(i)w(i)}}$ in Example 4.2

introduced into the stability analysis and controller synthesis. Via piecewise Lyapunov functions, two approaches to the piecewise-fuzzy state feedback controller and observer-based output feedback controller design are proposed in terms of linear matrix inequalities, which guarantee the closed-loop system to be asymptotically stable with a prescribed $\mathcal{H}_\infty$ performance level. Simulation examples are presented to demonstrate the effectiveness and advantages of the developed methods. However, it should be noted that the controller design for type-1 fuzzy-model-based systems are considered in present work. As an extension, the interval type-2 T-S fuzzy model, which effectively captures the system nonlinearities and parameter uncertainties, has attracted plenty of research interests in recent years. Thus, the problem of stability analysis and controller synthesis for interval type-2 fuzzy systems deserves further investigation under piecewise-Lyapunov-function-based framework.

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REFERENCES

- [1] C. C. Lee, "Fuzzy logic in control systems: fuzzy logic controller-part I," *IEEE Transactions on System, Man, and Cybernetics*, vol. 20, no. 2, pp. 404-418, 1990.
- [2] H. K. Lam, "A review on stability analysis of continuous-time fuzzy-model-based control systems: from membership-function-independent to membership-function-dependent analysis," *Engineering Applications of Artificial Intelligence*, vol. 67, pp. 390-408, 2018.
- [3] T. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. New York: Wiley, 2001.
- [4] G. Feng, *Analysis and Synthesis of Fuzzy Control Systems-A Model Based Approach*. Boca Raton, FL: CRC, 2010.
- [5] T. Hessburg and M. Tomizuka, "Fuzzy logic control for lateral vehicle guidance," *IEEE Control Systems*, vol. 14, no. 4, pp. 55-63, 1994.
- [6] L.-B. Wu, X.-Q. He, and D.-Q. Zhang, "Cooperative adaptive fuzzy control for a class of uncertain nonlinear multi-agent systems with time delays," *Journal of Control and Decision*, vol. 4, no. 3, pp. 131-152, 2017.

- [7] Z. Gao, Y. Wang, J. Xiong, and Y. Pan, "Stabilising control for a class of chaotic systems based on adaptive logic systems," *Journal of Control and Decision*, vol. 3, no. 3, pp. 165-178, 2016.
- [8] E. H. Mamdani, "Application of fuzzy logic to approximate reasoning using linguistic synthesis," *IEEE Transactions on Computers*, vol. 12, pp. 1182-1191, 1977.
- [9] A. Sala, T. M. Guerra, and R. Babuška, "Perspectives of fuzzy systems and control," *Fuzzy Sets and Systems*, vol. 156, no. 3, pp. 432-444, 2005.
- [10] J. A. Meda-Campaña, A. Grande-Meza, J. J. Rubio, R. Tapia-Herrera, T. Hernández-Cortés, A. Curtidor-López, L. A. Páramo-Carranza, I. O. Cázares-Ramírez, "Design of stabilizers and observers for a class of multivariable TS fuzzy models on the basis of new interpolation functions," *IEEE Transactions on Fuzzy Systems*, DOI: 10.1109/TFUZZ.2017.2786244.
- [11] J. J. Rubio, "Discrete time control based in neural networks for pendulums," *Applied Soft Computing*, vol. 68, pp. 821-832, 2018.
- [12] L. A. Páramo-Carranza, J. A. Meda-Campaña, J. J. Rubio, R. Tapia-Herrera, A. V. Curtidor Lopez, A. Grande Meza, I. Czares-Ramirez, "Discrete-time Kalman filter for Takagi-Sugeno fuzzy models," *Evolving Systems*, vol. 8, no. 3, pp. 211-219, 2017.
- [13] G. Feng, "A survey on analysis and design of model-based fuzzy control systems," *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 5, pp. 676-697, Oct. 2006.
- [14] S. Tong, C. Liu, Y. Li, and H. Zhang, "Adaptive fuzzy decentralized control for large-scale nonlinear systems with time-varying delays and unknown high-frequency gain sign," *IEEE Transactions on Systems, Man, Cybernetics: Part B-Cybernetics*, vol. 41, no. 2, pp. 474-485, Apr. 2011.
- [15] T. M. Guerra and L. Vermeiren, "LMI-based relaxed nonquadratic stabilization conditions for nonlinear systems in the Takagi-Sugeno's form," *Automatica*, vol. 40, no. 5, pp. 823-829, 2004.
- [16] H. K. Lam and H. Li, "Output-feedback tracking control for polynomial fuzzy-model-based control systems," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 12, pp. 5830-5840, Dec. 2013.
- [17] Y. Zhao and H. Gao, "Fuzzy-model-based control of an overhead crane with input delay and actuator saturation," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 1, pp. 181-186, Aug. 2012.
- [18] Y. Li, S. Sui, and S. Tong, "Adaptive fuzzy control design for stochastic nonlinear switched systems with arbitrary switchings and unmodeled dynamics," *IEEE Transactions on Cybernetics*, vol. 47, no. 2, pp. 403-414, Feb. 2017.
- [19] H. Wang and G.-H. Yang, "Decentralized state feedback control of uncertain affine fuzzy large-scale system with unknown interconnections," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 5, pp. 1134-1146, Oct. 2016.
- [20] X.-J. Li and G.-H. Yang, "Neural-network-based adaptive decentralized fault-tolerant control for a class of interconnected nonlinear systems," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 1, pp. 144-155, Jan. 2018.
- [21] X.-J. Li and G.-H. Yang, "Fuzzy approximation-based global pinning synchronization control of uncertain complex dynamical networks," *IEEE Transactions on Cybernetics*, vol. 47, no. 4, pp. 873-883, Apr. 2017.
- [22] Y. Wei, J. Qiu, and H. R. Harimi, "Reliable output feedback control of discrete-time fuzzy affine systems with actuator faults," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 64, no. 1, pp. 170-181, Jan. 2017.
- [23] T. Tanaka, T. Ikeda, and H. O. Wang, "Robust stabilization of a class of uncertain nonlinear systems via fuzzy control: quadratic stabilizability, $\mathcal{H}_\infty$ control theory, and linear matrix inequalities," *IEEE Transactions on Fuzzy Systems*, vol. 4, no. 1, pp. 1-13, Feb. 1996.
- [24] Y.-Y. Cao and P. M. Frank, "Robust $\mathcal{H}_\infty$ disturbance attenuation for a class of uncertain discrete-time fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 4, pp. 406-415, Aug. 2000.
- [25] E. Kim and H. Lee, "New approaches to relaxed quadratic stability condition of fuzzy control systems," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 5, pp. 523-533, Oct. 2000.
- [26] X. Liu and Q. Zhang, "New approaches to $\mathcal{H}_\infty$ controller designs based on fuzzy observers for T-S fuzzy systems via LMI," *Automatica*, vol. 39, pp. 1571-1582, 2003.
- [27] H. O. Wang, K. Tanaka, and M. F. Griffin, "An approach to fuzzy control of nonlinear systems: stability and design issues," *IEEE Transactions on Fuzzy Systems*, vol. 4, no. 1, pp. 14-23, Feb. 1996.
- [28] M. Johansson, A. Rantzer, and K.-E. Årzén, "Piecewise quadratic stability of fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 7, no. 6, pp. 713-722, Dec. 1999.
- [29] Z. Feng, W. X. Zheng, and L. Wu, "Reachable set estimation of T-S fuzzy systems with time-varying delay," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 4, pp. 878-891, Jul. 2017.
- [30] J. Qiu, Y. Wei, H. R. Harimi, and H. Gao, "Reliable control of discrete-time piecewise-affine time-delay systems via output feedback," *IEEE Transactions on Reliability*, vol. 67, no. 1, pp. 79-91, Mar. 2018.
- [31] C. Zhang, J. Hu, J. Qiu, and Q. Chen, "Reliable output feedback control for T-S fuzzy systems with decentralized event triggering communication and actuator failures," *IEEE Transactions on Cybernetics*, vol. 47, no. 9, pp. 2592-2606, 2017.
- [32] H. Li, C. Wu, S. Yin, and H.-K. Lam, "Observer-based fuzzy control for nonlinear networked systems under unmeasurable premise variable," *IEEE Transactions on Fuzzy Systems*, vol. 24, no. 5, pp. 1233-1245, Oct. 2017.
- [33] D. J. Choi and P. Park, " $\mathcal{H}_\infty$ state-feedback controller design for discrete-time fuzzy systems using fuzzy weighting-dependent Lyapunov functions," *IEEE Transactions on Fuzzy Systems*, vol. 11, no. 2, pp. 271-278, Apr. 2003.
- [34] M. Narimani and H. K. Lam, "Relaxed LMI-based stability conditions for Takagi-Sugeno fuzzy control systems using regional-membership function-shape-dependent analysis approach," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 5, pp. 1121-1228, Oct. 2009.
- [35] J. C. Lo, and J. R. Wan, "Studies on linear matrix inequality relaxations for fuzzy control systems via homogeneous polynomials," *IET Control Theory Applications*, vol. 4, no. 11, pp. 2293-2302, Nov. 2010.
- [36] A. Sala and C. Ariño, "Relaxed stability and performance LMI conditions for Takagi-Sugeno fuzzy systems with polynomial constraints on membership function shapes," *IEEE Transactions on Fuzzy Systems*, vol. 16, no. 5, pp. 1328-1336, Oct. 2008.
- [37] A. Sala and C. Ariño, "Relaxed stability and performance conditions for Takagi-Sugeno fuzzy systems with knowledge on membership function overlap," *IEEE Transactions on Systems, Man, Cybernetics-Part B: Cybernetics*, vol. 37, no. 3, pp. 727-732, Jun. 2007.
- [38] H. K. Lam and M. Narimani, "Stability analysis and performance design for fuzzy-model-based control under imperfect premise matching," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 4, pp. 949-961, Aug. 2009.
- [39] H. K. Lam and F. H. F. Leung, "Stability analysis of fuzzy control systems subject to uncertain grades of membership," *IEEE Transactions on Systems, Man, Cybernetics-Part B: Cybernetics*, vol. 35, no. 6, pp. 1322-1325, Dec. 2005.
- [40] H. K. Lam and M. Narimani, "Quadratic-stability analysis of fuzzy-model-based control systems using staircase membership functions," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 1, pp. 125-137, Feb. 2010.
- [41] L. El Ghaoui, F. Oustry, and M. AitRami, "A cone complementarity linearization algorithm for static output-feedback and related problems," *IEEE Transactions on Automatic Control*, vol. 42, no. 8, pp. 1171-1176, Aug. 1997.
- [42] J. Qiu, G. Feng, and H. Gao, "Asynchronous output-feedback control of networked nonlinear systems with multiple packet dropouts: T-S fuzzy affine model-based approach," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 6, pp. 1014-1030, Dec. 2011.
- [43] K. Tanaka and M. Sano, "A robust stabilization problem of fuzzy control systems and its application to backing up of a truck-trailer," *IEEE Transactions on Fuzzy Systems*, vol. 2, no. 2, pp. 119-134, 1994.
- [44] X.-J. Zeng and M. G. Singh, "Approximation theory of fuzzy systems-SISO case," *IEEE Transactions on Fuzzy Systems*, no. 2, vol. 2, pp. 162-176, 1994.

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