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Resource Allocation for URLLC in 5G
Mission-Critical IoT Networks

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Abstract—Ultra-reliable and low-latency communication (URLLC) is one of three pillar applications that should be supported by the fifth generation (5G) communications. The research on this topic is still in its infancy due to the difficulties in guaranteeing extremely high reliability (say 10−9) and low latency (say 1 ms) simultaneously. The achievable data rate under the short packet transmission is a complicated function of the transmission power, the blocklength and the decoding error probability. In this paper, we consider resource allocation problem in a factory automation scenario, where the central controller aims for transmitting different packets to two devices (e.g., a robot and an actuator). Two transmission schemes are considered: orthogonal multiple access (OMA) and relay-assisted transmission. We aim to jointly optimize the blocklength and power allocation to minimize the error probability of the actuator subject to reliability requirement of the robot as well as the latency constraints. We develop low-complexity algorithms to address the optimization problems for each transmission scheme. Simulation results demonstrate that the relay-assisted transmission significantly outperforms the OMA scheme.

I. INTRODUCTION

Mission-critical internet of things (IoT) is one of three pillar use cases that should be supported by the fifth generation (5G) networks [1]. However, its research is still in its infancy, applications of mission-critical tasks include factory automation (FA), autonomous driving, remote surgery and smart grid automation, etc. These applications require sub-millisecond latency with much lower packet error probability (e.g., 10−9).

In URLLC, the packet size should be extremely low (say 20 bytes) to support the low-latency transmission [2]. As a result, the Shannon’s capacity are not applicable since the law of large numbers cannot be used. Recently, the achievable data rate in finite blocklength regime has been derived in [3], which is a complicated function of signal-to-noise (SNR), blocklength, and decoding error probability. Unfortunately, this expression is neither convex nor concave with respect to the blocklength and the transmission power. Hence, the globally optimal solution is hard to obtain.

Recently, the resource allocation problems for short-packet URLLC have been studied in [4]–[6]. In particular, [4] considered the short packet transmission for a two-user downlink non-orthogonal multiple access (NOMA) system, with an aim to maximize the throughput of user 1 subject to the throughput requirements for user 2. However, the decoding error probability requirement for user 2 has not been considered, and the throughput is less important in URRLC as only control signals with small packet size are transmitted in URLLC. In [5], She et al. jointly optimized the uplink and downlink transmission blocklengths to minimize the required total bandwidth based on statistical channel state information (CSI). However, the optimization is based on the simplified expression of the rate for short packet transmission. Several approximations are involved in the derivation of the error probability for each user due to the fact that only statistical CSI is available. Most recently, Hu et al. [6] considered simultaneous wireless information and power transfer (SWIPT) in relay-assisted URLLC systems, where the reliability performance is optimized. However, the error probability at the relay cannot be guaranteed and the power is assumed to be fixed in [6].

In this paper, we consider a typical mission-critical scenario, i.e., an FA scenario where the central controller needs to transmit a certain amount of different data to two devices within a given transmission time and with a very low error probability. One device named actuator is located far away from the controller, while the other device named robot can move between the controller and the actuator. Two multiple access schemes are considered, namely orthogonal multiple access (OMA) and relay-assisted transmission. In these two access schemes, we aim for jointly optimizing the blocklength and power allocation for these two devices to minimize the decoding error probability for the actuator while guaranteeing the error probability for the robot.

The contributions of this paper are summarized as follows:

1) For the OMA scheme, we first prove that both the error probability and energy constraints hold with equality at the optimal point. Then, we derive the tight bounds for the blocklength to reduce the search complexity. A low-complexity algorithm is proposed to find the optimal solution. In one special case, we prove that the blocklength allocation problem for the actuator is convex under certain condition, which allows the application of low-complexity bisection search method.

2) For the relay-assisted scheme, we also derive the tight bounds for the blocklength of the controller. The original complex optimization is reduced to a one-dimension search problem. To additionally reduce the complexity, we show that the objective function is a convex function under some certain conditions, which enables the application of low-complexity bisection search method.

3) Extensive simulation results are provided to evaluate the performance of the proposed schemes, and the results show that the relay-assisted scheme significantly outperforms the OMA scheme, which demonstrates the effectiveness of relying techniques in enhancing the
where symbols or channel uses. The transmission time corresponds to be the same, and are denoted as \( D \) bits.

The transmission of these two packets is subject to a latency constraint, i.e., the transmission should be finished within \( M \) symbols; the robot should satisfy its reliability requirement; the total consumed energy should be kept within \( E_{\text{tot}} \). Two transmission schemes are studied: the OMA and the relay-assisted transmission.

### A. OMA transmission

The OMA scheme means that the controller serves robot and actuator in two different orthogonal time slots. In the first phase, the controller transmits signal \( x_1 \) to robot with \( m_1 \) blocklength. The received signal at the robot is

\[
y_1 = \sqrt{p_1} h_1 x_1 + n_1,
\]

where \( p_1 \) is the transmit power of the robot, \( n_1 \) is the zero-mean AWGN with variance \( \sigma_1^2 \), \( x_1 \) carries information knowledge for robot with packet size \( D \). Hence, the coding rate at robot is given by \( D/m_1 \).

The received SNR at robot is given by

\[
\gamma_1 = \frac{p_1 h_1}{\sigma_1^2}.
\]

where \( h_1 = |\hat{h}_1|^2/\sigma_1^2 \). Then, according to (2), the decoding error probability of \( x_1 \) at robot is given by

\[
\varepsilon_1 = Q(f(\gamma_1, m_1, D)).
\]

In the second phase, the controller transmits signal \( x_2 \) to the actuator with \( m_2 \) blocklength. The corresponding error probability at the actuator is derived as

\[
\varepsilon_2 = Q(f(\gamma_2, m_2, D)),
\]

where \( \gamma_2 = p_2 h_2 \) with \( p_2 \) as the transmission power of the actuator and \( \gamma_1 = |\hat{h}_2|^2/\sigma_2^2 \). Without loss of generality (w.l.o.g.), we assume that in this paper robot has higher normalized channel gain than actuator, i.e., \( h_1 > h_2 \).

The resource allocation problem for the OMA transmission can be formulated as:

\[
\begin{align*}
\min_{\{m_1, m_2, p_1, p_2\}} & \quad \varepsilon_2 \\
\text{s.t.} & \quad \varepsilon_1 \leq \varepsilon_1^{\text{max}}, \\
& \quad m_1 p_1 + m_2 p_2 \leq E_{\text{tot}}, \\
& \quad m_1 + m_2 \leq M, \\
& \quad m_1, m_2 \in \mathbb{Z},
\end{align*}
\]

where constraint in (7b) is the error probability requirement of the robot, \( E_{\text{tot}} = E_{\text{tot}}/T \), and constraint (7e) means that the blocklength for each phase is integer.

To solve the optimization problem, we first provide the following lemmas.
Lemma 1: Constraints (7b) and (7c) hold with equality at the optimum solution.

Proof: This can be proved by using contradiction method, details of which are omitted.

1) Calculate the upper and lower bound of $m_1$ and $m_2$: Since $m_1$ and $m_2$ are integer, the exhaustive search method can be used to find the optimal solution. To reduce the search complexity when $M$ is large, we can shorten the search range of $m_1$ and $m_2$. In general, $\varepsilon_{\text{max}}$ is a very small value, which is much less than 10\(^{-1}\). Hence, a necessary condition for constraint (7b) to hold is that $\log_2 (1 + p_1 h_1) > D/m_1$, which leads to: $p_1 > (2^{\frac{D}{m_1}} - 1)/h_1$. On the other hand, based on the energy constraint (7c), we have: $p_1 < E_{\text{tot}}/m_1$. Thus, the blocklength allocation of the robot $m_1$ should satisfy the following inequality:

$$h_1 E_{\text{tot}} > m_1 2^{\frac{D}{m_1}} - m_1 \triangleq g(m_1). \quad (8)$$

The first and second derivative of function $g(m_1)$ w.r.t. $m_1$ can be calculated as

$$g'(m_1) = 2^{\frac{D}{m_1}} - 1 - \ln 2 \cdot \frac{D}{m_1} 2^{\frac{D}{m_1}},$$

$$g''(m_1) = (\ln 2)^2 \cdot \frac{D^2}{m_1^2} 2^{\frac{D}{m_1}} \geq 0.$$  

Hence, $g'(m_1)$ is a monotonically increasing function of $m_1$, and we have:

$$g'(m_1) \leq \lim_{m_1 \to +\infty} g'(m_1) = 0. \quad (9)$$

Hence, function $g(m_1)$ is a monotonically decreasing function of $m_1$. Then, we can find the lower bound of $m_1$ that satisfies the inequality (8), which is denoted as $m_1^{lb}$. Similarly, to guarantee the meaningfulness of $\varepsilon_2$, the inequality $\log_2 (1 + p_2 h_2) > D/m_2$ should hold. By using the same derivation as that of the robot, we can obtain the lower bound of $m_2$, which is denoted as $m_2^{lb}$. By using constraint (7d), we can obtain the upper bound of $m_1$, which is given by $m_1^{ub} = M - m_2^{lb}$. As a result, the search range has been shortened from $1 \leq m_1 < M$ to $m_1^{lb} \leq m_1 \leq m_1^{ub}$, which significantly reduces the search complexity.

2) Algorithm to solve Problem (7): Based on the above analysis, the algorithm to solve Problem (7) is given in Algorithm 1. The main idea can be summarized as follows. For each given integer value of $m_1$ that satisfies $m_1^{lb} \leq m_1 \leq m_1^{ub}$, we calculate the value of $\varepsilon_1$ when $p_1$ is set as $E_{\text{tot}}/m_1$. If $\varepsilon_1 > \varepsilon_{\text{max}}$, then the value of $m_1$ is not feasible, and we increase the value of $m_1$ by one and continue to check the updated $m_1$. Otherwise, we apply the bisection search method to find the value of $p_1$ such that $\varepsilon_1 = \varepsilon_{\text{max}}$ due to the monotonically decreasing property of decoding error probability $\varepsilon_1$ w.r.t. $p_1$ [4]. By using Lemma 1, we have $m_2 p_2 = E_{\text{tot}} - m_1 p_1$. The search range of $m_2$ is given by $m_2^{lb} \leq m_2 \leq M - m_1$. For each given $m_2$, the corresponding $p_2$ is given by $p_2 = (E_{\text{tot}} - m_1 p_1)/m_2$, and we can calculate the value of $\varepsilon_2$. For each feasible $m_1$, we can find the optimal solutions for $m_2$ and $p_2$ that yield the minimum value of $\varepsilon_2$, respectively. At last, we check all feasible of $m_1$ in the range of $m_1^{lb} \leq m_1 \leq m_1^{ub}$, and choose the final globally optimal solution.

Algorithm 1: Algorithm for Problem (7)

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Calculate $m_1^{lb}$ and $m_2^{lb}$;</td>
</tr>
<tr>
<td>2.</td>
<td>For $m_1 = m_1^{lb}$: $m_1^{ub}$ do</td>
</tr>
<tr>
<td>3.</td>
<td>Set $p_1 = E_{\text{tot}}/m_1$, and calculate the value of $\varepsilon_1$.</td>
</tr>
<tr>
<td>4.</td>
<td>If $\varepsilon_1 &gt; \varepsilon_{\text{max}}$ then</td>
</tr>
<tr>
<td>5.</td>
<td>The current $m_1$ is not feasible, and return to the next $m_1$;</td>
</tr>
<tr>
<td>6.</td>
<td>Else</td>
</tr>
<tr>
<td>7.</td>
<td>Apply the bisection search method to find the value of $p_1$ such that $\varepsilon_1 = \varepsilon_{\text{max}}$;</td>
</tr>
<tr>
<td>8.</td>
<td>For $m_2 = m_2^{lb}, M - m_2$ do</td>
</tr>
<tr>
<td>9.</td>
<td>Calculate $p_2 = (E_{\text{tot}} - m_1 p_1)/m_2$, and the value of $\varepsilon_2$, denoted as $\varepsilon_2(m_2, m_1)$.</td>
</tr>
<tr>
<td>10.</td>
<td>Given $m_1$, find the blocklength $m_2$ with the minimum value of $\varepsilon_2(m_2, m_1)$:</td>
</tr>
<tr>
<td>11.</td>
<td>$m_2^# = \arg \min_{m_2^{lb} \leq m_2 \leq m_2^{ub}} \varepsilon_2(m_2, m_1)$, and calculate the value of $\varepsilon_2(m_2^#, m_1)$;</td>
</tr>
<tr>
<td>12.</td>
<td>$m_1^\star = \arg \min_{m_1^{lb} \leq m_1 \leq m_1^{ub}} \varepsilon_2(m_1, m_2^#)$,</td>
</tr>
<tr>
<td>13.</td>
<td>$m_2^#</td>
</tr>
</tbody>
</table>

3) Reduce Search Complexity in Algorithm 1: In steps 8-10 of Algorithm 1, one has to calculate the value of $\varepsilon_2$ for each $m_2$. To further reduce the complexity, it is worthwhile to derive the closed-form solution for the optimal value of $m_2$. In URLLC, to guarantee the high reliability of the transmission, the SNR for each user is normally very high. In this case, the channel dispersion $V$ in (2) can be approximated as one, i.e., $V \approx 1$. Then, the decoding error probability in (2) can be approximated as

$$\tilde{\varepsilon} = Q \left( \tilde{\gamma} (\gamma, m, D) \right), \quad (10)$$

where $\tilde{\gamma} (\gamma, m, D) = \ln 2 \sqrt{m} \left( \log_2 (1 + \gamma) - \frac{D}{m} \right)$.

For given $m_1$ and $p_1$, the product of $m_2$ and $p_2$ should satisfy $m_2 p_2 = E_{\text{tot}} - m_1 p_1 \triangleq E_2$ according to Lemma 1. Then, the original problem defined in (7) can be transformed to the following optimization problem:

$$\min_{m_2^{lb} \leq m_2 \leq M - m_1, m_2 \in \mathbb{Z}} Q \left( \tilde{\gamma} (\gamma_2, m_2, D) \right).$$

Since Q-function is a decreasing function, the above problem is equivalent to the following problem by substituting $p_2 = E_2/m_2$ into it as

$$\max_{m_2^{lb} \leq m_2 \leq M - m_1, m_2 \in \mathbb{Z}} \ln 2 \sqrt{m_2} \left( \log_2 \left( 1 + \frac{E_2 h_2}{m_2} \right) - \frac{D}{m_2} \right). \quad (11)$$

To solve the above problem, we first relax the integer variable $m_2$ to a continuous variable, and define

$$\tilde{g}(m_2) \triangleq \sqrt{m_2} \left( \log_2 \left( 1 + \frac{E_2 h_2}{m_2} \right) - \frac{D}{m_2} \right). \quad (12)$$

In the following theorem, we provide a sufficient condition for $\tilde{g}(m_2)$ to be a concave function.
Theorem 1: Function \( \hat{g}(m_2) \) is a concave function when the condition \( \frac{E_{lb}}{M-m_1} \geq e - 1 \) is satisfied, where \( e \) is a natural constant.

Proof: Please see Appendix A.

In URLLC, the blocklength \( M \) is usually small and the optimal blocklength allocated for the actuator is its upper bound, i.e., \( m_2 = M - m_1 \). Therefore, \( \frac{E_{lb}}{M-m_1} \) can be regarded as the SNR for the actuator. In URLLC, to achieve very low error probability, the SNR is usually very large and in general much larger than \( e - 1 \). Hence, the condition in Theorem 1 holds with high probability.

When the condition in Theorem 1 is satisfied, Problem (11) is a convex optimization problem. If \( \hat{g}'(m_2^p) \leq 0 \), the optimal \( m_2 \) is given by \( m_2 = m_2^p \). If \( \hat{g}'(M-m_1) \geq 0 \), the optimal \( m_2 \) is \( m_2 = M - m_1 \). Otherwise, the optimal \( m_2^p \) satisfies \( \hat{g}'(m_2) = 0 \), and the low-complexity bisection search method can be used to find \( m_2^p \). The final optimal integer \( m_2 \) is the one with lower objective value for its two neighbor integers, i.e., \( \lfloor m_2^p \rfloor \) and \( \lceil m_2^p \rceil + 1 \).

B. Relay-assisted transmission

In this scheme, the robot acts as a relay that assists the transmission for actuator, where decode-and-forward (DF) relay is assumed at the robot. The packet ID inserted in the packet header for each device to differentiate their corresponding data information. The whole blocklength is divided into two phases, the broadcast phase with blocklength \( m_1 \) and relay phase with blocklength \( m_2 \), which satisfy the constraint \( m_1 + m_2 \leq M \).

In the first phase, the controller broadcasts a large packet that is a combination of two packets to both devices, where the combined packet size is \( 2D \). The received signals at both devices are given by

\[
\begin{align*}
y_{1,1} &= \sqrt{P_s}h_1x_1 + n_1, \\
y_{1,2} &= \sqrt{P_s}h_2x_2 + n_2,
\end{align*}
\]

where \( P_s \) denotes the power allocated to the combined packet, \( x_1 \) carries the data information of the combined packet with coding rate \( 2D/m_1 \). Then, the SNR for robot to decode the combined packet is given by \( \gamma_1 = P_s h_1 \), and the error probability at robot is given by

\[
\varepsilon_1 = Q(\gamma_1, m_1, 2D).
\]

Since robot acts as the relay based on the DF mode, if robot successfully decodes the combined packet, it will forward the actuator’s packet to the actuator with coding rate \( D/m_2 \) in the second phase. Then, in the second phase, the received signal at the actuator is given by

\[
y_{2,2} = \sqrt{p_r}h_3x_2 + n_3,
\]

where \( p_r \) is the transmission power at the actuator. The received SNR is

\[
\gamma_2 = P_r h_3,
\]

where \( h_3 \) is the normalized channel gain given by \( h_3 = \frac{|h_3|}{\sigma_2^2} \). The error probability is given by

\[
\varepsilon_2 = Q(\gamma_2, m_2, D).
\]

There is a possibility that the actuator cannot decode its packet under two cases. On one hand, with probability \( \varepsilon_1 \), the robot is not able to correctly decode the combined packet and will not forward anything to the actuator. On the other hand, with probability \( 1 - \varepsilon_1 \), the robot correctly decodes the combined packet and forwards the packet to the actuator. However, with probability \( \varepsilon_2 \), the actuator fails to decode the packet. Under the above two cases, the actuator will have to decode the combined packet by using the received signal from the first phase, i.e., \( y_{1,2} \). The achieved SNR of the actuator for decoding the combined packet is given by

\[
\hat{\gamma}_2 = p_s h_2,
\]

and the corresponding decoding error probability is given by

\[
\hat{\varepsilon}_2 = \begin{cases} Q(\hat{\gamma}_2, m_1, 2D) , & \text{if } \frac{2D}{m_1} \leq \log_2(1 + \hat{\gamma}_2) , \\
1 & \text{otherwise.}
\end{cases}
\]

As a result, the expected error probability of the actuator decoding its packet under this scheme is given by

\[
\varepsilon_2 = ((1 - \varepsilon_1) \varepsilon_2 + \varepsilon_1) \hat{\varepsilon}_2.
\]

Then, the resource allocation problem is formulated as

\[
\begin{align*}
\min_{\{m_1, m_2, p_s, p_r\}} & \varepsilon_2 \\
\text{s.t.} & \varepsilon_1 \leq \varepsilon_{1,\text{max}}, \\
& m_1 p_s + m_2 p_r \leq E_{\text{tot}}, \\
& m_1 + m_2 \leq M, \\
& m_1, m_2 \in \mathbb{Z}.
\end{align*}
\]

By using the contradiction method, we can easily prove that constraint (21c) holds with equality at the optimal solution. However, in contrast to the above two transmission modes, the error probability constraint (21b) may not hold with equality at the optimal solution since the objective function may also decrease with \( \varepsilon_1 \). The algorithms proposed for the above two transmission modes cannot be applied.

To make \( \varepsilon_1 \) and \( \varepsilon_2 \) meaningful, we must have

\[
\begin{align*}
p_s & \geq \frac{1}{h_1} \left( \frac{2D}{m_1} - 1 \right), \\
p_r & \geq \frac{1}{h_3} \left( \frac{2D}{m_2} - 1 \right).
\end{align*}
\]

From (23), we must have \( E_{\text{tot}} h_3 > m_2 (\frac{2D}{m_2} - 1) \). By using the similar derivations as for the OMA transmission case, we can obtain this lower bound \( m_2^{lb} \). Then the upper bound of \( m_1 \) is given by \( M - m_2^{lb} \). Similarly, from (22), the lower bound of \( m_1 \) is the minimum integer that satisfies \( E_{\text{tot}} h_1 > m_1 (\frac{2D}{m_1} - 1) \), which is denoted as \( m_1^{lb} \). As a result, we search for the optimal \( m_1 \) within the range of \( m_1^{lb} \leq m_1 \leq M - m_2^{lb} \). For each given \( m_1 \), the minimum transmission power \( p_s \) is the solution to the equation \( \varepsilon_1 = \varepsilon_{1,\text{max}} \), which is denoted as \( p_s^{lb} \). Then, we can obtain the upper bound of \( m_2 p_r \), which is given by \( m_2 p_r \leq E_{\text{tot}} - m_1 p_s^{lb} \). On the other hand, to make \( \varepsilon_2 \)
meaningful, the following inequalities should hold:

\[
\frac{D}{m_2} < \log_2 (1 + p_r h_3) \leq \log_2 \left(1 + \frac{E_{\text{tot}} - m_1 p_{\text{ub}}^{\text{lb}}}{m_2} h_3\right),
\]

which leads to

\[
m_2 \left(2 \frac{h_3}{m_1} - 1\right) \leq (E_{\text{tot}} - m_1 p_{\text{ub}}^{\text{lb}}) h_3.
\]

To guarantee there exists at least one \( m_2 \) within the range of \( m_2^{\text{lb}} \leq m_2 \leq M - m_1 \) that satisfies (25), the following conditions should hold:

\[
(M - m_1) \left(2 \frac{h_3}{m_1} - 1\right) \leq (E_{\text{tot}} - m_1 p_{\text{ub}}^{\text{lb}}) h_3,
\]

due to the monotonically decreasing property of the left hand side of (25) w.r.t. \( m_2 \). Hence, if the above inequality is not satisfied, the current \( m_1 \) is not feasible and increase it by one. Otherwise, for given \( m_1 \), find the minimum \( m_2 \) that satisfies inequality (25), which is denoted as \( m_2^{\text{lb}} \). Then, \( m_2^{\text{lb}} \) must be smaller than \( m_2^{\text{ub}} \). Hence, the feasible range of \( m_2 \) is updated as \( m_2^{\text{lb}} \leq m_2 \leq M - m_1 \). We then search each \( m_2 \) within the updated feasible range.

In the following, we study the optimization under fixed \( m_1 \) and \( m_2 \) and only need to optimize the power allocation \( p_s \) and \( p_r \). For each given \( m_2 \), we can obtain the lower bound of \( p_r \) to make \( \varepsilon_2 \) meaningful: \( p_r \geq (2D/m_2 - 1)/h_3 \triangleq p_r^{\text{ub}} \). Then, the upper bound of \( p_s \) can be derived as

\[
p_s \leq \frac{E_{\text{tot}}}{m_1} - \frac{m_2 p_{\text{ub}}^{\text{lb}}}{m_2} \triangleq p_s^{\text{up}}.
\]

Then, the feasible region of \( p_s \) is given by \( p_s^{\text{lb}} \leq p_s \leq p_s^{\text{ub}} \). When \( p_s \) is given, \( p_r \) can be calculated as \( p_r = (E_{\text{tot}} - m_1 p_s)/m_2 \). Then, the original optimization problem reduces to a one-dimension optimization problem as

\[
\min_{p_s} \bar{\varepsilon}_2 \quad \text{s.t.} \quad p_s^{\text{lb}} \leq p_s \leq p_s^{\text{ub}},
\]

Hence, the one dimension line search method can be used to find the optimal solution of the above problem.

Note that if \( \log_2 (1 + \gamma_2) < 2D/m_1 \), \( \bar{\varepsilon}_2 = 1 \) and the objective function reduces to \( \bar{\varepsilon}_2 = (1 - \varepsilon_1) \varepsilon_2 + \varepsilon_1 \). When the following condition holds

\[
\log_2 (1 + p_s^{\text{ub}} h_2) < \frac{2D}{m_1},
\]

then \( \varepsilon_2 = 1 \) for all feasible \( p_s \) within \( p_s^{\text{lb}} \leq p_s \leq p_s^{\text{ub}} \), and the objective function is always equal to \( \bar{\varepsilon}_2 = (1 - \varepsilon_1) \varepsilon_2 + \varepsilon_1 \). By using the similar derivations as in Appendix F of [4], the objective function can be proved to be a convex function. Then, the optimization problem is a convex optimization problem. If \( \frac{\partial \bar{\varepsilon}_2}{\partial p_s} \bigg|_{p_s = p_s^{\text{ub}}} \leq 0 \), the optimal \( p_s \) is given by

\[
p_s = p_s^{\text{ub}}.
\]

If \( \frac{\partial \bar{\varepsilon}_2}{\partial p_s} \bigg|_{p_s = p_s^{\text{ub}}} \geq 0 \), the optimal \( p_s^{\ast} \) satisfies the equation \( \frac{\partial \bar{\varepsilon}_2}{\partial p_s} \bigg|_{p_s = p_s^{\ast}} = 0 \). In practical communication systems, when the actuator is located at the cell edge, its channel gain \( h_2 \) is very low, which makes condition (29) hold with high probability. Hence, the optimization problem over the power allocation \( p_s \) is a convex problem with high probability.

In summary, we provide Algorithm 2 to solve Problem (21).

**Algorithm 2: Algorithm for Problem (21)**

1. \( h_1, h_2, D, M, \varepsilon_1^{\text{max}}, E_{\text{tot}} \)
2. \( p_s^{\ast}, p_r^{\ast}, m_1^{\ast}, m_2^{\ast} \)
3. **for** \( m_1 = m_1^{\ast} \) **do**
4. **if** Condition (26) holds **then**
5. **Find the minimum \( m_2 \) that satisfies inequality (25), which is denoted as \( m_2^{\ast} \)
6. **for** \( m_2 = m_2^{\ast} \) **do**
7. **Calculate the upper bound of \( p_s \) as \( p_s^{\ast} \) in (27), and solve Problem (28). Calculate the objective value \( \varepsilon_2(m_1, m_2) \)
8. **end**
9. **Given** \( m_1 \), find the blocklength \( m_2 \) with the minimum value of \( \varepsilon_2(m_1, m_2) \):
10. \( m_2^{\ast} = \arg \min_{m_2} \varepsilon_2(m_1, m_2) \)
11. **end**
12. **Return**

\[
m_1^{\ast} = \arg \min_{m_1} \varepsilon_2\left(m_1, m_2^{\ast}\right), m_2^{\ast} = m_2^{\ast}\bigg|_{m_1^{\ast}}
\]

and the corresponding \( p_s^{\ast} \) and \( p_r^{\ast} \).

### IV. Simulation Results

In this section, simulation results are provided to evaluate the performance of the proposed methods. The distance between the controller and the actuator is set to 500 m, and the distance between the controller and the robot ranges from 50 m to 450 m. The robot is served as the relay. The system bandwidth is set to \( B = 1 \) MHz, and the blocklength is set to \( M = 100 \) symbols. Hence, the downlink transmission delay duration is 100 us [2]. The noise power spectral density is \(-173\) dBm/Hz. The packet loss probability requirement for the robot is set to \( 10^{-9} \). The pathloss model is \( 35.3 + 37.6 \log_{10} d \) dB. We assume the channel gain is only determined by the path-loss.

In Fig. 2, we first study the impact of blocklength \( (M) \) on the packet error probabilities of relay-assisted scheme and OMA scheme. As expected, it is observed that larger \( M \) leads to much better reliability performance, and the error probability achieved by the relay protocol can be reduced from \( 10^{-5} \) to \( 10^{-9} \) when \( M \) increases from 50 bits to 100 bits. More importantly, the relay-assisted transmission is observed to outperform the OMA scheme for \( M = 50, 100 \) symbols, which verifies the effectiveness of relay-assisted transmission in improving the reliability performance. It is interesting to find that when the robot moves from the controller to the actuator, the packet error probability achieved by the OMA scheme always decreases. The main reason is that the channel gain from controller to robot is decreasing, and more energy and blocklength are required for the robot to guarantee its error probability requirement. As a result, the available energy and blocklength for the actuator will decrease. On the other hand, for the case of \( M = 100 \) symbols, the packet error probability
achieved by the relay-assisted scheme first increases and then decreases when the robot moves in the line. This can be explained as follows. When robot moves from 50 m to 200 m, the channel gain from the robot to the actuator is weak, which is the performance bottleneck. However, when the robot continues to move towards the actuator, the transmission link from the controller to the robot becomes the bottleneck link, and finally the optimization problem becomes infeasible when the robot is near the actuator.

We next investigate the impact of the packet size $D$ on the packet error probability of the two schemes in Fig. 3. As expected, a bigger packet size leads to a higher error probability for both schemes. In addition, the relay-assisted scheme show again the significant performance advantage over the OMA scheme. It is again observed that the error probability achieved by the OMA scheme always increases with the distance from controller to the robot, while that achieved by the relay-assisted scheme first increases and then decreases with the distance. Hence, for the latter scheme, it is better to place the relay in the middle of the controller and the actuator.

V. CONCLUSION

This work studied the resource allocation for OMA and relay-assisted transmission in short packet transmission for critical-mission IoT to achieve low latency and high reliability.

We formulated an optimization problem to minimize the decoding error probability for the device with lower channel gain while guaranteeing that the other user achieved a low error probability target. To facilitate the optimal design of the blocklength and power allocation, the analysis on the constraints and tight bounds on the blocklengths was proposed. Simulation results demonstrated that relay-assisted transmission significantly outperforms the OMA scheme in terms of lower packet error probability.

VI. ACKNOWLEDGEMENT

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APPENDIX A

PROOF OF THEOREM 1

The first and second derivative of function $g(m_2)$ w.r.t. $m_2$ can be calculated as

$$
\dot{g}(m_2) = \frac{1}{2 \ln 2 \sqrt{m_2}} \left( 1 + \frac{E_2 h_2}{m_2} \right)^{-\frac{3}{2}} \frac{1}{\ln 2 \sqrt{m_2}} \frac{E_2 h_2}{m_2 + E_2 h_2},
$$

$$
\ddot{g}(m_2) = -\ln \left( 1 + \frac{E_2 h_2}{m_2} \right) + \frac{E_2 h_2}{4 \ln 2 \sqrt{m_2} (m_2 + E_2 h_2)^2} \ln 2 \sqrt{m_2} \left( \frac{E_2 h_2}{m_2 + E_2 h_2} \right)^{-1} 3 \frac{D m_2^{\frac{1}{2}}}{4} < 0.
$$

Obviously, the last term of $\ddot{g}(m_2)$ is negative, we only need to prove that the sum of the first two terms is negative under the condition of $\frac{E_2 h_2}{m_2 + E_2 h_2} \geq e - 1$.

Since $m_2^{\hat{b}} \leq m_2 \leq M - m_1$, we have

$$
\frac{E_2 h_2}{m_2} \geq \frac{E_2 h_2}{M - m_1} \geq e - 1.
$$

(30)

Then, the following inequality follows:

$$
4 \leq \left( \frac{E_2 h_2}{m_2} + 2 + \frac{m_2}{E_2 h_3} \right) \ln \left( 1 + \frac{E_2 h_2}{m_2} \right).
$$

(31)

By rearranging the terms of the above inequality, we can prove that the sum of the first two terms is negative, which completes the proof.

REFERENCES


