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Towards Modelling Multi-Arm Robots: Eccentric Arrangement of Concentric Tubes

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Abstract—This paper presents and experimentally evaluates a quasistatic mechanics-based model that describes the shape of concentric tube robotic (CTR) arms when they are eccentrically arranged along an also-flexible backbone. The model can estimate the shape of both the backbone and CTR arms, and can accommodate an arbitrary number of CTR arms arranged in an eccentric position with regards to the backbone's neutral axis. Experimental evaluation with a prototype system on the benchtop highlights the promise of the end-to-end proposed modelling approach, as the error between model and experiment is around 11% of the manipulator's overall length. The theory is the first step towards modeling multi-arm concentric tube robots, a class of continuum robots that is increasingly being considered for single-port surgical applications.

I. INTRODUCTION

Robot-assisted single port surgery is envisioned as the next step in minimally invasive surgery (MIS), as it promises lower morbidity, improved cosmesis due to elimination of peripheral ports, reduced trauma, and shorter hospitalization time [1], [2]. To deliver optimal therapies, especially when deep seated pathologies are targeted, single port surgery requires instrumentation with increased flexibility and dexterity to overcome the challenges of limited surgical workspace, lack of articulation, and the danger of instrument clash.

Continuum robots show promise when anatomical pathways need to be traversed [3],[4], as they are able to steer along 3D curves in confined spaces and dexterously handle tissue. Therefore, they show advantages when deployed in the field of single port surgery. Concentric tube robots (CTRs) are a representative continuum robot comprising a series of precurved elastic tubes that can be translated and rotated with respect to each other to control the shape of the robot and tip pose. CTRs were originally envisioned as “single-arm” robots [5], [6] for a variety of surgical applications (*e.g.*, heart [7], lung [6], brain [8], [9], eye [10], [11], kidney [12]), but are nowadays being considered as parts of “multi-arm” systems that offer improved dexterity and manipulability in confined spaces (*e.g.*, prostate [13], brain [9]). Increasingly, CTRs are explored as systems tailored for single port surgery, *i.e.* multiple CTR arms arranged on also-flexible steerable backbones.

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Fig. 1. Main components and architecture of the microsurgical robot motivating this research: (1) Main body - outer tube, (2) Main body - inner tube, (3) Main body - filling material, (4) Camera holder - outer tube, (5) Camera holder - inner tube, (6) Camera head with integrated illumination, (7) Right instrument - outer tube, (8) Right instrument - inner tube, (9) Right instrument - end-effector, (10) Left instrument - outer tube, (11) Left instrument - inner tube, and (12) Left instrument - end-effector.

Mechanics-based models have been developed that can describe the shape of the CTR comprising any number of tubes, with any precurvature, in the presence of arbitrary external loading [14], [5], [15]. Friction has also been considered [16], and, recently, the effect of the manufacturing tolerances that govern the gap between subsequent concentric tubes has also been theoretically and experimental evaluated [17].

Limited work, however, has considered multi-arm CTRs; their modeling is the focus of this paper. Contrary to prior work that considers the backbone that houses multiple robotic arms as “decoupled” (due to large stiffness) from the motion of the flexible arms [18], [19], [13], the proposed work accounts for the interactions between multiple CTR arms and the flexible backbone. The theory is developed for a multi-arm continuum robot comprising a flexible CTR backbone that houses several CTR arms as end-effectors. The proposed mechanics-based model takes into account the eccentric arrangement of multiple CTRs housed inside a flexible backbone thus it considers the interaction of the motion of the backbone to the motion of each arm.

The paper is arranged as follows. In Sec. II, we present the robot design that motivates the modeling research. In Sec. III the model of multi-arm robot as an eccentric arrangement of several CTRs is presented. In Sec. IV, we present the experimental evaluation of the proposed model using a

prototype system. Concluding remarks appear in Sec. V.

II. MULTI-ARM CONCENTRIC TUBE ROBOT DESIGN

The robot that motivates the modeling research is being designed for neurosurgical applications. Three robotic flexible arms are eccentrically arranged with respect to the neutral robot axis: two hold tools for bimanual manipulation, and one holds a camera (1.2mm diameter, Enable, Inc., United States). They are housed within an actuated flexible body that acts as the robot backbone. For all arms and the backbone, flexibility and steerability are achieved through concentric tube technology. As is most common in the literature, all tubes are made from NiTi to take advantage of its excellent elasticity and recovery capabilities [20].

In our current design, the backbone CTR is a 3 Degree-of-Freedom (DoF) variable stiffness section [5], comprising two tubes of identical precurvature and stiffness, and is used for global positioning and navigation of the arms. Following the terminology of [5], the backbone acts as the “navigation section”, while the arms act as the “manipulation sections”. Each robotic arm comprises two tubes, therefore being a 5DoF robot when accounting for the capability of tool roll. The considerations below relate to the introduction of a filling material that retains the separation of the flexible manipulation sections within the robot’s navigation backbone. A rendering of the multi-arm system is depicted in Fig. 1.

The introduction of a filling material in the inner tube of the navigation section is required to avoid contact between the manipulation instruments and camera, both with respect to one other and the navigation backbone. This filling material should maintain the desired arrangement of manipulation instruments and their eccentric placement within the robot body, especially when the robot segments are independently actuated. The filling material must satisfy the requirements of high flexibility, high tear resistance, and low friction. Multi-lumen catheters made of materials such as PVC, PTFE, or FEP are therefore ideal candidates given their compatible material properties.

The manipulation sections are spatially constrained within the navigation section by the filling material. Therefore, when the navigation section’s shape changes, the shape of the manipulation arms changes as well - and vice versa. The stiffnesses of all tube constituents within the robot affect the overall shape, and these interactions need to be accounted for by any model. The development of the theory that describes the tube mechanics when eccentrically arranged is the focus of the next section.

III. MODELLING ECCENTRICALLY ARRANGED CONCENTRIC TUBES

The model for the statics of a single CTR has been derived in [5], [14]. We extend the previous work to develop a model, based on the Cosserat-rod theory, for multi-arm CTR as an eccentric arrangement of multiple CTRs. The assumptions of the classical elastic-rod theory of Kirchhoff are employed [14]. The model assumes that only one module is moving at a time, and is therefore quasistatic. The following notation is used

throughout the paper: x , $\mathbf{x}$, and $\mathbf{X}$ denote a scalar, a vector, and a matrix, respectively. The prime denotes derivation with respect to the spatial coordinate s . Subscript $i = 1, \dots, N$ denotes the parameters and variables of the i th tube, while $j = 1, \dots, N$ denotes the arm of the robot that the equation is referred to. For simplicity and without loss of generality, we present the model for the designed robot, *i.e.*, $i = \{1, 2\}$ (two tubes per section, 1 denoting the inner tube, and 2 denoting the outer tube), and $j = \{1, 2, 3, 4\}$ (1 – 3 for the manipulation sections and 4 for the navigation section).

As was presented in Sec. II, the navigation backbone of the robot comprises two NiTi tubes that can carry three manipulation arms, each composed of two NiTi tubes. The filling material placed inside navigation backbone, holds the three manipulation arms with an offset with respect to its centerline (see Fig. 1).

Each tube is modeled as a long, slender, one-dimensional Cosserat rod endowed with a continuous homogeneous transformation matrix attached to every point on its arc. The shape of each tube is determined by the position vector $\mathbf{r} = \mathbf{r}(s)$, where the coordinate $s \in [0, \ell]$ is the arc length, and ℓ is the tube’s length. The rotation matrix of the frame moving along the tube’s arc is $\mathbf{R}(s)$, and, therefore, the continuous homogeneous transformation describing the robot shape, $\mathbf{g}(s)$, is established as

$$\mathbf{g}(s) = \begin{bmatrix} \mathbf{R}(s) & \mathbf{r}(s) \\ \mathbf{0}^T & 1 \end{bmatrix}. \quad (1)$$

Moreover, it is known that the local curvature of a tube is obtained as

$$\mathbf{u}(s) = (\mathbf{R}^T(s)\mathbf{R}'(s))^V, \quad (2)$$

and its shape can be computed by integrating

$$\mathbf{g}'(s) = \mathbf{g}(s)\hat{\xi}, \quad (3)$$

where $\hat{\xi}(s) = [\mathbf{e}_3^T \quad \mathbf{u}^T(s)]^T$ and $\mathbf{e}_3 = [0 \ 0 \ 1]^T$, or, in terms of the position vector and the rotation matrix:

$$\mathbf{r}'(s) = \mathbf{R}(s)\mathbf{e}_3, \quad (4)$$

$$\mathbf{R}'(s) = \mathbf{R}(s)\hat{\mathbf{u}}, \quad (5)$$

where $\hat{\mathbf{u}}$ is a skew symmetric matrix, [14], [21].

Now, we break the robot into several segments between transitions points, *i.e.* points where a tube goes from straight to curved or points where a tube ends (see Fig. 2). A segment can contain from 1 to j arms. Segments are later connected to each other by enforcing the continuity of shape and internal moment. In our approach, we estimate the deformed shape of the moving section in each segment. Next, the shape of the moving section is used to compute the shape of the navigation section. Without loss of generality, it is assumed that manipulation section 1 is the moving section. The equilibrium shape of all tubes that comprise a segment should be identical, *i.e.*, $\mathbf{r}_1^{(j)}(s) = \mathbf{r}_2^{(j)}(s)$. Now, we introduce the angle $\theta_i^{(j)}(s)$ to parameterize tubes’ twist in a single section as the difference between the rotation of the outer

Fig. 2. Illustration of the concentric tube manipulation arms and the material filling the navigation section.

tube with respect to the inner one. Using this parametrization, we get

$$\mathbf{R}_{i=1,2}^{(j=1,2,3,4)} = \mathbf{R}_1^{(j=1,2,3,4)} \mathbf{R}_{\theta_{i=1,2}}^{(j=1,2,3,4)}, \quad (6)$$

where, based on definition, $\theta_1^{(j=1,2,3,4)} \equiv 0$ and $\theta_2^{(j=1,2,3,4)}$ is the rotation of the outer tube of each section. The rotation of the inner tube of the navigation section can be related to the inner tube of the moving manipulation section by

$$\mathbf{R}_{i=1}^{(j=4)} = \mathbf{R}_1^{(j=1)} \mathbf{R}_{\phi j=1} \mathbf{R}_{\theta_{i=1}}^{(j=4)}, \quad (7)$$

where $\mathbf{R}_{\phi j}$ is the rotation matrix that expresses the eccentricity of the outer tube of section $j = 1, 2, 3$ with respect to the outer tube of the navigation section frame. It is assumed that the twist of the manipulations section's outer tube with respect to the filling material is negligible. Now, the shape of the navigation section is given by

$$\mathbf{r}_i^{j=4} = \mathbf{r}_i^{j=1} + \mathbf{R}_1^{j=4} d_e^1, \quad (8)$$

where d_e^j is the eccentricity offset vector of section $j = 1, 2, 3$ in the frame of the outer tube of manipulation section j . Also, the shape of the fixed arms can be computed using

$$\mathbf{r}_i^{j=2,3} = \mathbf{r}_i^{j=4} + d_e^j, \quad (9)$$

Equations (6), (7), (8), (9) can be used to right the shape and rotation of all tubes as a function of the moving section's shape and orientation (section 1 in the examined scenario). Now, using (6) - (7) the curvature of the outer tube of each module can be related to the curvature of the moving inner tube using the definition of $\mathbf{u}_i$ as

$$\mathbf{u}_i^{(j)} = (\mathbf{R}_i^T \mathbf{R}_i')^\vee = \mathbf{R}_k^T \mathbf{u}_1^{(j=2,3,4)} + \theta_{i=1,2}' \mathbf{e}_3, \quad (10)$$

where $\mathbf{R}_k$ for the manipulation sections is

$$\mathbf{R}_k = \mathbf{R}_{\theta_{i=2}}^{(j=2,3,4)}, \quad (11)$$

while for the navigation section is

$$\mathbf{R}_k = \mathbf{R}_{\phi} \mathbf{R}_{\theta_{i=1}}^{(j=4)}. \quad (12)$$

As mentioned before, variable $\theta_i^{(j)}$ provides a parameterization of the torsion of outer tube of each section, which is free to vary independently as

$$\theta_i^{(j=1,2,3,4)} = u_{i,z}^{(j=1,2,3,4)} - u_{1,z}^{(j=1,2,3,4)}. \quad (13)$$

Taking the derivative of (10) with respect to s we obtain

$$\mathbf{u}_i'^{(j)} = \theta_i'^{(j)} \frac{d\mathbf{R}_{\theta_i}^{T(j)}}{d\theta_i^{(j)}} \mathbf{R}_{\phi}^T \mathbf{u}_1^{(j)} + \mathbf{R}_k^T \mathbf{u}_1'^{(j)} + \theta_i''^{(j)} \mathbf{e}_3. \quad (14)$$

In the case that (14) is referred to the navigation section, *i.e.*, $j = 4$, then $\mathbf{R}_{\phi}^T \equiv \mathbf{I}$ and the respective $\mathbf{R}_k$ is given by (11) - (12).

Taking into account that the model is quasi-static and the filling material is incompressible, the external force and moment exerted on the navigation section of the robot and on each manipulation section that it houses are the same. Writing the force/moment equilibrium equations with respect to s , we obtain

$$\begin{aligned} \sum_{i=1}^n (\mathbf{n}_i'^{(j)} + \mathbf{f}_i^{(j)}) &= 0 \\ \sum_{i=1}^n (\mathbf{m}_i'^{(j)} + \mathbf{e}_3 \times \mathbf{n}_i^{(j)} + \mathbf{l}_i^{(j)}) &= 0 \end{aligned} \quad (15)$$

Using a linear constitutive law to relate the strains to moments and (4), (5), (14) and (15), the derivative of curvatures can be derived as

$$\begin{aligned} \mathbf{u}_1'^{(j)} \Big|_{x,y} &= -\mathbf{K}^{-1} \sum_{i=1}^n \mathbf{R}_k (\mathbf{K}_i^{(j)} (\theta_i'^{(j)} \frac{d\mathbf{R}_{\theta_i}^{T(j)}}{d\theta_i^{(j)}} \mathbf{R}_{\phi}^T \mathbf{u}_1^{(j)} - \mathbf{u}_i'^{* (j)}) \\ &\quad + \hat{u}_i^{(j)} \mathbf{K}_i^{(j)} (\mathbf{u}_i^{(j)} - \mathbf{u}_i^{* (j)}) \Big|_{x,y} \\ &\quad - \mathbf{K}^{-1} (\mathbf{e}_3 \times \mathbf{R}_1^{T(j)} \int_s \mathbf{f}_s(\sigma) d\sigma + \mathbf{R}_1^{T(j)} \mathbf{l}) \Big|_{x,y}, \end{aligned} \quad (16)$$

where $\mathbf{K} = \sum_{i=1}^n \mathbf{K}_i^{(j)}$.

As far as torsion is concerned, it can be derived from the third component of the equation that describes the curvature for a single tube, [14],

$$\begin{aligned} u_{i,z}'^{(j)} &= u_{i,z}'^{* (j)} + \frac{E_i^{(j)} I_i^{(j)}}{G_i^{(j)} J_i^{(j)}} (u_{i,x}^{(j)} u_{i,y}'^{(j)} - u_{i,y}^{(j)} u_{i,x}'^{(j)}) + \\ &\quad \frac{G_i'^{(j)} I_i^{(j)}}{G_i^{(j)} J_i^{(j)}} (u_{i,z}'^{(j)} - u_{i,z}^{(j)}) - \frac{1}{G_i^{(j)} J_i^{(j)}} \mathbf{e}_3 \mathbf{R}_i^{T(j)} \mathbf{l}. \end{aligned} \quad (17)$$

Fig. 3. Experimental setup: (1) Manipulation section - Instrument inner rod, (2) Manipulation section - Instrument outer tube, (3) Navigation section - PVC Multi-lumen catheter, (4) Catheter rotation control, (5), (6), (7) Rotation and translation control for the manipulation section, and (8) Orthogonal camera system.

Finally, for each transition point, the continuity of shape and internal moment must be enforced. To preserve shape continuity, it is enforced that

$$\mathbf{g}_i(s-) = \mathbf{g}_i(s+) \quad (18)$$

Additionally, taking into account that there is no point load at the transition points, the static equilibrium imposes

$$\sum_{i=1}^n \mathbf{m}_i(s-) = \sum_{i=1}^n \mathbf{m}_i(s+) \quad (19)$$

Moreover, at the tip of the manipulation sections the sum of the moments is considered zero:

$$\sum_{i=1}^n \mathbf{m}_i(s=l) = 0 \quad (20)$$

The system of differential equations given in (4), (5), (13), (16), and (17) can be solved simultaneously to calculate the shape of the moving manipulation section and the navigation section in equilibrium.

IV. EXPERIMENTAL EVALUATION

We present the experimental setup and protocol that is used to evaluate the developed theory. In summary, a rotating PVC catheter is used as the flexible robot navigation backbone, and a pair of NiTi tubes, eccentrically arranged, form a manipulation arm.

A. Experimental Setup

The experimental setup emulates the behavior of the proposed robot, the primary difference being that the rig only replicates the presence of a single manipulation arm.

The setup features a multilumen catheter made of Polyvinyl Chloride (PVC) that emulates the presence of a flexible navigation section with its filling material. This section has an Outer Diameter (OD) of 9.5 mm, with three lamina arranged throughout its circumference at an angle of 120° . The Inner Diameter (ID) of each lumen is approximately 3.2 mm, allowing for the insertion of NiTi manipulation arms.

TABLE I
CONSTANT KNOWN PARAMETERS OF THE MODEL

Manipulation Section - Outer Tube					
EI [Nm^2]	GJ [Nm^2]	l [m]	r_{in} [m]	r_{out} [m]	u^* [m^{-1}]
0.0643	0.0495	164.3e-3	1.9e-3	2.45e-3	4.5
Manipulation Section - Inner Tube					
EI [Nm^2]	GJ [Nm^2]	l [m]	r_{in} [m]	r_{out} [m]	u^* [m^{-1}]
0.0057	0.0044	300e-3	0	1.5e-3	16
Navigation Section - PVC Catheter					
EI [Nm^2]	GJ [Nm^2]	l [m]	r_{in} [m]	r_{out} [m]	u^* [m^{-1}]
0.8259	0.6353	90e-3	1.5e-3	4.8e-3	0

TABLE II
EXPERIMENTAL DATA AND PREDICTION ON TRANSITION POINTS AND OVERALL SHAPE

	Mean (mm)	Standard Deviation (mm)
e_b	14.05	5.94
% e_b per unit length	15.61	6.61
e_c	21.18	5.86
% e_c per unit length	20.19	5.18
e_d	25.23	13.75
% e_d per unit length	16.47	9.05
e_{shape}	16	4.08
% e_{shape} per unit length	10.52	3.023

The manipulation arm comprises two tubes and is inserted through one of the three eccentrically arranged hollow lamina on the PVC catheter. The outermost tube has an OD of 2.45 mm, and carries an inner rod with a diameter of 1.5 mm. The precarvatures are known, and are given in Table I along with the remaining design specifications of the CTR.

The three tubular components of the setup (one for navigation, two for manipulation) are each mounted on rotational stages at their base. Angle markers indicate the base rotation angles. Further, the manipulation section's tubes are mounted on two carriages that lie on the same linear guide rail to allow for manual linear translation within an 80 mm range. The multi-lumen navigation section cannot translate in this implementation.

To measure the shape of the robot, two orthogonally arranged cameras (Logitech C930e, Logitech International S.A., Lausanne, Switzerland) observing the robot workspace from the top and the side were used. The top camera view is referred to as the XY plane, while the side camera view is referred to as the YZ plane. The pixel-to-millimetre conversion rate of the cameras was calibrated prior to the experiments using a known checkerboard pattern. The experimental setup is shown in Fig. 3.

The proposed modeling theory focuses on predicting the position of the manipulation arm tip and the overall shape of all sections. The modeling challenge is represented by the large number of components simultaneously working and

Fig. 4. Comparison of experimental and simulated results for five experiments. The XZ, and YZ planes, are shown in the top, and bottom row, respectively.

Fig. 5. Experimentally examined configurations.

interacting with each other, leading to an increasing amount of variables to model and control. For example, pushing the instrument inner rod through one channel of the multi-lumen catheter influences the position of both its paired instrument tube, but also the catheter.

Fig. 6. Tip and shape error for 10 experimental trials.

B. Experiments

We carried out 10 experiments covering a range of translations and rotations of all tubular components, therefore testing different configurations within the rig's workspace. The experimental curvatures and lengths of the rig's tubular components were calculated using the orthogonal images. Points were captured along the entire length of the tubes, and a parametric polynomial curve was fit to approximate the total shape that was used as the ground truth. The curve was converted from 2D into 3D using the camera calibration matrix and triangulation of the selected points. The refraction caused by the PVC catheter, however, is a potential source of error. All examined configurations are indicated in Fig. 5, where the shapes are derived from our model. Moreover, since the actual values for the moduli of the tubes and the wire are uncertain (for the NiTi they are listed as 28-83 GPa, while for the PVC the Young's modulus varies from 2.3 to 4.8 GPa), a calibration process was followed by solving an optimization problem for the parameter set $P = \{E_i^j I_i^j\}$ that identifies the moduli that minimise the error between the experiment and simulation.

Table II reports the mean error, the percentage of the error per unit length, and the standard deviation for transitions points (b) and (c), and the mechanism's tip (d). The error in predicting the overall shape is shown as well. We report errors for each transition point on the mechanism as the error there will reach its maximum value for each tube and the NiTi rod as well. The average error at the distal end of the navigation section (PVC catheter) is lower than in the other positions as it is closer to the base. At that point, the overall stiffness is greater and, as a result, the deviation between the experimental and simulated results is smaller. According to Table II, the maximum error appears at the tip of the manipulation section, which is the most distal point of the mechanism. Here, the stiffness is the lowest and the distance

from that point to the base is the greatest.

Fig. 4 reports the experimental and simulated results for four experiments. The manipulation section has a curvature in the x- axis. This fact is depicted on the error of the XZ plane of each experiment. As can be seen, the error in that plane is lower compared to the error in the YZ plane.

Figure 6 shows the error on the distal end of the mechanism and on its overall shape for the ten different experiments that were carried out. Figure 6 shows that the error at the mechanism's tip is between 2.8 mm and around 20 mm apart. The second experiment presents an error of 50 mm, which could be considered an outlier result. Overall, the error length ranges from 7 to 20 mm. Considering the error of the overall shape of the continuum mechanism, it represents the 5-11% of its total length as shown in Table II - such a value is reasonable for CTRs.

V. CONCLUSIONS

In this paper, we presented a new approach to model the behavior of pairs of concentric tubes that are eccentric with respect to their backbone. The developed theory relates the motion of each arm of the concentric tube robot to its overall shape via an end-to-end quasistatic model. Such a model is a first step towards investigating the intricacies of multi-arm continuum robot systems based on concentric tube technology. The model showed promising results, as the average error was not larger than 11% of the entire manipulator length.

Our current research involves creating the robot illustrated in Fig. 1 to evaluate the model when multiple manipulation sections are present. Further, we will employ EM tracking for estimating the ground truth of the shape, as it was identified that the refraction of the PVC catheter distorts the image and potentially leads to an increase in the reported error.

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