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No chiral light bending by clumps of axion-like particles

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We study the propagation of light in the presence of a parity-violating coupling between photons and axion-like particles (ALPs). Naively, this interaction could lead to a split of light rays into two separate beams of different polarization chirality and with different refraction angles. However, by using the eikonal method we explicitly show that this is not the case and that ALP clumps do not produce any spatial birefringence. This happens due to non-trivial variations of the photon's frequency and wavevector, which absorb time-derivatives and gradients of the ALP field. We argue that these variations represent a new way to probe the ALP-photon coupling with precision frequency measurements.

1. INTRODUCTION

Astrophysical data accumulated over the past decades provide an overwhelming evidence for the existence of a cold matter component of non-baryonic origin. The leading explanation is to complete the Standard model of particle physics (SM) with new particles, referred to as dark matter (DM).

Regardless of astrophysical data, the SM has its own puzzles, which make DM candidates addressing them particularly appealing. One such candidate is the quantum – chromodynamics (QCD) axion, which appears in the Peccei-Quinn solution to the strong CP problem [1–6]. More generically, many extensions of the SM predict light particles with similar properties to the QCD axion, but unrelated to the strong CP problem. These are known as axion-like particles (ALPs), and appear in particular in low energy-effective theories of string theory [7, 8]. In the following we will refer to both cases as ALPs.

Another motivation for ALPs is that they may resolve the difficulties of heavier DM candidates to explain observations on galactic scales, see e.g. [9, 10]. This topic has recently attracted significant attention in the context of

the so-called fuzzy dark matter, a hypothetical ALP with extremely light mass $m \sim 10^{-22}$ eV [10, 11]. Different observations disfavor these extremely small masses, while larger masses are still viable [12–15].

A peculiarity of the ALP field (denoted by a) is its coupling to photons,

$$\mathcal{L}_{\text{int}} = \frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (1)$$

where $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and A_μ is the photon field. Many observational effects of this interaction have been scrutinized in various astronomical and experimental settings [16–45].

Naively, the coupling (1) suggests that an ALP background might act as a usual optically active chiral medium. Recall that when a linearly polarized light beam enters such a medium, it splits into two rays of different chirality. The difference between refractive indices makes these waves acquire different phases as they propagate through the chiral fluid. Importantly, the two rays are *spatially separated* and propagate independently from each other. If the waves are recombined at the observation site, the resulting light should exhibit a rotation of the polarization plane as compared to the initial state. This effect is known as “natural optical rotation” [46].

The presence of the polarization plane rotation induced by the ALP-photon coupling was originally pointed out by

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Harari and Sikivie in Ref. [38]. Remarkably, for the coupling (1) the optical rotation depends only on the initial and final configurations of the ALP field [38, 42]. Pushing the analogy between the ALP field and a chiral medium, one might expect that inhomogeneities of the ALP field can generate a spatial split (“spatial birefringence”) between two photon’s polarizations.

Since the paper by Harari and Sikivie, anomalous optical properties of the ALP fluid were studied in many works, but only few of them discussed the spatial separation of the photons [18–20, 47]. In the present paper we fill this gap and scrutinize the effect of light deflections by ALP clumps. To this end we derive the light tracing equation with the eikonal method, which is valid for generic (mildly) time-dependent and refractive media.

2. EIKONAL CALCULATION

Consider the following Lagrangian density for an ALP of mass m interacting with photons:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu a \partial^\mu a - m^2 a^2) + \frac{ag_{a\gamma\gamma}}{4}F_{\mu\nu}\tilde{F}^{\mu\nu}. \quad (2)$$

The equations of motion are¹

$$\begin{aligned} \square A_\nu - \partial_\nu \partial_\mu A_\mu - g_{a\gamma\gamma} \epsilon^{\mu\nu\lambda\rho} \partial_\mu (a \partial_\lambda A_\rho) &= 0, \\ -\square a - m^2 a + \frac{g_{a\gamma\gamma}}{4} F_{\mu\nu} \tilde{F}^{\mu\nu} &= 0. \end{aligned} \quad (3)$$

Introducing the electric and magnetic components as $F_{ij} = -\epsilon_{ijk} H_k$, $F_{0i} = E_i$, we find the following pair of modified Maxwell equations,

$$\begin{aligned} \partial_0 E_i - \epsilon_{ijk} \partial_j H_k + g_{a\gamma\gamma} [\partial_0 (a H_i) + \epsilon_{ijk} \partial_j (a E_k)] &= 0, \\ \partial_i E_i + g_{a\gamma\gamma} \partial_i (a H_i) &= 0. \end{aligned} \quad (4)$$

The Bianchi identities take the standard form,

$$\partial_i H_i = 0, \quad \partial_0 H_i + \epsilon_{ijk} \partial_j E_k = 0, \quad (5)$$

while the ALP equation of motion reads

$$(-\partial_0^2 + \Delta - m^2)a - g_{a\gamma\gamma} E_i H_i = 0. \quad (6)$$

We study the propagation of electromagnetic fluctuations in a *fixed* ALP astrophysical background. In particular, we ignore any back-reaction from the photons on the

background. For simplicity, the results of this paper are derived assuming a flat Minkowski metric.

We will work in the ray optics (eikonal) approximation,

$$|\partial_0 a/a|, |\partial_i a/a| \ll \omega, \quad (7)$$

where ω is the photon frequency. Note that we do not assume any hierarchy between the temporal and spatial variations of the ALP field and keep all powers of $g_{a\gamma\gamma}$. From (7) we can neglect terms with two derivatives acting on a and find the following wave equations:

$$\begin{aligned} (\partial_0^2 - \Delta)E_i + g_{a\gamma\gamma} [\partial_0 a \partial_0 H_i - \partial_j a \partial_j H_i] &= 0, \\ (\partial_0^2 - \Delta)H_i - g_{a\gamma\gamma} [\partial_0 a \partial_0 E_i - \partial_j a \partial_j E_i] &= 0. \end{aligned} \quad (8)$$

To derive the photon path we follow the general eikonal formalism of Ref. [48]. First, we rewrite the wave equations (8) in the following operator form:

$$\hat{M}(t, x^i) * (\vec{E}, \vec{H})^T = 0. \quad (9)$$

The zeroth order eikonal approximation amounts to using an ansatz $E_i, H_i \propto e^{iS}$, where the phase S formally defines the photon’s frequency and wavevector along the ray orbit as

$$\omega(t, x^j(t)) \equiv -\partial_0 S, \quad k_i(t, x^j(t)) \equiv \partial_i S. \quad (10)$$

The eigenvalues of the operator \hat{M} are given by

$$D_0^\pm = \omega^2 - k^2 \pm g_{a\gamma\gamma} (\omega \partial_0 a + k^i \partial_i a), \quad (11)$$

where \pm corresponds to right and left circular polarizations. Using the standard equations for the photon’s orbit,

$$\begin{aligned} \frac{dx^i}{dt} &= -\frac{\partial D_0 / \partial k^i}{\partial D_0 / \partial \omega}, \quad \frac{dk^i}{dt} = \frac{\partial D_0 / \partial x^i}{\partial D_0 / \partial \omega}, \\ \frac{d\omega}{dt} &= -\frac{\partial D_0 / \partial t}{\partial D_0 / \partial \omega}, \end{aligned} \quad (12)$$

we find:

$$\frac{dx^i}{dt} = \frac{k^i \mp g_{a\gamma\gamma} \partial_i a / 2}{\omega \pm g_{a\gamma\gamma} \partial_0 a / 2}, \quad (13a)$$

$$\frac{dk^i}{dt} = \pm g_{a\gamma\gamma} \frac{\omega \partial_i \partial_0 a + k^j \partial_j \partial_i a}{2\omega \pm g_{a\gamma\gamma} \partial_0 a}, \quad (13b)$$

$$\frac{d\omega}{dt} = \mp g_{a\gamma\gamma} \frac{\omega \partial_0^2 a + k^i \partial_i \partial_0 a}{2\omega \pm g_{a\gamma\gamma} \partial_0 a}. \quad (13c)$$

Note that Eq. (13a) is a general formula for a group velocity. Taking a second total time derivative of (13a) and using Eq. (13) we arrive at

$$\frac{d^2 x^i}{dt^2} = 0, \quad (14)$$

¹ Note that $\epsilon^{0123} = -\epsilon_{0123} = 1$, and we use the following short-hands: $\Delta = \partial_i \partial^i$, $\square = \partial^\mu \partial_\mu$.

which formally holds true to all orders² in $g_{a\gamma\gamma}$. This implies that light is not bent by the ALP clumps. We stress that the result (14) holds true *for any* ALP configuration, even for unphysical time-independent profiles.

Importantly, Eq. (14) dictates that

$$\left| \frac{dx^i}{dt} \right|^2 = \frac{|k^i \mp g_{a\gamma\gamma} \partial_i a/2|^2}{(\omega \pm g_{a\gamma\gamma} \partial_0 a/2)^2} = 1, \quad (15)$$

which means that the variations of the photon's frequency and wavevector identically cancel time derivatives and gradients of the ALP field. This implies that the photon's frequency and wavevector must vary between the observer and the source sites. The total variation is obtained by integrating Eqs. (13b) and (13c) along the ray path (keeping terms of order $\mathcal{O}(g_{a\gamma\gamma})$),

$$\Delta k^i = \pm \frac{g_{a\gamma\gamma}}{2} (\partial_i a(t_d, \mathbf{x}_d) - \partial_i a(t_e, \mathbf{x}_e)), \quad (16a)$$

$$\Delta \omega = \mp \frac{g_{a\gamma\gamma}}{2} (\partial_0 a(t_d, \mathbf{x}_d) - \partial_0 a(t_e, \mathbf{x}_e)), \quad (16b)$$

where the subscripts “e” and “d” denote the moments of emission and detection, respectively. This implies that oscillations of the ALP field with period $2\pi/m$ induce an oscillating component in the photon's frequency, which varies with the same period (see also [49]). This effect is suppressed by m/ω compared to the polarization plane rotation (Harari-Sikivie effect) [38]. The typical frequency shifts for the hypothetical Galactic ALP can be crudely estimated as

$$\frac{\Delta \omega}{\omega} \sim 10^{-16} \left(\frac{g_{a\gamma\gamma}}{10^{-10} \text{GeV}^{-1}} \right) \left(\frac{1 \text{GHz}}{\omega} \right) \sqrt{\frac{\rho_{\text{DM}}}{0.3 \text{GeV}/\text{cm}^3}}, \quad (17)$$

where $\rho_{\text{DM}} = m^2 a^2/2$ is the local dark matter density. Such relative frequency shifts are similar to the accuracy of modern atomic clocks [50]. This suggests a possible new way to use atomic clocks to constrain ALPs, complementary to [51–56]. An even stronger effect can be generated by a passage of a dense ALP minicluster through Earth (see Ref. [57] for a similar idea in the context of domain walls). We leave the exploration of these possibilities for future work.

² Note that terms of order $\mathcal{O}(g_{a\gamma\gamma}^2)$ also contain two derivatives of the ALP field. Strictly speaking, the corrections to geometric optics (which we did not consider) are not negligible in this situation.

3. CONCLUSIONS

We have shown that the ALP-photon coupling does not lead to light bending. This result does not seem trivial to us, as it requires taking into account the variations of photon's frequency and wavevector, which are produced by fluctuations of the ALP field in space and time. The vanishing of the r.h.s. in Eq. (14) is a result of a delicate cancellation between these effects. On a positive note, our analysis justifies the approach of previous works on the ALP-induced polarization plane rotation, which explicitly neglected all effects related to photon's refraction through the axion clumps.

The chiral deflections by ALP clumps were studied in Refs. [47, 58], though their results were derived using the assumption that the photon's frequency is conserved. From Eq. (12), one sees that this assumption is not justified.

Qualitatively, the absence of chiral light bending may be traced back to the particular form of the axion-photon coupling (1), in which the axion field multiplies the total derivative of the electromagnetic field. Heuristically, in geometric optics $a \approx \text{const}$, in which case this operator will affect physics only via boundary effects. The result (14) can also be understood from the dispersion relation (11), which dictates that the group velocity is always unity regardless of the polarization state.

Finally, we have remarked that the absence of spatial birefringence implies that the photon's frequency and wavevector must exhibit time-periodic components. We plan to explore their possible influence in very precise frequency measurements in the future.

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