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Stability analysis and output feedback control for stochastic networked systems with multiple communication delays and nonlinearities using fuzzy control technique

Zhiming Zhang¹, Wei Zheng^{1*}, H.K. Lam², Shuhuan Wen¹, Fuchun Sun³, Ping Xie¹

1 School of Electrical Engineering, Yanshan University, Qinhuangdao, 066004, P.R. China

2 Department of Engineering, Kings College London, Strand Campus, Strand, London, WC2R 2LS, United Kingdom

3 School of Computer Science and Technology, Tsinghua University, Beijing, 100084, P.R. China

**Corresponding e-mail: weizheng@stumail.yzu.edu.cn*

Abstract This paper addresses the H-infinity Takagi-Sugeno (T-S) fuzzy control for a class of T-S fuzzy discrete networked control systems with random interval communication delays and random sector nonlinearities. Firstly, the T-S fuzzy model is employed to approximate the discrete networked control system and the ℓ th-order Rice fading channels model is introduced in the system model. Secondly, the T-S fuzzy dynamic output feedback controller with ℓ th-order Rice fading channels output is designed for the T-S fuzzy discrete networked control system. Thirdly, the discrete delay-dependent Lyapunov-Krasovskii functional, stochastic system theory and Bernoulli probability distribution are employed to derive the stability conditions in terms of linear matrix inequalities (LMIs). Compared with previous works, the fading channels in the signal transmission are described clearly by setting the different channels coefficients of the ℓ th-order Rice fading channels model. The closed-loop system is exponentially mean-square stable and prescribed H-infinity performance is guaranteed by designing the T-S fuzzy dynamic output feedback controller. The factorizations in the polynomial and the congruence transformation matrices are introduced to solve the LMIs, such that the controller gain matrices are determined. Finally, simulation examples are presented to show the effectiveness of proposed methods.

Keywords stability analysis; Lyapunov-Krasovskii functional; Takagi-Sugeno (T-S) fuzzy model; linear matrix inequalities (LMIs); communication delays.

1. Introduction

It is well-known that time delay is unavoidable in a large amount of dynamical systems because of the finite speed of information processing, which often leads to one of the main factors of poor performance or even instability [1-3]. Besides, the time delays may induce the undesirable dynamic behaviors, such as the overshoot, oscillation, instability and so on [4-6]. The nonlinearities are the common phenomenon due to the fault of the data interfaces, the sudden changes of the working environments and the long distance data transmissions [7, 8]. Sometimes, the networked control systems lead to two major problems, inevitable random time-varying delays and unexpected nonlinearities, which may degrade the system performances and causes the system instability [9]. Thus, it's important to analyse and synthesise the factors and minimise the effects of the factors in the networked control systems [9]. In particular, the Jensen's and Wirtinger integral inequalities are employed to simplify the integral terms which appeared in the derivation of stabilization results [10]. Then, if the obtained LMIs are feasible, the corresponding parameters of the controller are determined [10]. For example, a new inequality which is the modified version of free-matrix-based integral inequality is derived, and then by aid of this new inequality, two novel lemmas which are relaxed conditions for some matrices in a Lyapunov-Krasovskii functional are proposed [11]. Besides, unlike the construction of augmented Lyapunov functional and multiple integral Lyapunov functional, novel three Lyapunov functionals are suggested which are delay product type functions and lead to less conservative results [12].

The dissipative synchronization control problem for the Markovian jump networked control system is addressed with fully considering the time-varying delays and the fragility problem in the process of implementing the gain-scheduled controller [13]. By utilizing some improved integral inequalities and constructing a proper Lyapunov-Krasovskii functional, several delay-dependent synchronization criteria with less conservatism are established to ensure the networked control system is stable [14]. It is generally known that the advantages of networked control systems are low maintenance costs, easy installation, high flexibility and so on [15]. Moreover, in the communication channel of network, the existence of many network-induced phenomena is inevitable and the two frequently encountered cases of them are signal quantization and packet dropouts [16]. Thus, making an indepth study on networked control systems is necessary, and some researchers have put forth efforts to investigate such systems. Specifically, the fuzzy model has the nice ability to approximate the system plant [17-20], and the fuzzy model is one of the most important multimodel methods to denote the system plant as some linear subsystems [21-23]. Particularly, the Takagi-Sugeno (T-S) fuzzy model provides the better semi-linear characteristic [17, 18]. For example, the stability conditions were derived for the T-S fuzzy system by employing the Lyapunov-Krasovskii functional [24]. Besides, the membership functions can be designed arbitrarily and the design conditions were relaxed [25, 26]. Moreover, the adaptive T-S fuzzy controller was designed for a class of nonlinear systems to improve the haptic feedback fidelity [27], and the stability conditions were derived by employing

the Lyapunov-Krasovskii functional [28]. The fuzzy asynchronous dissipative filtering issue for Markov jump nonlinear systems subject to fading channels is discussed in [29], where the Rice fading model is employed to characterize the fading channels phenomenon in the system measurements for the first time. However, due to the complexity and compactness in the mechanical structure, in practice, not all the states of the system can be directly measured [30]. In most practical situations, the desired state variables of considered systems are usually partially measurable or even unmeasurable [31], thus an asynchronous output feedback controller was designed, which ensures that the closed-loop system is generalised stochastically dissipative [31]. Moreover, the fading channels reflect the complexity and randomness of the wireless channel, which is the basic problem to determine the performance of the networked control systems, thus the stochastic system theory was employed to describe the fading channels phenomenon [32]. In this paper, the conservative conditions will limit the flexibility of the controller design. However, the T-S fuzzy model provides a distinctive framework to denote the system plant as some linear subsystems [24, 25], such that the uncertainties are approximated effectively.

The problem of extended dissipativity-based state feedback control was solved for a Markov jump systems established upon Fornasini-Marchesini local state-space model, in which a kind of state delays is thoroughly taken into account [33]. The passive synchronization issue for Markov jump neural networks subject to randomly occurring gain variations was solved, in which the event-triggered mechanism is employed to save the limited communication resource [34]. In practice, it may be difficult to obtain all the information of the state variables, then the output feedback control has been proposed [35-38]. The main results in [38-40] mean that the state variables are unavailable in the measurement process, thus the output feedback control has been investigated. From the above analysis, it can be seen that the output feedback is effective than the state feedback [41-43], but some strict design conditions should be considered in static feedback [44, 45]. Thus, the dynamic output feedback was proposed [46-48]. Meanwhile, how to effectively use the network bandwidth becomes very important in the application of networked control systems [49]. All the above literatures are very important for us and point out the research direction in this paper. The controller design can affects the stability and H-infinity performance of the discrete-time closed-loop system directly. Thus, it should be mentioned that the design process of the proposed T-S fuzzy controller in this paper has more generality than the usual controller, i.e., the controller designing arithmetic in some existing literatures cannot be available in the H-infinity output-feedback controller design for a class of T-S fuzzy systems with random interval communication delays and random sector nonlinearities. The motivations in this paper come from: (1) the T-S fuzzy model has the better ability than the conventional fuzzy model to approximate the system [50]; (2) the dynamic output feedback has the better dynamic characteristic than the static output feedback to reflect the internal features [51]; (3) most of the existing literatures are investigated based on the Lyapunov-Krasovskii functional and few literatures employ stochastic system theory [52-54]; (4) γ is the H-infinity performance index and often used to investigate the H-infinity stability analysis of the minimum sensitivity [55]. The introductions of the stochastic system theory and Bernoulli probability distribution are the keys in solving the addressed problem. However, the introductions of the stochastic system theory and Bernoulli probability distribution are very important for the addressed problem in this paper. Obviously, the proposed controller work better than the conventional methods in dealing with the occurrence probability of random interval communication delays and random sector nonlinearities, and the deterioration of the H-infinity performance can be reduced effectively.

The contributions in this paper are summarized as follows.

(1) Not only the external disturbance, but also the random interval communication delays and random sector nonlinearities are considered in the discrete networked control system. The T-S fuzzy model provides the distinctive framework to approximate the nonlinear system plant as an average weighted sum of some linear subsystems in this paper, then the uncertainties are approximated effectively. Besides, the stochastic system theory and Bernoulli probability distribution are more suitable to describe the random phenomena in the signal transmission. Thus, the stochastic system theory and Bernoulli probability distribution are employed, then the random interval communication delays and random sector nonlinearities are described clearly in the signal transmission.

(2) The ℓ th-order Rice fading channels model is introduced, then the fading channels in the signal transmission are described clearly by setting the different channels coefficients. The dynamic output feedback control has the nice dynamic characteristic to reflect the internal features of the control system, thus the T-S fuzzy dynamic output feedback controller with ℓ th-order Rice fading channels output is designed for the T-S fuzzy discrete networked control system. With the help of the proposed controller, the closed-loop system is exponentially mean-square stable and the prescribed H-infinity performance is guaranteed.

(3) The discrete Lyapunov-Krasovskii functional is introduced in this paper, then the less conservative stability conditions are derived for the closed-loop system. The stochastic system theory and Bernoulli probability distribution are employed in the stability analysis, then the stability conditions in terms of LMIs are derived for the closed-loop system. The factorizations in the polynomial and the congruence transformation matrices are introduced to solve the LMIs, then the controller gain matrices are determined and the computation complexity of solving LMIs is reduced.

This paper is organized as follows. The T-S fuzzy model is employed to approximate the system in **Section 2**. The T-S fuzzy dynamic output feedback controller with ℓ th-order Rice fading channels output is designed in **Section 3**. The stability conditions in terms of LMIs are derived in **Section 4**. Simulation examples are presented to show the effectiveness in **Section 5**. The conclusions are presented in **Section 6**.

Notations: R^n denotes the n -dimensional Euclidean space, $l_2[0, \infty)$ denotes the space of the square-integrable vector function. $A < 0 (\leq 0)$ and $A > 0 (\geq 0)$ denote the negative definite (semi-negative definite) matrix and positive definite (semi-positive definite) matrix with appropriate dimensions, respectively. The superscripts “ T ” and “ -1 ” denote the matrix transposition and matrix inverse, respectively. $\|M\|$ denotes the Euclidean norm of the matrix “ M ”, “ $*$ ” denotes the term that is induced by symmetry. $\text{Prob}\{\cdot\}$ denotes the Bernoulli probability distribution of “ $\cdot$ ”, $\text{diag}\{r_1, r_2, \dots, r_n\}$ denotes the block diagonal matrix with diagonal terms $r_1, r_2, \dots, r_n$.

2. System formulation

By employing the T-S fuzzy model [56], the T-S fuzzy discrete networked control system with external disturbance is described as follows

Plant rule i : if $\hat{x}_1(k)$ is $\mathcal{M}_{i_1}$, $\hat{x}_2(k)$ is $\mathcal{M}_{i_2}$, ..., and $\hat{x}_g(k)$ is $\mathcal{M}_{i_g}$, then

$$\begin{cases} x(k+1) = A_{i_i}x(k) + B_i u(k) \\ y(k) = E_i x(k) + D_{i_i} \omega(k) \\ z(k) = D_{2i} x(k) \\ x(k) = \phi(k), \quad k \in \mathbb{H} \triangleq \{-h, -h+1, \dots, 0\} \end{cases} \quad (1)$$

where $\hat{x}_1(k), \hat{x}_2(k), \dots, \hat{x}_g(k)$ are the premise variables of the system, $\mathcal{M}_{i_g}$ ($i=1, 2, \dots, r, j=1, 2, \dots, g$) is the fuzzy set, r is the number of the fuzzy rules, and g is the number of the premise variables. $x(k) \in R^n$, $y(k) \in R^{n_y}$, $z(k) \in R^{n_z}$ and $u(k) \in R^{n_u}$ are the state variable, measured output, control output and control input of the system, respectively. $\omega(k) \in l_2[0, \infty)$ is the external disturbance, $\phi(k)$ is the initial condition for $k \in \mathbb{H} \triangleq \{-h, -h+1, \dots, 0\}$. $A_{i_i}, B_i, D_{i_i}, D_{2i}$ and E_i are the system gain matrices.

Note that the random interval communication delays $\tau_m(k)$ and random sector nonlinearities $g(x(k))$ are considered in (1), thus the state variable $x(k+1)$ is rewritten as follows

$$x(k+1) = A_{i_i}x(k) + A_{2i} \sum_{m=1}^h \beta_m(k) x(k - \tau_m(k)) + B_i u(k) + r(k) g(x(k)), \quad m=1, 2, \dots, h \quad (2)$$

where A_{2i} is the system gain matrices. $\beta_m(k)$ and $r(k)$ are the stochastic variables to describe $\tau_m(k)$ and $g(x(k))$, respectively. $g(x(k))$ is the random sector nonlinearities, $\tau_m(k)$ are the random interval communication delays such that

$$\begin{cases} 0 < \tau_l \leq \tau_m(k) \leq \tau_u \\ \Delta \tau_m(k) \leq \tau^* \end{cases} \quad (3)$$

where $\Delta \tau_m(k)$ is the forward difference of $\tau_m(k)$. τ_l, τ_u and τ^* are the lower bound of $\tau_m(k)$, upper bound of $\tau_m(k)$ and upper bound of $\Delta \tau_m(k)$, respectively.

From (2), it can be seen that the stochastic variables $\beta_m(k)$ and $r(k)$ are employed to describe $\tau_m(k)$ and $g(x(k))$, respectively. Thus, according to the stochastic system theory and Bernoulli probability distribution, one has

$$\begin{cases} \beta_m(k) = \begin{cases} 1, & \tau_m(k) \text{ is available} \\ 0, & \tau_m(k) \text{ is unavailable} \end{cases} \\ r(k) = \begin{cases} 1, & g(x(k)) \text{ is available} \\ 0, & g(x(k)) \text{ is unavailable} \end{cases} \end{cases} \quad (4)$$

with

$$\begin{cases} Prob\{\beta_m(k)=1\} = E\{\beta_m(k)=1\} = \bar{\beta}_m \\ Prob\{\beta_m(k)=0\} = E\{\beta_m(k)=0\} = 1 - \bar{\beta}_m \\ Prob\{r(k)=1\} = E\{r(k)=1\} = \bar{r} \\ Prob\{r(k)=0\} = E\{r(k)=0\} = 1 - \bar{r} \end{cases} \quad (5)$$

where $Prob\{\bullet\}$ is the Bernoulli probability distribution of “ $\bullet$ ”, $0 \leq \bar{\beta}_m \leq 1$ and $0 \leq \bar{r} \leq 1$ are the scalars.

Thus, substituting (2) into (1), the system (1) is rewritten as follows

$$\begin{cases} x(k+1) = A_{1i}x(k) + A_{2i} \sum_{m=1}^h \beta_m(k)x(k - \tau_m(k)) + B_i u(k) + r(k)g(x(k)) \\ y(k) = E_i x(k) + D_{1i} \omega(k) \\ z(k) = D_{2i} x(k) \\ x(k) = \phi(k) \end{cases} \quad (6)$$

where $A_{2i} \sum_{m=1}^h \beta_m(k)x(k - \tau_m(k))$ is the vector term of random interval communication delays, and $r(k)g(x(k))$ is the vector term of random sector nonlinearities. A_{2i} is the gain matrix of $A_{2i} \sum_{m=1}^h \beta_m(k)x(k - \tau_m(k))$, $\beta_m(k)$ and $r(k)$ are the stochastic variables of vector terms $A_{2i} \sum_{m=1}^h \beta_m(k)x(k - \tau_m(k))$ and $r(k)g(x(k))$, respectively. Compared (1) and (6), it can be seen that the random interval communication delays and random sector nonlinearities are considered in the system (1), thus the state variable $x(k+1)$ in (1) is rewritten in the form of (6), i.e., $x(k+1) = A_{1i}x(k) + A_{2i} \sum_{m=1}^h \beta_m(k)x(k - \tau_m(k)) + B_i u(k) + r(k)g(x(k))$, as shown in the first line in (6).

For $g(x(k))$ in (6), **Assumption 1** and **Assumption 2** are presented as follows.

Assumption 1 [57]. For the random sector nonlinearities $g(x(k)): R^n \rightarrow R^n$, where $g(0)=0$, the following sector bounded conditions hold

$$\begin{cases} \|g(x(k))\|^2 \leq \lambda \|Gx(k)\|^2 \\ (g(x(k)) - M_1 x(k))^T (g(x(k)) - M_2 x(k)) \leq 0 \end{cases} \quad (7)$$

where $\lambda > 0$ is a scalar. G , M_1 and M_2 are the known matrices such that $0 \leq M_2 < M_1$.

Assumption 2 [58]. For the random sector nonlinearities $g(x(k)): R^n \rightarrow R^n$, where $g(0)=0$, $g(x(k))$ is decomposed into a linear term and a nonlinear term as follows

$$g(x(k)) = M_1 x(k) + g_s(x(k)) \quad (8)$$

where $M_1 x(k)$ is the linear term, $g_s(x(k)) \in G_s$ is the nonlinear term, G_s is the set of $g_s(x(k))$ and described as follows

$$G_s = \{g_s(x(k)) : g_s^T(x(k))(g_s(x(k)) - Mx(k)) \leq 0\} \quad (9)$$

where

$$M = M_1 - M_2 > 0 \quad (10)$$

Remark 1. Due to the random sudden changes in the environmental circumstances such as repairs of the components, random failures, and changes in the interconnections of subsystems, the nonlinear disturbances may occur in a probabilistic way, which rise to the randomly varying nonlinearities [59, 60]. For example, the leader-following consensus problem of multi-agent systems with randomly varying nonlinearities and stochastic disturbances under switching topologies has been investigated in [60]. In the real world, when sensors are employed to measure the system outputs, the sector nonlinearities of the sensors are usually caused by the severe environments such as uncontrollable elements (e.g., variations in flow rates, temperature, etc.) and aggressive conditions (e.g., corrosion, erosion, and fouling, etc.) [58]. Thus, the sector nonlinearities of the sensors are often investigated based on the **Assumption 1** and **Assumption 2**.

Remark 2. From **Assumption 1**, one knows that the phenomena of the sensor nonlinearities $g(x(k))$ as well as the fading channels satisfy the conditions (7), where the sensor signal is saturated subject to sector-like bounds and then sent to the actuator

via the fading channels [57, 58]. From **Assumption 2**, one knows that the random sector nonlinearities are decomposed into a linear term and a nonlinear term as well as the conditions (8) holds. The randomly varying nonlinearities including linear and nonlinear terms are introduced to reflect more realistic dynamical behaviors caused by the noisy environment [60]. For the sensor nonlinearities, the nonlinear function is often divided into linear and nonlinear terms [58]. Thus, the random sector nonlinearities are decomposed into a linear term and a nonlinear term in this paper to reduce the computation complexity of solving LMIs.

By employing the T-S fuzzy inference [56], one has

$$\begin{cases} x(k+1) = \sum_{i=1}^r h_i(\hat{x}(k)) \left(A_{1i}x(k) + A_{2i} \sum_{m=1}^h \beta_m(k) x(k - \tau_m(k)) + B_i u(k) + r(k) g(x(k)) \right) \\ y(k) = \sum_{i=1}^r h_i(\hat{x}(k)) (E_i x(k) + D_{1i} \omega(k)) \\ z(k) = \sum_{i=1}^r h_i(\hat{x}(k)) (D_{2i} x(k)) \end{cases} \quad (11)$$

where $\hat{x}(k) = [\hat{x}_1(k) \ \hat{x}_2(k) \ \dots \ \hat{x}_g(k)]^T$, $h_i(\hat{x}(k))$ is the membership function of the system (11) and

$$h_i(\hat{x}(k)) = \frac{\prod_{j=1}^g \mathcal{M}_{ij}(\hat{x}_j(k))}{\sum_{i=1}^r \prod_{j=1}^g \mathcal{M}_{ij}(\hat{x}_j(k))} \quad (12)$$

with

$$\begin{cases} h_i(\hat{x}(k)) \geq 0 \\ \sum_{i=1}^r h_i(\hat{x}(k)) = 1 \end{cases} \quad (13)$$

where $\mathcal{M}_{ij}(\hat{x}_j(k))$ is the grade of the membership function.

$\tau_m(k)$ and $g(x(k))$ are considered in the system (11), the fading channels may arise in the signal transmission [61]. From [61], one knows that the ℓ th-order Rice fading channels model can be employed to describe the fading channels in the signal transmission. Thus, the ℓ th-order Rice fading channels model is introduced to describe the fading channels [61]

$$\xi(k) = \sum_{l=0}^{\ell} \alpha_l(k) y(k-l), \quad l = 0, 1, \dots, \ell \quad (14)$$

with

$$\begin{cases} E\{\alpha_l(k)\} = \bar{\alpha}_l \\ E\{\alpha_l^2(k)\} = \bar{\alpha}_l^2 \end{cases} \quad (15)$$

where ℓ is the order of the Rice fading channels model, $\xi(k)$ is the ℓ th-order Rice fading channels output. $\alpha_l(k)$ are the channels coefficients and obey the Normal distributions, i.e., $\alpha_l(k) \sim N(\bar{\alpha}_l, \bar{\alpha}_l^2)$ [61]. Compared with the ℓ th-order Rice fading channels model in [61], the Gaussian disturbance is not considered in (14).

Remark 3. The stochastic mean-square stability analysis and non-rational output feedback control were proposed for a class of linear continuous semi-Markovian jump systems with time-varying delays and time-varying disturbance [62], without considering random interval communication delays. The Mittag-leffler stability analysis and adaptive fuzzy fractional order control were proposed for a class of nonlinear continuous neural networks systems with input nonlinearities and unmodeled dynamics [63], without considering random sector nonlinearities. Compared with [62, 63], not only $\omega(k)$, but also $\tau_m(k)$ and $g(x(k))$ are considered for the discrete networked control system in this paper.

Remark 4. The T-S fuzzy model provides the distinctive framework to approximate the nonlinear system plant [50, 55]. Thus, the T-S fuzzy model is employed to approximate the discrete networked control system in this paper. Besides, the stochastic system theory and Bernoulli probability distribution are more suitable to describe the random phenomena in the signal transmission [56], such as the random interval communication delays $\tau_m(k)$ and random sector nonlinearities $g(x(k))$. Thus, the stochastic variables $\beta_m(k)$, $r(k)$ and the Bernoulli probability distribution $Prob\{\cdot\}$ are employed to describe the random interval communication delays $\tau_m(k)$ and random sector nonlinearities $g(x(k))$ in this paper. Moreover, the fading channels in the signal transmission can be described clearly by setting the different channels coefficients of the ℓ th-order Rice fading channels model [61]. Thus, the ℓ th-order Rice fading channels model is introduced to describe the fading channels in this paper.

3. Controller design

By employing the T-S fuzzy model [56], the T-S fuzzy dynamic output feedback controller with ℓ th-order Rice fading channels output is designed as follows.

Controller rule i : if $\hat{\theta}_1(k)$ is $\mathcal{M}_{i1}$, $\hat{\theta}_2(k)$ is $\mathcal{M}_{i2}$, ..., and $\hat{\theta}_g(k)$ is $\mathcal{M}_{ig}$, then

$$\begin{cases} x_c(k+1) = A_{ci}x_c(k) + B_{ci}\xi(k) \\ u(k) = C_{ci}x_c(k) \end{cases} \quad (16)$$

where $\hat{\theta}_1(k)$, $\hat{\theta}_2(k)$, ..., $\hat{\theta}_g(k)$ are the premise variables of the controller, $\mathcal{M}_{ig}$ ($i=1, 2, \dots, r$, $j=1, 2, \dots, g$) is the fuzzy set, r is the number of the fuzzy rules, and g is the number of the premise variables. $x_c(k) \in R^n$ is the state variable of the controller.

$\xi(k)$ is the ℓ th-order Rice fading channels output and defined in (14). A_{ci} , B_{ci} and C_{ci} are the controller gain matrices to be determined.

By employing the T-S fuzzy inference [56], one has

$$\begin{cases} x_c(k+1) = \sum_{i=1}^r h_i(\hat{\theta}(k)) (A_{ci}x_c(k) + B_{ci}\xi(k)) \\ u(k) = \sum_{i=1}^r h_i(\hat{\theta}(k)) (C_{ci}x_c(k)) \end{cases} \quad (17)$$

where $\hat{\theta}(k) = [\hat{\theta}_1(k) \ \hat{\theta}_2(k) \ \dots \ \hat{\theta}_g(k)]^T$, $h_i(\hat{\theta}(k))$ is the membership function of the controller (17) and

$$h_i(\hat{\theta}(k)) = \frac{\prod_{j=1}^g \mathcal{M}_{ij}(\hat{\theta}_j(k))}{\sum_{i=1}^r \prod_{j=1}^g \mathcal{M}_{ij}(\hat{\theta}_j(k))} \quad (18)$$

with

$$\begin{cases} h_i(\hat{\theta}(k)) \geq 0 \\ \sum_{i=1}^r h_i(\hat{\theta}(k)) = 1 \end{cases} \quad (19)$$

where $\mathcal{M}_{ij}(\hat{\theta}_j(k))$ is the grade of the membership function.

Applying (17) to (11), one has

$$\begin{cases} \eta(k+1) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j \left\{ \bar{A}_{1ij} \eta(k) + \bar{A}_{2i} \sum_{m=1}^h \bar{\beta}_m Z \eta(k - \tau_m(k)) + \bar{A}_{2i} \sum_{m=1}^h \tilde{\beta}_m(k) Z \eta(k - \tau_m(k)) + \mathcal{D}_{1ij} \tilde{\omega}(k) + \bar{r} \mathcal{F}(\eta(k)) + \tilde{r}(k) \mathcal{F}(\eta(k)) \right. \\ \quad \left. + \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) + \sum_{l=0}^{\ell} \tilde{\alpha}_l(k) \bar{B}_j Z \eta(k-l) + \sum_{l=1}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) + \sum_{l=0}^{\ell} \tilde{\alpha}_l(k) \tilde{B}_j g_s(Z \eta(k-l)) \right\} \\ z(k) = \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\varepsilon_i \eta(k) + \mathcal{D}_{2j} \tilde{\omega}(k)) \end{cases} \quad (20)$$

with

$$\eta(k) = \begin{bmatrix} x^T(k) & x_c^T(k) \end{bmatrix}^T, \tilde{\omega}(k) = \begin{bmatrix} \omega^T(k) & 0 \end{bmatrix}^T, \mathcal{F}(\eta(k)) = \begin{bmatrix} g(x(k)) \\ 0 \end{bmatrix}, \bar{A}_{1ij} = \begin{bmatrix} A_{1i} & B_{1i} C_{cj} \\ \bar{\alpha}_l B_{cj} M_1 & A_{cj} \end{bmatrix}, \bar{A}_{2i} = \begin{bmatrix} A_{2i} \\ 0 \end{bmatrix}, \bar{B}_j = \begin{bmatrix} 0 \\ B_{cj} M_1 \end{bmatrix}, \tilde{B}_j = \begin{bmatrix} 0 \\ B_{cj} \end{bmatrix},$$

$$\mathcal{D}_{1ij} = \begin{bmatrix} D_{1i} & 0 \\ 0 & B_{cj} \end{bmatrix}, \mathcal{D}_{2j} = \begin{bmatrix} D_{2j} & 0 \end{bmatrix}, Z = \begin{bmatrix} I & 0 \end{bmatrix}, \varepsilon_i = \begin{bmatrix} E_i & 0 \end{bmatrix}, \tilde{\beta}_m(k) = \beta_m(k) - \bar{\beta}_m, \tilde{r}(k) = r(k) - \bar{r}, \tilde{\alpha}_l(k) = \alpha_l(k) - \bar{\alpha}_l.$$

where I is the identity matrix with appropriate dimension and

$$\begin{cases} E\{\tilde{\beta}_m(k)\} = 0, & E\{\tilde{\beta}_m^2(k)\} = \bar{\beta}_m(1 - \bar{\beta}_m) \triangleq \tilde{\beta}_m^2 \\ E\{\tilde{r}(k)\} = 0, & E\{\tilde{r}^2(k)\} = \bar{r}(1 - \bar{r}) \triangleq \tilde{r}^2 \\ E\{\tilde{\alpha}_l(k)\} = 0, & E\{\tilde{\alpha}_l^2(k)\} \triangleq \tilde{\alpha}_l^2 \end{cases} \quad (21)$$

where $\tilde{\beta}_m^2$, $\tilde{r}^2$ and $\tilde{\alpha}_l^2$ are the scalars.

According to **Assumption 1**, it can be seen that $\mathcal{F}(\eta(k))$ in (20) satisfies

$$\|\mathcal{F}(\eta(k))\|^2 \leq \lambda \|\mathcal{G}\eta(k)\|^2 \quad (22)$$

where $\mathcal{G} = [G \ 0]$.

For the problem formulated, **Definition 1** and **Lemmas 1-4** are presented to derive the main results in **Section 4**.

Definition 1 (Exponential mean-square stability) [64]. For any initial condition $\phi(k)$ and $\tilde{\omega}(k) = 0$, if there exist the scalars $\delta > 0$ $\nu \geq 0$ and $0 < \rho < 1$ such that

$$E\left\{\|\eta(k)\|^2\right\} \leq \delta \rho^\nu \sup_{k \in \mathbb{H}} E\left\{\|\phi(k)\|^2\right\}, \quad \tilde{\omega}(k) = 0 \quad (23)$$

then the closed-loop system is said to be exponentially mean-square stable.

Lemma 1 [65]. For the given matrix X_{ij} , if there exists the matrix S with appropriate dimension such that

$$S = S^T > 0 \quad (24)$$

then the following inequality holds

$$\sum_{i=1}^r \sum_{j=1}^r \sum_{s=1}^r \sum_{t=1}^r h_i h_j h_s h_t X_{ij}^T S X_{st} \leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j X_{ij}^T S X_{ij} \quad (25)$$

Lemma 2 (Mean-value theorem) [66]. For the given vectors $x \in R^n$ and $y \in R^n$, the following inequality holds

$$2xy \leq x^2 + y^2 \quad (26)$$

Lemma 3 [67]. For the given vectors $x \in R^n$ and $y \in R^n$, if there exist the scalar ϖ and matrix P with appropriate dimension such that

$$\begin{cases} \varpi > 0 \\ P > 0 \end{cases} \quad (27)$$

then the following inequality holds

$$2x^T y \leq \varpi x^T P x + \varpi^{-1} y^T P y \quad (28)$$

Lemma 4. (Schur complement) [68]. For the given matrices $S_{11} = S_{11}^T$ and $S_{22} = S_{22}^T$, the following inequality

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21}^T & S_{22} \end{bmatrix} < 0 \quad (29)$$

is equivalent to

$$\begin{cases} S_{22} < 0 \\ S_{11} - S_{12} S_{22}^{-1} S_{21}^T < 0 \end{cases} \quad (30)$$

Remark 5. The objective in this paper is to design the controller (17) for the system (11) such that

(i) for any initial condition $\phi(k)$ and $\tilde{\omega}(k) = 0$, the closed-loop system (20) is exponentially mean-square stable according to

Definition 1.

(ii) for the zero initial condition $\phi(0) = 0$ and $\tilde{\omega}(k) \neq 0$, the prescribed H-infinity performance of the closed-loop system (20) is guaranteed, i.e., the control output $z(k)$ satisfies

$$\sum_{k=0}^{\infty} E\left\{\|z(k)\|^2\right\} \leq \gamma^2 \sum_{k=0}^{\infty} E\left\{\|\tilde{\omega}(k)\|^2\right\}, \quad \tilde{\omega}(k) \neq 0 \quad (31)$$

where $\gamma > 0$ is the H-infinity performance index.

Remark 6. The asymptotic stability analysis was proposed for a class of linear continuous switched systems with constant time delay and time-varying disturbance [69], without considering H-infinity stability analysis. The H-infinity passivity stability analysis was proposed for a class of nonlinear continuous singular systems with constant time delay and time-varying disturbance [70], without considering T-S fuzzy dynamic output feedback control. Compared with [69, 70], the H-infinity stability analysis and the T-S fuzzy dynamic output feedback control with ℓ th-order Rice fading channels output are considered for the T-S fuzzy discrete networked control system in this paper.

Remark 7. The T-S fuzzy model has the nice ability to provide the better semi-linear characteristic in the stability analysis if the system plant contains the nonlinear uncertainties [71]. Besides, the static output feedback is easy to implement, but some strict design conditions should be considered [72]. The dynamic output feedback is more flexible and the required design conditions are less conservative [72]. Moreover, the fading channels in the signal transmission can be described by setting the different channels coefficients of ℓ th-order Rice fading channels model [61]. Then, the ℓ th-order Rice fading channels output is introduced in this

paper. Combining the T-S fuzzy model and dynamic output feedback, the T-S fuzzy dynamic output feedback controller with ℓ th-order Rice fading channels output is designed for the T-S fuzzy discrete networked control system in this paper.

4. Main results

In this section, the main results are presented for the closed-loop system in **Theorems 1-2**.

4.1. Stability conditions

Theorem 1. For the given scalars $\gamma > 0$, $\psi > 0$ and $\varphi > 0$, if there exist the matrices $P > 0$, $Q_m > 0$ and $R_\ell > 0$ with appropriate dimensions for $m=1, 2, \dots, h$ and $\ell=0, 1, \dots, \ell$, such that

$$\begin{cases} R_{ii}^1 < 0, & i = j \\ \frac{1}{r-1} R_{ii}^1 + \frac{1}{2} (R_{ij}^1 + R_{ji}^1) < 0, & 1 \leq i < j \end{cases} \quad (32)$$

with

$$R_{ij}^1 = \begin{bmatrix} A_{11}^1 & \Omega_{12ij}^1 & 0 \\ * & A_{22}^1 & \Omega_{23ij}^1 \\ * & * & A_{33}^1 \end{bmatrix} \quad (33)$$

where

$$\begin{cases} A_{11}^1 = \text{diag}\{-P, P_{\ell+1}, -I, -\varphi I\} \\ A_{22}^1 = \text{diag}\{-P, -Q_h, -R_\ell, -\varphi I, -\psi I, -\gamma^2 I\} \\ A_{33}^1 = \text{diag}\{-P_h, -P, -Q_m, -R_\ell, -\psi I_{\ell+1}\} \end{cases} \quad (34)$$

$$\left\{ \begin{aligned} \Omega_{12ij}^1 &= \begin{bmatrix} P\bar{A}_{1ij} & P(\bar{\Theta}_1 \otimes \bar{A}_{2i}) & P(\bar{\Theta}_2 \otimes \bar{B}_j) & \bar{r}P & P(\bar{\Theta}_3 \otimes \tilde{B}_j) & P\mathcal{D}_{1ij} \\ 0 & 0 & P_{\ell+1}(\tilde{\Theta}_3 \otimes \bar{B}_j) & 0 & P_{\ell+1}(\tilde{\Theta}_3 \otimes \tilde{B}_j) & 0 \\ \varepsilon_i & 0 & 0 & 0 & 0 & \mathcal{D}_{2j} \\ \varphi\sqrt{\lambda}\mathcal{G} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \Omega_{23ij}^1 &= \begin{bmatrix} 0 & 0 & Z^T & Z_\ell^T & 0 \\ (\tilde{\Theta}_1 \otimes \bar{A}_{2i}^T)P_h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi\mathcal{M}_{\ell+1}^T \\ 0 & \bar{r}P & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \end{aligned} \right. \quad (35)$$

in which

$$\begin{cases} \mathcal{P}_{\ell+1} = \text{diag}\{P, P, \dots, P\}_{\ell+1}, & \mathcal{P}_h = \text{diag}\{P, P, \dots, P\}_h \\ I_{\ell+1} = \text{diag}\{I, I, \dots, I\}_{\ell+1}, & Z_\ell = \text{diag}\{Z, Z, \dots, Z\}_\ell \\ \mathcal{M}_{\ell+1} = \text{diag}\{M, M, \dots, M\}_{\ell+1}, & \mathcal{R}_\ell = \text{diag}\{I, R_1, R_2, \dots, R_\ell\} \\ Q_h = \text{diag}\{Q_1, Q_2, \dots, Q_h\}, & \tilde{\Theta}_1 = \text{diag}\{\tilde{\beta}_1, \tilde{\beta}_2, \dots, \tilde{\beta}_h\} \\ \tilde{\Theta}_3 = \text{diag}\{\tilde{\alpha}_0, \tilde{\alpha}_1, \dots, \tilde{\alpha}_\ell\}, & \bar{\Theta}_1 = [\bar{\beta}_1 \ \bar{\beta}_2 \ \dots \ \bar{\beta}_h] \\ \bar{\Theta}_2 = [0 \ \bar{\alpha}_1 \ \bar{\alpha}_2 \ \dots \ \bar{\alpha}_\ell], & \bar{\Theta}_3 = [\bar{\alpha}_0 \ \bar{\alpha}_1 \ \dots \ \bar{\alpha}_\ell] \\ \bar{Q}_m = -\sum_{m=1}^h (\tau_u - \tau_l + 1) Z^T Q_m Z \end{cases} \quad (36)$$

then the closed-loop system (20) is exponentially mean-square stable and prescribed H-infinity performance is guaranteed.

Proof. Note that the proof in **Theorem 1** is divided into two steps, i.e., **Step 1** and **Step 2**.

Step 1. The proof of the objective (i) in **Remark 5**: for any initial condition $\phi(k)$ and $\tilde{\omega}(k)=0$, the closed-loop system (20) is exponentially mean-square stable according to **Definition 1**.

For (20), consider the discrete delay-dependent Lyapunov-Krasovskii functional

$$V(x(k)) = V_1(x(k)) + V_2(x(k)) + V_3(x(k)) + V_4(x(k)) \quad (37)$$

where

$$\begin{cases} V_1(x(k)) = \eta^T(k) P \eta(k) \\ V_2(x(k)) = \sum_{m=1}^h \sum_{i=k-\tau_m(k)}^{k-1} \eta^T(i) Z^T Q_m Z \eta(i) \\ V_3(x(k)) = \sum_{m=1}^h \sum_{n=1-\tau_u}^{-\tau_l} \sum_{i=k+n}^{k-1} \eta^T(i) Z^T Q_m Z \eta(i) \\ V_4(x(k)) = \sum_{l=1}^{\ell} \sum_{i=k-l}^{k-1} \eta^T(i) Z^T \mathcal{R}_l Z \eta(i) \end{cases} \quad (38)$$

Taking the forward difference of (37) along (20), one has

$$\begin{aligned} \Delta V(x(k)) &= V(x(k+1)) - V(x(k)) \\ &= \Delta V_1(x(k)) + \Delta V_2(x(k)) + \Delta V_3(x(k)) + \Delta V_4(x(k)) \end{aligned} \quad (39)$$

Then, taking the mathematical expectation of (39) along (20), one has

$$\begin{aligned} E\{\Delta V(x(k))\} &= E\{V(x(k+1)) - V(x(k))\} \\ &= E\{\Delta V_1(x(k))\} + E\{\Delta V_2(x(k))\} + E\{\Delta V_3(x(k))\} + E\{\Delta V_4(x(k))\} \end{aligned} \quad (40)$$

The vector term $E\{\Delta V_1(x(k))\}$ in (40) can be rewritten

$$\begin{aligned} E\{\Delta V_1(x(k))\} &= E\{V_1(x(k+1)) - V_1(x(k))\} \\ &= E\left\{ \sum_{i=1}^r \sum_{j=1}^r \sum_{s=1}^r h_i h_j h_s \left(\eta^T(k) \bar{A}_{ij}^T P \bar{A}_{1st} \eta(k) + 2\eta^T(k) \bar{A}_{ij}^T P \bar{A}_{2s} \sum_{m=1}^h \bar{\beta}_m Z \eta(k - \tau_m(k)) \right. \right. \\ &\quad + 2\eta^T(k) \bar{A}_{ij}^T P \mathcal{D}_{1st} \tilde{\omega}(k) + 2\eta^T(k) \bar{A}_{ij}^T P \bar{r} \mathcal{F}(\eta(k)) + 2\eta^T(k) \bar{A}_{ij}^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_l Z \eta(k-l) \\ &\quad + 2\eta^T(k) \bar{A}_{ij}^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \bar{B}_l g_s(Z \eta(k-l)) + \sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2t}^T P \bar{A}_{2s} \sum_{m=1}^h \bar{\beta}_m Z \eta(k - \tau_m(k)) \\ &\quad + 2 \sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2t}^T P \mathcal{D}_{1st} \tilde{\omega}(k) + 2 \sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2t}^T P \bar{r} \mathcal{F}(\eta(k)) \\ &\quad + 2 \sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2t}^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_l Z \eta(k-l) + 2 \sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2t}^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \bar{B}_l g_s(Z \eta(k-l)) \\ &\quad + \sum_{m=1}^h \tilde{\beta}_m(k) Z \eta^T(k - \tau_m(k)) \bar{A}_{2t}^T P \bar{A}_{2s} \sum_{m=1}^h \tilde{\beta}_m(k) Z \eta(k - \tau_m(k)) + \tilde{\omega}^T(k) \mathcal{D}_{1ij}^T P \mathcal{D}_{1st} \tilde{\omega}(k) + 2\tilde{\omega}^T(k) \mathcal{D}_{1ij}^T P \bar{r} \mathcal{F}(\eta(k)) \\ &\quad + 2\tilde{\omega}^T(k) \mathcal{D}_{1ij}^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_l Z \eta(k-l) + 2\tilde{\omega}^T(k) \mathcal{D}_{1ij}^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \bar{B}_l g_s(Z \eta(k-l)) + \bar{r} \mathcal{F}^T(\eta(k)) P \bar{r} \mathcal{F}(\eta(k)) \\ &\quad + 2\bar{r} \mathcal{F}^T(\eta(k)) P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_l Z \eta(k-l) + 2\bar{r} \mathcal{F}^T(\eta(k)) P \sum_{l=0}^{\ell} \bar{\alpha}_l \bar{B}_l g_s(Z \eta(k-l)) + \bar{r}(k) \mathcal{F}^T(\eta(k)) P \bar{r}(k) \mathcal{F}(\eta(k)) \\ &\quad + \sum_{l=1}^{\ell} \bar{\alpha}_l \eta^T(k-l) Z^T \bar{B}_l^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_l Z \eta(k-l) + 2 \sum_{l=1}^{\ell} \bar{\alpha}_l \eta^T(k-l) Z^T \bar{B}_l^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \bar{B}_l g_s(Z \eta(k-l)) \\ &\quad + \sum_{l=0}^{\ell} \tilde{\alpha}_l(k) \eta^T(k-l) Z^T \bar{B}_l^T P \sum_{l=0}^{\ell} \tilde{\alpha}_l(k) \bar{B}_l (Z \eta(k-l)) + 2 \sum_{l=0}^{\ell} \tilde{\alpha}_l(k) \eta^T(k-l) Z^T \bar{B}_l^T P \sum_{l=0}^{\ell} \tilde{\alpha}_l(k) \tilde{B}_l g_s(Z \eta(k-l)) \\ &\quad \left. + \sum_{l=0}^{\ell} \tilde{\alpha}_l(k) g_s^T(Z \eta(k-l)) \tilde{B}_l^T P \sum_{l=0}^{\ell} \tilde{\alpha}_l(k) \tilde{B}_l g_s(Z \eta(k-l)) \right) - \eta^T(k) P \eta(k) \Big\} \end{aligned} \quad (41)$$

Applying **Lemma 1** to (41), one has

$$\begin{aligned}
E\{\Delta V_1(x(k))\} &= E\{V_1(x(k+1)) - V_1(x(k))\} \\
&= E\left\{\sum_{i=1}^r \sum_{j=1}^r h_j h_j \left(\eta^T(k) \bar{A}_{ij}^T P \bar{A}_{ij} \eta(k) + \bar{r} \mathcal{F}^T(\eta(k)) P \bar{r} \mathcal{F}(\eta(k)) + 2\eta^T(k) \bar{A}_{ij}^T P \bar{A}_{2i} \sum_{m=1}^h \bar{\beta}_m Z \eta(k - \tau_m(k)) \right. \right. \\
&\quad + 2\eta^T(k) \bar{A}_{ij}^T P \mathcal{D}_{ij} \tilde{\omega}(k) + 2\eta^T(k) \bar{A}_{ij}^T P \bar{r} \mathcal{F}(\eta(k)) \\
&\quad + 2 \sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2i}^T P \mathcal{D}_{ij} \tilde{\omega}(k) + 2 \sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2i}^T P \bar{r} \mathcal{F}(\eta(k)) \\
&\quad + \sum_{l=1}^{\ell} \bar{\alpha}_l \eta^T(k-l) Z^T \bar{B}_j^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) + 2 \sum_{l=1}^{\ell} \bar{\alpha}_l \eta^T(k-l) Z^T \bar{B}_j^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) \\
&\quad + 2\eta^T(k) \bar{A}_{ij}^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) + 2 \sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2i}^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) \\
&\quad + 2\eta^T(k) \bar{A}_{ij}^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) + 2 \sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2i}^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) \\
&\quad + 2\tilde{\omega}^T(k) \mathcal{D}_{ij}^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) + 2\bar{r} \mathcal{F}^T(\eta(k)) P \sum_{l=0}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) + \tilde{r}(k) \mathcal{F}^T(\eta(k)) P \tilde{r}(k) \mathcal{F}(\eta(k)) \\
&\quad + \sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2i}^T P \bar{A}_{2i} \sum_{m=1}^h \bar{\beta}_m Z \eta(k - \tau_m(k)) + \sum_{m=1}^h \bar{\beta}_m(k) Z \eta^T(k - \tau_m(k)) \bar{A}_{2i}^T P \bar{A}_{2i} \sum_{m=1}^h \bar{\beta}_m(k) Z \eta(k - \tau_m(k)) \\
&\quad + \tilde{\omega}^T(k) \mathcal{D}_{ij}^T P \mathcal{D}_{ij} \tilde{\omega}(k) + 2\tilde{\omega}^T(k) \mathcal{D}_{ij}^T P \bar{r} \mathcal{F}(\eta(k)) + 2\tilde{\omega}^T(k) \mathcal{D}_{ij}^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) \\
&\quad + 2\bar{r} \mathcal{F}^T(\eta(k)) P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) + \sum_{l=0}^{\ell} \bar{\alpha}_l(k) \eta^T(k-l) Z^T \bar{B}_j^T P \sum_{l=0}^{\ell} \bar{\alpha}_l(k) \bar{B}_j (Z \eta(k-l)) \\
&\quad + 2 \sum_{l=0}^{\ell} \bar{\alpha}_l(k) \eta^T(k-l) Z^T \bar{B}_j^T P \sum_{l=0}^{\ell} \bar{\alpha}_l(k) \tilde{B}_j g_s(Z \eta(k-l)) + \sum_{l=0}^{\ell} \bar{\alpha}_l(k) g_s^T(Z \eta(k-l)) \tilde{B}_j^T P \sum_{l=0}^{\ell} \bar{\alpha}_l(k) \tilde{B}_j g_s(Z \eta(k-l)) \Big) \\
&\quad - \eta^T(k) P \eta(k) \Big\}
\end{aligned}$$

Applying **Lemma 2** to the above inequality

$$\begin{aligned}
E\{\Delta V_1(x(k))\} &\leq E\left\{\sum_{i=1}^r \sum_{j=1}^r h_j h_j \left(\eta^T(k) (\bar{A}_{ij}^T P \bar{A}_{ij} - P) \eta(k) + 2\eta^T(k) \bar{A}_{ij}^T P \bar{A}_{2i} \sum_{m=1}^h \bar{\beta}_m Z \eta(k - \tau_m(k)) + 2\eta^T(k) \bar{A}_{ij}^T P \mathcal{D}_{ij} \tilde{\omega}(k) \right. \right. \\
&\quad + 2\eta^T(k) \bar{A}_{ij}^T P \bar{r} \mathcal{F}(\eta(k)) + 2\eta^T(k) \bar{A}_{ij}^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) + 2\eta^T(k) \bar{A}_{ij}^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) \\
&\quad + \left(\sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2i} \right)^T P \bar{A}_{2i} \sum_{m=1}^h \bar{\beta}_m Z \eta(k - \tau_m(k)) + 2 \left(\sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2i} \right)^T P \mathcal{D}_{ij} \tilde{\omega}(k) \\
&\quad + 2 \left(\sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2i} \right)^T P \bar{r} \mathcal{F}(\eta(k)) + 2 \left(\sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2i} \right)^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) \\
&\quad + 2 \left(\sum_{m=1}^h \bar{\beta}_m Z \eta^T(k - \tau_m(k)) \bar{A}_{2i} \right)^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) + \left(\sum_{m=1}^h \bar{\beta}_m(k) Z \eta^T(k - \tau_m(k)) \bar{A}_{2i} \right)^T P \bar{A}_{2i} \sum_{m=1}^h \bar{\beta}_m(k) Z \eta(k - \tau_m(k)) \\
&\quad + \tilde{\omega}^T(k) \mathcal{D}_{ij}^T P \mathcal{D}_{ij} \tilde{\omega}(k) + 2\tilde{\omega}^T(k) \mathcal{D}_{ij}^T P \bar{r} \mathcal{F}(\eta(k)) + 2\tilde{\omega}^T(k) \mathcal{D}_{ij}^T P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) + 2\tilde{\omega}^T(k) \mathcal{D}_{ij}^T P \sum_{l=0}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) \\
&\quad + \bar{r} \mathcal{F}^T(\eta(k)) P \bar{r} \mathcal{F}(\eta(k)) + 2\bar{r} \mathcal{F}^T(\eta(k)) P \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) + 2\bar{r} \mathcal{F}^T(\eta(k)) P \sum_{l=0}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) \\
&\quad + \tilde{r}^2 \mathcal{F}^T(\eta(k)) P \mathcal{F}(\eta(k)) + \sum_{l=1}^{\ell} \bar{\alpha}_l \eta^T(k-l) Z^T \bar{B}_j^T P + \sum_{l=1}^{\ell} \bar{\alpha}_l \bar{B}_j Z \eta(k-l) + 2 \sum_{l=1}^{\ell} \bar{\alpha}_l \eta^T(k-l) Z^T \bar{B}_j^T \\
&\quad \times P \sum_{l=0}^{\ell} \bar{\alpha}_l \tilde{B}_j g_s(Z \eta(k-l)) + \sum_{l=0}^{\ell} \bar{\alpha}_l^2 \eta^T(k-l) Z^T \bar{B}_j^T P \bar{B}_j Z \eta(k-l) + 2 \sum_{l=0}^{\ell} \bar{\alpha}_l^2 \eta^T(k-l) Z^T \bar{B}_j^T P \tilde{B}_j g_s(Z \eta(k-l)) \\
&\quad + \sum_{l=0}^{\ell} \bar{\alpha}_l^2 g_s^T(Z \eta(k-l)) \tilde{B}_j^T P \tilde{B}_j g_s(Z \eta(k-l)) \Big) \Big\} \tag{42}
\end{aligned}$$

Via the similar method, the vector terms $E\{\Delta V_2(x(k))\}$, $E\{\Delta V_3(x(k))\}$ and $E\{\Delta V_4(x(k))\}$ in (40) can be rewritten as follows

$$E\{\Delta V_2(x(k))\} \leq E\left\{\eta^T(k) Z^T Q_m Z \eta(k) - \eta^T(k - \tau_m(k)) Z^T Q_m Z \eta(k - \tau_m(k)) + \sum_{i=k-\tau_a+1}^{k-\tau_l} \eta^T(i) Z^T Q_m Z \eta(i)\right\} \tag{43}$$

$$E\{\Delta V_3(x(k))\} \leq E\left\{\sum_{m=1}^h \left((\tau_u - \tau_l) \eta^T(k) Z^T Q_m Z \eta(k) - \sum_{i=k-\tau_u+1}^{k-\tau_l} \eta^T(i) Z^T Q_m Z \eta(i) \right)\right\} \quad (44)$$

$$E\{\Delta V_4(x(k))\} = E\left\{\sum_{l=1}^{\ell} (\eta^T(k) Z^T \mathcal{R}_l Z \eta(k) - \eta^T(k-l) Z^T \mathcal{R}_l Z \eta(k-l))\right\} \quad (45)$$

Let

$$\begin{cases} \eta(k-\tau) = [\eta^T(k-\tau_1(k)) Z^T & \eta^T(k-\tau_2(k)) Z^T & \dots & \eta^T(k-\tau_m(k)) Z^T]^T \\ \tilde{\eta}(k) = [\eta^T(k) & \eta^T(k-l) & \eta_\ell^T(k) & \mathcal{F}^T(\eta(k)) & \mathcal{G}_\ell^T(k) & \tilde{\omega}^T(k)]^T \\ \bar{\eta}(k) = [\eta^T(k) & \eta^T(k-l) & \eta_\ell^T(k) & \mathcal{F}^T(\eta(k)) & \mathcal{G}_\ell^T(k)]^T \\ \eta_\ell(k) = [\eta^T(k) Z^T & \eta^T(k-1) Z^T & \dots & \eta^T(k-\ell) Z^T]^T \\ \mathcal{G}_\ell(k) = [g_s^T(Z\eta(k)) & g_s^T(Z\eta(k-1)) & \dots & g_s^T(Z\eta(k-\ell))]^T \end{cases} \quad (46)$$

then from (9), (22), (42)-(46), one has

$$\begin{aligned} E\{\Delta V(x(k))\} \leq & E\{\Delta V_1(x(k)) + \Delta V_2(x(k)) + \Delta V_3(x(k)) + \Delta V_4(x(k)) - \phi(\mathcal{F}^T(\eta(k)) \mathcal{F}(\eta(k)) - \lambda \eta^T(k) \mathcal{G}^T \mathcal{G} \eta(k)) \\ & - 2\psi(\mathcal{G}_\ell^T(k) \mathcal{G}_\ell(k) - \psi \mathcal{G}_\ell^T(k) (I_{\ell+1} \otimes M^T M) \eta_\ell(k))\} \end{aligned} \quad (47)$$

Applying **Lemma 1** and **Lemma 3** to (47)

$$E\{\Delta V(x(k))\} \leq E\left\{\sum_{i=1}^r \sum_{j=1}^r h_i h_j \bar{\eta}^T(k) \Gamma_{lij} \bar{\eta}(k)\right\} \quad (48)$$

with

$$\Gamma_{lij} = \begin{bmatrix} \Gamma_{11ij} + \phi \lambda \mathcal{G}^T \mathcal{G} & \Gamma_{12ij} & \Gamma_{13ij} & \Gamma_{14ij} & \Gamma_{15ij} \\ * & \Gamma_{22ii} + I & \Gamma_{23ij} & \Gamma_{24ii} & \Gamma_{25ij} \\ * & * & \Gamma_{33jj} + \psi I_{\ell+1} \otimes (M^T M) & \Gamma_{34jj} & \Gamma_{35jj} \\ * & * & * & \Gamma_{44} - \phi I & \Gamma_{45jj} \\ * & * & * & * & \Gamma_{55jj} - \psi I \end{bmatrix} \quad (49)$$

where

$$\begin{cases} \Gamma_{11ij} = \bar{A}_{1ij}^T P \bar{A}_{1ij} + \tilde{\alpha}_0^2 Z^T \bar{A}_{2i}^T P \bar{A}_{2i} Z + \sum_{l=1}^{\ell} Z^T R_l Z^T + Z^T Z + \sum_{m=1}^h (\tau_u - \tau_l + 1) Z^T Q_m Z - P \\ \Gamma_{12ij} = [\bar{\beta}_1 \bar{A}_{1ij}^T P \bar{A}_{2i} \quad \dots \quad \bar{\beta}_h \bar{A}_{1ij}^T P \bar{A}_{2i}] \\ \Gamma_{13ij} = [0 \quad \bar{\alpha}_1 \bar{A}_{1ij}^T P \bar{B}_j \quad \dots \quad \bar{\alpha}_\ell \bar{A}_{1ij}^T P \bar{B}_j] \\ \Gamma_{14ij} = \bar{r} \bar{A}_{1ij}^T P \\ \Gamma_{15ij} = [\bar{\alpha}_0 \bar{A}_{1ij}^T P \tilde{B}_j \quad \dots \quad \bar{\alpha}_\ell \bar{A}_{1ij}^T P \tilde{B}_j] \end{cases} \quad (50)$$

$$\begin{cases} \Gamma_{22ii} = [\bar{\Theta} \otimes \bar{A}_{2i}]^T P [\bar{\Theta}_1 \otimes \bar{A}_{2i}] + [\bar{\Theta}_1 \otimes \bar{A}_{2i}]^T \mathcal{P}_h [\bar{\Theta}_1 \otimes \bar{A}_{2i}] - Q_h \\ \Gamma_{23ij} = [\bar{\Theta}_1 \otimes I]^T [0 \quad \bar{\alpha}_1 \bar{A}_{2i}^T P \bar{B}_j \quad \dots \quad \bar{\alpha}_\ell \bar{A}_{2i}^T P \bar{B}_j] \\ \Gamma_{24ii} = [\bar{\beta}_1 \delta \bar{A}_{2i}^T P \quad \dots \quad \bar{\beta}_h \delta \bar{A}_{2i}^T P] \\ \Gamma_{25ij} = [\bar{\Theta}_1 \otimes I]^T [0 \quad \bar{\alpha}_1 \bar{A}_{2i}^T P \tilde{B}_j \quad \dots \quad \bar{\alpha}_\ell \bar{A}_{2i}^T P \tilde{B}_j] \end{cases} \quad (51)$$

$$\begin{cases} \Gamma_{33jj} = [\bar{\Theta}_2 \otimes \bar{B}_j]^T P [\bar{\Theta}_2 \otimes \bar{B}_j] + [\bar{\Theta}_3 \otimes \bar{B}_j]^T \mathcal{P}_{\ell+1} [\bar{\Theta}_3 \otimes \bar{B}_j] - \mathcal{R}_\ell \\ \Gamma_{34jj} = [0 \quad \bar{\alpha}_1 \delta \bar{B}_j^T P \quad \dots \quad \bar{\alpha}_\ell \delta \bar{B}_j^T P] \\ \Gamma_{35jj} = [\bar{\Theta}_2 \otimes \bar{B}_j]^T P [\bar{\Theta}_3 \otimes \tilde{B}_j] + [\bar{\Theta}_3 \otimes \bar{B}_j]^T \mathcal{P}_{\ell+1} [\bar{\Theta}_3 \otimes \tilde{B}_j] \end{cases} \quad (52)$$

$$\begin{cases} \Gamma_{44} = \bar{r}^2 P + \tilde{r}^2 P \\ \Gamma_{45jj} = [\bar{\alpha}_0 \delta P \tilde{B}_j \quad \bar{\alpha}_1 \delta P \tilde{B}_j \quad \dots \quad \bar{\alpha}_\ell \delta P \tilde{B}_j] \end{cases} \quad (53)$$

$$\Gamma_{55jj} = [\bar{\mathcal{O}}_3 \otimes \tilde{B}_j]^T P [\bar{\mathcal{O}}_3 \otimes \tilde{B}_j] + [\bar{\mathcal{O}}_3 \otimes \tilde{B}_j]^T \mathcal{P}_{\ell+1} [\bar{\mathcal{O}}_3 \otimes \tilde{B}_j] \quad (54)$$

Applying **Lemma 4** and considering the inequalities (32), one can obtain $\Gamma_{1ij} < 0$, i.e., $E\{\Delta V(x(k))\} < 0$. Thus, the closed-loop system (20) is exponentially mean-square stable, i.e., the objective (i) in **Remark 5** is achieved. The similar proof was shown in [73]. The proof in **Step 1** is completed. **(Q. E. D.)**

Step 2. The proof of the objective (ii) in **Remark 5**: for the zero initial condition $\phi(0) = 0$ and $\tilde{\omega}(k) \neq 0$, the prescribed H-infinity performance of the closed-loop system (20) is guaranteed, i.e., the control output $z(k)$ satisfies the inequality (31).

For (20), consider the prescribed H-infinity performance function

$$J(n) = E \left\{ \sum_{k=0}^n (z^T(k)z(k) - \gamma^2 \tilde{\omega}^T(k)\tilde{\omega}(k)) \right\}, \quad \tilde{\omega}(k) \neq 0 \quad (55)$$

where n is an integer scalar. Obviously, the objective in **Step 2** is to prove $J(n) \leq 0$.

From (20) and (48)

$$\begin{aligned} J(n) &\leq E \left\{ \sum_{k=0}^n (z^T(k)z(k) - \gamma^2 \tilde{\omega}^T(k)\tilde{\omega}(k)) + \Delta V(k) \right\} \\ &\leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j (\tilde{\eta}^T(k) \Gamma_{2ij} \tilde{\eta}(k)) \end{aligned} \quad (56)$$

with

$$\Gamma_{2ij} = \begin{bmatrix} \Gamma_{11ij} + \phi \lambda \mathcal{G}^T \mathcal{G} + \varepsilon_i^T \varepsilon_i & \Gamma_{12ij} & \Gamma_{13ij} & \Gamma_{14ij} & \Gamma_{15ij} & \Gamma_{16ij} \\ * & \Gamma_{22ii} + I & \Gamma_{23ij} & \Gamma_{24ii} & \Gamma_{25ij} & \Gamma_{26ij} \\ * & * & \Gamma_{33jj} + \psi I_{\ell+1} \otimes (M^T M) & \Gamma_{34jj} & \Gamma_{35jj} & \Gamma_{36ij} \\ * & * & * & \Gamma_{44} - \phi I & \Gamma_{45jj} & \Gamma_{46ij} \\ * & * & * & * & \Gamma_{55jj} - \psi I & \Gamma_{56ij} \\ * & * & * & * & * & \Gamma_{66ij} + \mathcal{D}_{2j}^T \mathcal{D}_{2j} - \gamma^2 I \end{bmatrix} \quad (57)$$

where

$$\begin{cases} \Gamma_{16ij} = \bar{A}_{1ij}^T P \mathcal{D}_{1ij}, & \Gamma_{26ij} = [\bar{\beta}_1 \mathcal{D}_{1ij}^T P \bar{A}_{2i} \quad \dots \quad \bar{\beta}_h \mathcal{D}_{1ij}^T P \bar{A}_{2i}]^T \\ \Gamma_{36ij} = [0 \quad \bar{\alpha}_1 \mathcal{D}_{1ij}^T P \bar{B}_j \quad \dots \quad \bar{\alpha}_\ell \mathcal{D}_{1ij}^T P \bar{B}_j]^T, & \Gamma_{46ij} = \bar{r} P \mathcal{D}_{1ij} \\ \Gamma_{56ij} = [\bar{\alpha}_0 \mathcal{D}_{1ij}^T P \bar{B}_j \quad \dots \quad \bar{\alpha}_\ell \mathcal{D}_{1ij}^T P \bar{B}_j], & \Gamma_{66ij} = \mathcal{D}_{1ij}^T P \mathcal{D}_{1ij} \end{cases} \quad (58)$$

Applying **Lemma 4** and considering the inequalities (32) and (33), one can obtain $J(n) < 0$. From the proof in **Step 1**, it can be seen that the closed-loop system (20) is exponentially mean-square stable, then let $n \rightarrow \infty$ in (55), one has

$$E \left\{ \sum_{k=0}^{\infty} \|z(k)\|^2 \right\} \leq \gamma^2 E \left\{ \sum_{k=0}^{\infty} \|\tilde{\omega}(k)\|^2 \right\}, \quad \tilde{\omega}(k) \neq 0 \quad (59)$$

from (59), it is easy to verify that

$$J(n) = E \left\{ \sum_{k=0}^{\infty} \|z(k)\|^2 - \gamma^2 \sum_{k=0}^{\infty} \|\tilde{\omega}(k)\|^2 \right\} \leq 0, \quad \tilde{\omega}(k) \neq 0 \quad (60)$$

From (60), it can be seen that the prescribed H-infinity performance is guaranteed, i.e., the objective (ii) in **Remark 5** is achieved. Thus, the proof in **Step 2** is completed. **(Q. E. D.)**

From the proof in **Step 1** and **Step 2**, it can be seen that the whole proof in **Theorem 1** is completed. **(Q. E. D.)**

Remark 8. From **Theorem 1**, it can be seen that the closed-loop system (20) is exponentially mean-square stable and the prescribed H-infinity performance is guaranteed by employing the discrete delay-dependent Lyapunov-Krasovskii functional, stochastic system theory and Bernoulli probability distribution. Besides, in **Theorem 1**, it can be seen that the stability conditions are derived by employing the Schur complement, i.e., **Lemma 4**. It should be noticed that the stability conditions are presented for the closed-loop system via a set of LMIs (32) in **Theorem 1**. However, the controller gain matrices A_{ci} , B_{ci} and C_{ci} are not determined in **Theorem 1**. Thus, **Theorem 2** is presented to determine the controller gain matrices A_{ci} , B_{ci} and C_{ci} .

4.2. Controller gain matrices

Theorem 2. For the given scalars $\gamma > 0$, $\psi > 0$ and $\varphi > 0$, if there exist the matrices $S > 0$, $T > 0$, $Q_m > 0$, $R_l > 0$, H , K_{1j} , K_{2j} and K_{3j} with appropriate dimensions for $m=1, 2, \dots, h$, $l=0, 1, \dots, \ell$ and $j=1, 2, \dots, r$, such that

$$\begin{cases} R_{ii}^3 < 0 & i = j \\ \frac{1}{r-1}R_{ii}^3 + \frac{1}{2}(R_{ij}^3 + R_{ji}^3) < 0, & 1 \leq i < j \end{cases} \quad (61)$$

$$\begin{bmatrix} T^{-1} & -H \end{bmatrix}^T T \begin{bmatrix} T^{-1} & -H \end{bmatrix} \geq 0 \quad (62)$$

with

$$R_{ij}^3 = \begin{bmatrix} A_{11}^3 & \Omega_{12ij}^3 & 0 \\ * & A_{22}^3 & \Omega_{23ij}^3 \\ * & * & A_{33}^3 \end{bmatrix} \quad (63)$$

where

$$\begin{cases} A_{11}^3 = \text{diag}\{\hat{P}, \hat{P}_{\ell+1}, -I, -\varphi I\} \\ A_{22}^3 = \text{diag}\{-\tilde{P}, -Q_h, -R_\ell, -\varphi I, -\psi I, -\gamma^2 I\} \\ A_{33}^3 = \text{diag}\{-\tilde{P}_h, -\tilde{P}, -\bar{Q}_m, -R_\ell, -\psi I_{\ell+1}\} \end{cases} \quad (64)$$

$$\begin{cases} \Omega_{12ij}^3 = \begin{bmatrix} \bar{A}_{1ij} & \bar{\Theta}_1 \otimes \bar{A}_{2i} & \bar{\Theta}_2 \otimes \bar{B} & \bar{r}\bar{W}_2 & \bar{\Theta}_3 \otimes \tilde{B} & \bar{D}_{1ij} \\ 0 & 0 & \tilde{\Theta}_3 \otimes \tilde{B} & 0 & \tilde{\Theta}_3 \otimes \tilde{B} & 0 \\ \bar{\varepsilon}_i & 0 & 0 & 0 & 0 & \mathcal{D}_{2j} \\ \varphi\sqrt{\lambda}\bar{G} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \Omega_{23ij}^3 = \begin{bmatrix} 0 & 0 & Z^T \bar{Q}_m & Z^T \otimes R_\ell & 0 \\ \tilde{\Theta}_1 \otimes \tilde{A}_{2i}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi \mathcal{M}_{\ell+1}^T \\ 0 & \bar{r}T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{cases} \quad (65)$$

in which

$$\begin{cases} \hat{P}_{\ell+1} = \text{diag}\{\hat{P}, \hat{P}, \dots, \hat{P}\}, & \tilde{P}_h = \text{diag}\{\tilde{P}, \tilde{P}, \dots, \tilde{P}\}, & T = \text{diag}\{T, I\} \\ \hat{P} = \begin{bmatrix} \bar{T} & -I \\ * & -S \end{bmatrix}, & \tilde{P} = \begin{bmatrix} T & T \\ * & S \end{bmatrix}, & \bar{W}_2 = \begin{bmatrix} I & 0 \\ S & 0 \end{bmatrix} \\ \bar{T} = -H - H^T + H^T T H, & \bar{G} = \begin{bmatrix} G & 0 \\ 0 & G \end{bmatrix}, & \bar{\varepsilon}_i = \begin{bmatrix} E_i \\ E_i \end{bmatrix} \\ \bar{A}_{1ij} = \begin{bmatrix} \bar{A}_{1ij} + \bar{B}_i K_{3j} & \bar{A}_{1ij} \\ S\bar{A}_{1ij} + \bar{\alpha}_0 K_{2j} M_1 + K_{1j} & S\bar{A}_{1ij} + \bar{\alpha}_0 K_{2j} M_1 \end{bmatrix}, & \bar{A}_{2i} = \begin{bmatrix} \bar{A}_{2i} \\ S\bar{A}_{2i} \end{bmatrix}, & \tilde{A}_{2i} = \begin{bmatrix} T\bar{A}_{2i} \\ S\bar{A}_{2i} \end{bmatrix} \\ \bar{B} = \begin{bmatrix} 0 \\ K_{2j} M_1 \end{bmatrix}, & \tilde{B} = \begin{bmatrix} 0 \\ K_{2j} \end{bmatrix}, & \bar{D}_{1ij} = \begin{bmatrix} \mathcal{D}_{1ij} & 0 \\ S\mathcal{D}_{1ij} & K_{2j} \end{bmatrix} \end{cases} \quad (66)$$

then the closed-loop system (20) is exponentially mean-square stable and prescribed H-infinity performance is guaranteed. Further, the controller gain matrices A_{ci} , B_{ci} and C_{ci} can be determined

$$\begin{cases} A_{ci} = X_{12} \left(K_{1j} (T - S)^{-1} X_{12} - S B_i C_{cj} \right) \\ B_{ci} = X_{12}^{-1} K_{2j} \\ C_{ci} = K_{3j} (T - S)^{-1} X_{12} \end{cases} \quad (67)$$

with

$$I - ST^{-1} = X_{12}Y_{12}^T < 0 \quad (68)$$

where X_{12} and Y_{12} are the factorizations in the polynomial (68). The factorizations X_{12} and Y_{12} will be used in (72).

Proof. For the closed-loop system (20), the objective in this section is to determine A_{ci} , B_{ci} and C_{ci} , such that the inequalities (32) hold. For the problem formulated, P and P^{-1} in **Theorem 1** is rewritten

$$\left\{ P = \begin{bmatrix} S & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} T^{-1} & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix} \right. \quad (69)$$

where X_{22} and Y_{22} are the matrices with appropriate dimensions.

Let us define

$$\left\{ W_1 = \begin{bmatrix} T^{-1} & I \\ Y_{12}^T & 0 \end{bmatrix}, \quad W_2 = \begin{bmatrix} I & S \\ 0 & X_{22}^T \end{bmatrix} \right. \quad (70)$$

From (69) and (70), it is easy to verify that

$$\begin{cases} PW_1 = W_2 \\ W_1^T P W_1 = W_1^T W_2 \end{cases} \quad (71)$$

From (67) and (68)

$$\begin{cases} K_{1j} = (SB_{ci}C_{cj} + X_{12}A_{ci})Y_{12}^T T \\ K_{2j} = X_{12}B_{ci} \\ K_{3j} = C_{ci}M_1^T T \end{cases} \quad (72)$$

Taking the congruence transformation of (33) by employing the congruence transformation matrix $\text{diag}\left\{W_1, \underbrace{W_1, W_1, \dots, W_1}_{\ell+1}, I, I, W_1, I_h, I_{\ell+1}, I, I, I, \underbrace{W_1, W_1, \dots, W_1}_h, W_1, I, I_{\ell+1}, I_{\ell+1}\right\}$, where

$I_h = \text{diag}\left\{I, I, \dots, I\right\}_h$, one has

$$\begin{bmatrix} A_{11}^2 & \Omega_{12ij}^2 & 0 \\ * & A_{22}^2 & \Omega_{23ij}^2 \\ * & * & A_{33}^2 \end{bmatrix} < 0 \quad (73)$$

with

$$\begin{cases} A_{11}^2 = \text{diag}\{-\bar{P}, -\bar{P}_{\ell+1}, -I, -\phi I\} \\ A_{22}^2 = \text{diag}\{-\bar{P}, -Q_h, -R_\ell, -\phi I, -\psi I, -\gamma^2 I\} \\ A_{33}^2 = \text{diag}\{-\bar{P}_h, -\bar{P}, -\bar{Q}_m^{-1}, -R_\ell^{-1}, -\psi I_{\ell+1}\} \end{cases} \quad (74)$$

$$\begin{cases} \Omega_{12ij}^2 = \begin{bmatrix} \tilde{A}_{1ij} & \bar{\Theta}_1 \otimes \bar{A}_{2i} & \bar{\Theta}_2 \otimes \bar{B} & \bar{r}\bar{W}_2 & \bar{\Theta}_3 \otimes \bar{B} & \bar{D}_{1ij} \\ 0 & 0 & \bar{\Theta}_3 \otimes \bar{B} & 0 & \bar{\Theta}_3 \otimes \bar{B} & 0 \\ \tilde{\varepsilon}_i & 0 & 0 & 0 & 0 & D_{2j} \\ \phi\sqrt{\lambda}\tilde{G} & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \Omega_{23ij}^2 = \begin{bmatrix} 0 & 0 & Z^T T^{-1} & Z^T T^{-1} \otimes I_\ell & 0 \\ \bar{\Theta}_1 \otimes \bar{A}_{2i}^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \psi \mathcal{M}_{\ell+1}^T \\ 0 & \bar{r}W_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{cases} \quad (75)$$

where

$$\begin{cases} I_\ell = \text{diag}\left\{I, I, \dots, I\right\}_\ell, \quad \bar{P}_h = \text{diag}\left\{\bar{P}, \bar{P}, \dots, \bar{P}\right\}_h, \quad \bar{P}_{\ell+1} = \text{diag}\left\{\bar{P}, \bar{P}, \dots, \bar{P}\right\}_{\ell+1} \\ \bar{P} = \begin{bmatrix} T^{-1} & I \\ * & S \end{bmatrix}, \quad \tilde{A}_{1ij} = \begin{bmatrix} (\bar{A}_{1ij} + \bar{B}_i K_{3j})T^{-1} & \bar{A}_{1ij} \\ (S\bar{A}_{1ij} + \bar{\alpha}_0 K_{2j} M_1 + K_{1j})T^{-1} & S\bar{A}_{1ij} + \bar{\alpha}_0 K_{2j} M_1 \end{bmatrix}, \quad \tilde{\varepsilon}_i = [E_i T^{-1} \quad E_i], \quad \tilde{G} = [GT^{-1} \quad G] \end{cases} \quad (76)$$

Next, from (62), one has

$$-T^{-1} \leq -H - H^T + H^T T H \quad (77)$$

Via the similar method, taking the congruence transformation of (73) by employing the congruence transformation matrix

$$\text{diag} \left\{ I, \quad I_{\ell+1}, \quad I, \quad I, \quad T, \quad I_h, \quad I_{\ell+1}, \quad I, \quad I, \quad I, \quad \underbrace{T, \quad T, \quad \dots, \quad T}_h, \quad T, \quad \bar{Q}_m, \quad I_{\ell+1}, \quad \mathcal{R}_{\ell+1} \right\}, \text{ where } I_h = \text{diag} \left\{ I, \quad I, \quad \dots, \quad I \right\},$$

$\mathcal{R}_{\ell+1} = \text{diag} \{ I, \quad R_1, \quad R_2, \quad \dots, \quad R_{\ell+1} \}$, one has

$$\begin{bmatrix} A_{11}^3 & \Omega_{12ij}^3 & 0 \\ * & A_{22}^3 & \Omega_{23ij}^3 \\ * & * & A_{33}^3 \end{bmatrix} < 0 \quad (78)$$

If the inequalities (61) and (62) hold, the inequality (78) will hold. Thus, there exist the feasible solutions for (32). This means the stability conditions (32) are effective for (20). Besides, from **Theorem 2**, one knows that $S > 0$ and $T > 0$. Thus, if there exist the

feasible solutions for (61), one can obtain $\begin{bmatrix} -T & -T \\ -T & -S \end{bmatrix} < 0$, i.e., $\begin{bmatrix} T^{-1} & I \\ I & S \end{bmatrix} > 0$. This means the stability conditions (61) are effective

for (20). Moreover, from $PP^{-1} = I$, one knows that there exist the factorizations X_{12} and Y_{12} satisfying (68) [74]. Thus, A_{ci} , B_{ci} and C_{ci} can be determined by solving the equalities (72), such that the closed-loop system is exponentially mean-square stable and prescribed H-infinity performance is guaranteed. The proof in **Theorem 2** is completed. **(Q. E. D.)**

Remark 9. From **Theorem 2**, it can be seen that A_{ci} , B_{ci} and C_{ci} are determined in (67). Besides, the factorizations in the polynomial can be used to decompose a polynomial into the product of some integer expressions in the real number range [75]. Moreover, the congruence transformation is an equivalent transformation, and has the nice ability to derive the equivalent LMIs by introducing the congruence transformation matrices [76]. Thus, the factorizations in the polynomial and the congruence transformation matrices are introduced to solve the LMIs in this paper, such that A_{ci} , B_{ci} and C_{ci} are determined and computation complexity is reduced.

5. Simulation examples

In this section, simulation examples are presented to show the effectiveness of the proposed methods, respectively.

5.1. Example 1

Consider the 2-dimensional discrete networked control system in the form of (6) as follows

$$\begin{cases} x(k+1) = A_{1i}x(k) + A_{2i} \sum_{m=1}^h \beta_m(k) x(k - \tau_m(k)) + B_i u(k) + r(k)g(x(k)) \\ y(k) = E_i x(k) + D_{1i} \omega(k) \\ z(k) = D_{2i} x(k) \\ x(k) = \phi(k), \quad k \in \mathbb{H} \triangleq \{-h, -h+1, \dots, 0\}, \quad m=1, 2, \dots, h \end{cases} \quad (79)$$

The 2-rules T-S fuzzy model is employed in this example. Then, the overall T-S fuzzy discrete networked control system is described

$$\begin{cases} x(k+1) = \sum_{i=1}^r h_i(\hat{x}(k)) \left(A_{1i}x(k) + A_{2i} \sum_{m=1}^h \beta_m(k) x(k - \tau_m(k)) + B_i u(k) + r(k)g(x(k)) \right) \\ y(k) = \sum_{i=1}^r h_i(\hat{x}(k)) (E_i x(k) + D_{1i} \omega(k)) \\ z(k) = \sum_{i=1}^r h_i(\hat{x}(k)) (D_{2i} x(k)), \quad r=2 \end{cases} \quad (80)$$

Fuzzy rule 1: if $\hat{x}_1(k)$ is $h_1(\hat{x}_1(k))$, then

$$\begin{cases} x(k+1) = A_{11}x(k) + A_{21}\sum_{m=1}^h \beta_m(k)x(k-\tau_m(k)) + B_1u(k) + r(k)g(x(k)) \\ y(k) = E_1x(k) + D_{11}\omega(k) \\ z(k) = D_{21}x(k) \end{cases} \quad (81)$$

Fuzzy rule 2: if $\hat{x}_1(k)$ is $h_2(\hat{x}_1(k))$, then

$$\begin{cases} x(k+1) = A_{12}x(k) + A_{22}\sum_{m=1}^h \beta_m(k)x(k-\tau_m(k)) + B_2u(k) + r(k)g(x(k)) \\ y(k) = E_2x(k) + D_{12}\omega(k) \\ z(k) = D_{22}x(k) \end{cases}$$

where A_{11} , A_{12} , A_{21} , A_{22} , B_1 , B_2 , D_{11} , D_{12} , D_{21} , D_{22} , E_1 and E_2 are given as follows

$$A_{11} = \begin{bmatrix} 0.77 & -0.45 \\ 0.13 & -0.70 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.85 & -0.43 \\ 0.19 & -0.62 \end{bmatrix}, A_{21} = \begin{bmatrix} 0.57 & 1.16 \\ 0 & -0.23 \end{bmatrix}, A_{22} = \begin{bmatrix} 0.26 & 2 \\ 0 & -0.35 \end{bmatrix}, B_1 = B_2 = [0.46 \quad 1.29]^T, D_{11} = D_{12} = [0.81 \quad 1.93]^T, D_{21} = D_{22} = 0.60, E_1 = [-0.85 \quad 0.47], E_2 = [-0.66 \quad 0.51].$$

The membership functions are given as follows

$$\begin{cases} h_1(\hat{x}(k)) = 1 - \frac{0.26}{1 + e^{(-x+1)}} \\ h_2(\hat{x}(k)) = \frac{0.26}{1 + e^{(-x+1)}} \end{cases} \quad (82)$$

According to **Assumptions 1-2**, the random sector nonlinearities $g(x(k))$ in (79) are given as follows

$$g(x(k)) = 0.5(M_1 + M_2)x(k) + 0.5(M_1 - M_2)\sin(x(k)) \quad (83)$$

$$\text{where } M_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix} \text{ and } M_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}.$$

According to **Theorems 1-2**, γ , ψ and φ are given as $\gamma = 1.16$, $\psi = 0.25$ and $\varphi = 0.30$.

By employing the T-S fuzzy model, the controller is designed

$$\begin{cases} x_c(k+1) = \sum_{i=1}^r h_i(\hat{\theta}(k))(A_{ci}x_c(k) + B_{ci}\xi(k)) \\ u(k) = \sum_{i=1}^r h_i(\hat{\theta}(k))(C_{ci}x_c(k)), \quad r = 2 \end{cases} \quad (84)$$

Using the LMIs toolbox, the controller gain matrices are solved

$$A_{c1} = \begin{bmatrix} -0.2432 & 0.2556 \\ -0.7346 & -0.5569 \end{bmatrix}, A_{c2} = \begin{bmatrix} -0.6449 & 0.1692 \\ -0.7391 & -0.1327 \end{bmatrix}, B_{c1} = \begin{bmatrix} -0.1482 & 0.3071 \\ 0.2139 & 0.8203 \end{bmatrix}, B_{c2} = \begin{bmatrix} -0.3150 & 1.3927 \\ 0.1839 & 1.3236 \end{bmatrix}, C_{c1} = 1.1825, C_{c2} = 1.0461.$$

Figure 1. Responses of x_1 and x_2 for the open-loop system.

Figure 2. Responses of x_1 and x_2 for the closed-loop system.

Figure 3. Responses of the control inputs.

Figure 4. Response of $\sqrt{\frac{\sum_{k=0}^{\infty} E\{\|z(k)\|^2\}}{\sum_{k=0}^{\infty} E\{\|\tilde{w}(k)\|^2\}}}$ with $\omega(k) = 0.35 \sin k$ for the closed-loop system.

Figure 5. Response of $\sqrt{\frac{\sum_{k=0}^{\infty} E\{\|z(k)\|^2\}}{\sum_{k=0}^{\infty} E\{\|\tilde{w}(k)\|^2\}}}$ with $\omega(k) = 0.2 + 0.2 \sin k$ for the closed-loop system.

Figure 6. Response of $\sqrt{\frac{\sum_{k=0}^{\infty} E\{\|z(k)\|^2\}}{\sum_{k=0}^{\infty} E\{\|\tilde{\omega}(k)\|^2\}}}$ with $\omega(k) = 0.6 + 0.16 \sin(0.1k)$ for the closed-loop system.

The membership functions are given as

$$\begin{cases} h_1(\hat{\theta}(k)) = 1 - \frac{\sin(\theta(k))}{2} \\ h_2(\hat{\theta}(k)) = \frac{\sin(\theta(k))}{2}, \quad 0 \leq \theta(k) \leq \pi \end{cases} \quad (85)$$

For the simulations, the parameter scalar h is given as $h=2$, i.e., $m=1, 2$. The random time-varying delays are $\tau_1(k) = 0.25(1 + \sin k)$ and $\tau_2(k) = 0.1(1 + \sin k)$. The Bernoulli probability distribution for $\tau_1(k)$ and $\tau_2(k)$ are $\text{Prob}\{\beta_1(k)=1\} = \bar{\beta}_1 = 0.60$ and $\text{Prob}\{\beta_2(k)=1\} = \bar{\beta}_2 = 0.20$, respectively. The Bernoulli probability distribution for $g(x(k))$ is $\text{Prob}\{r(k)=1\} = \bar{r} = 0.10$. The order of the ℓ th-Rice fading channels model is $\ell=2$, i.e., $l=0, 1, 2$. The mathematical expectations of the channels coefficients are $\bar{\alpha}_0 = 0.80$, $\bar{\alpha}_1 = 0.30$ and $\bar{\alpha}_2 = 0.10$. The variances of the channels coefficients are $\bar{\alpha}_0^2 = 0.16$, $\bar{\alpha}_1^2 = 0.25$ and $\bar{\alpha}_2^2 = 0.49$. The responses of x_1 and x_2 for the open-loop system are shown in **Figure 1**. The responses of x_1 and x_2 for the closed-loop system are shown in **Figure 2**. Note that the disturbance function $\omega(k) = 0.35 \sin k$ is considered while plotting the **Figure 2**. The responses of the control inputs are shown in **Figure 3**. The response of $\sqrt{\frac{\sum_{k=0}^{\infty} E\{\|z(k)\|^2\}}{\sum_{k=0}^{\infty} E\{\|\tilde{\omega}(k)\|^2\}}}$ with $\omega(k) = 0.35 \sin k$ is shown in **Figure 4**. The response of $\sqrt{\frac{\sum_{k=0}^{\infty} E\{\|z(k)\|^2\}}{\sum_{k=0}^{\infty} E\{\|\tilde{\omega}(k)\|^2\}}}$ with $\omega(k) = 0.2 + 0.2 \sin k$ is shown in **Figure 5**. The response of $\sqrt{\frac{\sum_{k=0}^{\infty} E\{\|z(k)\|^2\}}{\sum_{k=0}^{\infty} E\{\|\tilde{\omega}(k)\|^2\}}}$ with $\omega(k) = 0.6 + 0.16 \sin(0.1k)$ is shown in **Figure 6**. From **Figures 1-2**, it can be seen that the proposed methods are effective. From **Figures 1-3**, it can be seen that the convergence rates of the state variables are fast. From **Figures 4-6**, it can be seen that the responses of $\sqrt{\frac{\sum_{k=0}^{\infty} E\{\|z(k)\|^2\}}{\sum_{k=0}^{\infty} E\{\|\tilde{\omega}(k)\|^2\}}}$ are smaller than $\gamma = 1.16$. This means the strong robustness is obtained in the controller design.

5.2. Example 2

Consider the discrete chemical stirred tank reactor [77, 78]

$$\begin{cases} \dot{\hat{C}}_A = q\hat{V}^{-1}(\hat{C}_{Af_{A\kappa}} - \hat{C}_A) - a_0 \exp\left(-\frac{\hat{E}}{\hat{R}\hat{T}}\right)\hat{C}_A \\ \dot{\hat{T}} = q\hat{V}^{-1}(\hat{T}_{f_{d\kappa}} - \hat{T}) - a_1 \exp\left(-\frac{\hat{E}}{\hat{R}\hat{T}}\right)\hat{C}_A + a_2(\hat{T}_C - \hat{T}) \end{cases} \quad (86)$$

where $\hat{C}_A$, $\hat{T}$ and $\hat{T}_C$, are the reactant concentration, reactor temperature and coolant temperature, respectively. $\hat{V}$ and q are the reactor volume and feed flow rate, respectively. $\hat{R}$ and $\hat{E}$ are the gas constant and activation energy constant, respectively.

$\kappa=1, 2$ and $f_{si}=1, 2$ are the mode index and feed stream index, respectively. a_0, a_1 and a_2 are the nominal parameters [77]. The using details of the parameters in (86) were shown in [77]. Moreover, the schematic diagram of the chemical stirred tank reactor system is shown in **Figure 7**.

Figure 7. Schematic diagram of the chemical stirred tank reactor system.

The mode index and the feed stream index are $\kappa=1$ and $f_{si}=1$, then the system (86) is rewritten [77]

$$(\kappa=1, f_{si}=1): \begin{cases} \dot{C}_A = qV^{-1}(C_{A_{f_{si}1}} - C_A) - a_0 \exp\left(-\frac{E}{RT}\right)C_A \\ \dot{T} = qV^{-1}(T_{f_{si}1} - T) - a_1 \exp\left(-\frac{E}{RT}\right)C_A + a_2(T_C - T) \end{cases} \quad (87)$$

The state variables are defined as $x_1 = \hat{C}_A - \hat{C}_A^*$ and $x_2 = \hat{T} - \hat{T}^*$, where $\hat{C}_A^* = 0.5 \text{ mol/L}$, $\hat{T}^* = 350 \text{ K}$ and $\hat{T}_C^* = 280 \text{ K}$ are the desired values. Besides, $\hat{V} = 100$, $q = 50$, $\frac{\hat{E}}{R} = 8750$, $\kappa=1$, $f_{si}=1$, $a_0 = 7.2 \times 10^{10}$, $a_1 = -1.506 \times 10^{13}$ and $a_2 = 2.092$. Then, the system (87) is rewritten [77]

$$(\kappa=1, f_{si}=1): \begin{cases} x_1(k+1) = x_2(k) + 0.5(1.5 - x_1(k)) - a_0 x_1(k) e^{-8750/(x_2(k)+350)} - x_2(k) \\ x_2(k+1) = a_2 u(k) - 2.592 x_2(k) - a_1 x_1(k) e^{-8750/(x_2(k)+350)} - 104.6 \\ y = [x_1 \quad x_2]^T \end{cases} \quad (88)$$

For the problem formulated, the system (88) is transformed in the form of (6)

$$\begin{cases} x(k+1) = A_{1i}x(k) + A_{2i} \sum_{m=1}^h \beta_m(k) x(k - \tau_m(k)) + B_i u(k) + r(k)g(x(k)) \\ y(k) = E_i x(k) + D_{1i} \omega(k) \\ z(k) = D_{2i} x(k) \\ x(k) = \phi(k), \quad k \in \mathbb{H} \triangleq \{-h, -h+1, \dots, 0\}, \quad m=1, 2, \dots, h \end{cases} \quad (89)$$

Note that the system (89) is modified from the system model in [78] by adding the random interval communication delays $\tau_m(k)$ and random sector nonlinearities $g(x(k))$.

The 2-rules T-S fuzzy model is employed in this example. Then, the overall T-S fuzzy discrete chemical stirred tank reactor system with networked control is described as

$$\begin{cases} x(k+1) = \sum_{i=1}^r h_i(\hat{x}(k)) \left(A_{1i}x(k) + A_{2i} \sum_{m=1}^h \beta_m(k) x(k - \tau_m(k)) + B_i u(k) + r(k)g(x(k)) \right) \\ y(k) = \sum_{i=1}^r h_i(\hat{x}(k)) (E_i x(k) + D_{1i} \omega(k)) \\ z(k) = \sum_{i=1}^r h_i(\hat{x}(k)) (D_{2i} x(k)), \quad r=2 \end{cases} \quad (90)$$

Fuzzy rule 1: if $\hat{x}_1(k)$ is $h_1(\hat{x}_1(k))$, then

$$\begin{cases} x(k+1) = A_{11}x(k) + A_{21}\sum_{m=1}^h \beta_m(k)x(k-\tau_m(k)) + B_1u(k) + r(k)g(x(k)) \\ y(k) = E_1x(k) + D_{11}\omega(k) \\ z(k) = D_{21}x(k) \end{cases} \quad (91)$$

Fuzzy rule 2: if $\hat{x}_1(k)$ is $h_2(\hat{x}_1(k))$, then

$$\begin{cases} x(k+1) = A_{12}x(k) + A_{22}\sum_{m=1}^h \beta_m(k)x(k-\tau_m(k)) + B_2u(k) + r(k)g(x(k)) \\ y(k) = E_2x(k) + D_{12}\omega(k) \\ z(k) = D_{22}x(k) \end{cases}$$

where A_{11} , A_{12} , A_{21} , A_{22} , B_1 , B_2 , D_{11} , D_{12} , D_{21} , D_{22} , E_1 and E_2 are given

$$A_{11} = \begin{bmatrix} -1.5806 & 0.5160 \\ 2.8050 & -0.7008 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -\bar{a} & 0.1370 \\ 1.0198 & -0.2571 \end{bmatrix}, \quad A_{21} = \begin{bmatrix} 4.4800 & 210.2019 \\ -0.0468 & -1.8996 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -0.9868 & 30.1905 \\ -0.2328 & -1.2400 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 2.0921 \\ 0.2305 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -b+6 \\ 0.3549 \end{bmatrix}, \quad D_{11} = D_{12} = 0.01, \quad D_{21} = D_{22} = [0.02 \quad 0.05], \quad E_1 = E_2 = [0.2 \quad -0.1].$$

where $\bar{a}$ and $\bar{b}$ are the system parameters and used in the solving process of the stability regions, as shown in **Figures 8-11**.

The membership functions are given as

$$\begin{cases} h_1(\hat{x}(k)) = 0.5 - \frac{0.5}{1 + e^{(-x+1)}} \\ h_2(\hat{x}(k)) = 0.5 + \frac{0.5}{1 + e^{(-x+1)}} \end{cases} \quad (92)$$

According to **Assumptions 1-2**, $g(x(k))$ in (89) are given as

$$g(x(k)) = 0.3016(M_1 + M_2)x(k) + 0.2166(M_1 - M_2)\sin(x(k)) \quad (93)$$

$$\text{where } M_1 = \begin{bmatrix} 0.2795 & 0 \\ 0 & 0.4501 \end{bmatrix} \text{ and } M_2 = \begin{bmatrix} 0.1299 & 0 \\ 0 & 0.1076 \end{bmatrix}.$$

According to **Theorems 1-2**, γ , ψ and φ are given as $\gamma = 0.8601$, $\psi = 0.3168$ and $\varphi = 0.5899$.

By employing the T-S fuzzy model, the controller is designed as

$$\begin{cases} x_c(k+1) = \sum_{i=1}^r h_i(\hat{\theta}(k))(A_{ci}x_c(k) + B_{ci}u(k)) \\ u(k) = \sum_{i=1}^r h_i(\hat{\theta}(k))(C_{ci}x_c(k)), \quad r = 2 \end{cases} \quad (94)$$

Using the LMIs toolbox, the controller gain matrices in (94) are solved

$$A_{c1} = \begin{bmatrix} 0.2857 & -0.2014 \\ -0.4547 & 0.8322 \end{bmatrix}, \quad A_{c2} = \begin{bmatrix} 0.4610 & -0.3702 \\ -0.3668 & 0.6111 \end{bmatrix}, \quad B_{c1} = 0.3668, \quad B_{c2} = 0.1970, \quad C_{c1} = \begin{bmatrix} -4.4615 & 1.5350 \\ -5.1306 & 0.4109 \end{bmatrix}, \quad C_{c2} = \begin{bmatrix} -5.4603 & 7.9990 \\ -0.2039 & 0.1773 \end{bmatrix}.$$

The membership functions are given as

$$\begin{cases} h_1(\hat{\theta}(k)) = 1 - \frac{\sin(\theta(k))}{2} \\ h_2(\hat{\theta}(k)) = \frac{\sin(\theta(k))}{2}, \quad 0 \leq \theta(k) \leq \pi \end{cases} \quad (95)$$

Case 1. The system parameters intervals of the stability regions are chosen as $6 \leq \bar{a} \leq 16$ and $10 \leq \bar{b} \leq 22$. The comparison results of the stability regions for $6 \leq \bar{a} \leq 16$ and $10 \leq \bar{b} \leq 22$ by employing methods in [56] (‘ O ’) and **Theorem 2** (‘ □ ’) are shown in **Figure 8**. The comparison results of the stability regions for $6 \leq \bar{a} \leq 16$ and $10 \leq \bar{b} \leq 22$ by employing methods in [73] (‘ O ’) and **Theorem 2** (‘ □ ’) are shown in **Figure 9**. From **Figures 8-9**, it can be seen that there exist the feasible solution regions for the closed-loop system, and the stability regions can be obtained by employing the methods in [56, 73] and **Theorem 2**,

respectively. Besides, compared ‘ O ’ with ‘ $\square$ ’ in **Figures 8-9**, it can be seen that the larger stability regions are obtained by employing **Theorem 2** than [56, 73].

Case 2. The system parameters intervals of the stability regions are chosen as $20 \leq \bar{a} \leq 30$ and $16 \leq \bar{b} \leq 28$. The comparison results of the stability regions for $20 \leq \bar{a} \leq 30$ and $16 \leq \bar{b} \leq 28$ by employing methods in [56] (‘ O ’) and **Theorem 2** (‘ $\square$ ’) are shown in **Figure 10**. The comparison results of the stability regions for $20 \leq \bar{a} \leq 30$ and $16 \leq \bar{b} \leq 28$ by employing methods in [73] (‘ O ’) and **Theorem 2** (‘ $\square$ ’) are shown in **Figure 11**. From **Figures 10-11**, it can be seen that there exist the feasible solution regions for the closed-loop system, and the stability regions can be obtained by employing the methods in [56, 73] and **Theorem 2**, respectively. Besides, compared ‘ O ’ with ‘ $\square$ ’ in **Figures 10-11**, it can be seen that the larger stability regions are obtained by employing **Theorem 2** than [56, 73]. Overall, it can be seen that the larger stability regions are obtained in **Theorem 2** whether $6 \leq \bar{a} \leq 16$, $10 \leq \bar{b} \leq 22$ or $20 \leq \bar{a} \leq 30$, $16 \leq \bar{b} \leq 28$.

Figure 8. Comparison results of stability regions for $6 \leq \bar{a} \leq 16$ and $10 \leq \bar{b} \leq 22$ by employing methods in [56] (‘ O ’) and Theorem 2 (‘ $\square$ ’).

Figure 9. Comparison results of stability regions for $6 \leq \bar{a} \leq 16$ and $10 \leq \bar{b} \leq 22$ by employing methods in [73] (‘ O ’) and Theorem 2 (‘ $\square$ ’).

Figure 10. Comparison results of stability regions for $20 \leq \bar{a} \leq 30$ and $16 \leq \bar{b} \leq 28$ by employing methods in [56] (‘ O ’) and Theorem 2 (‘ $\square$ ’).

Figure 11. Comparison results of stability regions for $20 \leq \bar{a} \leq 30$ and $16 \leq \bar{b} \leq 28$ by employing methods in [73] (‘ O ’) and Theorem 2 (‘ □ ’).

Table 1. Comparison results of upper bounds τ_u with different γ for $\tau^* = 0.20$ by employing methods in [56, 73, 79, 80] and Theorem 2.

Methods	γ					
	1.20	1.40	1.60	1.80	2.00	2.20
[56]	1.6421	2.5437	3.4016	4.5027	5.5592	6.5622
[73]	1.6762	2.5791	3.4392	4.5359	5.5864	6.5901
[79]	1.6799	2.5803	3.4415	4.5420	2.5927	6.5990
[80]	1.6896	2.5916	3.4486	4.5501	2.5962	6.6092
Theorem 2	1.6974	2.5989	3.4508	4.5533	5.6001	6.6137

Table 2. Comparison results of upper bounds τ_u with different γ for $\tau^* = 0.40$ by employing methods in [56, 73, 79, 80] and Theorem 2.

Methods	γ					
	1.20	1.40	1.60	1.80	2.00	2.20
[56]	1.4340	2.4491	3.3504	4.4031	5.4614	6.3514
[73]	1.4520	2.4663	3.3772	4.4200	5.4802	6.3712
[79]	1.4559	2.4692	3.3780	4.4209	5.4866	6.3751
[80]	1.4601	2.4700	3.3810	4.4308	5.4917	6.3849
Theorem 2	1.4698	2.4711	3.3819	4.4314	5.4970	6.3895

Table 3. Comparison results of lower bounds γ with different τ_u for $\tau^* = 0.20$ by employing methods in [56, 73, 79, 80] and Theorem 2.

Methods	τ_u					
	0.85	0.90	0.95	1.00	1.05	1.10
[56]	1.1083	1.1635	1.2204	1.2860	1.3435	1.4028
[73]	1.0830	1.1432	1.2074	1.2670	1.3208	1.3864
[79]	1.0816	1.1411	1.2000	1.2635	1.3197	1.3819
[80]	1.0760	1.1398	1.1989	1.2599	1.3163	1.3760
Theorem 2	1.0715	1.1360	1.1970	1.2584	1.3121	1.3691

Table 4. Comparison results of lower bounds γ with different τ_u for $\tau^* = 0.40$ by employing methods in [56, 73, 79, 80] and Theorem 2.

Methods	τ_u					
	0.85	0.90	0.95	1.00	1.05	1.10
[56]	1.1294	1.1946	1.2515	1.3071	1.3746	1.4364
[73]	1.1141	1.1832	1.2485	1.2970	1.3608	1.4215
[79]	1.1113	1.1805	1.2451	1.2952	1.3596	1.4201
[80]	1.1090	1.1786	1.2400	1.2919	1.3550	1.4197
Theorem 2	1.1030	1.1706	1.2370	1.2804	1.3521	1.4188

Case 3. The comparison results of upper bounds τ_u with different γ for $\tau^* = 0.20$ by employing methods in [56, 73, 79, 80] and **Theorem 2** are shown in **Table 1**. The comparison results of upper bounds τ_u with different γ for $\tau^* = 0.40$ by employing methods in [56, 73, 79, 80] and **Theorem 2** are shown in **Table 2**. From **Tables 1-2**, it can be seen that the larger upper bounds τ_u are obtained by employing **Theorem 2**. Thus, it can be seen that the proposed method is effective for the chemical stirred tank reactor system with networked control and the proposed method has the better ability to obtain larger upper bounds τ_u . In **Tables 1-2**, the different values of τ^* are considered and different values of upper bounds τ_u are obtained in the simulations. That is the values of τ^* can affect the values of τ_u in the solutions process of LMIs.

Case 4. The comparison results of lower bounds γ with different τ_u for $\tau^* = 0.20$ by employing methods in [56, 73, 79, 80] and **Theorem 2** are shown in **Table 3**. The comparison results of lower bounds γ with different τ_u for $\tau^* = 0.40$ by employing methods in [56, 73, 79, 80] and **Theorem 2** are shown in **Table 4**. From **Tables 3-4**, it can be seen that the smaller lower bounds γ are obtained by employing **Theorem 2**. Thus, it can be seen that the proposed method is effective for the chemical stirred tank reactor system with networked control and the proposed method has the better ability to obtain smaller lower bounds γ . In **Tables 3-4**, the different values of τ^* are considered and different values of lower bounds γ are obtained in the simulations. That is the values of τ^* can affect the values of γ in the solutions process of LMIs.

Remark 10. Some conservative conditions in the form of LMIs may arise in the stability analysis if the discrete networked control system contains more nonlinearities. The computational problem may arise if the size of LMIs gets bigger. However, the factorization in the polynomial is one of the most important identical transformations in the mathematics fields, and has the nice ability in the mathematical root mapping and solving the quadratic equations [75]. The congruence transformation employs a special structure of the matrices that result from Galerkin's discretization of piecewise linear functions and product approximation for the nonlinear term by decoupling the system of perturbation into independent subsystems [76]. Thus, the factorizations in the polynomial and the congruence transformation matrices are introduced in this paper to solve the LMIs for reducing the computation complexity of solving LMIs. Compared **Table 1** with **Table 2**, it can be seen that the upper bounds τ_u depend on the prescribed H-infinity performance index γ and upper bounds τ^* at the same time. Also, compared **Table 3** with **Table 4**, it can be seen that the lower bounds γ depend on the upper bounds τ_u and upper bounds τ^* at the same time. Thus, the synthesis function relationship of τ_u , γ and τ^* will be investigated for the T-S fuzzy discrete networked control systems in the future.

6. Conclusions

In this paper, the external disturbance, the random interval communication delays and random sector nonlinearities are considered in the discrete networked control system. The T-S fuzzy model provides the distinctive framework to approximate the nonlinear system plant as an average weighted sum of some linear subsystems in this paper, then the nonlinear uncertainties are approximated effectively. Besides, the stochastic system theory and Bernoulli probability distribution are more suitable to describe the random phenomena. Thus, the stochastic system theory and Bernoulli probability distribution are employed, and the random delays and random nonlinearities are described clearly in the signal transmission. The dynamic output feedback control has the nice dynamic characteristic to reflect the internal features, and the T-S fuzzy dynamic output feedback controller with ℓ th-order Rice fading channels output is designed for the T-S fuzzy discrete networked control system. With the help of the proposed method, the

closed-loop system is exponentially mean-square stable and prescribed H-infinity performance is guaranteed. Additionally, the Lyapunov-Krasovskii functional method often requires the time-delays to satisfy some conditions, such as $0 < \tau_l \leq \tau_m(k) \leq \tau_u$ and $\Delta \tau_m(k) \leq \tau^*$. However, the aforementioned restrictions from the Lyapunov-Krasovskii functional method can be avoided by using the Lyapunov-Razumikhin functional method. Thus, the Lyapunov-Razumikhin functional method will be considered for the controller design of the time-delays system in the future.

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Competing interests

All the authors declare that there is no conflict of interest for the publication of this paper.

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