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Finite Element Analysis of a Wideband Microwave Tomography System for Potential Medical Imaging

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Abstract—In this paper, we present two in-house finite element analysis methods for a wideband microwave tomography system setup. A hybrid vector three-dimensional (3D) formulation and a scalar two-dimensional (2D) formulation of the finite element method (FEM) for the forward model in microwave imaging are proposed. Simulations with dispersive materials of the system setup are performed in CST Microwave Studio and FEM formulations. We then compare the value of the S -parameters between the data from CST and FEM both in 3D and 2D, with the results showing a good agreement at different frequencies. To that end, we introduce a convenient calibration procedure to match the S -parameters and electric field values used in reconstruction algorithms.

Index Terms—microwave imaging; finite element method; S -parameters

I. INTRODUCTION

Microwave imaging for medical applications has regained the attention from the research community during the last decade [1], [2] because of the enhanced computational and measurement capabilities currently available. From a basic viewpoint, microwave tomography relies on three key aspects: the data acquisition system, the forward model and the inverse problem solver [3]. This paper focuses on the second one, specifically on the development of models based on the finite element method (FEM). This method has been used as a forward solver in microwave tomography as early as 1995 [4], but with a different implementation and for a different experimental system.

This paper reports the in-house development and assessment of FEM formulations [5], [6] suitable to analyze a wideband microwave tomography system prototype conceived for the development of future medical applications as breast tumor or stroke detection [7]. The first formulation consists of a hybrid vector three-dimensional (3D) FEM - modal analysis method and can achieve a full-wave characterization of the system. The second proposed formulation is a simplified scalar two-dimensional (2D) FEM model, which is

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useful to be included in reconstruction algorithms due to its inherent computational efficiency compared to a 3D scheme. Numerical results from commercial software CST Microwave Studio and the in-house FEM code are reported and compared with good agreement.

II. MICROWAVE TOMOGRAPHY SYSTEM SETUP

Fig. 1 shows a photograph of the experimental system setup to be numerically analyzed with FEM in this work. It consists of 8 equispaced coaxially-fed printed monopole antennas which are immersed in a coupling liquid, with an off-centered cylindrical target. In this setup, the coupling liquid is considered to be a mixture of 90% glycerin and 10% water, while the cylindrical target is supposed to be filled with a mixture of 60% glycerin and 40% water. The system is surrounded by an absorber material (ECCOSORB MS) after which a metallic layer is placed for shielding purposes. Fig. 2 shows the geometry implemented with commercial software CST Microwave Studio to model this microwave tomography system setup. This model rigorously includes all dispersive dielectric and magnetic properties of every material, including not only the immersion and target liquids, but also the absorber as well as the substrate for the antennas and their associated coaxial feeders' materials. All metals in the structure are modeled as perfect electric conductors. Further details of this system and its simulation with commercial software CST Microwave Studio can be found in [7].

III. FINITE ELEMENT FORMULATIONS

In order to analyze with the FEM [5], [6] the microwave tomography system described in section II, both 3D and 2D formulations have been developed. The in-house implementations have been carried out with the *FEniCS* open-source computing platform [8], [9] by means of its high-level *Python* interface.

A. Hybrid vector 3D FEM - modal analysis

The formulation used with this approach results in a full-wave characterization of the complete microwave tomography system setup in an analogous fashion as the one provided by

Fig. 1. Experimental microwave tomography system setup.

Fig. 2. CST Microwave Studio model of the microwave tomography system setup.

CST Microwave Studio. Specifically, the hybrid vector 3D FEM - modal analysis implemented in this work is a modified version of the one found in [10], but with straight first order vector elements [5]. Furthermore, in this case, the E -field formulation [6] instead of the original H -field formulation [10] is adopted, and therefore the field required at the coaxial ports is the transverse magnetic field instead of the transverse electric field. Only the fundamental mode, *i.e.* the transverse electromagnetic (TEM) mode, is considered at the coaxial ports. Thus, this formulation results in the computation of the impedance matrix $\mathbf{Z}$ relating the TEM modes at the coaxial ports as:

$$\mathbf{Z} = j\omega\mu_0\Delta^T\mathbf{B}^T(\mathbf{K} - k_0^2\mathbf{M} + jk_0\mathbf{R})^{-1}\mathbf{B}\Delta, \quad (1)$$

where μ_0 stands for the free-space magnetic permeability, ω is the angular frequency and k_0 is the free-space wavenumber for this frequency, while superscript T stands for the transpose operation. In (1), $\mathbf{K}$ and $\mathbf{M}$ are the usual matrices arising in FEM after the discretization of the inhomogeneous vectorial Helmholtz equation, $\mathbf{R}$ is a matrix which accounts for the first

Fig. 3. 3D mesh of the microwave tomography system to analyzed with FEM. Left: complete overview with 1st order absorbing boundary conditions in light green and perfect electric conductor boundary conditions in dark blue for the outer surfaces, while the internal edges of elements corresponding to the immersion liquid and the cylindrical target are depicted in orange and light blue respectively. Right: detailed view of the meshing of one of the 8 printed monopole antennas of the system.

order absorbing boundary conditions used in this formulation, $\mathbf{B}$ is a matrix which contains the inner products of the FEM basis functions and the magnetic fields at the coaxial ports and Δ is a diagonal matrix used for normalization purposes. From the impedance matrix $\mathbf{Z}$ the 8×8 scattering matrix $\mathbf{S}$ is readily available through standard microwave engineering operations.

A model of the microwave tomography system setup described in section II has been created and meshed with the open-source software GMSH [11]. The resulting mesh is shown in Fig. 3, and it is made up of 145406 tetrahedra. Adding the respective FEM basis functions to this mesh results in a system of equations with 351542 degrees of freedom. In this case, a direct solver has been used, so when the factorization of the matrix to be inverted in (1) is achieved, the solution for all 8 TEM coaxial excitations can be computed straightaway, rather than solving a system of equations through an iterative solver for every different excitation.

B. Simplified scalar 2D FEM model

The use of a complete and detailed 3D model of the microwave tomography system is unconceivable to be inserted as a forward model in an iterative procedure, such as the distorted Born iterative method [12, ch. 9], which attempts to solve the inverse scattering problem and thus to reconstruct the dielectric profile in the imaging domain. Therefore, the need to build simpler models, both 3D and 2D, arises. In this work, a simplified scalar 2D formulation is used by assuming transverse magnetic (TM) excitation and an infinite constant cross-sectional geometry along the longitudinal dimension of the cylindrical target and the cylindrical bore of the microwave tomography system. Under these assumptions the equation to be solved with FEM is the ϵ_r -inhomogeneous scalar wave equation [5]:

$$\Delta_t E_z + k_0^2 \epsilon_r E_z = j\omega\mu_0 J_z, \quad (2)$$

Fig. 4. 2D mesh used in the simplified scenario. Blue: immersion liquid. Red: cylindrical target.

where Δ_t stands for the transverse Laplacian operator, ε_r is the relative dielectric permittivity profile, E_z is the longitudinal electric field component of a TM wave and J_z is the longitudinal current density of the sources. In this case, J_z corresponds to infinitely long current wires located at the same points where the antennas are actually placed in the microwave tomography system.

Fig. 4 shows the 2D mesh used in this simple scenario. It consists of 25218 triangles, which result in 12791 degrees of freedom for first order elements as the ones used in this work. The overall circular domain is bounded by second order analytical absorbing boundary conditions [6] which mimic the absorbing material in the actual microwave tomography system setup. A detailed derivation of the matrix constructions (except for the absorbing boundary conditions and the source term J_z) required for a 2D scalar FEM procedure can be found in [13].

IV. NUMERICAL RESULTS

A. Hybrid vector 3D FEM - modal analysis

First, results from the 3D hybrid vector FEM - modal analysis are presented and compared with those from CST Microwave Studio. Fig. 5 shows two plots corresponding respectively to the magnitude of the S_{2j} and S_{3j} parameters, where a high level of agreement can be seen between them for the simulations from the in-house FEM code and the commercial software tool. It should be highlighted how very low levels (in the order of -120 dB) are accurately computed.

In the next figure, Fig. 6, two plots corresponding respectively to the phase of the S_{7j} and S_{8j} parameters are shown. In this case all phase results exhibit a very good agreement with results from CST Microwave Studio, except for the ones corresponding to S_{ii} parameters (this has been cross-checked with the rest of calculations). Although this issue is still under investigation, the fact that backscattered measurements are not usually taken into account in microwave imaging [14] makes it possible to use the developed hybrid vector 3D FEM - modal analysis formulation for the characterization and data generation in this microwave tomography system.

Finally, it should be borne in mind that, because of the configuration and materials in the microwave tomography sy-

Fig. 5. Results from the hybrid vector 3D FEM - modal analysis (solid) and CST Microwave Studio (dashed) for the magnitude of the S_{2j} (left) and S_{3j} (right) parameters.

Fig. 6. Results from the hybrid vector 3D FEM - modal analysis (solid) and CST Microwave Studio (dashed) for the phase of the S_{7j} and S_{8j} parameters.

stem setup, reciprocity, *i.e.* $S_{ij} = S_{ji}$, is expected. However, because of the off-centered position of the target which breaks up symmetry, no relation or assumption can be made between S_{ij} and S_{kj} when $k \neq i$, as is deduced from the figures.

B. Simplified scalar 2D FEM model

As previously stated, it is important to have simplified models to be efficiently included as a part of an overall reconstruction algorithm. However, these simplified models should ideally still be able to capture the behavior or trend of actual measurements. Notwithstanding, the first thing to deal with when using these models is the fact that S -parameters cannot be directly derived from them. This is the case of the 2D scalar model proposed in this work, since it is based on the computation of the electric field received at the point where antenna i is placed when the point where antenna j acts as the source, namely $E_z(i, j)$. Thus, for a fair assessment of the comparison of results from CST Microwave Studio and the 2D scalar FEM, a *calibration* or *balance* procedure should be carried out to somehow match the S -parameters with the E_z values. In this work, we have chosen the following approach:

$$S_{ij}^{FEM}(\omega) = E_z(i, j, \omega) \frac{S_{i,i-1}^{CST}(\omega_0)}{E_z(i, i-1, \omega_0)}, i \neq j, \quad (3)$$

where ω_0 stands for the angular frequency at the central frequency of the band, *i.e.* 2 GHz. In the case $i = 1$,

Fig. 7. Results from CST Microwave Studio (dashed) and *calibrated* results using (3) from the simplified 2D scalar FEM (solid) for the magnitude of the $S_{1j}, j \neq 1$ (left) and $S_{6j}, j \neq 6$ (right) parameters.

Fig. 8. Results from CST Microwave Studio (dashed) and *calibrated* results using (3) from the simplified 2D scalar FEM (solid) for the phase of the $S_{4j}, j \neq 4$ (left) and $S_{5j}, j \neq 5$ (right) parameters.

then subindex $i - 1$ is replaced by 8. Looking at Fig. 2 one can easily see that this approach amounts to equating to the corresponding S -parameter the value of the received field by the immediately preceding antenna to the one that is acting as the emitter at the central frequency of the operation band. It should also be highlighted that S_{ii} parameters are straightforwardly discarded based on the facts that on the one hand the 2D model is not meant to provide accurate results on reflection and on the other hand, as previously stated, they are not usually considered in actual dielectric reconstruction algorithms [14].

Fig. 7 shows the magnitude of the $S_{1j}, j \neq 1$ (left) and $S_{6j}, j \neq 6$ (right) parameters computed using CST Microwave Studio and computed using (3) from the in-house 2D simplified scalar FEM code. As expected, correlation is not as good as in the full-wave 3D hybrid vector FEM - analysis modal formulation, but it provides a fair estimation of the actual trend of the CST Microwave Studio data. Fig. 8 shows, for the same both computational approaches (CST Microwave Studio and 2D scalar FEM), the phase values for the $S_{4j}, j \neq 4$ (left) and $S_{5j}, j \neq 5$ (right) parameters, where again similar conclusions can be inferred.

V. CONCLUSION

This work has shown the application of two in-house FEM implementations for the analysis of a wideband microwave tomography system setup. The first formulation, namely a hybrid vector 3D FEM - modal analysis method, results in a full-wave characterization of the system. The second proposed formulation, a simplified scalar 2D FEM model, is suitable to be included in reconstruction algorithms. Results for both formulations have been presented and compared to those provided by commercial software CST Microwave Studio. Overall, the agreement is good, but preprocessing of the electric field solutions arising from the simplified 2D scalar FEM must be carried out since the S -parameters are not readily available from it. Results of reconstructed images using the proposed formulations as well as CST Microwave Studio and experimentally measured data from this system will be shown and discussed at the conference.

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