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Sliding Mode Control of T-S Fuzzy Systems Under Weighted Try-Once-Discard Protocol

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Abstract—This paper investigates the sliding mode control (SMC) problem for a class of T-S fuzzy systems with communication constraints. The weighted try-once-discard (WTOD) protocol is utilized to arrange the transmission order of actuator nodes, under which only one node is permitted to obtain access to the communication network at any time. A key issue is how to integrate the WTOD protocol into the design of SMC system. To this end, an update rule is proposed for the actuators to obtain the actual actuator signals. And then, a membership-function-dependent (MFD) sliding surface and a token-dependent sliding mode controller are designed. By taking the information of membership functions into stability and reachability analysis, MFD sufficient conditions are derived such that both the asymptotic stability of the closed-loop fuzzy systems and the reachability of the specified sliding surface can be guaranteed. Moreover, the cone complementary linearization (CCL) algorithm is employed to convert nonlinear inequalities into a minimization problem with linear constraints. Finally, a numerical example is utilized to illustrate the proposed SMC design approach.

Index Terms—T-S fuzzy system, WTOD protocol, sliding mode control, membership-function-dependent conditions.

I. INTRODUCTION

NETWORK control systems (NCSs) have been received considerable attention in the past two decades because of their low cost, reduced weight and power, easy installation and so on. The main feature of NCSs is that system components, e.g. actuators, controllers and sensors, are connected via shared communication networks, where the bandwidth is usually limited. Consequently, data collisions and network congestions may happen. To coordinate the network traffic, scheduling protocols are utilized to determine which sensor/actuator node can gain access to communication networks at each given transmission opportunity. Such protocols include, but are not limited to, the round-robin (RR) protocol [1]–[3], the stochastic communication (SC) protocol [4]–[6] and the WTOD protocol [7], [8]. Among these protocols, the WTOD protocol, first proposed in [9], allocates the network resources based on the “Maximum-error-first” policy, i.e., the node with the

largest weighted error from the last reported value will win and use the network. The realization of this protocol could base on the mature hardware technology of controller area network (CAN), which is specially designed for bitwise arbitration [9].

To date, under the WTOD protocol, some researchers have investigated the control design [8], [10] and filter design [11], [12] problem in the framework of discrete-time [11] and hybrid [13] systems. It has been shown in [7], [10] that the properties of the WTOD protocol can be characterized by quadratic functions, which makes it feasible to analyze and synthesize the linear and nonlinear systems based on the Lyapunov stability theory. Furthermore, in these existing results, the WTOD protocol was used to arrange the communication from sensors to controllers (S/C transmission), and the current scheduling decision is available for the controller. However, if the WTOD protocol is used to coordinate the communication from controllers to actuators (C/A transmission), the current scheduling signal will be unavailable for the controller. The design for a suitable controller for such cases is an open question which motivates us to address this issue in this paper.

As an effective robust control strategy, SMC has many attracted features, such as fast response, strong robustness, good transient performance and so on [14]–[17]. The main idea of the SMC approach is to drive the system states onto a specified sliding surface in finite time by a discontinuous control law, and then to keep the system states moving along the sliding surface toward the origin with desired performance. More recently, the SMC approach has been applied to investigate the control problem of NCSs under protocols. For example, Song, et al. [18] addressed the SMC problem for a class of uncertain NCSs under the SC protocol and synthesized a token-dependent sliding mode controller such that the closed-loop system is stochastically stable. When the system state is unmeasurable, Song, et al. also considered the output feedback SMC problem in [19] for a class of uncertain nonlinear systems under the RR protocol.

On the other hand, as a powerful tool for handling the nonlinear system, T-S fuzzy system can represent the nonlinear plant by the “blending” of several local linear subsystems [20]–[29]. Consequently, the stability analysis results on linear systems can be extended to nonlinear systems. To date, the most widely used stability analysis method concerning T-S fuzzy system is the Lyapunov stability theory. Basic stability conditions are usually constructed in terms of linear matrix inequalities (LMIs). To obtain less conservative stability conditions, researchers have provided many valuable stability analysis approaches [30], [31]. Especially, a novel MFD stability analysis, where the information of membership functions was

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included into stability conditions, was firstly attempted in [32] by Lam et. al, which significantly relax the stability analysis results. Since then, various membership function information involved into stability conditions can be found in the literature proposed by Lam and his co-workers, for example, staircase membership functions [33], piecewise membership functions [34], Taylor-series membership functions [35] and so on.

When it comes to control strategies of T-S fuzzy system, a rich body of literature has been available. The common control approaches include output feedback control [36], state feedback control [37], [38], H_∞ control [39], SMC [14], sampled control [40] and so on. Recently, some works have been reported on SMC for T-S fuzzy systems in the framework of NCSs. The event-triggered SMC scheme was developed in [41], [42] and the quantization effects were considered in [43], [44]. It should be pointed out that the above works made an implicit assumption that all the sensors/controllers can transmit their data simultaneously. Apparently, it is not always true in the practical communication process due to the limited channels.

In response to the aforementioned discussions, a seemingly interesting research topic is the SMC design problem for the T-S fuzzy system under the WTOD protocol, especially that under C/A transmission. This is a nontrivial problem with the following challenges: 1) how to design a desirable SMC law by considering differences between the WTOD protocol on the C/A transmission and the S/C transmission? 2) how to analyze the asymptotic stability for the resultant closed-loop systems? 3) how to guarantee the reachability of the specified sliding mode surface? These are the additional motivations of this work.

In this paper, we aim to address the problem of SMC for T-S fuzzy systems subject to communication constraints. To prevent the transmission data from collisions, the WTOD protocol is used to determine which actuator node can gain access to the shared network. A token-dependent SMC law is synthesized to ensure both the asymptotic stability of the closed-loop fuzzy system and the reachability of the specified sliding surface. The main contributions of this work are summarized as follows:

- By considering the differences (stated in Paragraph 1 in this section) among the WTOD protocols implemented on the C/A transmission and S/C transmission, a token-dependent SMC law is synthesized. In this work, the SMC law relies on the information of token at the last instant $k - 1$ instead of the current instant k in the existing results [7], [8], [10], which makes the proposed SMC method implementable for the WTOD protocols implemented on the C/A transmission.
- By taking the information of membership functions into stability and reachability analysis, MFD sufficient conditions are derived so that both asymptotic stability of the closed-loop fuzzy system and reachability of the specified sliding surface can be guaranteed.

Notations: In this paper, $\mathbb{R}$ is the set of reals and $\|\cdot\|$ denotes the Euclidean norm. For a real symmetric matrix, $P > 0$ (≥ 0) is used to represent that P is a positive definite (or positive semidefinite) matrix. I denotes an identity matrix with proper

dimensions. The transpose of the matrix A is represented by A^T and $\text{tr}A$ is the trace of A . In symmetric matrices, $(*)$ is an ellipsis for a block matrix that can be induced by symmetry. $\lambda_{\max}(Q)$ ($\lambda_{\min}(Q)$) denotes the largest (smallest) eigenvalue of Q . Matrices, if not explicitly stated, are assumed to have compatible dimensions. Moreover, in this work, we denote $\mathbb{A}(h) = \sum_{i=1}^s h_i(x(k))\mathbb{A}_i$, $\mathbb{P}(h+) = \sum_{i=1}^s h_i(x(k+1))\mathbb{P}_i$ and $\mathbb{A}_i \in \{A_i, B_i, P_i, R_i, Q_i\}$, $\mathbb{P}_i \in \{P_i, R_i, Q_i\}$.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a discrete-time T-S fuzzy system described by s rules of the following format:

Rule i : IF $f_1(x(k))$ is F_1^i , and $\dots$, $f_\beta(x(k))$ is F_β^i , THEN

$$x(k+1) = A_i x(k) + B_i(u(k) + f(k, x(k))), \quad i = 1, 2, \dots, s \quad (1)$$

where $f_j(x(k))$ is the premise variable, and F_j^i is the fuzzy set; $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ are the state and the control input, respectively; $A_i \in \mathbb{R}^{n \times n}$ and $B_i \in \mathbb{R}^{n \times m}$ are known real constant matrices. The disturbance $f(k, x(k))$ satisfies $\|f(k, x(k))\| \leq \mu \|x(k)\|$ with $\mu \geq 0$ being a known scalar; In practice, $f(k, x(k))$ could result from modeling errors, aging, uncertainties and so on, which exist in many realistic problem [45]. The defuzzified output of the T-S fuzzy system described by the rules in the form (1) is represented as follows:

$$x(k+1) = \sum_{i=1}^s h_i(x(k)) [A_i x(k) + B_i(u(k) + f(k, x(k)))] \\ \triangleq A(h)x(k) + B(h)(u(k) + f(k, x(k))) \quad (2)$$

where the membership functions are given by

$$h_i(x(k)) = \frac{\prod_{j=1}^\beta \mu_{ij}(f_j(x(k)))}{\sum_{i=1}^s \prod_{j=1}^\beta \mu_{ij}(f_j(x(k)))}, \quad i = 1, 2, \dots, s$$

with $0 \leq \mu_{ij}(f_j(x(k))) \leq 1$ being the grade of membership of $f_j(x(k))$ in F_j^i . According to the theory of fuzzy sets, we have

$$\sum_{i=1}^s h_i(x(k)) = 1, \quad 0 \leq h_i(x(k)) \leq 1, \quad i = 1, 2, \dots, s$$

Fig. 1. The control systems under the WTOD protocol

The network configuration under the WTOD protocol is described in Fig 1. To simplify the analysis and design

problem, it is assumed that the state variables $x(k)$ can always be received by the controllers. The control signals are then transmitted via a shared communication channel toward the actuators, where the WTOD protocol is adopted to decide which actuator node can gain access to the network at any time. As a dynamic protocol, the WTOD protocol determines the selected node $\sigma(k) \in \mathbb{M} \triangleq \{1, 2, \dots, m\}$ based on the maximum-error-first policy as follows:

$$\sigma(k) \triangleq \arg \max_{1 \leq p \leq m} \left(v_p(k) - u_p(k-1) \right) \Phi_p \times \left(v_p(k) - u_p(k-1) \right), \quad (3)$$

where $v_p(k)$ is the p th control signal at instant k , $u_p(k-1)$ is the previously received signal by the p th actuator, and $\Phi_p > 0$ ($p \in \mathbb{M}$) is the weight matrix with $\sum_{p=1}^m \Phi_p = 1$. According to the WTOD protocol (3), the actuator node p with the maximum difference between the current control value and the last received value will obtain the access token. If two or more nodes have the same maximal values, one of them is chosen arbitrarily in this work. Define $u(-1) = 0$.

A compact form of the WTOD protocol (3) can be given as follows:

$$\sigma(k) \triangleq \arg \max_{1 \leq p \leq m} \left(v(k) - u(k-1) \right)^T \bar{\Phi}_p \times \left(v(k) - u(k-1) \right), \quad (4)$$

where $v(k) \triangleq [v_1^T(k) \ v_2^T(k) \ \dots \ v_m^T(k)]^T$, $u(k) \triangleq [u_1^T(k) \ u_2^T(k) \ \dots \ u_m^T(k)]^T$, $\bar{\Phi}_p \triangleq \bar{\Phi} \Delta_p$, $\bar{\Phi} \triangleq \text{diag}\{\Phi_1, \Phi_2, \dots, \Phi_m\}$, $\Delta_p \triangleq \text{diag}\{\delta_p^1, \delta_p^2, \dots, \delta_p^m\}$, $p = 1, 2, \dots, m$ with $\delta_a^b = \delta(a-b)$ and $\delta(\cdot)$ taking values in $\{0, 1\}$ denotes the Kronecker delta function.

According to the above WTOD protocol, only one actuator node can gain access to the shared network and obtain new information at each instant. This implies that the other actuators cannot receive any information at this time. Aiming at the above phenomenon, this work proposes the following update rule for the actuators:

$$u_p(k) = \begin{cases} v_p(k), & \text{if } \sigma(k) = p, \\ u_p(k-1), & \text{otherwise,} \end{cases}$$

which means that if one actuator node does not obtain the access token, it will utilize the previously received information. Thus, the actual actuator signal at the plant side is given as:

$$u(k) = \Delta_{\sigma(k)} v(k) + (I - \Delta_{\sigma(k)}) u(k-1) \quad (5)$$

with $\Delta_{\sigma(k)} \in \{\Delta_1, \Delta_2, \dots, \Delta_m\}$.

We will achieve the following objectives in the following sections: i) design a desirable SMC law involving in scheduling signal such that the closed-loop fuzzy system is asymptotically stable. ii) analyze both the reachability and the stability of the closed-loop T-S fuzzy system under the WTOD protocol; iii) provide a feasible solving algorithm on the design of T-S fuzzy system with SMC law.

III. MAIN RESULTS

In this work, the following MFD sliding function is constructed:

$$s(k) = Gx(k) - GA(h)x(k-1), \quad (6)$$

where $G \in \mathbb{R}^{m \times n}$ is a given matrix. It follows from (2), (5) and (6) that

$$s(k+1) = GB(h)\Delta_{\sigma(k)}v(k) + GB(h)(I - \Delta_{\sigma(k)}) \times u(k-1) + GB(h)f(k, x(k)). \quad (7)$$

It is noted from the WTOD protocol (4) that the selection of token $\sigma(k)$ at instant k depends on the control signal $v(k)$. This means that at time instant k , only the information of token $\sigma(k-1)$, not $\sigma(k)$, may be available at the controller side. Therefore, we design the token $\sigma(k-1)$ -dependent SMC law $v(k)$ as follows:

$$v(k) = K_{\sigma(k-1)}(h)x(k) - \mu \xi_{\sigma(k-1)} \|x(k)\| \text{sgn}(s(k)) \quad (8)$$

with $K_{\sigma(k-1)}(h) \triangleq \sum_{i=1}^s h_i(x(k)) K_{i, \sigma(k-1)}$. The matrices $K_{i, \sigma(k-1)}$ and the positive scalar $\xi_{\sigma(k-1)}$ will be designed later.

By means of (2), (5) and (8), we obtain the resulting closed-loop system as:

$$x(k+1) = \mathcal{A}_{\sigma(k), \sigma(k-1)}(h)x(k) + B(h)f_s(k, x(k)) + B(h)(I - \Delta_{\sigma(k)})u(k-1) \quad (9)$$

with $\mathcal{A}_{\sigma(k), \sigma(k-1)}(h) \triangleq A(h) + B(h)\Delta_{\sigma(k)} K_{\sigma(k-1)}(h)$ and $f_s(k, x(k)) \triangleq f(k, x(k)) - \mu \xi_{\sigma(k-1)} \|x(k)\| \Delta_{\sigma(k)} \text{sgn}(s(k))$ satisfying $\|f_s(k, x(k))\| \leq \mu(1 + \xi_{\sigma(k-1)})\|x(k)\|$.

Remark 1: In literature [7], [8], [10], the WTOD protocol was used to regulate the S/C transmission, and the designed controllers depended on the token $\sigma(k)$. However, the WTOD scheduling behavior in this work happens on the C/A transmission, which makes the scheduling influence on the design of controllers different from the one on S/C transmission. Actually, the token $\sigma(k)$ under S/C transmission is available for the controller at instant k , so the control signal at instant k can be computed according to the newest scheduling signal $\sigma(k)$. However, the token $\sigma(k)$ under C/A transmission is not available for the controller at instant k . This means that we can only utilize the last token $\sigma(k-1)$ to design the controller as in (8), which not only makes the present design different from the above existing results, but also brings great complexity in stability and reachability conditions due to the coupled existence of $\sigma(k)$ and $\sigma(k-1)$.

A. Stability Analysis

In this part, a sufficient condition is constructed to guarantee the asymptotic stability of the closed-loop T-S fuzzy system under the WTOD protocol.

Theorem 1: Consider the T-S fuzzy system (2) with the updating rule (5) and the token-dependent SMC law (8). For given scalars $\varepsilon_{\sigma(k)r} \geq 0$ ($\sigma(k) \in \mathbb{M}$, $r = 1, 2, \dots, m$) with $\sum_{r=1}^m \varepsilon_{\sigma(k)r} = 1$, if there exist matrices $P_i > 0$, $R_i > 0$, $K_{i, \sigma(k-1)}$, and positive scalars λ , ρ , $\xi_{\sigma(k-1)}$ satisfying

$$B_i^T P_l B_i \leq \lambda I, \quad i, l = 1, 2, \dots, s \quad (10)$$

$$R_l \leq \rho I, \quad l = 1, 2, \dots, s \quad (11)$$

$$\Upsilon(h) \triangleq \begin{bmatrix} \Upsilon_{11}(h) & \Upsilon_{12}(h) \\ * & \Upsilon_{22}(h) \end{bmatrix} < 0 \quad (12)$$

with

$$\begin{aligned} \Upsilon_{11}(h) &\triangleq 2\mathcal{A}_{\sigma(k),\sigma(k-1)}^T(h)P(h+)\mathcal{A}_{\sigma(k),\sigma(k-1)}(h)-P(h) \\ &\quad + 2K_{\sigma(k-1)}^T(h)\left(\Delta_{\sigma(k)}R(h+)\Delta_{\sigma(k)}+\|\Sigma_{\sigma(k)}\|\right) \\ &\quad \times K_{\sigma(k-1)}(h)+2\mu^2\left(\lambda(1+\xi_{\sigma(k-1)})^2\right. \\ &\quad \left.+\rho\xi_{\sigma(k-1)}^2+m\xi_{\sigma(k-1)}^2\|\Sigma_{\sigma(k)}\|\right)I, \\ \Upsilon_{12}(h) &\triangleq 2\mathcal{A}_{\sigma(k),\sigma(k-1)}^T(h)P(h+)B(h)(I-\Delta_{\sigma(k)}) \\ &\quad + 2K_{\sigma(k-1)}^T(h)\Delta_{\sigma(k)}R(h+)(I-\Delta_{\sigma(k)}) \\ &\quad - 2K_{\sigma(k-1)}^T(h)\Sigma_{\sigma(k)}, \\ \Upsilon_{22}(h) &\triangleq 2(I-\Delta_{\sigma(k)})\left(B^T(h)P(h+)B(h)+R(h+)\right) \\ &\quad (I-\Delta_{\sigma(k)})+2\Sigma_{\sigma(k)}-R(h) \end{aligned}$$

and

$$\Sigma_{\sigma(k)} = \bar{\Phi} \sum_{r=1}^m \varepsilon_{\sigma(k)r} (\Delta_{\sigma(k)} - \Delta_r), \quad \sigma(k) \in \mathbb{M}, \quad (13)$$

the closed-loop system (9) is asymptotically stable.

Proof. Define the following MFD Lyapunov function candidate

$$\begin{aligned} V_1(x(k), u(k-1)) &= x^T(k)P(h)x(k) \\ &\quad + u^T(k-1)R(h)u(k-1). \end{aligned} \quad (14)$$

By means of (9) and (10), one has

$$\begin{aligned} &x^T(k+1)P(h+)x(k+1) \\ &\leq 2x^T(k)\mathcal{A}_{\sigma(k),\sigma(k-1)}^T(h)P(h+)\mathcal{A}_{\sigma(k),\sigma(k-1)}(h)x(k) \\ &\quad + 2u^T(k-1)(I-\Delta_{\sigma(k)})B^T(h)P(h+)B(h) \\ &\quad \times (I-\Delta_{\sigma(k)})u(k-1) + 4x^T(k)\mathcal{A}_{\sigma(k),\sigma(k-1)}^T(h) \\ &\quad \times P(h+)B(h)(I-\Delta_{\sigma(k)})u(k-1) \\ &\quad + 2\lambda\mu^2(1+\xi_{\sigma(k-1)})^2x^T(k)x(k). \end{aligned} \quad (15)$$

It follows from (5) and (8) that

$$\begin{aligned} u(k) &= \Delta_{\sigma(k)}K_{\sigma(k-1)}(h)x(k) + (I-\Delta_{\sigma(k)})u(k-1) \\ &\quad - \mu\xi_{\sigma(k-1)}\|x(k)\|\Delta_{\sigma(k)}\text{sgn}(s(k)), \end{aligned} \quad (16)$$

which leads to

$$\begin{aligned} &u^T(k)R(h+)u(k) \\ &\leq 2x^T(k)K_{\sigma(k-1)}^T(h)\Delta_{\sigma(k)}R(h+)\Delta_{\sigma(k)}K_{\sigma(k-1)}(h)x(k) \\ &\quad + 2u^T(k-1)(I-\Delta_{\sigma(k)})R(h+)(I-\Delta_{\sigma(k)})u(k-1) \\ &\quad + 4x^T(k)K_{\sigma(k-1)}^T(h)\Delta_{\sigma(k)}R(h+)(I-\Delta_{\sigma(k)})u(k-1) \\ &\quad + 2\rho\mu^2\xi_{\sigma(k-1)}^2x^T(k)x(k). \end{aligned} \quad (17)$$

According to the WTOD principle (4), we have for all $r \in \mathbb{M}$:

$$\begin{aligned} &\left(v(k)-u(k-1)\right)^T \bar{\Phi}(\Delta_{\sigma(k)}-\Delta_r) \\ &\times \left(v(k)-u(k-1)\right) \geq 0. \end{aligned} \quad (18)$$

Noting (13) and substituting (8) into (18) yield

$$\begin{aligned} &\left(v(k)-u(k-1)\right)^T \Sigma_{\sigma(k)}\left(v(k)-u(k-1)\right) \\ &\leq 2x^T(k)K_{\sigma(k-1)}^T(h)\|\Sigma_{\sigma(k)}\|K_{\sigma(k-1)}(h)x(k) \\ &\quad + 2u^T(k-1)\Sigma_{\sigma(k)}u(k-1) \\ &\quad - 4x^T(k)K_{\sigma(k-1)}^T(h)\Sigma_{\sigma(k)}u(k-1) \\ &\quad + 2m\mu^2\xi_{\sigma(k-1)}^2\|\Sigma_{\sigma(k)}\|x^T(k)x(k). \end{aligned} \quad (19)$$

Note the following fact:

$$\begin{aligned} &\Delta V_1(x(k), u(k-1)) \\ &\leq x^T(k+1)P(h+)x(k+1) + u^T(k)R(h+)u(k) \\ &\quad - x^T(k)P(h)x(k) - u^T(k-1)R(h)u(k-1) \\ &\quad + \left(v(k)-u(k-1)\right)^T \Sigma_{\sigma(k)}\left(v(k)-u(k-1)\right). \end{aligned}$$

By substituting (15), (17) and (19) into the above expression, we have

$$\Delta V_1(x(k), u(k-1)) \leq \bar{x}^T(k)\Upsilon(h)\bar{x}(k) \quad (20)$$

with $\bar{x}(k) \triangleq [x^T(k) \ u^T(k-1)]^T$. Thus, if the condition (12) is feasible, one has $\Delta V_1(x(k), u(k-1)) < 0$. Hence, the closed-loop system (9) is asymptotic stable under the WTOD protocol scheduling. $\square$

B. Reachability Analysis

In the sequel, we shall further analyze the reachability of the specified sliding surface $s(k) = 0$ under the token-dependent SMC law (8) with the WTOD protocol. It shall be shown that the state trajectories can be driven into a domain around the sliding surface $s(k) = 0$ in finite time.

Theorem 2: Consider the T-S fuzzy system (2) with the update rule (5) and the token-dependent SMC law (8). For given scalars $\varepsilon_{\sigma(k)r} \geq 0$ ($\sigma(k) \in \mathbb{M}$, $r = 1, 2, \dots, m$) with $\sum_{r=1}^m \varepsilon_{\sigma(k)r} = 1$, if there exist matrices $P_i > 0$, $Q_i > 0$, $R_i > 0$, $K_{i,\sigma(k-1)}$, and positive scalars $\lambda, \rho, \nu, \xi_{\sigma(k-1)}$ satisfying (10)-(11) in Theorem 1 and the following inequalities:

$$B_i^T G^T Q_l G B_i \leq \nu I, \quad i, l = 1, 2, \dots, s \quad (21)$$

$$\hat{\Upsilon}(h) \triangleq \begin{bmatrix} \hat{\Upsilon}_{11}(h) & \hat{\Upsilon}_{12}(h) \\ * & \hat{\Upsilon}_{22}(h) \end{bmatrix} < 0 \quad (22)$$

with

$$\begin{aligned} \hat{\Upsilon}_{11}(h) &\triangleq \Upsilon_{11}(h) + 2K_{\sigma(k-1)}^T(h)\Delta_{\sigma(k)}B^T(h)G^T \\ &\quad \times Q(h+)GB(h)\Delta_{\sigma(k)}K_{\sigma(k-1)}(h), \\ \hat{\Upsilon}_{12}(h) &\triangleq \Upsilon_{12}(h) + 2K_{\sigma(k-1)}^T(h)\Delta_{\sigma(k)}B^T(h)G^T \\ &\quad \times Q(h+)GB(h)(I-\Delta_{\sigma(k)}), \\ \hat{\Upsilon}_{22}(h) &\triangleq \Upsilon_{22}(h) + 2(I-\Delta_{\sigma(k)})B^T(h)G^T \\ &\quad \times Q(h+)GB(h)(I-\Delta_{\sigma(k)}), \end{aligned} \quad (23)$$

the state trajectories will enter the following domain $\mathbb{O}$:

$$\mathbb{O} \triangleq \left\{s(k) \mid \|s(k)\| \leq d(k)\right\}, \quad (24)$$

where

$$d(k) \triangleq \sqrt{\frac{2\nu}{(1-\alpha) \min_i \lambda_{\min}(Q_i)}} \mu(1 + \max_{\sigma(k-1)} \xi_{\sigma(k-1)}) \times \|x(k)\| \quad (25)$$

with α being a given scalar satisfying $0 < \alpha < 1$.

Proof. Define the following Lyapunov function candidate:

$$V_2(x(k), u(k-1), s(k)) = V_1(x(k), u(k-1)) + s^T(k)Q(h)s(k), \quad (26)$$

where the function $V_1(x(k), u(k-1))$ is defined as in (14). Substituting (8) into (7) leads to

$$\begin{aligned} s(k+1) = & GB(h)\Delta_{\sigma(k)}K_{\sigma(k-1)}(h)x(k) \\ & + GB(h)(I - \Delta_{\sigma(k)})u(k-1) \\ & + GB(h)f_s(k, x(k)). \end{aligned} \quad (27)$$

Then, one has

$$\begin{aligned} & s^T(k+1)Q(h+)s(k+1) \\ \leq & 2x^T(k)K_{\sigma}^T(h)\Delta_{\sigma(k)}B^T(h)G^TQ(h+)GB(h) \\ & \times \Delta_{\sigma(k)}K_{\sigma(k-1)}(h)x(k) + 2u^T(k-1)(I - \Delta_{\sigma(k)}) \\ & \times B^T(h)G^TQ(h+)GB(h)(I - \Delta_{\sigma(k)})u(k-1) \\ & + 4x^T(k)K_{\sigma}^T(h)\Delta_{\sigma(k)}B^T(h)G^TQ(h+) \\ & \times GB(h)(I - \Delta_{\sigma(k)})u(k-1) \\ & + 2\nu\mu^2(1 + \xi_{\sigma(k-1)})^2x^T(k)x(k), \end{aligned} \quad (28)$$

in which the condition (21) is utilized. Noting the following fact:

$$\begin{aligned} & \Delta V_2(x(k), u(k-1), s(k)) \\ \leq & V_2(x(k+1), u(k), s(k+1)) - V_2(x(k), u(k-1), s(k)) \\ & + (v(k) - u(k-1))^T \Sigma_{\sigma(k)}(v(k) - u(k-1)) \end{aligned}$$

and using the expressions (15), (17), (19), (28), we have

$$\begin{aligned} & \Delta V_2(x(k), u(k-1), s(k)) \\ \leq & \bar{x}^T(k)\hat{\Upsilon}(h)\bar{x}(k) - \alpha s^T(k)Q(h)s(k) \\ & - \left[(1-\alpha) \min_i \lambda_{\min}(Q_i) \|s(k)\|^2 \right. \\ & \left. - 2\nu\mu^2(1 + \max_{\sigma(k-1)} \xi_{\sigma(k-1)})^2 x^T(k)x(k) \right]. \end{aligned} \quad (29)$$

By using (22) and noting that $Q_i > 0$, when $\|s(k)\| > d(k)$, one has $\Delta V_2(x(k), u(k-1), s(k)) \leq \bar{x}^T(k)\hat{\Upsilon}(h)\bar{x}(k) - \alpha s^T(k)Q(h)s(k) < 0$, which implies that the state trajectory will be driven into the domain $\mathbb{O}$ around the sliding surface $s(k) = 0$ in finite time. $\square$

In the sequel, we further give a sufficient condition to guarantee the stability conditions in Theorem 1 and the reachability conditions in Theorem 2 simultaneously.

Theorem 3: The conditions in Theorems 1 and 2 can be guaranteed simultaneously by (10), (11), (21) and the following inequality:

$$\tilde{\Upsilon}(h) \triangleq \begin{bmatrix} \tilde{\Upsilon}_{11}(h) & \tilde{\Upsilon}_{12}(h) & \tilde{\Upsilon}_{13}(h) \\ * & \tilde{\Upsilon}_{22}(h) & 0_{3 \times 3} \\ * & * & \tilde{\Upsilon}_{33}(h) \end{bmatrix} < 0 \quad (30)$$

with

$$\tilde{\Gamma}_{11}(h) \triangleq \begin{bmatrix} -P(h) & -2K_{\sigma(k-1)}^T(h)\Sigma_{\sigma(k)} \\ * & -R(h) + 2\Sigma_{\sigma(k)} \\ * & * \\ K_{\sigma(k-1)}^T(h)\Delta_{\sigma(k)}B^T(h)G^TQ(h+) \\ (I - \Delta_{\sigma(k)})B^T(h)G^TQ(h+) \\ -\frac{1}{2}Q(h+) \end{bmatrix},$$

$$\tilde{\Gamma}_{12}(h) \triangleq \begin{bmatrix} \mathcal{A}_{\sigma(k), \sigma(k-1)}^T(h)P(h+) \\ (I - \Delta_{\sigma(k)})B^T(h)P(h+) \\ 0 \\ K_{\sigma(k-1)}^T(h)\Delta_{\sigma(k)}R(h+) & K_{\sigma(k-1)}^T(h)\sqrt{\|\Sigma_{\sigma(k)}\|} \\ (I - \Delta_{\sigma(k)})R(h+) & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Gamma}_{13}(h) \triangleq \begin{bmatrix} \mu(1 + \xi_{\sigma(k-1)})I & \mu\xi_{\sigma(k-1)}I \\ 0 & 0 \\ 0 & 0 \\ \mu\sqrt{\|m\Sigma_{\sigma(k)}\|}\xi_{\sigma(k-1)} \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Gamma}_{22}(h) \triangleq \text{diag}\{-\frac{1}{2}P(h+), -\frac{1}{2}R(h+), -\frac{1}{2}I\},$$

$$\tilde{\Gamma}_{33}(h) \triangleq \text{diag}\{-\frac{1}{2}\lambda^{-1}I - \frac{1}{2}\rho^{-1}I - \frac{1}{2}I\}.$$

Proof. Applying Schur complement, it follows from (30) that

$$\tilde{\Upsilon}(h) \triangleq \begin{bmatrix} \Upsilon_{11}(h) & \Upsilon_{12}(h) & \tilde{\Upsilon}_{13}(h) \\ * & \Upsilon_{22}(h) & \tilde{\Upsilon}_{23}(h) \\ * & * & -\frac{1}{2}Q(h+) \end{bmatrix} < 0 \quad (31)$$

with

$$\tilde{\Upsilon}_{13}(h) \triangleq K_{\sigma(k-1)}^T(h)\Delta_{\sigma(k)}B^T(h)G^TQ(h+),$$

$$\tilde{\Upsilon}_{23}(h) \triangleq (I - \Delta_{\sigma(k)})B^T(h)G^TQ(h+).$$

Then, it is obtained from (31) that conditions (12) and (22) hold. Hence, the stability conditions in Theorem 1 and the reachability conditions in Theorem 2 can be guaranteed simultaneously by (10), (11), (21) and (30). $\square$

It is noted that the stability condition (30) contains membership functions, which may be difficultly solved, since there are an infinite number of inequalities to be examined for all values of $x(k)$. A simple way is to remove these membership functions in these equalities, i.e., to change them into the membership-function-independent (MFI) conditions initialed in [36]. However, it should be pointed out that the MFI stability conditions may bring great conservatism, due to ignoring characteristic information of the nonlinear plant expressed via membership functions. Hence, we shall adopt the MFD analysis approaches as in [34], [46]–[48] to deal with the membership-functions in (30) by approximating the original membership functions $h_{ij}(x(k)) \triangleq h_i(x(k))h_j(x(k))$ with the piecewise linear membership functions (PLMFs) $\hat{h}_{ij}(x(k))$ [34], [46], which approximate these infinite number of inequalities (30) by finite ones. The idea of constructing PLMFs is to sample the original membership functions, and

then, linear interpolation is employed to approximate the original membership functions based on sample points. The concepts and detailed procedures are given as follows [34], [46].

Divide the state space Ω into q connected sub-state spaces Ω_η , $\eta = 1, 2, \dots, q$, that is, $\Omega = \bigcup_{\eta=1}^q \Omega_\eta$. Then, PLMFs $\hat{h}_{ij}(x(k))$ can be expressed as

$$\hat{h}_{ij}(x(k)) = \sum_{\eta=1}^q \sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 \prod_{\tau=1}^n v_{\tau i_\tau \eta}(x_\tau(k)) \times \hat{h}_{ij}(x_{i_1 \dots i_n \eta}), \quad \forall i, j \quad (32)$$

$$0 \leq \hat{h}_{ij}(x(k)) \leq 1, \quad \forall i, j \quad (33)$$

$$0 \leq \hat{h}_{ij}(x_{i_1 \dots i_n \eta}) \leq 1, \quad \forall i, j \quad (34)$$

where $\hat{h}_{ij}(x_{i_1 \dots i_n \eta})$ is a sample point of the function $h_{ij}(x(k))$ at a chosen $x(k) \in \Omega$; $v_{\tau i_\tau \eta}(x_\tau(k))$ is a given function satisfying $0 \leq v_{\tau i_\tau \eta}(x_\tau(k)) \leq 1$ and $\sum_{i_\tau=1}^2 v_{\tau i_\tau \eta}(x_\tau(k)) = 1$ for all τ, η and $x(k) \in \Omega_\eta$, otherwise, if $x(k) \notin \Omega_\eta$, we have $v_{\tau i_\tau \eta}(x_\tau(k)) = 0$. Then, one has

$$\sum_{\eta=1}^q \sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 \prod_{\tau=1}^n v_{\tau i_\tau \eta}(x_\tau(k)) = 1. \quad (35)$$

The approximation errors $\Delta h_{ij}(x(k)) = h_{ij}(x(k)) - \hat{h}_{ij}(x(k))$ satisfy

$$\Delta h_{ij} \leq \Delta h_{ij}(x(k)) \leq \bar{\Delta} h_{ij}, \quad (36)$$

where Δh_{ij} and $\bar{\Delta} h_{ij}$ can be obtained numerically.

Now, we shall utilize the approximating of PLMFs to give a sufficient condition.

Theorem 4: Consider the T-S fuzzy system (2) with the updating rule (5) and the token-dependent SMC law (8). For given scalars $\varepsilon_{\sigma(k)r} \geq 0$ ($\sigma(k) \in \mathbb{M}$, $r = 1, 2, \dots, m$) with $\sum_{r=1}^m \varepsilon_{\sigma(k)r} = 1$, if there exist matrices $X_i, \bar{Q}_i, Y_i, K_{i,\sigma(k-1)}, Y_{ijl} = Y_{ijl}^T \geq 0$ and scalars $\hat{\lambda}, \hat{\rho}, \nu, \xi_{\sigma(k-1)} > 0$ satisfying

$$\sum_{i=1}^s \sum_{j=1}^s \left((\hat{h}_{ij}(x_{i_1 \dots i_n \eta}) + \Delta h_{ij}) \Gamma_{ijl\sigma(k), \sigma(k-1)} + (\Delta \bar{h}_{ij} - \Delta h_{ij}) Y_{ijl} \right) < 0, \quad \forall i, \dots, i_n, \eta, l, \sigma(k), \sigma(k-1) \quad (37)$$

$$\begin{bmatrix} -\hat{\lambda} I & \hat{\lambda} B_i^T \\ * & -X_l \end{bmatrix} \leq 0, \quad \begin{bmatrix} -\hat{\rho} I & \hat{\rho} I \\ * & -Y_l \end{bmatrix} \leq 0, \quad \forall i, l \quad (38)$$

$$\begin{bmatrix} -\nu I & B_i^T G^T \\ * & -\bar{Q}_l \end{bmatrix} \leq 0, \quad \forall i, l \quad (39)$$

$$\Gamma_{ijl\sigma(k), \sigma(k-1)} \leq Y_{ijl}, \quad \forall i, j, l, \sigma(k), \sigma(k-1) \quad (40)$$

where

$$\Gamma_{ijl\sigma(k), \sigma(k-1)} \triangleq \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} \\ * & \Gamma_{22} & 0_{3 \times 3} \\ * & * & \Gamma_{33} \end{bmatrix}, \quad Y_{ijl} \triangleq \begin{bmatrix} Y_{11ijl} & \cdots & Y_{19ijl} \\ \vdots & \ddots & \vdots \\ * & \cdots & Y_{99ijl} \end{bmatrix},$$

$$\Gamma_{11} \triangleq \begin{bmatrix} -X_i^{-1} & -2K_{j\sigma(k-1)}^T \Sigma_{\sigma(k)} \\ * & -Y_i^{-1} + 2\Sigma_{\sigma(k)} \\ * & * \end{bmatrix}, \quad \begin{bmatrix} K_{j\sigma(k-1)}^T \Delta_{\sigma(k)} B_i^T G^T \\ (I - \Delta_{\sigma(k)}) B_i^T G^T \\ -\frac{1}{2} \bar{Q}_l \end{bmatrix},$$

$$\Gamma_{12} \triangleq \begin{bmatrix} \mathcal{A}_{ij\sigma(k), \sigma(k-1)}^T & K_{j\sigma(k-1)}^T \Delta_{\sigma(k)} \\ (I - \Delta_{\sigma(k)}) B_i^T & I - \Delta_{\sigma(k)} \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} K_{j\sigma(k-1)}^T \sqrt{\|\Sigma_{\sigma(k)}\|} \\ 0 \\ 0 \end{bmatrix},$$

$$\Gamma_{13} \triangleq \begin{bmatrix} \mu(1 + \xi_{\sigma(k-1)}) I & \mu \xi_{\sigma(k-1)} I \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} \mu \sqrt{\|m \Sigma_{\sigma(k)}\|} \xi_{\sigma(k-1)} \\ 0 \\ 0 \end{bmatrix},$$

$$\Gamma_{22} \triangleq \text{diag}\left\{-\frac{1}{2} X_l, -\frac{1}{2} Y_l, -\frac{1}{2} I\right\},$$

$$\Gamma_{33} \triangleq \text{diag}\left\{-\frac{1}{2} \hat{\lambda} I, -\frac{1}{2} \hat{\rho} I, -\frac{1}{2} I\right\}$$

with $\mathcal{A}_{ij\sigma(k), \sigma(k-1)} \triangleq A_i + B_i \Delta_{\sigma(k)} K_{j\sigma(k-1)}$, the reachability and the asymptotic stability of the closed-loop T-S fuzzy system can be guaranteed simultaneously.

Proof. Let $X_i = P_i^{-1}$, $Y_i = R_i^{-1}$, $\bar{Q}_i = Q_i^{-1}$, $\hat{\lambda} = \lambda^{-1}$, $\hat{\rho} = \rho^{-1}$. It can be shown that inequalities (10), (11) and (21) can be ensured by conditions (38) and (39). Thus, according to Theorem 3, it is only required to prove that the condition (30) can be ensured. To this end, define

$$\tilde{\Gamma}(h) \triangleq \sum_{l=1}^s h_l(x(k+1)) \sum_{i=1}^s \sum_{j=1}^s h_{ij}(x(k)) \times \Gamma_{ijl\sigma(k), \sigma(k-1)}. \quad (41)$$

We have

$$\begin{aligned} \tilde{\Gamma}(h) &= \sum_{l=1}^s h_l(x(k+1)) \left[\sum_{i=1}^s \sum_{j=1}^s \left(h_{ij}(x(k)) - \hat{h}_{ij}(x(k)) + \hat{h}_{ij}(x(k)) \right) \Gamma_{ijl\sigma(k), \sigma(k-1)} \right. \\ &\quad \left. + \sum_{i=1}^s \sum_{j=1}^s (\Delta h_{ij} - \Delta \bar{h}_{ij}) \Gamma_{ijl\sigma(k), \sigma(k-1)} \right]. \end{aligned}$$

Noting $\Delta h_{ij}(x(k)) = h_{ij}(x(k)) - \hat{h}_{ij}(x(k))$, one has

$$\begin{aligned} \tilde{\Gamma}(h) &= \sum_{l=1}^s h_l(x(k+1)) \left(\sum_{i=1}^s \sum_{j=1}^s \hat{h}_{ij}(x(k)) \right. \\ &= \sum_{l=1}^s h_l(x(k+1)) \left(\sum_{i=1}^s \sum_{j=1}^s (\hat{h}_{ij}(x(k)) \right. \\ &\quad \left. + \Delta h_{ij}) \Gamma_{ijl\sigma(k), \sigma(k-1)} \right. \\ &\quad \left. + \sum_{i=1}^s \sum_{j=1}^s (\Delta h_{ij} - \Delta \bar{h}_{ij}) \Gamma_{ijl\sigma(k), \sigma(k-1)} \right). \quad (42) \end{aligned}$$

By means of inequalities $0 \leq \Delta h_{ij} - \Delta \bar{h}_{ij} \leq \Delta \bar{h}_{ij} - \Delta h_{ij}$ and $\Gamma_{ijl\sigma(k,k-1)} \leq Y_{ijl}$, we obtain from (42) that

$$\begin{aligned} \tilde{\Gamma}(h) \leq & \sum_{l=1}^s h_l(x(k+1)) \sum_{i=1}^s \sum_{j=1}^s \left((\hat{h}_{ij}(x(k)) \right. \\ & \left. + \Delta \bar{h}_{ij}) \Gamma_{ijl\sigma(k),\sigma(k-1)} + (\Delta \bar{h}_{ij} - \Delta h_{ij}) Y_{ijl} \right). \end{aligned} \quad (43)$$

Substituting (32) into (43) yields

$$\begin{aligned} \tilde{\Gamma}(h) \leq & \sum_{l=1}^s h_l(x(k+1)) \sum_{\eta=1}^q \sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 \prod_{\tau=1}^n v_{\tau i_\tau \eta}(x_\tau(k)) \\ & \sum_{i=1}^s \sum_{j=1}^s \left((\hat{h}_{ij}(x_{i_1 \dots i_n \eta}) + \Delta \bar{h}_{ij}) \Gamma_{ijl\sigma(k),\sigma(k-1)} \right. \\ & \left. + (\Delta \bar{h}_{ij} - \Delta h_{ij}) Y_{ijl} \right), \end{aligned}$$

in which the property in (35) is utilized. Thus, we have $\tilde{\Gamma}(h) < 0$ from (37). Define $\Lambda = \text{diag}\{I, I, Q_l, P_l, R_l, I, I, I, I\}$, it can be shown that $\tilde{\Gamma}(h)$ in (30) and $\tilde{\Gamma}(h)$ in (41) have the following relation:

$$\begin{aligned} \tilde{\Gamma}(h) = & \sum_{l=1}^s h_l(x(k+1)) \sum_{i=1}^s \sum_{j=1}^s h_{ij}(x(k)) \\ & \times \Lambda \Gamma_{ijl\sigma(k),\sigma(k-1)} \Lambda = \Lambda \tilde{\Gamma}(h) \Lambda. \end{aligned} \quad (44)$$

Therefore, we have $\tilde{\Gamma}(h) < 0$ from $\tilde{\Gamma}(h) < 0$, that is, the condition (30) holds. $\square$

Remark 2: It should be pointed out that the PLMFs in (32) take all of the state variables into consideration, which is the most complicated case. In practical applications, the number of involved state variables may be less than n (such as the example in simulation). In such cases, one can just remove the additional system states form (32).

Remark 3: In this work, the MFD stability conditions are derived based on the PLMFs method. Compared with the MFI results, the MFD conditions will be of less conservativeness, since more information of the nonlinear plant is taken into consideration. The authors in [20], [34], [47] have explained this by both theoretical analysis and simulation results.

However, it is noted that this MFD approach will increase the number of stability conditions and decision variables, thus bring more increasing computational demand on finding a feasible solution. In practical applications, if less computational demand is required, the MFI conditions i.e., $\Gamma_{ijl\sigma(k),\sigma(k-1)} < 0$ combining with (38)-(39), can be utilized. If no feasible solution is found for the MFI conditions, the MFD method can be applied by beginning with less amount of sample points. More sample points can be added for relaxed stability conditions if no feasible solution is found.

Remark 4: The number of decision variables and inequalities for the MFD conditions in Theorem 4, and the number of MFI conditions stated in Remark 3 are given in Table I: where the variable m_c , $c = 1, 2, \dots, n$, denotes the number of sample points of state variables x_c , $c = 1, 2, \dots, n$.

TABLE I
NUMBER OF DECISION VARIABLES AND INEQUALITIES

	number of decision variables	number of inequalities
MFD conditions	$45s^3 + (m+3)(s+1)$	$(s^3 + \prod_{c=1}^n m_c s) m^2 + 2s^2 + s$
MFI conditions	$(m+3)(s+1)$	$s^3 m^2 + 2s^2 + s$

C. Computational Algorithm

It is noted that the conditions (37)-(40) are nonlinear due to these variables X_i , X_i^{-1} , Y_i and Y_i^{-1} . Hence, by using the CCL algorithm [49] to deal with the nonlinear matrix inequalities (37) and (40), we convert the conditions (37)-(40) in Theorem 4 into the following minimization problem with linear matrix inequality constraints:

$$\min \text{tr} \left(\sum_{i=1}^s P_i X_i \right) + \text{tr} \left(\sum_{i=1}^s R_i Y_i \right) \quad (45)$$

subject to the inequalities (38), (39) and

$$\begin{aligned} \sum_{i=1}^s \sum_{j=1}^s \left((\hat{h}_{ij}(x_{i_1 \dots i_n \eta}) + \Delta \bar{h}_{ij}) \hat{\Gamma}_{ijl\sigma(k),\sigma(k-1)} \right. \\ \left. + (\Delta \bar{h}_{ij} - \Delta h_{ij}) Y_{ijl} \right) < 0, \\ \forall i_1, \dots, i_n, \eta, l, \sigma(k), \sigma(k-1) \end{aligned} \quad (46)$$

$$\hat{\Gamma}_{ijl\sigma(k),\sigma(k-1)} \leq Y_{ijl}, \quad \forall i, j, l, \sigma(k), \sigma(k-1) \quad (47)$$

$$\begin{bmatrix} X_i & I \\ I & P_i \end{bmatrix} \geq 0, \quad \begin{bmatrix} Y_i & I \\ I & R_i \end{bmatrix} \geq 0, \quad \forall i \quad (48)$$

with

$$\hat{\Gamma}_{ijl\sigma(k),\sigma(k-1)} \triangleq \begin{bmatrix} \hat{\Gamma}_{11} & \Gamma_{12} & \Gamma_{13} \\ * & \Gamma_{22} & 0_{3 \times 3} \\ * & * & \Gamma_{33} \end{bmatrix},$$

$$\hat{\Gamma}_{11} \triangleq \begin{bmatrix} -P_i & -2K_{j\sigma(k-1)}^T \Sigma_{\sigma(k)} \\ * & -R_i + 2\Sigma_{\sigma(k)} \\ * & * \\ K_{j\sigma(k-1)}^T \Delta_{\sigma(k)} B_i^T G^T \\ (I - \Delta_{\sigma(k)}) B_i^T G^T \\ -\frac{1}{2} Q_l \end{bmatrix}.$$

The design procedure of the proposed strategy is outlined as follows:

- 1) Choose sliding parameter G in (6) and parameters $\varepsilon_{\sigma(k)r}$ ($\sigma(k) \in \mathbb{M}$) satisfying $\sum_{r=1}^m \varepsilon_{\sigma(k)r} = 1$ arbitrarily.
- 2) Solve the minimization problem (45) via CCL algorithm:
 - (i) Find a set of feasible solutions $(P_i^{(0)}, R_i^{(0)}, X_i^{(0)}, Y_i^{(0)}, \bar{Q}_i^{(0)}, K_{i,\sigma(k-1)}^{(0)}, Y_{ijl}^{(0)}, \hat{\lambda}^{(0)}, \hat{\rho}^{(0)}, \hat{\nu}^{(0)})$ to (38), (39), (46)-(48). Set $q = 0$.
 - (ii) Solve the following minimization problem:

$$\min \text{tr} \left(\sum_{i=1}^s P_i X_i \right) + \text{tr} \left(\sum_{i=1}^s R_i Y_i \right)$$

subject to (38), (39), (46)-(48)

for the decision variables $(P_i, R_i, X_i, Y_i, \bar{Q}_i, K_{i,\sigma(k-1)}, Y_{ijl}, \hat{\lambda}, \hat{\rho}, \hat{\nu})$. Set $P_i^{(q+1)} = P_i$, $X_i^{(q+1)} = X_i$, $R_i^{(q+1)} = R_i$, $Y_i^{(q+1)} = Y_i$.

(iii) If (37)-(40) are feasible for the above solutions $(X_i, Y_i, \bar{Q}_i, K_{i,\sigma(k-1)}, Y_{ijl}, \hat{\lambda}, \hat{\rho}, \hat{\nu})$, then go to iv). Otherwise, if the iteration time q is less than the maximum iteration number N , let $q = q + 1$ and return to (ii). If $q \geq N$, inequalities (37)-(40) have no feasible solutions. EXIT.

(iv) Output the feasible solutions $(P_i^{(q)}, R_i^{(q)}, X_i^{(q)}, Y_i^{(q)}, \bar{Q}_i^{(q)}, K_{i,\sigma(k-1)}^{(q)}, Y_{ijl}^{(q)}, \hat{\lambda}^{(q)}, \hat{\rho}^{(q)}, \hat{\nu}^{(q)})$.

- 3) Calculate the SMC law $v(k)$ by (8), the token $\sigma(k)$ by selection principle (3), the actual control $u(k)$ by (5) sequentially.

Remark 5: In order to obtain control gains, the CCL algorithm is utilized to cope with the nonlinear inequalities in Theorem 4 with X_i , X_i^{-1} , Y_i and Y_i^{-1} as decision variables. An alternative method is to employ the inequality $-X_i^{-1} \leq X_i - 2I$ (or $-Y_i^{-1} \leq Y_i - 2I$) to decouple X_i and X_i^{-1} (or Y_i and Y_i^{-1}) [50]. However, this method will inevitably introduce some conservatism.

For comparison purposes, the SMC problem for T-S fuzzy system (2) without protocol will be further considered. The SMC law is designed as

$$v(k) = K(h)x(k) - \mu\xi\|x(k)\|\text{sgn}(s(k)), \quad (49)$$

where ξ is a positive scalar to be designed.

By following similar lines as SMC design under the WTOD protocol, the stability and reachability conditions are further given in the following lemma.

Lemma 1: If there exist matrices X_i , $\bar{Q}_i$, K_i , $Y_{ijl} = Y_{ijl}^T \geq 0$ and scalars $\hat{\lambda}$, ν , $\xi > 0$ solving the following minimization problem:

$$\min \text{tr} \left(\sum_{i=1}^s P_i X_i \right) \quad (50)$$

subject to:

$$\sum_{i=1}^s \sum_{j=1}^s \left((\hat{h}_{ij}(x_{i_1 \dots i_n \eta}) + \Delta h_{ij}) \Theta_{ijl} + (\Delta \bar{h}_{ij} - \Delta h_{ij}) Y_{ijl} \right) < 0, \quad \forall i_1, \dots, i_n, \eta, l \quad (51)$$

$$\begin{bmatrix} -\hat{\lambda}I & \hat{\lambda}B_i^T \\ * & -X_l \end{bmatrix} \leq 0, \quad \Theta_{ijl} \leq Y_{ijl}, \quad \forall i, j, l \quad (52)$$

$$\begin{bmatrix} -\nu I & B_i^T G^T \\ * & -\bar{Q}_l \end{bmatrix} \leq 0, \quad \begin{bmatrix} X_i & I \\ I & P_i \end{bmatrix} \geq 0, \quad \forall i, l \quad (53)$$

with

$$\Theta_{ijl} \triangleq \begin{bmatrix} -P_i & K_j^T B_i^T G^T & (A_i + B_i K_j)^T & \mu(1 + m\xi)I \\ * & -\frac{1}{2}\bar{Q}_l & 0 & 0 \\ * & * & -\frac{1}{2}X_l & 0 \\ * & * & * & -\frac{1}{2}\hat{\lambda}I \end{bmatrix},$$

the T-S fuzzy system (2) synthesized by (49) is asymptotically stable, and the system states will be driven into the following sliding domain:

$$\mathbb{D} \triangleq \left\{ s(k) \mid \|s(k)\| \leq \bar{d}(k) \right\}, \quad (54)$$

where

$$\bar{d}(k) \triangleq \sqrt{\frac{2\nu}{(1-\alpha) \min_i \lambda_{\min}(\bar{Q}_i)}} \mu(1 + m\xi) \|x(k)\|$$

with α being a given scalar satisfying $0 < \alpha < 1$.

IV. SIMULATION

Consider a T-S fuzzy system with two rules in the form of (1), and the corresponding parameters are given as follows:

$$A_1 = \begin{bmatrix} 0.9 & 0.2 & 0.1 \\ 0 & 0 & 0.8 \\ 0.2 & 0 & 0.8 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.9 & 0 & 0.2 \\ 0 & 1 & 0.1 \\ 0.1 & 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$f(k, x(k)) = \begin{bmatrix} \mu\sqrt{2\|x_1(k)x_2(k)\|} \\ \mu\sqrt{\|x_3(k)\|} \end{bmatrix}$$

with $\mu = 0.02$. The membership functions are chosen as

$$h_1(x_1(k)) = \frac{M_2 - x_1(k)}{M_2 - M_1}, \quad M_1 = -2, \\ h_2(x_1(k)) = \frac{x_1(k) - M_1}{M_2 - M_1}, \quad M_2 = 2.$$

The parameters $\varepsilon_{\sigma(k)r}$ ($\sigma(k) \in \mathbb{M}$) satisfying $\sum_{r=1}^m \varepsilon_{\sigma(k)r} = 1$ are given as $\varepsilon_{11} = 0.3$, $\varepsilon_{12} = 0.2$, $\varepsilon_{21} = 0.1$, $\varepsilon_{22} = 0.4$. Moreover, the weighting matrix in (4) and the sliding parameter in (6) are

$$\bar{\Phi} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \text{ and } G = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \text{ respectively.}$$

To solve the minimization problem (45), we need to define the PLMFs as in (32). As $h_{ij}(x_1(k)) = h_i(x_1(k))h_j(x_1(k))$ depends on $x_1(k)$ in this example, the PLMFs can be constructed as $\hat{h}_{ij}(x_1(k)) = \sum_{\eta=1}^q \sum_{i_1=1}^2 v_{1i_1\eta}(x_1(k)) \hat{h}_{ij}(x_{i_1\eta})$ by considering only $x_1(k)$. Considering $x_1(k) \in [M_1, M_2]$, we choose $v_{11\eta}(x_1(k)) = \frac{x_{2\eta} - x_1(k)}{x_{2\eta} - x_{1\eta}}$ and $v_{12\eta}(x_1(k)) = 1 - v_{11\eta}(x_1(k))$ with $x_{1\eta} = \eta - 3$ and $x_{2\eta} = \eta - 2$, i.e., $x_{1\eta} = -2, -1, 0, 1$ and $x_{2\eta} = -1, 0, 1, 2$. Thus, the state space is divided into 4 connected sub-state spaces, i.e., $q = 4$. Moreover, $\hat{h}_{ij}(x_{i_1\eta})$ are the sample points of $h_{ij}(x_1(k))$ in $x_{1\eta}$ and $x_{2\eta}$. According to the PLMFs $\hat{h}_{ij}(x_1(k))$ and the original membership functions $h_{ij}(x_1(k))$, it can be found that $\Delta \bar{h}_{11} = 0$, $\Delta \bar{h}_{12} = 0.0150$, $\Delta \bar{h}_{22} = 0$ and $\Delta \bar{h}_{11} = -0.0150$, $\Delta \bar{h}_{12} = 0$, $\Delta \bar{h}_{22} = -0.0150$.

By solving the minimization problem (45), we obtain the control gains in (8):

$$K_{11} = \begin{bmatrix} -0.0270 & 0.0256 & -0.1647 \\ -0.1101 & -0.0312 & -0.1022 \end{bmatrix},$$

$$\begin{aligned}
K_{12} &= \begin{bmatrix} -0.0243 & 0.0230 & -0.1482 \\ -0.0991 & -0.0281 & -0.0920 \end{bmatrix}, \\
K_{21} &= \begin{bmatrix} -0.0283 & 0.1983 & -0.0129 \\ -0.1424 & -0.0126 & -0.1547 \end{bmatrix}, \\
K_{22} &= \begin{bmatrix} -0.0312 & 0.2181 & -0.0142 \\ -0.1566 & -0.0139 & -0.1701 \end{bmatrix}, \\
\xi_1 &= 3.4744, \quad \xi_2 = 2.7795.
\end{aligned}$$

For initial state $x(0) = [-2 \ 1.5 \ -1.5]^T$, we give the simulation results in Figs. 2-3. Obviously, the response of the open-loop fuzzy system is unstable as plotted in Fig. 2. Furthermore, it is observed from Fig. 3 that the closed-loop fuzzy system is asymptotically stable and the specified sliding surface $s(k) = 0$ is reached by using the proposed control strategy under the WTOD protocol.

Fig. 2. State trajectories $x(k)$ under the WTOD protocol for open-loop case

Fig. 3. The WTOD scheduling sequence $\sigma(k)$, state trajectories $x(k)$, sliding variable $s(k)$ and control signal $u(k)$ under the WTOD protocol

As comparison, simulation results without the WTOD protocol will be provided. By solving the minimization problem (50) in Lemma 1, the following control gains can be obtained:

$$K_1 = \begin{bmatrix} -0.0829 & 0.0600 & -0.3681 \\ -0.1466 & -0.0483 & -0.1519 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} -0.1603 & 0.4336 & -0.1425 \\ -0.2414 & 0.0641 & -0.2659 \end{bmatrix}, \quad \xi = 3.4744.$$

The evolution of system states $x(k)$ and control signal $u(k)$ are shown in Fig. 4 for the same initial state. By comparing these two cases with and without the WTOD protocol, we find that the control performances, e.g., the convergence speed, of the latter case are better than the former ones as more information can be received by actuators. In other words, the WTOD protocol alleviates the communication burden at the cost of degrading certain performances of the controlled fuzzy systems.

Fig. 4. State variables $x(k)$ and control signal $u(k)$ without protocol

V. CONCLUSION

This paper has investigated the problem of SMC for a class of T-S fuzzy systems. The WTOD protocol has been utilized to determine which actuator node can gain access to the shared network at each transmission instant. By taking the influence of the WTOD protocol into account, a token-dependent sliding mode controller has been designed such that the asymptotic stability and the reachability can be ensured. A numerical example has illustrated the effectiveness of the proposed SMC approach.

REFERENCES

- [1] H. Shen, S. Huo, J. Cao, and T. Huang, "Generalized state estimation for Markovian coupled networks under round-robin protocol and redundant channels," *IEEE Trans. Cybern.*, vol. 49, no. 4, pp. 1292–1301, Apr. 2019.
- [2] M. C. F. Donkers, W. P. M. H. Heemels, N. van de Wouw, and L. Hetel, "Stability analysis of networked control systems using a switched linear systems approach," *IEEE Trans. Autom. Control*, vol. 56, no. 9, pp. 2101–2115, Sep. 2011.
- [3] H. Zhao, Y. Niu, and T. Jia, "Security control of cyber-physical switched systems under round-robin protocol: Input-to-state stability in probability," *Inf. Sci.*, vol. 508, pp. 121–134, 2020.
- [4] B. Li, Z. Wang, Q. Han, and H. Liu, "Input-to-state stabilization in probability for nonlinear stochastic systems under quantization effects and communication protocols," *IEEE Trans. Cybern.*, vol. 49, no. 9, pp. 3242–3254, Sep. 2019.
- [5] J. Zhang, C. Peng, M.-R. Fei, and Y.-C. Tian, "Output feedback control of networked systems with a stochastic communication protocol," *J. Frankl. Inst.*, vol. 354, no. 9, pp. 3838–3853, 2017.

- [6] K. Liu, A. Selivanov, and E. Fridman, "Survey on time-delay approach to networked control," *Annu. Rev. Control*, to be published, doi: 10.1016/j.arcontrol.2019.06.005.
- [7] L. Zou, Z. Wang, Q. Han, and D. Zhou, "Ultimate boundedness control for networked systems with try-once-discard protocol and uniform quantization effects," *IEEE Trans. Autom. Control*, vol. 62, no. 12, pp. 6582–6588, Dec. 2017.
- [8] K. Liu, A. Seuret, E. Fridman, and Y. Xia, "Improved stability conditions for discrete-time systems under dynamic network protocols," *Int. J. Robust Nonlin. Control*, vol. 28, no. 15, pp. 4479–4499, 2018.
- [9] G. C. Walsh, Hong Ye, and L. G. Bushnell, "Stability analysis of networked control systems," *IEEE Trans. Control Syst. Technol.*, vol. 10, no. 3, pp. 438–446, May 2002.
- [10] Y. Song, Z. Wang, D. Ding, and G. Wei, "Robust H_2/H_∞ model predictive control for linear systems with polytopic uncertainties under weighted mef-tod protocol," *IEEE Trans. Syst., Man, and Cybern., Syst.*, vol. 49, no. 7, pp. 1470–1481, Jul. 2019.
- [11] L. Zou, Z. Wang, and H. Gao, "Set-membership filtering for time-varying systems with mixed time-delays under round-robin and weighted try-once-discard protocols," *Automatica*, vol. 74, pp. 341–348, 2016.
- [12] L. Liu, Y. Wang, L. Ma, J. Zhang, and Y. Bo, "Robust finite-horizon filtering for nonlinear time-delay Markovian jump systems with weighted try-once-discard protocol," *Syst. Sci. Control Eng.*, vol. 6, no. 1, pp. 180–194, 2018.
- [13] K. Liu, E. Fridman, and L. Hetel, "Networked control systems in the presence of scheduling protocols and communication delays," *SIAM J. Control Optim.*, vol. 53, no. 4, pp. 1768–1788, 2015.
- [14] D. W. C. Ho and Y. Niu, "Robust fuzzy design for nonlinear uncertain stochastic systems via sliding-mode control," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 3, pp. 350–358, Jun. 2007.
- [15] Y. Niu and D. W. C. Ho, "Design of sliding mode control subject to packet losses," *IEEE Trans. Autom. Control*, vol. 55, no. 11, pp. 2623–2628, Nov. 2010.
- [16] Z. Cao and Y. Niu, "Finite-time sliding mode control of Markovian jump systems subject to actuator nonlinearities and its application to wheeled mobile manipulator," *J. Frankl. Inst.*, vol. 355, no. 16, pp. 7865–7894, 2018.
- [17] Z. Cao, Y. Niu, and J. Song, "Sliding mode control of Markovian jump cyber-physical systems against randomly occurring injection attacks," *IEEE Trans. Autom. Control*, to be published, doi: 10.1109/TAC.2019.2926156.
- [18] J. Song, Z. Wang, and Y. Niu, "On H_∞ sliding mode control under stochastic communication protocol," *IEEE Trans. Autom. Control*, vol. 64, no. 5, pp. 2174–2181, May 2019.
- [19] J. Song, Z. Wang, and Y. Niu, "Static output-feedback sliding mode control under round-robin protocol," *Int. J. Robust Nonlin. Control*, vol. 28, no. 18, pp. 5841–5857, 2018.
- [20] H. K. Lam, "A review on stability analysis of continuous-time fuzzy-model-based control systems: From membership-function-independent to membership-function-dependent analysis," *Eng. Appl. Artif. Intell.*, vol. 67, pp. 390–408, 2018.
- [21] X. Zhao, Y. Yin, B. Niu, and X. Zheng, "Stabilization for a class of switched nonlinear systems with novel average dwell time switching by T-S fuzzy modeling," *IEEE Trans. Cybern.*, vol. 46, no. 8, pp. 1952–1957, Aug. 2016.
- [22] H. K. Lam, F. H. F. Leung, and P. K. S. Tam, "Stable and robust fuzzy control for uncertain nonlinear systems," *IEEE Trans. Syst., Man, and Cybern., Syst.*, vol. 30, no. 6, pp. 825–840, Nov. 2000.
- [23] W. Sun, S. Su, Y. Wu, J. Xia, and V. Nguyen, "Adaptive fuzzy control with high-order barrier lyapunov functions for high-order uncertain nonlinear systems with full-state constraints," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2018.2890256.
- [24] H. K. Lam, L. Wu, and J. Lam, "Two-step stability analysis for general polynomial-fuzzy-model-based control systems," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 3, pp. 511–524, Jun. 2015.
- [25] Z. Wu, S. Dong, P. Shi, D. Zhang, and T. Huang, "Reliable filter design of Takagi-Sugeno fuzzy switched systems with imprecise modes," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2018.2885505.
- [26] S. Dong, M. Fang, P. Shi, Z. Wu, and D. Zhang, "Dissipativity-based control for fuzzy systems with asynchronous modes and intermittent measurements," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2018.2887060.
- [27] M. Zhang, C. Shen, Z. Wu, and D. Zhang, "Dissipative filtering for switched fuzzy systems with missing measurements," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2019.2908430.
- [28] Z. Zhang, Y. Niu, and J. Song, "Input-to-state stabilization of interval type-2 fuzzy systems subject to cyber attacks: an observer-based adaptive sliding mode approach," *IEEE Trans. Fuzzy Syst.*, to be published, doi: 10.1109/TFUZZ.2019.2902105.
- [29] X. Song, M. Wang, C. K. Ahn, and S. Song, "Finite-time H_∞ asynchronous control for nonlinear Markov jump distributed parameter systems via quantized fuzzy output-feedback approach," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2019.2936827.
- [30] E. Kim and H. Lee, "New approaches to relaxed quadratic stability condition of fuzzy control systems," *IEEE Trans. Fuzzy Syst.*, vol. 8, no. 5, pp. 523–534, Oct. 2000.
- [31] A. Sala and C. Arino, "Asymptotically necessary and sufficient conditions for stability and performance in fuzzy control: Applications of polya's theorem," *Fuzzy Sets Syst.*, vol. 24, no. 158, pp. 2671–2686, 2007.
- [32] H. K. Lam and F. H. F. Leung, "Stability analysis of fuzzy control systems subject to uncertain grades of membership," *IEEE Trans. Syst., Man, Cybern., Cybern.*, vol. 35, no. 6, pp. 1322–1325, Dec. 2005.
- [33] H. K. Lam and M. Narimani, "Quadratic-stability analysis of fuzzy-model-based control systems using staircase membership functions," *IEEE Trans. Fuzzy Syst.*, vol. 18, no. 1, pp. 125–137, Feb. 2010.
- [34] H. K. Lam, "Polynomial fuzzy-model-based control systems: stability analysis via piecewise-linear membership functions," *IEEE Trans. Fuzzy Syst.*, vol. 19, no. 3, pp. 588–593, Jun. 2011.
- [35] H. K. Lam, C. Liu, L. Wu, and X. Zhao, "Polynomial fuzzy-model-based control systems: Stability analysis via approximated membership functions considering sector nonlinearity of control input," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 6, pp. 2202–2214, Dec. 2015.
- [36] L. James and S. Zhou, "Dynamic output feedback H_∞ control of discrete-time fuzzy systems: a fuzzy-basis-dependent Lyapunov function approach," *Int. J. Syst. Sci.*, vol. 38, no. 1, pp. 25–37, 2007.
- [37] Z. Lian, Y. He, C. Zhang, and M. Wu, "Stability and stabilization of T-S fuzzy systems with time-varying delays via delay-product-type functional method," *IEEE Trans. Cybern.*, to be published, doi: 10.1109/TCYB.2018.2890425.
- [38] Z. Wang, N. Rong, and H. Zhang, "Finite-time decentralized control of IT2 T-S fuzzy interconnected systems with discontinuous interconnections," *IEEE Trans. Cybern.*, vol. 49, no. 9, pp. 3547–3556, Sep. 2019.
- [39] J. Song, Y. Niu, J. Lam, and H. K. Lam, "Fuzzy remote tracking control for randomly varying local nonlinear models under fading and missing measurements," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 3, pp. 1125–1137, Jun. 2018.
- [40] Z. Wu, P. Shi, H. Su, and R. Lu, "Dissipativity-based sampled-data fuzzy control design and its application to truck-trailer system," *IEEE Trans. Fuzzy Syst.*, vol. 23, no. 5, pp. 1669–1679, Oct. 2015.
- [41] S. Wen, T. Huang, X. Yu, M. Z. Q. Chen, and Z. Zeng, "Aperiodic sampled-data sliding-mode control of fuzzy systems with communication delays via the event-triggered method," *IEEE Trans. Fuzzy Syst.*, vol. 24, no. 5, pp. 1048–1057, Oct. 2016.
- [42] X. Su, F. Xia, J. Liu, and L. Wu, "Event-triggered fuzzy control of nonlinear systems with its application to inverted pendulum systems," *Automatica*, vol. 94, pp. 236–248, 2018.
- [43] Y. Xue, B. Zheng, and X. Yu, "Robust sliding mode control for T-S fuzzy systems via quantized state feedback," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 4, pp. 2261–2272, Aug. 2018.
- [44] S. Ran, Y. Xue, B.-C. Zheng, and Z. Wang, "Quantized feedback fuzzy sliding mode control design via memory-based strategy," *Appl. Math. Comput.*, vol. 298, pp. 283–295, 2017.
- [45] H. K. Khalil, *Nonlinear systems*. New York: Macmillan, 1996, pp. 95–96.
- [46] H. K. Lam, H. Li, C. Deters, E. L. Secco, H. A. Wurdemann, and K. Althoefer, "Control design for interval type-2 fuzzy systems under imperfect premise matching," *IEEE Trans. Ind. Electron.*, vol. 61, no. 2, pp. 956–968, Feb. 2014.
- [47] H. K. Lam, L. Wu, and Y. Zhao, "Linear matrix inequalities-based membership-function-dependent stability analysis for non-parallel distributed compensation fuzzy-model-based control systems," *IET Control Theory Appl.*, vol. 8, no. 8, pp. 614–625, 2014.
- [48] H. K. Lam, B. Xiao, Y. Yu, X. Yin, H. Han, S.-H. Tsai, and C.-S. Chen, "Membership-function-dependent stability analysis and control synthesis of guaranteed cost fuzzy-model-based control systems," *Int. J. Fuzzy Syst.*, vol. 18, no. 4, pp. 537–549, 2016.
- [49] Y. S. Moon, P. Park, W. H. Kwon, and Y. S. Lee, "Delay-dependent robust stabilization of uncertain state-delayed systems," *Int. J. Control*, vol. 74, no. 14, pp. 1447–1455, 2001.
- [50] Q. Zhou, D. Liu, K. Sun, C. Wu, and X. Xing, "Design of observer-based controller for T-S fuzzy systems with intermittent measurements," *Neurocomputing*, vol. 174, pp. 689–697, 2016.

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