Abstract—This paper deals with the extended-dissipativity-based adaptive event-triggered (AET) control problem for stochastic polynomial fuzzy singular systems (SPFSSs). The purpose of this work is to design a state-feedback controller through the AET mechanism, which not only makes the closed-loop system admissible and extended dissipative, but also saves the network resources. Firstly, with a suitable Lyapunov-Krasovskii functional and an integral inequality in stochastic setting, an AET control criterion of SPFSSs considering asynchronous premise is established to ensure the mean square admissibility and extended dissipativity of the close-loop singular system. Additionally, a simple lemma is employed to approximate the non-convex sum-of-squares (SOS) conditions with the convex SOS conditions due to the difficulty in solving the non-convex SOS design conditions. Then both the desired feedback controller gain and event-triggered weighting polynomial matrix are co-designed based on the derived criterion. Finally, simulation examples are provided to illustrate the effectiveness of the proposed approaches.

Index Terms—Stochastic polynomial fuzzy singular systems (SPFSSs), adaptive event-triggered (AET), extended dissipativity, sum-of-squares (SOS).

I. INTRODUCTION

In the process of practical mathematical modeling, the relationships of some variables are dynamic, while those of other variables are static. In this situation, it is necessary to use singular system model to describe such systems. Up to now, a lot of valuable theoretical results about singular systems have been reported, such as stability [1], dissipativity [2], reachable set estimation [3]. However, in practice cases, most practical systems are also affected by stochastic factors and nonlinear terms [4], [5], which undoubtedly arouses scholars’ interest in the study of nonlinear stochastic singular systems. For nonlinear stochastic singular systems, it is difficult to find a suitable Lyapunov-Krasovskii (L-K) functional, and there is a lack of systematic control synthesis methods. Thanks to the development of Takagi-Sugeno (T-S) fuzzy model, nonlinear stochastic singular systems can be approximated by a family of local linear stochastic singular subsystems blended by membership functions. The mature theories of linear stochastic systems and singular systems can be used to study complex nonlinear stochastic singular systems in fuzzy framework. For stochastic T-S fuzzy singular systems, mean-square admissibility and dissipativity are dealt with in [6] and [7], respectively, and the effectiveness of the method is verified by a single-species bio-economic model. In [8] and [9], the sliding mode control problem is investigated and the sliding mode observer based fault tolerant control is also considered in [10] for stochastic T-S fuzzy singular systems.

During the recent decade, a great deal of attention has been paid to studying the polynomial fuzzy system, which is the extension of the traditional T-S fuzzy system. Compared with the T-S fuzzy model, the polynomial fuzzy model allows polynomial terms to exist in subsystem matrices, which has potential to reduce the number of fuzzy rules. However, if the polynomial matrix is involved in the local state space expression, the linear matrix inequality (LMI) toolbox cannot be used to solve the system stability or stabilization problems. Instead, the obtained stability analysis or synthesis conditions can be expressed in the form of sum-of-squares (SOS), which can be solved directly by SOSTOOLS [11]. The application of SOS technology in fuzzy control system was proposed for the first time in [12]. Several examples show that compared with the existing LMI-based methods, the SOS-based methods can provide more relaxed stability and controller design conditions [13]. When membership-function-dependent analysis is employed, the stability analysis results can be further relaxed [14]. Since then, various control synthesis problems of polynomial fuzzy systems have been actively discussed, such as $H_\infty$ stabilization [15], sampled-data output-feedback stabilization [16], networked tracking control [17] and observer-based tracking control [18]. As far as we know, only a few results about polynomial fuzzy singular systems have been published, especially concerning the influence of stochastic factors. Because of the complexity of the real-world systems, it is necessary to consider the control synthesis of stochastic polynomial fuzzy singular systems (SPFSSs).

On the other hand, in order to make more effective use of network resources and reduce the transmission burden of broadband network, the event-triggered control mechanism arises at the historic moment, which can choose “necessary” data to transmit through the network. In recent years, the co-design method of controller gains and a trigger matrix has been widely concerned [19], and fruitful results have...
been achieved. In addition, the co-design method has been successfully extended to networked stochastic systems [20–22] and T-S fuzzy systems [23]. Since singular systems are more common than normal systems, the application of event-triggered scheme in network singular systems have more profound significance, and event-triggered control problems of singular systems have been considered in [24–26]. However, there are only a few results on event-triggered control of polynomial fuzzy systems [17], [27]. It should be pointed out that the threshold parameters of the event-triggered mechanism in the above literature are given in advance and cannot be adjusted adaptively according to the change of system state. Therefore, an adaptive event-triggered (AET) mechanism is developed to design controllers and fault detection for T-S fuzzy systems in [28–31]. However, due to some special properties of stochastic singular systems, it is difficult to analyze their stability. Therefore, how to establish an effective AET control method for SPFSSs becomes more meaningful and challenging.

Moreover, as we all know, dissipativity includes some important performance indices, such as $H_{\infty}$ passivity. A novel performance of extended dissipativity, is proposed in [32], which includes not only $H_{\infty}$ passivity and dissipativity, but also $L_2 - L_\infty$ performance. After that, the extended dissipativity analysis problem of various systems is studied, such as Markov jump systems [33], neural networks [34], interval type-2 fuzzy systems [35]. For T-S fuzzy singular systems, extended dissipativity analysis has been carried out in [36], but the controller design based on extended dissipativity is not mentioned. To the best of our knowledge, the extended dissipativity AET control for SPFSSs has not been paid much attention to, which is the motivation behind this work. It is worth noting that due to some special properties of the closed-loop stochastic singular systems, how to develop a convex SOS admissibility criterion based on extended dissipativity is a challenging problem.

Motivated by the above discussion, in this paper we consider the problem of extended dissipativity AET control for SPFSSs. The main contributions of this work lie in the following points:

1) There are few results published about extended dissipativity of singular systems such as [26], [36], the admissibility criterion in [36] is difficult to utilize for feedback control problem. There is a non-strict LMI in the admissibilization and extended dissipativity criterion in [26]. We propose a new SOS-based event-triggered control condition for SPFSSs, which transforms a non-strict LMI that cannot be solved directly by LMI solver into an SOS that can be solved by SOS solver.

2) Inspired by [21], an integral inequality is established about stochastic singular systems, which is more tighter than Jensen inequality in [37], [38], and the result derived is less conservative.

3) Compared with the AET condition in [31], the trigger matrix is given in the form of polynomials related to states, which improves the flexibility of the design.

The rest of this paper is organized as follows. Section II formulates the problem, some useful definitions and lemmas. The extended-dissipativity-based AET control is carried out via the SOS method for SPFSSs in Section III. Two examples are provided to show the effectiveness of our developed scheme in Section IV. Finally, Section V concludes this paper.

**Notation:** $\mathcal{F}$ is the set of all SOS polynomials; if $p(x) = \sum_{i=1}^{l} q_i(x)$, then $p(x)$ is SOS, where $l$ is a nonnegative integer and $q_i(x)$ is a polynomial; sym$[A]$ is defined as $A + A^T$; $\| \cdot \|$ denotes the Euclidean norm of a vector or the induced norm of a matrix; $\mathbb{E}\{ \cdot \}$ stands for the mathematical expectation operator; $E^+$ stands for that the Moore-Penrose pseudo inverse of the matrix $E$; $*$ means the symmetric terms in symmetric matrix.

**II. PRELIMINARIES**

Consider the following SPFSS:

**Plant Rule i:** If $\varrho_i(t) = G_{i1}$ and $\cdots$ and $\varrho_j(t) = G_{ij}$, then

$$
\begin{align*}
Edz(t) &= \left[ A_i(x(t))z(t) + B_i(x(t))u(t) + D_{1i}(x(t))v(t) \right]dt + J_i(x(t))x(t)dw(t), \\
& \quad \text{where } i = 1, 2, \cdots, l, l \text{ is the number of if-then rules; } \\
& \quad G_{is} \text{ is the fuzzy set of rule } i \text{ corresponding to the premise variables } \varrho_s(t), s = 1, 2, \cdots, g; x(t) \in \mathbb{R}^n \text{ is the state vector,} \\
& \quad u(t) \in \mathbb{R}^m \text{ is the control input vector and } z(t) \in \mathbb{R}^p \text{ is the controlled output vector; } \\
& \quad v(t) \in \mathbb{R}^q \text{ is the disturbance vector belonging to } L_2[0, \infty); \\
& \quad w(t) \text{ is a one-dimensional Wiener process having properties of } \mathbb{E}\{dw(t)\} = 0 \text{ and } \\
& \quad \mathbb{E}\{dw^2(t)\} = dt; A_i(x(t)), B_i(x(t)), D_{1i}(x(t)), J_i(x(t)), \\
& \quad C_i(x(t)) \text{ and } D_{2i}(x(t)) \text{ are polynomial matrices with appropriate dimensions; } \\
& \quad E \text{ is a degenerate matrix and satisfies } r(E) = r \leq n. \\
& \quad \text{Furthermore, the overall model of system (1) can be inferred as:}
\end{align*}
$$

(2)

where $h_i(\varrho(t)) = \mu_i(\varrho(t))/\sum_{i=1}^{l} \mu_i(\varrho(t)) \geq 0, \mu_i(\varrho(t)) = \prod_{j=1}^{g} G_{ij}(\varrho_j(t)) \geq 0, \sum_{i=1}^{l} h_i(\varrho(t)) = 1$ and $G_{ij}(\varrho_j(t))$ is the membership grade of $\varrho_j(t)$ in $G_{ij}$.

In order to reduce the burden of network communication, inspired by the work in [28], an AET mechanism with a fixed period $h$ is considered to determine whether the sampled data is transmitted at the current time. The transmission sequence $\{t_k, h, k \in \mathbb{N}_+\}$ based on an AET mechanism is given by

$$
t_{k+1}h = t_kh + \min\{\ell h | (e_{xk}(t))^T \Omega(x(t)) e_{xk}(t) > \sigma(t) x_{x}(t_k) \Omega(x(t_k)) x(t_k)\},
$$

where $e_{xk}(t_k) = x(t_kh + \ell h) - x(t_kh), \ell \in \mathbb{N}_+$ is the error between the current sampled data and the latest triggered data. $\Omega(x(t)) > 0$ is the polynomial matrix to be designed. The threshold parameter $\sigma(t)$ is designed as follows [31]:

$$
\dot{\sigma} = -\rho \sigma^2(t) (e_{xk}(t))^T \Omega(x(t)) e_{xk}(t), \sigma(0) \in (0, 1),
$$

where $\rho > 0$ is a given constant. If (3) is satisfied, this implies that the sampled data will be released and transmitted to the controller as $x(t_{k+1}h)$; otherwise it will be abandoned.

**Remark 1:** When $\rho = 0$ and $0 < \sigma(t) < 1$, it is clear that $\sigma(t)$ becomes a constant, and this AET mechanism becomes...
the traditional event-triggered mechanism as shown in [19–21], [23]. When \( \sigma(0) = 0 \), this AET mechanism becomes the traditional time-triggered mechanism. In addition, the adaptive law used in this paper can ensure that \( \sigma(t) \) is always kept within \((0, 1]\). However, the adaptive law adopted in [28], [39] does not guarantee that \( \sigma(t) > 0 \) in all cases.

Assuming that the sample data released by satisfying the AET is not lost or disordered during the transmission of the communication channel, but network-induced delay is unavoidable. Taking the role of the zero-order holder and the network-induced delay into account. Define \( \eta_k \in [\eta_m, \eta_M] \) as the network-induced delay, where \( \eta_m \) and \( \eta_M < h \) are known constants. Then, the following fuzzy controller can be expressed as

**Plant Rule**: If \( \dot{\vartheta}_j(t_k h) = \mathcal{G}_j l \) and \( \cdots \) and \( \vartheta_j(t_k h) = \mathcal{G}_j g \), then

\[
u(t) = K_j x(t_k h), t \in [t_k h + \eta_k, t_{k+1} h + \eta_{k+1}],
\]

where \( K_j \) is the constant feedback gain to be determined for each \( j = 1, 2, \ldots, l \).

Under the AET mechanism (3), in order to apply time-delay method to analyze the stability of the system, it is necessary to transform the SPFSS (1) with feedback controller (5) into the system with time-varying delay. Inspired by [40], the subintervals on \([t_k h + \eta_k, t_{k+1} h + \eta_{k+1}]\) can be express as \((\bigcup_{k=0}^{l_k-1} I_k) \cup [t_k h + (\ell_k - 1) h + \eta_k, t_{k+1} h + \eta_{k+1}]\), where \( I_k = [t_k h + \kappa h + \eta_k, t_k h + (\kappa + 1) h + \eta_k], \) \( \ell_k \) is the minimum value that makes condition (3) hold. Define piecewise time-varying delay \( \tau(t) = t - t_k h - \kappa h \), the error vector \( e_x^k(t) = x(t_k h + \kappa h) - x(t_k h), \kappa = 0, 1, \ldots, \ell_k - 1 \).

Then overall fuzzy controller can be represented by

\[
u(t) = \sum_{j=1}^{l} h_j(\vartheta(t_k h))K_j x(t - \tau(t)) - e_x^k(t).
\]

Accroding to the above definition about \( \tau(t) \), it yields that \( \dot{\tau}(t) = 1 \), and

\[
\tau_1 \triangleq \eta_m \leq \eta_k \leq \tau(t) < h + \max\{\eta_k, \eta_{k+1}\} \leq h + \eta_M \triangleq \tau_2.
\]

Notable is, the premise variables of polynomial fuzzy system and the event-triggered fuzzy controller rules are asynchronous. Inspired by [23] and [41]–[43], the fuzzy basis function are described by

\[
\begin{align*}
  h_j(\vartheta(t_k h)) &= \alpha_j h_j(\vartheta(t)), \\
  | h_j(\vartheta(t_k h)) - h_j(\vartheta(t)) | &\leq \delta_j,
\end{align*}
\]

where \( \alpha_j > 0 \), \( \delta_j \geq 0 \) (\( j = 1, 2, \ldots, l \)). Using the same way in [23] and [43], based on the asynchronous constraints (7), it is clear that

\[
\frac{1}{\delta_j} \leq \frac{\alpha_j}{h_j(\vartheta(t))} \leq \alpha_j \leq 1 + \frac{\delta_j}{h_j(\vartheta(t))} \leq \frac{\alpha_j}{\delta_j} \leq \frac{\alpha_j}{\alpha_j} \leq \frac{\alpha_j}{\alpha_j} \leq 1,
\]

then, we have

\[
\frac{\alpha_j}{\alpha_j} \leq \min\{\alpha_j\} \leq \max\{\alpha_j\} \leq \alpha_j \leq 1 + \frac{\delta_j}{h_j(\vartheta(t))} \leq \frac{\alpha_j}{\delta_j} \leq \frac{\alpha_j}{\alpha_j} \leq 1,
\]

Setting \( \phi_1 = \min\{\phi_1\} \) and \( \phi_2 = \max\{\phi_2\} \), we can get that

\[
\frac{1}{\delta_j} \leq \frac{\alpha_j}{\phi_j} \leq \frac{\alpha_j}{\phi_j} \leq \frac{\alpha_j}{\alpha_j} \leq \frac{\alpha_j}{\phi_2} \leq \frac{\alpha_j}{\phi_2} \leq 1.
\]

Substituting (6) into (2) yields

\[
\begin{align*}
  Edx(t) &= f(t)dt + g(t)dw(t), \\
  z(t) &= \sum_{i=1}^{l_i} h_i(\vartheta(t))[C_i x(t)x(t) + D_2i x(t) v(t)],
\end{align*}
\]

where \( f(t) = \sum_{i=1}^{l_i} \sum_{j=1}^{l_j} h_i(\vartheta(t))h_j(\vartheta(t_k h)) [A_i(\vartheta(t)x(t)] + B_i(\vartheta(t))K_j x(t) (1 - \tau(t)) - B_i(\vartheta(t))K_j e_x^k(t) + D_1i(\vartheta(t)) v(t)] \) and \( g(t) = \sum_{i=1}^{l_i} h_i(\vartheta(t))J_i x(t)x(t)\).

The following two definitions and several lemmas are presented to facilitate development of the main results.

**Assumption 1**: [32] For given real symmetric matrices \( \Psi_1 \leq 0, \Psi_3 \) and \( \Psi_4 \geq 0 \), real matrix \( \Psi_2 \) satisfy the following conditions:

1) \( \|D_21\| \cdot \|\Psi_4\| = 0 \),
2) \( \langle \|\Psi_1\| + \|\Psi_2\| \rangle = 0 \),
3) \( D_21^2 \Psi_1 D_21 + D_21^2 \Psi_2 + \Psi_2^T D_21 + \Psi_3 > 0 \).

**Definition 1**: [32] For given matrices \( \Psi_1, \Psi_2, \Psi_3, \Psi_4 \) satisfying Assumption 1, system (8) is said to be extended dissipative if there exists a scalar \( \gamma \) such that the following inequality holds for any \( t_f \geq 0 \) and all \( v(t) \in L_2(0, \infty) \):

\[
\begin{align*}
  \mathbb{E}\left\{ \int_0^{t_f} \mathcal{J}(\mathcal{J}(t)) dt \right\} &\leq \sup_{0 \leq t \leq t_f} \mathbb{E}\{z(t)^T \Psi_4 z(t) \} \geq \gamma,
\end{align*}
\]

where \( \mathcal{J}(t) = z(t)^T \Psi_4 z(t) + 2z(t)^T \Psi_2 v(t) + \nu^T(t) \Psi_3 v(t) \).

Consider the following unforced stochastic polynomial singular system

\[
\begin{align*}
  Edx(t) &= \sum_{i=1}^{l_i} h_i(\vartheta(t)) [A_i x(t)x(t)dt + J_i x(t)v(t)dw(t)], \\
  z(t) &= \sum_{i=1}^{l_i} h_i(\vartheta(t)) C_i x(t) x(t).
\end{align*}
\]

**Definition 2**: [6], [44]

1) The system (10) is said to be regular and impulse free if \( \det[s E - \sum_{i=1}^{l_i} h_i(\vartheta(t)) A_i x(t)] \) is not identically zero and \( \text{deg}\{\det[s E - \sum_{i=1}^{l_i} h_i(\vartheta(t)) A_i x(t)]\} = \text{rank}(E) \).

2) The system (10) is said to be mean square stable if for any \( \varepsilon > 0 \), there exists a scalar \( \delta(\varepsilon) > 0 \) such that \( \mathbb{E}\{\|x(t)\|^2\} \leq \varepsilon \) for \( t > 0 \), with \( \mathbb{E}\{\|x(0)\|^2\} < \delta(\varepsilon) \).

3) The system (10) is said to be mean-square admissible if both (1) and (2) are satisfied.

**Lemma 1**: Consider the stochastic singular system

\[
Edx(t) = \tilde{f}(t) dt + \tilde{g}(t) dw(t),
\]

where \( E \) is a singular matrix, and \( w(t) \) is a one-dimensional Wiener process. For scalars \( b > a > 0 \) and any constant matrix \( Z \in \mathcal{R}^{n \times n}, Z = Z^T > 0, Z = \text{diag}\{Z, 3Z\} \), then

\[
(a - b) \int_a^b \overline{f}^T(s) Z \overline{f}(s) ds 
\leq -\mu^T(a, b) Z \mu(a, b) - 2\mu^T(a, b) Z \nu(a, b),
\]

with

\[
\begin{align*}
  \mu^T(a, b) &= [\mu_1^T(a, b), \mu_2^T(a, b)], \\
  \nu^T(a, b) &= [\nu_1^T(a, b), \nu_2^T(a, b)], \\
  \mu_1(a, b) &= Ex(b) - Ex(a),
\end{align*}
\]
Lemma 2: Consider the stochastic singular system (11). For \( \tau_1 \leq \tau(t) \leq \tau_2 \), if there exists a matrix \( X \) in \( \mathbb{R}^{2n \times 2n} \) such that \( \begin{bmatrix} \dot{Z} & X \\ \ast & \dot{Z} \end{bmatrix} \) is positive definite, then the following inequality holds:

\[
-\tau_2 \int_{\tau_1}^{\tau_2} \tilde{f}(s) ds \leq \left[ \begin{array}{c} \dot{Z} \\ X \\ \ast \\ \dot{Z} \end{array} \right] \mu(t) - \left[ \begin{array}{c} \mu(t) \\ \ast \end{array} \right] \leq \left[ \begin{array}{c} \dot{Z} \\ X \\ \ast \\ \dot{Z} \end{array} \right] \mu(t).
\]

Remark 2: When \( E = I \), Lemma 1 and Lemma 2 degenerate to Lemma 1 and Lemma 2 in [21], respectively. Therefore, Lemma 1 and Lemma 2 can be regarded as generalizations of Wirtinger inequality in stochastic singular systems.

Lemma 3: [45] For a scalar \( \beta \) and matrices \( Y, P, W, U \) with appropriate dimensions, if the following inequality holds:

\[
\begin{bmatrix}
Y & (WU)^T \\
\ast & -\beta W - \beta W^T + \beta^2 P
\end{bmatrix} < 0,
\]

then we have

\[
Y + U^T P U < 0.
\]

In the following sections, for the simplicity, \( h_i(\vartheta(t)) \) will be represented as \( h_i \). In addition, the time \( t \) in the polynomial matrix will be omitted. For instance, \( A_i(x(t)) \) will be represented as \( A_i(x) \).

### III. MAIN RESULTS

In this section, the extended dissipativity analysis and event-triggered-mechanism-based fuzzy controller for the system (8) are derived within the frame of SOS.

Theorem 1: Given positive scalars \( \tau_1, \bar{\tau}, \lambda, \phi = \{ \phi_k, 0 \leq k \leq \bar{\tau} \} \) (\( \varphi \geq 1 \)), a matrix \( K_j \), and matrices \( \Psi_1, \Psi_2, \Psi_3, \Psi_4 \), and matrices \( \Phi_1, \Phi_2, \Phi_3, \Phi_4 \), and matrices \( \Psi_1, \Psi_2, \Psi_3, \Psi_4 \), and matrices \( \Omega(x) \) and matrices \( R, X = \begin{bmatrix} X_{i1} & X_{i2} \\ X_{i1} & X_{i2} \end{bmatrix} \), such that the following SOS-based conditions are satisfied for each \( i, j = 1, 2, \cdots, l, i < j, \) \( \psi = 1, 2, 3, 4, \)

\[
\begin{align*}
\phi_1^T (Z_{i} - \epsilon_1 I) \phi_1 & \in \mathcal{S}, \\
\phi_2^T (P_{i} - \epsilon_2 I) \phi_2 & \in \mathcal{S}, \\
\phi_3^T (Q - \epsilon_3 I) \phi_3 & \in \mathcal{S}, \\
\phi_4^T (\Omega(x) - \epsilon_4 I) \phi_4 & \in \mathcal{S},
\end{align*}
\]

where \( \phi_1, \phi_2, \phi_3, \phi_4 \) are arbitrary vectors independent of \( x(t) \) with appropriate dimensions; \( \epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5 \) are predefined scalars and greater than 0; and

\[
\begin{bmatrix}
\xi_{ij} & \Phi_1 \\
\ast & \Phi_1
\end{bmatrix},
\]

\[
\begin{bmatrix}
\Psi_{11i} & -2E^T Z_{3i} & P^T B_i(x) K_j & 0 \\
* & \Psi_{22} & \Psi_{23} & \Psi_{24} \\
* & * & \Psi_{33} & \Psi_{34} \\
* & * & * & \Psi_{44}
\end{bmatrix},
\]

\[
\begin{bmatrix}
\Omega(x) & 0 & 0 & 0 \\
0 & \Omega(x) & 0 & 0 \\
0 & 0 & \Omega(x) & 0 \\
(1 - \lambda) \Omega(x) & 0 & 0 & 0
\end{bmatrix},
\]

\[
\begin{bmatrix}
\tilde{Z}_4 & diag(Z_4, 3Z_4) \\
\Phi_1 & \begin{bmatrix} -D_{22}^T (x) \Psi_1 C_i(x) - \Psi_2^T C_i(x) + D_{11}^T (x) P^T, \\
0, \cdots, 0^T \end{bmatrix}
\end{bmatrix},
\]

\[
\begin{bmatrix}
\Psi_{11i} & -2E^T Z_{3i} & P^T B_i(x) K_j & 0 \\
* & \Psi_{22} & \Psi_{23} & \Psi_{24} \\
* & * & \Psi_{33} & \Psi_{34} \\
* & * & * & \Psi_{44}
\end{bmatrix},
\]

\[
\begin{bmatrix}
\Psi_{11i} & -2E^T Z_{3i} & P^T B_i(x) K_j & 0 \\
* & \Psi_{22} & \Psi_{23} & \Psi_{24} \\
* & * & \Psi_{33} & \Psi_{34} \\
* & * & * & \Psi_{44}
\end{bmatrix},
\]

\[
\begin{bmatrix}
\Omega(x) & 0 & 0 & 0 \\
0 & \Omega(x) & 0 & 0 \\
0 & 0 & \Omega(x) & 0 \\
(1 - \lambda) \Omega(x) & 0 & 0 & 0
\end{bmatrix},
\]

\[
\begin{bmatrix}
\Omega(x) & 0 & 0 & 0 \\
0 & \Omega(x) & 0 & 0 \\
0 & 0 & \Omega(x) & 0 \\
(1 - \lambda) \Omega(x) & 0 & 0 & 0
\end{bmatrix},
\]

in which

\[
\begin{bmatrix}
\tilde{Z}_4 & diag(Z_4, 3Z_4) \\
\Phi_1 & \begin{bmatrix} -D_{22}^T (x) \Psi_1 C_i(x) - \Psi_2^T C_i(x) + D_{11}^T (x) P^T, \\
0, \cdots, 0^T \end{bmatrix}
\end{bmatrix},
\]

\[
\begin{bmatrix}
\Psi_{11i} & -2E^T Z_{3i} & P^T B_i(x) K_j & 0 \\
* & \Psi_{22} & \Psi_{23} & \Psi_{24} \\
* & * & \Psi_{33} & \Psi_{34} \\
* & * & * & \Psi_{44}
\end{bmatrix},
\]

\[
\begin{bmatrix}
\Omega(x) & 0 & 0 & 0 \\
0 & \Omega(x) & 0 & 0 \\
0 & 0 & \Omega(x) & 0 \\
(1 - \lambda) \Omega(x) & 0 & 0 & 0
\end{bmatrix},
\]

\[
\begin{bmatrix}
\Psi_{11i} & -2E^T Z_{3i} & P^T B_i(x) K_j & 0 \\
* & \Psi_{22} & \Psi_{23} & \Psi_{24} \\
* & * & \Psi_{33} & \Psi_{34} \\
* & * & * & \Psi_{44}
\end{bmatrix},
\]

\[
\begin{bmatrix}
\Omega(x) & 0 & 0 & 0 \\
0 & \Omega(x) & 0 & 0 \\
0 & 0 & \Omega(x) & 0 \\
(1 - \lambda) \Omega(x) & 0 & 0 & 0
\end{bmatrix},
\]
where \( P = P_1 E + SR, S \in \mathbb{R}^{n \times (n-r)} \) is a full column rank matrix and satisfies \( E^T S = 0 \).

**Proof** This proof consists of two parts: one is admissibility of the SPFSS (8), the other part is extended dissipativity analysis of the SPFSS (8). For the first part, we choose two non-degenerate matrices \( M \) and \( N \) such that

\[
MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}
\]

(20)

\[
MA_i(x)N = \begin{bmatrix} A_{11i}(x) & A_{12i}(x) \\ A_{21i}(x) & A_{22i}(x) \end{bmatrix}
\]

Denote

\[
M^{-T}PN = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}
\]

(21)

From (18) or (19), we can obtain \( \mathcal{Y}_{11i} < 0 \). Due to \( Z_1 > 0 \) and \( \Psi_1 \leq 0 \), \( \mathcal{Y}_{11i} = \text{sym}(P^T A_i(x)) + J_i^T (x)(E^+)^T E^T P E^+ J_i(x) - AE^T Z_2 E < 0 \) is derived. From (13) and \( E^T S = 0 \), obviously, \( E^T P = P^T E \) is obtained. Then, by (20) and (21), we have \( P_{12} = 0 \). Pre- and post-multiplying \( \mathcal{Y}_{11i} \) by \( N_T \) and \( N \), respectively, a straightforward computation yields

\[
\begin{bmatrix} \circ & \circ \end{bmatrix} \text{sym}(P^T_{22i} A_{22i}(x)) < 0,
\]

(22)

where \( \circ \) represents content that is not relevant to the following analysis. Due to \( \sum_{i=1}^t h_i = 1 \) and \( h_i > 0 \), by (22), we obtain \( \sum_{i=1}^t h_i[\text{sym}(P^T_{22i} A_{22i}(x))] < 0 \). This means that \( \sum_{i=1}^t h_i A_{22i}(x) \) is nonsingular. Therefore, according to Definition 2, we conclude that the system in (8) is regular and impulse-free. Next, the following L-K functional is employed to prove the mean square stability of the system (8):

\[
V(t) = V_1(t) + V_2(t) + V_3(t),
\]

(23)

where

\[
V_1(t) = x^T(t) E^T P x(t) + (\tau_2 - \tau(t)) x^T(t - \tau(t)) Q x(t - \tau(t)) + \frac{1}{2\rho}(\frac{1}{\sigma(t)} - \lambda)^2,
\]

\[
V_2(t) = \int_{t-\tau_1}^t x^T(s) Z_1 x(s) ds + \int_{t-\tau_2}^{t-\tau_1} x^T(s) Z_2 x(s) ds,
\]

\[
V_3(t) = \tau_1 \int_{t-\tau_1}^t f^T(s) Z_3 f(s) ds + \int_{t-\tau_2}^{t-\tau_1} f^T(s) Z_4 f(s) ds d\theta + \bar{\tau} \int_{t-\tau_2}^{t-\tau_1} f^T(s) Z_4 f(s) ds d\theta.
\]

By the Theorem 2 of [46] and one-dimensional Itô’s formula, the weak infinitesimal operator \( L \) of the stochastic process \( x(t) \), is given by

\[
LV(t) = LV_1(t) + LV_2(t) + LV_3(t),
\]

(24)

where

\[
LV_2(t) = x^T(t) Z_1 x(t) + x^T(t - \tau_2)(Z_2 - Z_1) x(t - \tau_2)
\]

\[
- x^T(t - \tau(t)) Q x(t - \tau(t))
\]

\[
+ x^T(tk) \Omega(x) x(tk) + \lambda (e^x_{\theta^k}(t))^T \Omega(x) e^x_{\theta^k}(t),
\]

\[
v(t) = x^T(t) E^T P x(t) + (\tau_2 - \tau(t)) x^T(t - \tau(t)) Q x(t - \tau(t)) + \frac{1}{2\rho}(\frac{1}{\sigma(t)} - \lambda)^2,
\]

\[
LV_2(t) = x^T(t) Z_1 x(t) + x^T(t - \tau_2)(Z_2 - Z_1) x(t - \tau_2)
\]

\[
- x^T(t - \tau(t)) Q x(t - \tau(t))
\]

\[
+ x^T(tk) \Omega(x) x(tk) + \lambda (e^x_{\theta^k}(t))^T \Omega(x) e^x_{\theta^k}(t),
\]

(28)

\[
\Xi_{11i} + \Gamma_{11i}^T (\tau_2^2 Z_3 + \bar{\tau}^2 Z_4) \Gamma_{11i} \xi(t) + \Delta(dw(t))
\]

(29)

\[
\Xi_{11i} + \Gamma_{11i}^T (\tau_2^2 Z_3 + \bar{\tau}^2 Z_4) \Gamma_{11i} \xi(t) + \Delta(dw(t))
\]

(30)
According to (29) and (30), when \( \varpi = 1 \), i.e., \( \frac{\alpha_i}{\alpha_j} = 1 \), it thus yields
\[
\Xi_{1ij} + \Xi_{1ji} + \Gamma_{1ij}^T(r_1^2Z_3 + \bar{\tau}^2Z_4)\Gamma_{1ij} + \Gamma_{1ji}^T(r_1^2Z_3 + \bar{\tau}^2Z_4)\Gamma_{1ji} < 0. \tag{31}
\]
When \( \varpi > 1 \), denoting \( \varphi_1 = \frac{\varpi - \frac{\alpha_i}{\alpha_j}}{\varpi - 1} \) and \( \varphi_2 = \frac{\varpi - 1}{\varpi - \frac{1}{2}} \), we have
\[
\varphi_1(\Xi_{1ij} + \frac{1}{\varpi}\Xi_{1ji}) + \Gamma_{1ij}^T(r_1^2Z_3 + \bar{\tau}^2Z_4)\Gamma_{1ij} + \varphi_2(\Xi_{1ij} + \varpi\Xi_{1ji}) + \Gamma_{1ij}^T(r_1^2Z_3 + \bar{\tau}^2Z_4)\Gamma_{1ji} < 0,
\]
which implies that
\[
\Xi_{1ij} + \frac{\alpha_i}{\alpha_j}\Xi_{1ji} + \Gamma_{1ij}^T(r_1^2Z_3 + \bar{\tau}^2Z_4)\Gamma_{1ij} + \frac{\alpha_i}{\alpha_j}\Gamma_{1ji}^T(r_1^2Z_3 + \bar{\tau}^2Z_4)\Gamma_{1ji} < 0. \tag{32}
\]
Thus, it follows from (27), (28), (31) and (32) that \( \mathbb{E}\{LV(t)\} < 0 \), which deduces that the SPFSS in (8) is mean-square stable.

Let us prove that the system (8) is extended dissipative. Let \( v(t) \neq 0 \) and \( \eta(t) = [\xi^T(t), \nu^T(t)]^T \), according to (18) and (19), we have
\[
\mathbb{E}\{LV(t) - J(t)\} \\
\leq \mathbb{E}\left\{\sum_{i=1}^{l} \alpha_i h_i \xi_i^T(t)(\Xi_{ii} + \Gamma_{ii}^T(r_1^2Z_3 + \bar{\tau}^2Z_4)\Gamma_{ii})\xi(t) + \sum_{j=1}^{l} \alpha_j h_j \xi_j^T(t)\Xi_{jj} + \frac{\alpha_i}{\alpha_j}\Xi_{ij} + \Gamma_{ij}^T(r_1^2Z_3 + \bar{\tau}^2Z_4)\Gamma_{ji}\xi(t)\right\} < 0. \tag{33}
\]
To this end, the following two cases will be discussed, respectively. First, the case of \( \Psi_4 = 0 \) is considered. Taking consideration of (33), let \( -V(0) \geq \gamma \) the following inequality holds:
\[
\mathbb{E}\left\{\int_0^{t_f} J(s)ds\right\} \geq \mathbb{E}\{V(t) - V(0)\} \\
\geq \mathbb{E}\{x^T(t)E^TPx(t)\} + \gamma \\
\geq \gamma.
\]
Next, we consider the case of \( \Psi_4 > 0 \) and \( \|D_{21}\| = 0 \). Recalling (16), we have
\[
z^T(t)\Psi_4z(t) \leq \sum_{i=1}^{l} h_i(C_i x(t))^T\Psi_4 C_i x(t) \\
\leq \sum_{i=1}^{l} h_i x^T(t)E^TPx(t),
\]
Based on the above discussions, the system (8) is extended dissipative in the sense of Definition 1.

**Theorem 2:** Given positive scalars \( \varpi, \tau, \lambda, \phi = \{\varpi, \frac{\alpha_i}{\alpha_j}\} (\varpi \geq 1) \), \( a \) and matrices \( \Psi_1, \Psi_2, \Psi_3, \) and \( \Psi_4 \) satisfying Assumption 1, the SPFSS (8) under the AET mechanism (3) is mean-square admissible and extended dissipative if there exist exist symmetric matrices \( P_1, Q, Z_4, \Omega(x), G(x), G_1(x), G_2(x) \) and matrices \( W, H_j, R, X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \) with appropriate dimensions, such that (12)-(17) and the following SOS-based conditions are satisfied for each \( i,j = 1,2,\cdots,l, i < j, \varsigma = 1,2,3,4, \)
\[
\nu^T_4(G_4(x) - \epsilon_4(x)I)v_4 \in \mathcal{S}, \tag{34}
\]
\[
\nu^T_4(G_1(x) - \epsilon_1(x)I)v_4 \in \mathcal{S}, \tag{35}
\]
\[
\nu^T_4(G_2(x) - \epsilon_1(x)I)v_4 \in \mathcal{S}, \tag{36}
\]
\[
-\nu^T_4(\Sigma_{ii} + \epsilon_i(x)I)v_4 \in \mathcal{S}, \tag{37}
\]
\[
-\nu^T_4(\Sigma_{ij} + \epsilon_i(x)I)v_4 \in \mathcal{S}, \tag{38}
\]
where \( \nu_4 \) and \( \nu_5 \) are arbitrary vectors independent of \( x(t) \) with appropriate dimensions; \( \epsilon_4(x) > 0, \epsilon_1(x) > 0, \epsilon_1(x) > 0; \nu_1, \epsilon_i(x) \) and \( \epsilon_i(x) \) are defined in Theorem 1; and
\[
\Sigma_{ii} = \begin{bmatrix} \Theta_{ii} + \left[ \frac{\partial^2}{\partial \xi_i^2}G(x)\right] \xi_{2i} & \Xi_{12i} & 0 \\ \Xi_{12i}^T & 0 & \Sigma_{2i} \\ \Sigma_{2i} & 0 & \Sigma_{3i} \end{bmatrix},
\]
\[
\Sigma_{ij} = \begin{bmatrix} \xi_{ij}^T & 0 & 0 \\ 0 & \Sigma_{2j} & 0 \\ 0 & 0 & \Sigma_{3j} \end{bmatrix},
\]
\[
\Theta_{ii} = \begin{bmatrix} \Xi_{ii} & 0 \\ 0 & \Theta_{ii} \end{bmatrix},
\]
\[
\Theta_{ij} = \begin{bmatrix} \Xi_{ij} + \bar{\phi}\Xi_{jj} & \sqrt{\bar{\phi}}\Xi_{ij} \\ \bar{\phi}\Xi_{ij}^T & \Theta_{jj} \end{bmatrix},
\]
\[
\Sigma_{11ij} = \Theta_{ij} + \left[ \frac{\partial^2}{\partial \xi_i^2}G_1(x)\right] \xi_i + \left[ \frac{\partial^2}{\partial \xi_j^2}G_2(x)\right] \xi_j,
\]
\[
\Sigma_{12ij} = H_j^T B_1^T(x) B_1(x),
\]
\[
\Sigma_{22i} = \text{sym}(P_1^T(x)P_1(x)W),
\]
\[
\Theta_{22} = -a(P^T + a^2(\tau_1^2Z_3 + \bar{\tau}^2Z_4),
\]
where \( \Theta_i = \begin{bmatrix} 0_{n \times 2n} & 0_{n \times n} & -I_{n \times n} & 0_{n \times (4n+q_4)} \\ 0_{n \times n} & 0_{n \times (7n+q_4)} & I_{n \times n} \end{bmatrix}, \)
\[
\xi_{12} = \begin{bmatrix} \xi_1 & 0_{n \times n} & \xi_2 \end{bmatrix}, \xi_4 = \begin{bmatrix} \xi_2 & 0_{n \times n} \end{bmatrix}, \xi_5 = \phi \xi_3, \xi_6 = \phi \xi_4, \Gamma_{ij} = P^T \Gamma_{ij}, \Xi_{ij} \) and \( \Xi_{ij} \) can be obtained by replacing \( P^T B_1^T(x)K_j \) in \( \Xi_{ij} \) and \( \Xi_{ij} \) with \( B_1(x)H_j \), for \( i,j = 1,2,\cdots,l \). Moreover, the fuzzy control gains under the AET mechanism (3) are given by \( K_j = W^{-1}H_j \).
Adaptive parameter $0.485$

then applying Lemma 3 and Schur complement, we get that
\[
\Theta_{ii} + A_2^T G(x) A_2 + (W^{-1} H_i E_i)^T (W^{-1} H_i E_i)^T < 0.
\]

Similarly, the following inequality is also obtained
\[
\Theta_{ij} + A_1^T G_1(x) A_2 + A_2^T G_2(x) A_2 + (W^{-1} H_j E_j)^T (W^{-1} H_j E_j) < 0.
\]

Due to $\hat{\Theta}_{22} \geq -P^T (\tau_2^2 Z_3 + \tau_2^2 Z_4)^{-1} P$, from (39) and (40), it can ensure that the inequality (33) is satisfied, which ends the proof.

Remark 3: In the study of $L_2-L_{\infty}$ performance of singular systems, a non-strict LMI is often involved, but the LMI toolbox cannot solve it directly. However, the SOS method is no longer to solve this LMI, but to verify whether it is the SOS. The new admissibility criterion based on SOS in Theorem 2 can be easily checked by using SOSTOOLS.

IV. SIMULATION EXAMPLES

In this section, a numerical example and a practical example are used to show the effectiveness and the merit of the proposed method. In order to reduce the burden of symbols, the time $t$ associated with state $x(t)$ is omitted without ambiguity.

Example 1: Consider a two-rule SPFSS with following parameters:

\[
E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_1(x) = \begin{bmatrix} 2x_1 & 0.2 \\ -1.3 & -0.5 \end{bmatrix},
\]

\[
A_2(x) = \begin{bmatrix} -x_1 & 0.1 \\ -1.5 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -2 \\ 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -3 \\ 0.2 \end{bmatrix},
\]

\[
D_{11} = D_{12} = \begin{bmatrix} 0.05 \\ 0.01 \end{bmatrix}, \quad J_1 = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix},
\]

\[
J_2 = \begin{bmatrix} 0 & 0.2 \\ 0.05 & 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix},
\]

\[
D_{21} = 0.01, \quad D_{22} = 0.15, \quad S = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\]

\[
h_1(x) = 1 - \frac{1}{1 + e^{-x_1 - 2.5}}, \quad h_2(x) = 1 - h_1(x).
\]

We will consider $H_{\infty}$ performance, passivity and dissipativity and $L_2-L_{\infty}$ performance, respectively. Firstly, the values of each matrix in Assumption 1 are given in Table I for different performances. Giving scalars $\tau_1 = 0.01$, $\bar{\tau} = 0.02$, $\lambda = 2$, $a = 2$, $\rho = 0.05$, $\varpi = 1.1$, $h = 0.05$, $\epsilon = \epsilon_o(x) = \epsilon_1(x) = \epsilon_2(x) = \epsilon_3(x) = 10^{-7}$, for all $t = 1, 2, \ldots, T$, $a = 8, 9, 10, 11$, and the disturbance signal is chosen as

\[
v(t) = \begin{cases} 0 < t < 5, \\
0, \quad \text{else}
\end{cases}
\]

- $H_{\infty}$ Performance: $\chi_{\min} = 0.1507$ is obtained by iterative calculation. Let $\chi^2 = 0.06$, the corresponding trigger matrix and gain matrices of the controller can be computed by Theorem 2, which are listed as below:

\[
\Omega(x) = \begin{bmatrix} \Omega_{11}(x) & \Omega_{12}(x) \\ \Omega_{21}(x) & \Omega_{22}(x) \end{bmatrix},
\]

\[
K_1 = \begin{bmatrix} 1.1942 & -0.9219 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.2692 & -0.16 \end{bmatrix},
\]

where $\Omega_{11}(x) = -6.452 \times 10^{-8} x_1^4 + 2.356 \times 10^{-6} x_1^3 + 4.568 \times 10^{-3} x_1^2 - 4.203 \times 10^{-2} x_1 + 29.86$, $\Omega_{12}(x) = -3.425 \times 10^{-9} x_1^4 + 5.64 \times 10^{-7} x_1^3 + 1.204 \times 10^{-4} x_1^2 - 6.84 \times 10^{-3} x_1 - 1$, $\Omega_{22}(x) = 2.066 \times 10^{-8} x_1^4 + 8.26 \times 10^{-8} x_1^3 + 6.366 \times 10^{-4} x_1^2 - 9.161 \times 10^{-4} x_1 + 36.76$.

For the case of $H_{\infty}$ Performance, by using AET mechanism and state feedback controllers, the adaptive parameter $\sigma(t)$ is described in Fig. 1. Fig. 2 and Fig. 3 present state responses, control input, release instants and intervals with initial condition $x(0) = [0.2 - 0.5442]^T$, respectively.

- Passivity: Let $\chi^2 = 0.06$, the corresponding trigger matrix and gain matrices of the controller can be computed by

\[
\sigma = 0.01 + \frac{1}{1 + e^{-x_1 - 2.5}}.
\]
Theorem 2, which are listed as follows:

\[
\Omega(x) = \begin{bmatrix}
\Omega_{11}(x) & \Omega_{12}(x) \\
* & \Omega_{22}(x)
\end{bmatrix},
\]

\[K_1 = \begin{bmatrix} 1.2613 & -0.8985 \end{bmatrix},\]

\[K_2 = \begin{bmatrix} 0.2336 & -0.1509 \end{bmatrix},\]

where \(\Omega_{11}(x) = -4.078 \times 10^{-7} x_1^4 + 1.166 \times 10^{-5} x_1^3 + 2.431 \times 10^{-2} x_1^2 - 0.2079 x_1 + 149.3, \)
\(\Omega_{12}(x) = -2.323 \times 10^{-8} x_1^4 + 3.235 \times 10^{-6} x_1^3 + 9.336 \times 10^{-4} x_1^2 - 4.394 \times 10^{-2} x_1 - 6.567,\)
\(\Omega_{22}(x) = 8.787 \times 10^{-8} x_1^4 + 2.597 \times 10^{-7} x_1^3 + 3.419 \times 10^{-3} x_1^2 - 2.057 \times 10^{-3} x_1 + 176.\)

For the case of passivity, by using AET mechanism and state feedback controllers, the adaptive parameter \(\sigma(t)\) is described in Fig. 4. Fig. 5 and Fig. 6 present state responses, control input, release instants and intervals with initial condition \(x(0) = [0.2 - 0.7232]^T\), respectively.

- Dissipativity: The corresponding trigger matrix and gain matrices of the controller can be computed by Theorem 2, which are listed as below:

\[
\Omega(x) = \begin{bmatrix}
\Omega_{11}(x) & \Omega_{12}(x) \\
* & \Omega_{22}(x)
\end{bmatrix},
\]

\[K_1 = \begin{bmatrix} 1.1811 & -2.2014 \end{bmatrix},\]

\[K_2 = \begin{bmatrix} 0.308 & -0.3551 \end{bmatrix},\]

where \(\Omega_{11}(x) = -6.503 \times 10^{-7} x_1^4 + 1.913 \times 10^{-5} x_1^3 + 3.242 \times 10^{-2} x_1^2 - 0.2845 x_1 + 170.8, \)
\(\Omega_{12}(x) = -8.352 \times 10^{-8} x_1^4 + 1.107 \times 10^{-5} x_1^3 + 2.758 \times 10^{-3} x_1^2 - 0.1338 x_1 - 18.46, \)
\(\Omega_{22}(x) = 3.407 \times 10^{-8} x_1^4 + 1.654 \times 10^{-6} x_1^3 + 8.309 \times 10^{-3} x_1^2 - 1.605 \times 10^{-2} x_1 + 211.6.\)

For the case of dissipativity, by using AET mechanism and state feedback controllers, the adaptive parameter \(\sigma(t)\) is described in Fig. 7. Fig. 8 and Fig. 9 present state responses, control input, release instants and intervals with initial condition \(x(0) = [0.2 - 0.6035]^T\), respectively.

- \(L_2 - L_\infty\) Performance: Let \(\chi^2 = 0.06\), the corresponding
Adaptive parameter

State responses

0.15
0.25
0.35
0.05
0.15
0.42
0.25
0.44
0.46
0.48
-0.7
-0.6
-0.4
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0
0.2
0.4
0.6
Control input

0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Adaptive parameter

0 1 2 3 4 5 6 7 8 9 10
Times(s)
0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6 7 8 9 10
0 1 2 3 4 5 6 7 8 9 10

Fig. 7. Variation of $\sigma(t)$ in dissipativity case.

Fig. 8. Release instants and intervals in dissipativity case.

Fig. 9. State responses of $x(t)$ in dissipativity case.

Fig. 10. Variation of $\sigma(t)$ in $L_2-L_\infty$ case.

Fig. 11. Release instants and intervals in $L_2-L_\infty$ case.

2.794 \times 10^{-5} x_1^3 + 6.028 \times 10^{-3} x_1^2 - 0.2802 \times 10^{-4} x_1 - 33.2,
\Omega_{22}(x) = 1.049 \times 10^{-6} x_1^4 + 2.033 \times 10^{-7} x_1^3 + 2.493 \times 10^{-2} x_1^2 - 9.549 \times 10^{-3} x_1 + 1030.

For the case of $L_2-L_\infty$ Performance, by using AET mechanism and state feedback controllers, the adaptive parameter $\sigma(t)$ is described in Fig. 10. Fig. 11 and Fig. 12 present state responses, control input, release instants and intervals with initial condition $x(0) = [0.2 \ 0.7128]^T$, respectively. Through the above calculation and analysis, the method proposed in this paper is effective.

**Example 2:** Consider the following nonlinear stochastic single-species bio-economic model modified from [7]:

\[
\begin{align*}
dx(t) &= (-a_{11}x(t) + a_{12}y(t) + d_1v(t))dt + J_{12}y(t)dw(t) , \\
dy(t) &= (a_{21}x(t) - a_{22}y^2(t) - y(t)E(t) + d_2v(t) + u(t))dt , \\
0 &= E(t)(a_{33}y(t) - \bar{a}_{33}) - m + u(t) + d_3v(t) ,
\end{align*}
\]

where the actual meaning of parameters have been introduced in [6]. Let $x(t) = [x_1 \ x_2 \ x_3]^T = [x(t) \ y(t) \ E(t)]^T$, and the parameters of this model are $a_{11} = 0.7$, $a_{12} = 0.2$, $a_{21} = 0.05$, $a_{22} = 0.1$, $a_{33} = 1$, $\bar{a}_{33} = 30$, $d_1 = 0.1$, $d_2 = 0.1$, $d_3 = -0.2$, $J_{12} = 0.06$. We can obtain the one-rule stochastic polynomial singular system as follows:

\[
\begin{align*}
Edx(t) &= (A(x)x(t) + Bu(t) + D_1v(t))dt + Jx(t)dw(t) , \\
y(t) &= Cx(t) + D_2v(t) ,
\end{align*}
\]
where

\[
E = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
A(x) = \begin{bmatrix}
-0.7 & 0.2 & 0 \\
0.05 & -0.1x_2 & -x_2 \\
0 & 0 & x_2 - 30
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 \\
1 \\
1
\end{bmatrix},
\]

\[
D_1 = \begin{bmatrix}
0.1 \\
0.1 \\
-0.2
\end{bmatrix},
\]

\[
J = \begin{bmatrix}
0 & 0.06 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
0.2 & 0.1 & 0.1
\end{bmatrix},
\]

\[
D_2 = 0.2.
\]

Based on Theorem 2, we will check whether the resulting closed-loop stochastic polynomial singular single-species bi-economic system satisfies \( H_{\infty} \) performance, passivity and dissipativity and \( L_2 - L_{\infty} \) performance. Due to the limited space, we only consider \( H_{\infty} \) performance.

- \( H_{\infty} \) Performance: Let \( \Psi_1 = -1, \Psi_2 = 0, \Psi_3 = 0.2752^2, \Psi_4 = 0, \tau_1 = 0.01, \bar{\tau} = 0.02, \lambda = 2, \sigma = 1.1, a = 10, \rho = 5, h = 0.03, \) and \( \epsilon_i = \epsilon_i(x) = \epsilon_{11}(x) = 10^{-12}, \) for all \( i = 1, 2, \cdots, 7, o = 8, 9, \) then the corresponding trigger matrix and gain matrix of the controller can be computed by Theorem 2, which are listed as below:

\[
\Omega(x) = \begin{bmatrix}
\Omega_{11}(x) & \Omega_{12}(x) & \Omega_{13}(x) \\
\ast & \Omega_{22}(x) & \Omega_{23}(x) \\
\ast & \ast & \Omega_{33}(x)
\end{bmatrix},
\]

\[
K = \begin{bmatrix}
-0.02417 & -0.1726 & -0.3243
\end{bmatrix},
\]

where \( \Omega_{11}(x) = 4.264 \times 10^{-9}x_2^6 - 6.911 \times 10^{-7}x_2^4 + 2.552 \times 10^{-7}x_2^6 + 1.21, \Omega_{12}(x) = -2.864 \times 10^{-11}x_2^6 - 1.046 \times 10^{-7}x_2^4 + 3.589 \times 10^{-4}x_2^2 - 3.863 \times 10^{-2}x_2 + 1.492 \times 10^{-11}x_2^6 + 4.141 \times 10^{-9}x_2^4 - 2.782 \times 10^{-5}x_2^2 + 7.807 \times 10^{-3}, \Omega_{22}(x) = 4.163 \times 10^{-9}x_2^6 - 6.419 \times 10^{-7}x_2^4 + 3.264 \times 10^{-9}x_2^6 + 1.123, \Omega_{23}(x) = 1.197 \times 10^{-10}x_2^6 + 8.932 \times 10^{-9}x_2^4 - 1.509 \times 10^{-9}x_2^2 + 0.05103, \Omega_{33}(x) = 4.69 \times 10^{-9}x_2^6 - 1.304 \times 10^{-6}x_2^4 + 1.93 \times 10^{-3}x_2^2 + 1.291.
\]

Furthermore, the minimum \( H_{\infty} \) performance of \( \chi \) calculated by different methods is illustrated in Table II. The disturbance signal is chosen as \( v(t) = \sin(t)e^{-t}. \) Under the obtained trigger matrix and gain matrix, the trajectory of adaptive parameter \( \sigma(t) \) is described in Fig. 13, the release instants and release interval are depicted in Fig. 14. Fig. 15 and Fig. 16 provide state responses and control input of the closed-loop system, respectively, where \( x(0) = [0.75 2 -0.012979]^T. \) It can be seen from the figures that our method is effective, and the number of fuzzy rules is reduced compared with [6], [7]. The nonlinear stochastic singular single-species bi-economic model can be solved directly by Theorem 2 without fuzzification. Therefore, compared with the T-S fuzzy system, the polynomial fuzzy system can express the original nonlinear system more accurately in some cases.

**V. CONCLUSION**

In this paper, the extended dissipativity analysis and AET controller design of SPFSSs have been studied. Considering the asynchronous constraint of membership functions, an admissibilization condition in terms of SOS has been proposed to guarantee that the closed-loop is regular, impulse-free, mean square stable and extended dissipative by a suitable L-K functional and a novel integral inequality in stochastic.
setting. Then, the corresponding AET controller design has been carried out by a simple inequality. Finally, both a numerical example and a nonlinear stochastic single-species bio-economic model are provided to demonstrate the feasibility and practicability of our proposed design approach. It is noteworthy that this work solves the problem of non-stRICT LMIIs caused by the special properties of singular systems. Our future work will continue to focus on SOS methods to study fuzzy singular systems, such as the output feedback control based on event-triggered techniques.

**REFERENCES**


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