Event-Triggered Prescribed-Time Fuzzy Control for Space Teleoperation Systems Subject to Multiple Constraints and Uncertainties

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Abstract—Limited by the operation time window and working space, space teleoperation tasks need to be completed within an expected time while ensuring that the end effector meets the physical constraints. Meanwhile, the interaction with unknown environments would cause uncertainty in the closed-loop system, which brings great challenges to the control design. To solve the above problems, the control performance issue for a class of space teleoperation systems subject to multiple constraints and interaction uncertainties is investigated in this paper. The force interaction with the human operator/space environment is represented by interval type-2 (IT2) Takagi-Sugeno (T-S) fuzzy systems, where the uncertain equivalent mass and damping parameters can be effectively described and captured by IT2 membership functions. In order to reduce the communication burden and satisfy the constraints of settling time, transient-state performance and operating space, a time-varying threshold event-triggered control scheme together with exponential-type Lyapunov function (EBLF) is developed for the first time. We show that, with the proposed controller, the synchronization tracking errors are guaranteed to converge to a user-defined residual set within pre-assigned settling time, and never exceed the prescribed range despite unknown control direction and actuator faults, which solves the long-standing constraint issue with more flexibility due to the fact that the related constraints can be arbitrarily specific within the physically available range. Moreover, the convergence set is only dependent on fewer user-defined parameters rather than approximation errors, which provides an effective analysis technique to deal with the difficulty that the convergence accuracy is difficult to calculate quantitatively in the presence of unknown disturbance. Detailed simulation results are provided to show the effectiveness and merit of the proposed control strategy.

Index Terms—space teleoperation, IT2 fuzzy control, prescribed-time stability, event-triggered control, fault-tolerant control.

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I. INTRODUCTION

LIMITED by the cognition of complex space environment and the perception technology of space robots, the current space robots are suffering from low level of intelligent autonomy and insufficient emergency-handling capability. To address this issue, space teleoperation, which is composed of ground manipulator (master part) and space-robot (slave part) connected by communication channels, incorporates human judgment into the closed-loop system, and completes complex on-orbit operation tasks through human-machine cooperation, which thus is widely applied to both theoretical and engineering fields [1]–[3]. Due to complicated signal transmissions in the process of coordination of multiple ground stations, dynamic characteristics of Earth-space time delay will bring adverse effects to the system stability. Thus, long-distance time delay is a critical issue to be addressed in the control design of space teleoperation. Classic methods were proposed for stability analysis from the perspective of four-channel control [4]–[6], passivity [7], [8], and adaptive scheme [9]–[11]. Although the time delay problem is solved in the aforementioned works to some extent, updated state information needs to be transmitted all the time via the communication channel, resulting in high computational burden and the latency of packet loss.

Recently, event-triggered scheme is presented to solve this issue, where state updates are triggered by specific event-triggered mechanism, which is thus widely applied to the field of networked control systems (NCSs). The IT2 fuzzy-model-based control concept covering modeling, membership-function-dependent stability analysis and control synthesis was first proposed in [12], [13]. Based on [12], [13], an event-triggered membership-function-dependent control scheme was developed to tackle the stabilization issue of IT2 polynomial-fuzzy-model-based (PFMB) networked control systems [14]. In [15], a threshold event-triggered condition was set to determine the data transmission from the output of T-S fuzzy system to the reduced-order filter. Different from the locations of the above event-triggered mechanisms, an event-triggered scheme between the sensor and the controller was developed to reduce the released times [16]. However, limited works have discussed the event-triggered control issue of teleoperation systems, especially when the long-distance time delay (e.g., Earth-space time delay) exists. From the perspective of robot control, asymptotic stability of the tracking position errors was...
guaranteed for event-driven teleoperation systems subject to constant time delays [17], [18]. Nevertheless, only asymptotic convergence is realized in the literature concerned with event-triggered teleoperation. It indicates that the above controllers can achieve convergence performance only when the settling time approaches infinity, which cannot be directly applied to space teleoperation systems due to limited time window in on-orbit operation missions [2], [19]. Despite some works aiming to realize finite/fixed-time convergence of teleoperation systems [20]–[22], the obtained settling time is dependent on initial values or control parameters, and the so-called settling time is only the estimated upper bound rather than the actual time, which will cause difficulties for control engineers in parameter selection. Thus, we need to develop a prescribed-time method such that the settling time can be manually specified according to the task requirements rather than parameter-dependent. To the best knowledge of authors, no work has investigated prescribed-time convergence in the framework of event-triggered control for teleoperation systems.

Force interaction with uncertain environments is another important issue to be addressed in space teleoperation control design. Force interaction is difficult to model accurately, especially when the end-effector of the space-robot is in contact with an unknown environment, which will inevitably transform the passiveness of the system, thereby causing the degradation of system performance. It is well known that interval type-2 (IT2) fuzzy sets show advantages in dealing with uncertainties in terms of fuzzy-model-based (FMB) systems [23]. A series of seminal works were proposed aiming at stability analysis and control synthesis for IT2 T-S FMB systems from the perspective of imperfect premise matching [12], [13], [24], [25], output feedback control [26], [27], networked control systems [14], [28], fuzzy filter [29], and model reduction [30], [31]. Different from the above researches with respect to generalized linear systems, sampling input-output data based type-2 fuzzy model was utilized to capture system uncertainties in bilateral [6] and multilateral teleoperation [32]. Type-2 fuzzy neural network was employed to obtain desired position/velocity and force signals in terms of position-loop control [33].

However, constraint requirement on working space is not discussed in the aforementioned works, which is an important issue to be tackled in practical space teleoperation, especially when the space-robot travels through confined spaces and hazardous environments. For example, when space-robot fits through small gaps and circumnavigates obstacles, prescribed constraints on each joint and end-effector are required to avoid collision phenomenon for the sake of security. To this end, constraint requirement needs to be satisfied in a unified control framework. Classic back-stepping methods were proposed based on barrier Lyapunov function (BLF) to address the constraint requirements on output states [34]. BLF is a Lyapunov function that explicitly contains a given constraint function, which tends to increase infinity if the system states approach the constraint boundary. Following this line of thought, tangent-type BLF [35], [36] was subsequently developed and applied to improve restrictions on constraint types in traditional methods. With the exponential-type BLF (EBLF), the authors discussed constraint control problem of teleoperation systems [19], [20]. However, no work in the literature has been done to deal with BLF design of FMB systems. Hence, how to satisfy the prescribed constraint requirements of IT2 FMB teleoperation systems is still an open and challenging issue to be tackled.

The above discussion indicates that the control methods in reported works are difficult to deal with the constraints of settling time and operation space in practical space teleoperation. In addition, there is currently no unified approach to handle multiple uncertainties, including parametric uncertainties in operator/environment interactions, actuator failures, and signal transmission delays. In this paper, the enhanced control performance issue of space teleoperation systems subject to multiple constraints and uncertainties is investigated. Time-varying constraint requirements in terms of operation space and uncertain force interaction can be effectively handled in the framework of IT2 T-S fuzzy control. Furthermore, the settling time in the proposed control scheme can be arbitrarily specified within the physically achievable range, while the residual-set can be user-defined rather than dependent on any prior information of disturbances. The proposed control scheme effectively tackles the human-machine-environment interaction issue in space teleoperation subject to multiple constraints and uncertainties with less local communication burden. It breaks through the long-standing problem in engineering that control performance is dependent on parameter setting trick rather than arbitrarily specified by the engineer, which thus guarantees control performance indexes, including settling time and tracking accuracy, available to be estimated and specified within physically realizable range. Compared to the existing literature, the main contributions in this paper are presented as follows:

1) The practically prescribed-time convergence performance of the synchronization tracking errors is guaranteed, where the settling time can be user-defined arbitrarily within the physically achievable category according to the specific task requirements, which is essentially different from traditional finite/fixed-time control [20]–[22].

2) The constraint requirement on the system states is satisfied for the first time in the framework of IT2 T-S fuzzy control. A new control structure is developed together with EBLF technique and event-triggered mechanism to achieve prescribed transient performance and steady-state accuracy despite uncertain force interaction, where no prior information about disturbances is required in calculating the residual-set and convergence accuracy.

3) Unknown control direction and actuator faults can be effectively addressed in the proposed adaptive control scheme, where the upper and lower bounds of actuator faults are not required in the control design. Different from adaptive fuzzy control with the help of universal approximation theorem [22], [37], [38], the residual-set is only dependent on fewer user-defined parameters rather than approximation errors of actuator faults.

The remainder of this paper is organized as follows. In Sec. II, dynamics of space teleoperation systems, IT2 T-S fuzzy model describing the nonlinear dynamics with respect to force interaction with human operator and environments
subject to uncertainties, and necessary lemmas about practically prescribed-time stability are presented. The adaptive event-triggered fault-tolerant control scheme is proposed in Sec. III. In Sec. IV, simulation results show the effectiveness of the proposed controller, followed by the conclusion in Sec. V.

Notation. The subscript \( i = m, s \) denotes the master and slave manipulator, respectively. For \( \forall \alpha \in \mathbb{R}^{n \times n} \), \( \|A\| \) is the Euclidean norm of \( A \); \( \lambda_{\min} \) and \( \lambda_{\max} \) represent the minimum and maximum eigenvalue, respectively. \( \text{diag}(\{a_j\}) \) stands for the diagonal matrix with \( a_j \) as the \( j \)-th element; \( \text{col}(\{a_j\}) \) is the column vector with \( a_j \) as the \( j \)-th element. \( I \) is the identity matrix.

II. Preliminaries

A. Dynamics of Space Teleoperation Systems

Since the attitude of a satellite is required to keep stabilized in implementing on-orbit operation, the classic Euler-Lagrange model can be employed to describe the dynamic motion of space master-slave manipulator, decoupling from the satellite. The gravity item in conventional robotic dynamics is omitted due to the weightless environment in space. Then a class of \( n \) degree-of-freedom (DOF) space teleoperation systems can be formulated as follows

\[
M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + f_i(q_i) + h_i = d_iu_i^F + J_i^T(q_i)F_i^h
\]

(1)

where \( q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^n \) are the position, velocity, and acceleration signals defined in joint space, respectively. \( M_i(q_i) \in \mathbb{R}^{n \times n} \) is the inertia matrix; \( C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n \times n} \) is the matrix of Centripetal and Coriolis torques; \( f_i(q_i) \in \mathbb{R}^n \) and \( h_i \in \mathbb{R}^n \) represent the Coulomb friction and external non-homogeneous disturbance, respectively. The force generated by human operator (environments) is denoted as \( F_i^h \in \mathbb{R}^n \) \((F_i^s \in \mathbb{R}^n)\). \( J_i^T(q_i) \in \mathbb{R}^{n \times m} \) is the force Jacobian matrix such that \( \dot{x}_i = J_i(q_i)\dot{q}_i \), in which \( x_i \in \mathbb{R}^m \) contains the position and orientation of end-effector. \( d_i \in \{-1, 1\} \) stands for the uncertain control direction. \( u_i^F \in \mathbb{R}^n \) is the control input with actuator faults

\[
u_i = \phi_iu_i + \varphi_i
\]

(2)

where \( \phi_i = \text{diag}(\phi_{ik}) \in \mathbb{R}^{n \times n} \) is the multiplicative actuator failure with \( \phi_{ik} \in (0, 1) \) for \( k = 1, 2, \ldots, n \); \( u_i \in \mathbb{R}^n \) is the nominal control input; \( \varphi_i \in \mathbb{R}^n \) is the additive actuator failure. With the robot kinematics, the teleoperation dynamics (1) can be transformed into the form defined in Cartesian space

\[
\mathfrak{M}_i\ddot{x}_i + \mathcal{C}_i\dot{x}_i + \mathfrak{F}_i = d_i\phi_i\mathbf{U}_i + F_i^h
\]

(3)

in which \( \mathfrak{M}_i = J_i^T(q_i)M_i(q_i)J_i(q_i), \mathcal{C}_i = J_i^T(q_i)C_i(q_i, \dot{q}_i) - M_i(q_i)J_i(q_i) \dot{J}_i(q_i) + J_i^T(q_i)f_i(q_i) + h_i - d_i\varphi_i, \mathfrak{F}_i = J_i^T(q_i)u_i = J_i^T(q_i)u_i + (J_i^T(q_i))^T \) with \( J_i^T(q_i) \) being the Moore-Penrose inverse of \( J_i(q_i) \). The control structure of space teleoperation systems is presented in Fig. 1.

B. Ground Operator and Environmental Model

The interaction with operator and environment subject to time-varying parameters can be formulated as an inferred \( p \)-rule IT2 T-S fuzzy model (the state argument of all variables will be omitted)

Rule 1: IF \( G_1(\ddot{x}_i(t)) \) is \( Q_1^i \) AND...AND \( G_a(\ddot{x}_i(t)) \) is \( Q_a^i \)

\[
\text{THEN } f_i = f_i^h - M_i^*\dot{x}_i - C_i^*\dot{x}_i - K_i\dot{x}_i,
\]

(4)

where \( \ddot{x}_m(t) = \left[ x_m^T(t), x_m^T(t) - T_s, x_m^T(t) - x_m^T(t) \right]^T \) and \( \ddot{x}_s(t) = \left[ x_s^T(t), x_s^T(t) - T_s, x_s^T(t) - x_s^T(t) \right]^T \), where \( T_m \) and \( T_s \) stand for the bounded forward and backward time-varying delay between the ground station and space robot, respectively; \( Q_1^i \) is an IT2 fuzzy set of rule 1 corresponding to the function \( g_i(\ddot{x}_i), l = 1, 2, \ldots, p, \nu \), \( \nu \) is a positive integer representing the number of fuzzy sets; \( M_i^* \in \mathbb{R}^{m \times m}, C_i^* \in \mathbb{R}^{m \times m}, \) and \( K_i \in \mathbb{R}^{m \times m} \) are the equivalent positive-definite inertia, damping and stiffness matrices, respectively; \( f_i^h \) is the bounded exogenous force. Then combining (4) with (3) yields

Rule 1: IF \( G_1(\ddot{x}_i(t)) \) is \( Q_1^i \) AND...AND \( G_a(\ddot{x}_i(t)) \) is \( Q_a^i \)

\[
\text{THEN } M_i\ddot{x}_i + \mathcal{C}_i\dot{x}_i + K_i\dot{x}_i = d_i\phi_i\mathbf{U}_i + F_i^h
\]

(5)

where \( \mathfrak{M}_i = \mathfrak{M}_i + M_i^* + K_i, \mathcal{C}_i = \mathcal{C}_i + C_i^*, F_i^h = f_i^h - \mathfrak{F}_i, \) and \( \ddot{x}_i = \ddot{x}_i(t) \). The firing strength of the Rule 1 is of the following interval set

\[
W_i(\ddot{x}_i) = \left[ \omega_i(\ddot{x}_i), \bar{\omega}_i(\ddot{x}_i) \right],
\]

where \( \omega_i(\ddot{x}_i) = \prod_{l=1}^{p} \mu_{Q_1^i}(g_i(\ddot{x}_i)), \bar{\omega}_i(\ddot{x}_i) = \prod_{l=1}^{p} \bar{\mu}_{Q_1^i}(g_i(\ddot{x}_i)) \) in which \( \mu_{Q_1^i}(g_i(\ddot{x}_i)) \) and \( \bar{\mu}_{Q_1^i}(g_i(\ddot{x}_i)) \) denote the lower and upper membership function such that \( 1 \geq \mu_{Q_1^i}(g_i(\ddot{x}_i)) \geq \bar{\mu}_{Q_1^i}(g_i(\ddot{x}_i)) \geq 0 \); \( \omega_i(\ddot{x}_i) \) and \( \bar{\omega}_i(\ddot{x}_i) \) are the lower and upper grade of membership, respectively, satisfying \( \bar{\omega}_i(\ddot{x}_i) \geq \omega_i(\ddot{x}_i) \geq 0 \). As a result, the teleoperation dynamics represented by inferred IT2 T-S fuzzy model can be presented as

\[
\ddot{x}_i = \sum_{i=1}^{p} \omega_i(\ddot{x}_i)M_i^{-1}\left( d_i\phi_i\mathbf{U}_i + F_i^h - C_i\dot{x}_i - K_i\dot{x}_i \right),
\]

(6)

\[
\bar{\omega}_i(\ddot{x}_i) = \omega_i(\ddot{x}_i)b(\ddot{x}_i) + \bar{\omega}_i(\ddot{x}_i)b(\ddot{x}_i),
\]

(7)

in which \( b(\ddot{x}_i), \bar{b}(\ddot{x}_i) \in [0, 1] \) such that \( b(\ddot{x}_i) + \bar{b}(\ddot{x}_i) = 1 \).

The IT2 T-S fuzzy model (6) defined in Cartesian space shows important structural properties as follows [11], [39]

**Property 1**: \( M_i \) is symmetric and positive-definite such that \( \lambda_{\min}\{M_i\}I \leq M_i \leq \lambda_{\max}\{M_i\}I \).

**Property 2**: There exists a positive scalar \( \bar{c} \) satisfying \( \|C_i(q_i, \dot{q}_i)\| \leq \bar{c} \|\dot{q}_i\| \).

**Property 3**: For \( \forall y \in \mathbb{R}^m \), the dynamics can be written in linearly parameterizable form

\[
M_i\ddot{y} + C_i\dot{y} = \Psi_i(q_i, \dot{q}_i, \dot{y}, \ddot{y})\zeta_{it}
\]

(8)

where \( \Psi_i(q_i, \dot{q}_i, \dot{y}, \ddot{y}) \in \mathbb{R}^{m \times m_o} \) is a regressor matrix of known functions and \( \zeta_{it} \in \mathbb{R}^{m_o} \) is a vector of unknown parameters.

C. Practically Prescribed-Time Stability

**Definition 1** [19]. Consider a class of nonlinear systems

\[
\dot{z}(t) = f(z(t), t)
\]

(9)

where \( z(t) \in U_0 \subset \mathbb{R}^n \) is the system state and \( f:U_0 \times \mathbb{R}_+ \rightarrow \mathbb{R}^n \) is a continuous-differential function. Then the origin of (9)
is practically prescribed-time stable (PPTS) if \( \|z(t)\| \leq \bar{\varepsilon} \) for \( t \geq t_0 + T \), in which \( t_0 \) stands for the initial time, \( \bar{\varepsilon} \) and \( T \) are pre-specified, and \( 0 < T_r \leq T < +\infty \) with \( T_r \) being the time consuming of signal transmission and processor computing.

**Lemma 1** [19]. For the system (9), if there exists a positive-definite continuous-differential function \( V(z(t), t) : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}_+ \) and scalars \( \beta > 0, 0 \leq \gamma < +\infty, 0 \leq \eta < +\infty \) such that

\[
\dot{V} \leq -\beta V - 2|\dot{\varsigma}|V + \eta \varsigma + \gamma,
\]

in which the piece-wise time-varying function \( \varsigma(t) \) is defined as

\[
\varsigma(t) = \begin{cases} 
\exp(\alpha (t_0 + T - t)) - 1, & t \in [t_0, t_0 + T) \\
\exp(\alpha (t - t_0)) \tan(\alpha (t - t_0 + T)), & t \in [t_0 + T, +\infty)
\end{cases}
\]

where \( \exp(\cdot) \) and \( \tanh(\cdot) \) represent the exponential and hyperbolic tangent function, respectively; \( c_1 \triangleq |\dot{\varsigma}(t)|; \alpha \) and \( \mathcal{E} \) are positive tunable parameters, then the origin of (9) is PPTS.

**Remark 1**: Different from conventional finite/fixed-time control [20]–[22], a new control performance, featured by prescribed settling time and residual set, can be simultaneously realized via **Lemma 1**, leading to user-defined convergence accuracy and guaranteed transient-state performance. In particular, as an independent tuning parameter, the settling time can be pre-assigned within physically achievable range according to practical tasks rather than limited by control parameters, relaxing the requirement on initial values especially in fine on-orbit operation with insufficient sensors, such as rendezvous and docking.

### III. Control Law Design

Define the synchronization tracking errors in Cartesian space as \( e_m(t) = x_m(t) - x_s(t - T_m) \) and \( e_x(t) = x_s(t) - x_m(t - T_m) \). Then the control objective can be stated as follows. For a class of space teleoperation systems with actuator faults and uncertain interaction with human operator/environment (6), design an event-triggered controller such that the PPTS property of the closed-loop system can be guaranteed, and the synchronization tracking errors never exceed the predetermined time-varying boundary. To this end, we design the IT2 fuzzy control input with \( q \) rules as

\[
\text{Rule } r : \text{IF } D_1(\hat{x}_i) \text{ is } N_{w}^{r} \text{ AND...AND } D_c(\hat{x}_i) \text{ is } N_{c}^{r},
\]

\[
\text{THEN } u_i = u_{ir},
\]

where \( N_{w}^{r} \) is an IT2 fuzzy set of rule \( r \) corresponding to the function \( D_w(\hat{x}_i) \), \( r = 1, 2, ..., q \); \( w = 1, 2, ..., c \); \( c \) is a positive integer representing the number of fuzzy sets. The firing strength of the Rule \( r \) is of the following interval set

\[
M_{ir}(\hat{x}_i) = [m_{ir}(\hat{x}_i), \bar{m}_{ir}(\hat{x}_i)],
\]

where \( m_{ir}(\hat{x}_i) = \prod_{w=1}^{q} \mu_{N_{w}^{r}} D_w(\hat{x}_i), \bar{m}_{ir}(\hat{x}_i) = \prod_{w=1}^{q} \bar{\mu}_{N_{w}^{r}} D_w(\hat{x}_i) \), in which \( \mu_{N_{w}^{r}} D_w(\hat{x}_i) \) and \( \bar{\mu}_{N_{w}^{r}} D_w(\hat{x}_i) \) denote the lower and upper membership function such that \( 1 \geq \mu_{N_{w}^{r}} D_w(\hat{x}_i) \geq \bar{\mu}_{N_{w}^{r}} D_w(\hat{x}_i) \geq 0 \); \( m_{ir}(\hat{x}_i) \) and \( \bar{m}_{ir}(\hat{x}_i) \) are the lower and upper grade of membership, respectively, satisfying \( m_{ir}(\hat{x}_i) \geq m_{ir}(\hat{x}_i) \geq 0 \). Then the inferred IT2 fuzzy controller is represented by

\[
u_i = \sum_{r=1}^{q} \bar{m}_{ir}(\hat{x}_i) u_{ir},
\]

where

\[
\bar{m}_{ir}(\hat{x}_i) = \frac{\theta_{ir}(\hat{x}_i) m_{ir}(\hat{x}_i) + \bar{\theta}_{ir}(\hat{x}_i) \bar{m}_{ir}(\hat{x}_i)}{\sum_{r=1}^{q} \theta_{ir}(\hat{x}_i) m_{ir}(\hat{x}_i) + \bar{\theta}_{ir}(\hat{x}_i) \bar{m}_{ir}(\hat{x}_i)},
\]

with \( \bar{m}_{ir}(\hat{x}_i) \) is the grade of the embedded membership function such that \( \sum_{r=1}^{q} m_{ir}(\hat{x}_i) = 1, \theta_{ir}(\hat{x}_i), \bar{\theta}_{ir}(\hat{x}_i) \in [0, 1] \) are predefined functions satisfying \( \theta_{ir}(\hat{x}_i) + \bar{\theta}_{ir}(\hat{x}_i) = 1 \).

In order to reduce computing burden induced by long-distance signal transmission, we need to design a mechanism to determine whether to send updated status information to the slave manipulator. Thus, a time-varying threshold event-triggered control scheme is developed as follows

\[
u_i(t) = \tau_{ir}(t), \forall t \in \{t_k, t_{k+1}\}, k \in \mathbb{Z}_+
\]

\[
t_{k+1} = \inf \{ t > t_k \mid \tau_{ir}(t) - \nu_i(t) \geq \alpha_i \nu_i(t) + \varkappa_i \}
\]

where \( \alpha_i \in (0, 1) \) and \( \varkappa_i \in (0, 1) \) are positive design parameters; \( t_k \) is the update time; \( \nu_i(t) \) and \( \tau_{ir}(t) \) are the \( j \)th element of \( \nu_i(t) \) and \( \tau_{ir}(t) \), respectively, for \( j = 1, 2, ..., n \). Once the mechanism (15) is triggered, the control input \( \nu_i(t) \) will be updated by the intermediate control \( \tau_{ir}(t-k+1) \). Thus, for \( t \in \{t_k, t_{k+1}\} \), \( \nu_i(t) \) remains at \( \tau_{ir}(t) \) updated at the last moment such that

\[
\tau_{ir}(t) - \nu_i(t) \leq \alpha_i \nu_i(t) + \varkappa_i,
\]

which further indicates

\[
u_i(t) = \frac{\tau_{ir}(t) - \bar{\theta}_{ir}(t) \varkappa_i}{1 + \bar{\theta}_{ir}(t) \alpha_i},
\]

where \( \bar{\theta}_{ir}(t) \in [-1, 1] \) and \( \bar{\theta}_{ir}(t) \in [-1, 1] \) are the time-varying parameters. Denote \( \Gamma_{ir} = \text{diag \{1/1 + \bar{\theta}_{ir}(t) \varkappa_i\}} \) and \( \Omega_{ir} = \text{col\{ \bar{\theta}_{ir}(t) \varkappa_i/1 + \bar{\theta}_{ir}(t) \varkappa_i\}} \), one has a more compact form of (17) as

\[
u_i(t) = \Gamma_{ir} \tau_{ir}(t) - \Omega_{ir}.
\]
Combining (6), (13) with (18), we can obtain the IT2 fuzzy-model-based control system
\[
\dot{x}_i = \sum_{l=1}^{p} \omega_{il}(\dot{x}_i)M_{il}^{-1} \left( d_i \phi_i \sum_{r=1}^{q} \tilde{m}_{ir}(\dot{x}_i) \mathcal{U}_i + \mathcal{F}_{il} \right) - C_{il} \dot{x}_i - K_{il} x_i
\]
\[
= \sum_{l=1}^{p} \sum_{r=1}^{q} \tilde{h}_{ilr} M_{il}^{-1} \left( \Gamma_{ilr} \dot{\tau}_{ilr} + \Omega_{ilr} - C_{il} \dot{x}_i - K_{il} x_i \right)
\]
where \( \tilde{h}_{ilr} \) is a positive scalar to be estimated. Property 3 and \( \dot{\tau}_{ilr} \) are positive scalars; \( \Omega_{ilr} \) is utilized; \( \Gamma_{ilr} = d_i \phi_i \Gamma_{ilr} \); \( \Omega_{ilr} = F_{il} - d_i \phi_i \Omega_{ilr} \). Due to the boundedness of \( \phi_i \) and \( \Gamma_{ilr} \), the boundary of \( \Gamma_{ilr} \) can be obtained as \( \varepsilon_{ilr} \leq \| \Gamma_{ilr} \| \leq \varepsilon_{ilr} \), where \( \varepsilon_{ilr} \) is an unknown positive scalar to be estimated.

Define the following sliding mode
\[
S_i = \dot{e}_i + \kappa_{ii} e_i
\]
where \( \kappa_{ii} \) is a positive constant. With Property 3 and \( \dot{\tau}_{ilr} \), we can obtain from (19) that
\[
\dot{S}_i = \sum_{l=1}^{p} \sum_{r=1}^{q} \tilde{h}_{ilr} M_{il}^{-1} \left( \Gamma_{ilr} \dot{\tau}_{ilr} + \Omega_{ilr} - C_{il} S_i - K_{il} x_i \right)
\]
- \( \Psi_{ilr}(q_i, \dot{q}_i, v_i, \dot{v}_i) \dot{q}_{ilr} \).

Design the intermediate bilateral control input as
\[
\tau_{ilr} = - \frac{M_{ilr} S_i}{\sqrt{S_i^T \Psi_{ilr}^2 + e^2}} \tau_{ilr} + e^2,
\]
and \( \tau_{ilr} \) is designed as
\[
\tau_{ilr} = (\tilde{\mu}_i + \kappa_{ii} \| q_i \|) S_i - K_{ilr} M_{ilr}^{-1} x_i - \tau_{ilr}^*,
\]
in which \( \tilde{\mu}_i = \mu_i + \kappa_{ii}/\xi_i + \kappa_{ii} (k_i^2 - S_i^T S_i)^2 + \xi_i^2 / (2 || S_i ||^2 \xi_i) \) and
\[
\tau_{ilr}^* = M_{ilr}^{-1} \Psi_{ilr}(q_i, \dot{q}_i, v_i, \dot{v}_i) \dot{q}_{ilr} - \xi_i \psi_{ilr} - \frac{\xi_i^2 S_i}{\sqrt{S_i^T \Psi_{ilr}^2 + e^2}} S_i - \kappa_{ii} e_i,
\]
with the following adaptive update laws
\[
\dot{\psi}_{ilr} = -\delta_4 \xi_i \xi_i^2 S_i \tau_{ilr} - \delta_4 \dot{\psi}_{ilr},
\]
\[
\dot{\psi}_{ilr} = -\delta_3 \xi_i \xi_i^2 S_i \tau_{ilr} - \delta_3 \dot{\psi}_{ilr},
\]
where \( \delta_3, \delta_4, \delta_5, \delta_6, \delta_7 \) are positive scalars to be determined; \( \xi_i \) will be defined later; \( \mu_i = \sup \left( \left( \frac{\xi_i^2}{\xi_i} \right)^2 + e \right) \) being a positive constant; \( k_i \in \mathbb{R}_+ \) denotes the prescribed \( n \)-order differentiable time-varying constraint function such that \( k_i(t_0) > || S_i(t_0) || \); \( \dot{q}_{ilr}, \psi_{ilr}, \dot{\psi}_{ilr} \) are the estimation of \( q_{ilr}, \psi_{ilr}, \dot{\psi}_{ilr} \), respectively; \( \dot{\tau}_{ilr} = 1/\varepsilon_{ilr} \) and \( \psi_{ilr} \) will be defined later.

**Theorem 1:** For the IT2 T-S FMB system (19), if the bilateral control input (22) triggered by (14)-(15) and adaptive laws (25)-(27) are adopted and the condition (28) is satisfied,
\[
\begin{align*}
\delta_{1i} & \geq \delta_{11} \kappa_{ii} S_i + \kappa_{ii} + \sigma_i \\
\delta_{2i} & \geq \delta_{22} \kappa_{ii} S_i + \kappa_{ii} + \sigma_i \\
\delta_{3i} & \geq \delta_{33} \kappa_{ii} S_i + \kappa_{ii} + \sigma_i \\
\kappa_{ii} & \geq \frac{R_i}{2} + \kappa_{ii}/2 \\
\kappa_{ii} & \geq \frac{R_i}{2} + \kappa_{ii}/2
\end{align*}
\]
then the following properties will hold.

1. The origin of the system trajectory is PPTS, and all the signals of the closed-loop teleoperation system are bounded.
2. The constraint requirement on the synchronization tracking errors is satisfied. That is, the preassigned constraint will never be violated.
3. The uniformly ultimately bounded performance of force tracking can be guaranteed in the case of contact with the unknown environment.

**Proof.** Choose the following Lyapunov-Krasovskii functionals
\[
V_i = \sum_{i=m,s} \frac{1}{2} k_i^2 (\exp(k_i \circ S_i) - 1),
\]
\[
V_1 = \sum_{i=m,s} \frac{1}{2} k_i^2 (\exp(k_i \circ S_i) - 1),
\]
\[
V_2 = \sum_{i=m,s} \sum_{l=1}^{p} \sum_{r=1}^{q} \tilde{h}_{ilr} \dot{q}_{ilr} \psi_{ilr} + \tilde{h}_{ilr} \dot{\psi}_{ilr},
\]
\[
V_3 = \sum_{i=m,s} \frac{\kappa_{ii}}{2} \left( \int_{\tau - T_i}^{\tau} \left( \exp(\alpha^* (\tau - t + T_i)) - 1 \right) \chi_i(\tau) d\tau \right)
\]
\[
\times R_i \chi_i(\tau) d\tau + \epsilon_i^2 e_i,
\]
where \( k_i \circ S_i = \frac{S_i^T S_i}{(k_i^2 - S_i^T S_i)^2}; \) \( \dot{q}_{ilr} = \dot{q}_{ilr} - \dot{\bar{q}}_{ilr}, \)
\( \dot{\psi}_{ilr} = \dot{\psi}_{ilr} - \dot{\bar{\psi}}_{ilr}, \) and \( \dot{\bar{q}}_{ilr} = \dot{q}_{ilr} - \dot{\bar{q}}_{ilr} \) represent the estimation error of \( q_{ilr}, \psi_{ilr}, \) and \( \dot{\bar{q}}_{ilr}, \dot{\bar{\psi}}_{ilr}, \) respectively; \( \chi_i = [S_i^T, \dot{q}_{ilr}^T, \dot{\psi}_{ilr}^T, \dot{\bar{q}}_{ilr}, \dot{\bar{\psi}}_{ilr}^T]; R_i \) is a positive definite matrix; \( T_i = \max \{ T \}; \) \( \alpha^* \) is a positive continuously differentiable function to be designed. Then taking time derivative of \( V_1 \) gives
\[
\dot{V}_1 = \sum_{i=m,s} \dot{k}_i k_i (\exp(k_i \circ S_i) - 1) + k_i^2 \exp(k_i \circ S_i)
\]
\[
\times \frac{S_i^T S_i k_i^2 - S_i^T S_i k_i}{(k_i^2 - S_i^T S_i)^2}
\]
\[
\leq -\sum_{i=m,s} \dot{k}_i k_i - \frac{\dot{k}_i k_i^3 S_i^T S_i}{(k_i^2 - S_i^T S_i)^2} \exp(k_i \circ S_i)
\]
\[
+ k_i^4 \exp(k_i \circ S_i)
\]
In general, the constraint function is set to be monotonically decreasing for the sake of transient-state convergence perfor-
mance. Hence, with \( \mu_i = \sup \sqrt{\left( \frac{k_i}{\delta_i} \right)^2 + \epsilon} \), it follows from (33) that

\[
\dot{V}_1 \leq \sum_{i=m,s} \mu_i k_i^2 + \mu_i \xi_i S^T \dot{s}_i + \xi_i S^T \ddot{s}_i
\]

where \( \xi_i = k_i^2 \exp(k_i \circ S_i)/(k_i^2 - S^T \dot{s}_i)^2 \). Substituting (21) into (34) yields

\[
\dot{V}_1 \leq \sum_{i=m,s} \sum_{l=1}^p \sum_{\ell=1}^q \tilde{h}_{ilr} \left( \mu_i k_i^2 + \mu_i \xi_i S^T \dot{s}_i + \xi_i S^T \mathcal{M}^{-1}_{ilr} \right)
\]

\[
\tilde{\Gamma}_{li} \dot{s}_{ilr} + \tilde{\Omega}_{ilr} - C_{il r} - K_{ilr} \dot{x}_i - \mathcal{P}_{il}(q_i, \dot{q}_i, v_i, \dot{v}_i) \theta_{ilr}
\]

(35)

Note that we can obtain from (35) that

\[
\xi_i S^T \mathcal{M}^{-1}_{ilr} \tilde{\Omega}_{ilr} \leq \xi_i \|s_i\| \|\mathcal{M}^{-1}_{ilr}\| \|\tilde{\Omega}_{ilr}\|
\]

\[
\leq \psi_{ilr} \epsilon + \psi_{ilr} \frac{\xi_i^2 \|s_i\|}{\sqrt{\xi_i^2 \|s_i\|^2 + \epsilon^2}} + \frac{\xi_i \epsilon}{\lambda_{\min}\{\mathcal{M}_{ilr}\}} \|\tilde{q}_i\| \|s_i\|
\]

\[
- \xi_i S^T \mathcal{M}^{-1}_{ilr} C_{il r} \leq \frac{\xi_i \epsilon}{\lambda_{\min}\{\mathcal{M}_{ilr}\}} \|\tilde{q}_i\| \|s_i\|
\]

(36)

Then substituting (36) and (37) into (35) and further simplifying yield

\[
\dot{V}_1 \leq \sum_{i=m,s} \sum_{l=1}^p \sum_{\ell=1}^q \tilde{h}_{ilr} \left( \mu_i k_i^2 + \mu_i \xi_i S^T \dot{s}_i + \xi_i S^T \mathcal{M}^{-1}_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr} + \psi_{ilr} \epsilon + \psi_{ilr} \frac{\xi_i^2 \|s_i\|}{\sqrt{\xi_i^2 \|s_i\|^2 + \epsilon^2}} + \frac{\xi_i \epsilon}{\lambda_{\min}\{\mathcal{M}_{ilr}\}} \|\tilde{q}_i\| \|s_i\| - \xi_i S^T \mathcal{M}^{-1}_{ilr} \right)
\]

\[
(38)
\]

With (22)-(23), it can be concluded from (38) that

\[
\xi_i S^T \mathcal{M}^{-1}_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr} = - \xi_i S^T \mathcal{M}^{-1}_{ilr} \frac{\delta^2 \theta_{ilr}^2 S^T \dot{s}_i}{\sqrt{\xi_i^2 \|s_i\|^2 + \epsilon^2}} \tilde{\Gamma}_{li} \dot{s}_{ilr} + \epsilon^2
\]

\[
\leq - \tilde{\tau}_{ilr} \frac{\xi_i S^T \theta_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr}}{\sqrt{\xi_i^2 \|s_i\|^2 + \epsilon^2}} + \epsilon^2
\]

\[
\leq \tilde{\tau}_{ilr} \xi_i \epsilon - \tilde{\tau}_{ilr} \xi_i S^T \theta_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr}
\]

\[
\leq \frac{\xi_i^2}{2} + \frac{\xi_i^2 \epsilon^2}{2} - \tilde{\tau}_{ilr} \xi_i S^T \theta_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr}
\]

(39)

which indicates a compact form of (38)

\[
\dot{V}_1 \leq \sum_{i=m,s} \sum_{l=1}^p \sum_{\ell=1}^q \tilde{h}_{ilr} \left( \mu_i k_i^2 + \mu_i \xi_i S^T \dot{s}_i + \frac{\xi_i^2}{2} + \frac{\xi_i^2 \epsilon^2}{2}
\]

\[
- \tilde{\tau}_{ilr} \xi_i S^T \theta_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr} + \psi_{ilr} \epsilon + \psi_{ilr} \frac{\xi_i^2 \|s_i\|}{\sqrt{\xi_i^2 \|s_i\|^2 + \epsilon^2}} + \frac{\xi_i \epsilon}{\lambda_{\min}\{\mathcal{M}_{ilr}\}} \|\tilde{q}_i\| \|s_i\|
\]

\[
+ \psi_{ilr} \theta_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr}
\]

\[
(40)
\]

Then taking the time-derivative of \( V_2 \) and substituting it into (40), we have

\[
\dot{V}_2 \leq \sum_{i=m,s} \sum_{l=1}^p \sum_{\ell=1}^q \tilde{h}_{ilr} \left( \mu_i k_i^2 + \mu_i \xi_i S^T \dot{s}_i + \frac{\xi_i^2}{2} + \frac{\xi_i^2 \epsilon^2}{2}
\]

\[
- \tilde{\tau}_{ilr} \xi_i S^T \theta_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr} + \psi_{ilr} \epsilon + \psi_{ilr} \frac{\xi_i^2 \|s_i\|}{\sqrt{\xi_i^2 \|s_i\|^2 + \epsilon^2}} + \frac{\xi_i \epsilon}{\lambda_{\min}\{\mathcal{M}_{ilr}\}} \|\tilde{q}_i\| \|s_i\|
\]

\[
+ \psi_{ilr} \theta_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr}
\]

\[
(41)
\]

which implies that

\[
\dot{V}_1 + \dot{V}_2 \leq \sum_{i=m,s} \sum_{l=1}^p \sum_{\ell=1}^q \tilde{h}_{ilr} \left( \gamma_{ilr} + \mu_i \xi_i S^T \dot{s}_i + \frac{\xi_i^2}{2}
\]

\[
- \tilde{\tau}_{ilr} \xi_i S^T \theta_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr} + \psi_{ilr} \frac{\xi_i^2 \|s_i\|}{\sqrt{\xi_i^2 \|s_i\|^2 + \epsilon^2}} + \frac{\xi_i \epsilon}{\lambda_{\min}\{\mathcal{M}_{ilr}\}} \|\tilde{q}_i\| \|s_i\|
\]

\[
+ \psi_{ilr} \theta_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr}
\]

\[
(42)
\]

where \( \gamma_{ilr} = \mu_i k_i^2 + \frac{\xi_i^2}{2} \). Define \( \sigma_i = \max\{\tilde{h}_{ilr}, |\tilde{h}_{ilr}|\} \), then combining the adaptive updated laws (25)-(27) and the controller (23)-(24) with (42), more compact form of (42) can be obtained

\[
\dot{V}_1 + \dot{V}_2 \leq \sum_{i=m,s} \sum_{l=1}^p \sum_{\ell=1}^q \tilde{h}_{ilr} \left( \gamma_{ilr} + \mu_i \xi_i S^T \dot{s}_i + \frac{\xi_i^2}{2} - \xi_i
\]

\[
+ \psi_{ilr} \frac{\xi_i^2 \|s_i\|}{\sqrt{\xi_i^2 \|s_i\|^2 + \epsilon^2}} + \frac{\xi_i \epsilon}{\lambda_{\min}\{\mathcal{M}_{ilr}\}} \|\tilde{q}_i\| \|s_i\|
\]

\[
+ \psi_{ilr} \theta_{ilr} \tilde{\Gamma}_{li} \dot{s}_{ilr}
\]

\[
(43)
\]

which can be further simplified to

\[
\dot{V}_1 + \dot{V}_2 \leq \sum_{i=m,s} \sum_{l=1}^p \sum_{\ell=1}^q \tilde{h}_{ilr} \left( \gamma_{ilr} - \kappa_k \xi_i S^T \dot{s}_i - \kappa_k \xi_i S^T \epsilon_i
\]

\[
+ \frac{\xi_i \epsilon}{\lambda_{\min}\{\mathcal{M}_{ilr}\}} \|\tilde{q}_i\| \|s_i\|
\]

\[
+ \psi_{ilr} \frac{\xi_i^2 \|s_i\|}{\sqrt{\xi_i^2 \|s_i\|^2 + \epsilon^2}} + \frac{\xi_i \epsilon}{\lambda_{\min}\{\mathcal{M}_{ilr}\}} \|\tilde{q}_i\| \|s_i\|
\]

\[
(44)
\]
Note that it follows from (44) that
\[
- \frac{\delta_{i4}}{\sigma_{1i}} \bar{q}_{i4t} \bar{q}_{i4r} = - \frac{\delta_{i4}}{\sigma_{1i}} \bar{q}_{i4t} \bar{q}_{i4r} + \frac{\delta_{i4}}{2\sigma_{2i}} \bar{q}_{i4t} \bar{q}_{i4r} + \frac{\delta_{i4}}{2\sigma_{2i}} \bar{q}_{i4t} \bar{q}_{i4r}
\]
\[
\leq - \frac{\delta_{i4}}{\sigma_{1i}} \bar{q}_{i4t} \bar{q}_{i4r} + \frac{\delta_{i4}}{2\sigma_{2i}} \bar{q}_{i4t} \bar{q}_{i4r} + \frac{\delta_{i4}}{2\sigma_{2i}} \bar{q}_{i4t} \bar{q}_{i4r}.
\]
Similarly with (45), we have
\[
\frac{\delta_{i5}}{2\sigma_{2i}} \bar{q}_{i5t} \bar{q}_{i5r} \leq - \frac{\delta_{i5}}{2\sigma_{2i}} \gamma_{i5r}^2 \bar{q}_{i5r} + \frac{\delta_{i5}}{2\sigma_{2i}} \gamma_{i5r}^2 \bar{q}_{i5r}.
\]
\[
\frac{\tau_{l4t}}{2\sigma_{3i}} \bar{q}_{i6l} \bar{q}_{i6l} \leq - \frac{\tau_{l4t}}{2\sigma_{3i}} \gamma_{i6l}^2 \bar{q}_{i6l} + \frac{\tau_{l4t}}{2\sigma_{3i}} \gamma_{i6l}^2 \bar{q}_{i6l}.
\]
Replacing (45)-(47) with the last three terms of (44) and taking time-derivative of $V_3$, we can obtain
\[
\dot{V} = V_1 + V_2 + V_3
\]
\[
\leq \sum_{i=m,s} \sum_{l=1}^{p} \sum_{k=1}^{q} \hat{h}_{ilr} \left( \gamma_{ilr} - \frac{\delta_{i4} - \sigma_{i4}}{2\sigma_{1i}} \bar{q}_{i4t} \bar{q}_{i4r} - \frac{\delta_{i5} - \sigma_{i5}}{2\sigma_{2i}} \bar{q}_{i5r} \bar{q}_{i5r} - \frac{\tau_{l4t} \delta_{i6l} - \tau_{l4t} \sigma_{i6l}}{2\sigma_{3i}} \bar{q}_{i6l} \bar{q}_{i6l} - \frac{k_i^2}{2} \kappa_i \exp(k_i \circ S_i) + \frac{k_i}{2} \left( \exp(\alpha^* \bar{T}_i) - 1 \right) \chi_i^T R_i \chi_i + \frac{\eta}{\kappa} + \int_{-\bar{T}_i}^{t} \left( \exp(\alpha^*(\tau - t + \bar{T}_i)) \right) \chi_i^T R_i \chi_i \dot{\tau} \right)
\]
where $\gamma_{ilr} = \gamma_{ilr} + \frac{\delta_{i4}}{2\sigma_{1i}} \bar{q}_{i4t} \bar{q}_{i4r} + \frac{\delta_{i5}}{2\sigma_{2i}} \bar{q}_{i5r} \bar{q}_{i5r} + \frac{\tau_{l4t}}{2\sigma_{3i}} \bar{q}_{i6l} \bar{q}_{i6l}$. To further analyze the effect of the last term of (48) in terms of stability performance, we define the following functional operator
\[
\text{Proj}(\bar{T}, \tau^*) = \frac{\gamma \exp(\gamma \tau^*)}{\exp(\gamma \tau^*) - 1}
\]
where $\gamma \equiv \gamma(\tau^*) : U_1 \rightarrow \mathbb{R}_+$ with $U_1 = (0, \bar{T}]$ and $\gamma^* = \gamma - \gamma_{ilr} + \bar{T}_i$. It can be concluded from (49) that the following inequality holds
\[
\text{Proj}(\bar{T}, \tau^*) \geq \text{Proj}(\gamma, \tau^*) \geq \gamma, \forall \tau^* \in U_1,
\]
if $\bar{T} \geq \gamma$. Due to the strongly convex property as shown in (49), one can obtain $\nabla_T \text{Proj}(\bar{T}, \tau^*) \geq \frac{1}{2}$ according to L’Hospital’s rule, which means $2\text{Proj}(\alpha + \beta, \tau^*) \geq 2\text{Proj}(\alpha, \tau^*) + \beta$. In view of the continuity and monotonically increasing nature of (49), there exists a monotonically increasing functional $\alpha^*$ such that $\text{Proj}(\alpha^*, \tau^*) = 2\text{Proj}(\alpha + \beta, \tau^*)$. Note that when $\alpha$ and $\beta$ are constants, there also exists a constant $\alpha^*$ such that the above inequality holds. Thus, we have
\[
\text{Proj}(\alpha^*, \tau^*) - \frac{\alpha^* \tau^* \exp(\alpha^* \tau^*)}{\exp(\alpha^* \tau^*) - 1} = \frac{\alpha^* - \alpha^* \tau^* \exp(\alpha^* \tau^*)}{\exp(\alpha^* \tau^*) - 1} \geq 2\text{Proj}(\alpha, \tau^*) + \beta,
\]

which is equivalent to $(\alpha^\tau^* - \alpha^*) \exp(\alpha^\tau^*) \leq -\kappa_4 (\exp(\alpha^\tau^*) - 1)$. Then recalling $S_i = \dot{\epsilon}_i + \kappa_{1i} \epsilon_i$, (48) can be rewritten as
\[
\dot{V} \leq \sum_{i=m,s} \sum_{l=1}^{p} \sum_{k=1}^{q} \hat{h}_{ilr} \left( - \frac{\delta_{i4} - \sigma_{i4}}{2\sigma_{1i}} \gamma_{i4r}^2 \bar{q}_{i4r} - \frac{\delta_{i5} - \sigma_{i5}}{2\sigma_{2i}} \gamma_{i5r}^2 \bar{q}_{i5r} - \frac{\tau_{l4t} \delta_{i6l} - \tau_{l4t} \sigma_{i6l}}{2\sigma_{3i}} \gamma_{i6l}^2 \bar{q}_{i6l} - \frac{k_i^2}{2} \kappa_i \exp(k_i \circ S_i) + \frac{k_i}{2} \left( \exp(\alpha^*(\tau - t + \bar{T}_i)) \right) \chi_i^T R_i \chi_i \dot{\tau} \right)
\]
where $\bar{T}_i = (\exp(\alpha^* \bar{T}_i) - 1) \lambda_{\max}(R_i)$. With the condition (28), then we have
\[
\dot{V} \leq - (\beta + 2\frac{\gamma^{\frac{\tau}{2}}}{\kappa}) V + \frac{\eta}{\kappa} + \gamma,
\]
where $\gamma = \sum_{i=m,s} \sum_{l=1}^{p} \sum_{k=1}^{q} \hat{h}_{ilr} \gamma_{i4r}$. It indicates the PPTS property of the closed-loop system according to Lemma 1. Consequently, we have $\dot{\epsilon}_i, \bar{q}_{i4r}, \bar{q}_{i5r}, \bar{q}_{i6r} \in \mathbb{L}_\infty$, and $|S_i| \leq k_i$ for $t \in [t_0, t_0 + T]$. When $t \geq t_0 + T$, the system trajectory will converge to a prescribed convergence set such that $V_1 \leq V \leq \eta/\alpha$, which implies that the boundary of $S_i$ can be further compressed into
\[
|S_i| \leq \sqrt{\frac{k_i^2}{1 + \kappa_i^2} k_i}
\]
in terms of $S_i = \dot{\epsilon}_i + \kappa_{1i} \epsilon_i$, one has $|S_i| \leq |S_i| \leq k_i$ with the subscript $k = 1, 2, ..., n$, namely $-k_i - \kappa_{1i} \epsilon_i \leq \dot{\epsilon}_i \leq -k_i - \kappa_{1i} \epsilon_i + k_i$. Multiplying both sides of the above inequality by the integrating factor $\exp(\kappa_{1i} t)$, one can obtain
\[
\frac{d}{dt} (e^{\kappa_{1i} t} \dot{\epsilon}_i) \leq k_i e^{\kappa_{1i} t}.
\]
Integrating both sides of (54) from $t_0$ to $t$ yields $\dot{\epsilon}_i \leq f_{t_0}^{t} k_i(s) e^{\kappa_{1i} (s-t)} ds$. In light of the attenuation property of $k_i$, it is often set to the following form $k_i = P_i e^{-W_i t} + V_i$, where $P_i, W_i, \text{and} V_i$ are positive designed scalars. Then we have
\[
|\epsilon_i| \leq \frac{P_i}{\kappa_{1i} - W_i} e^{-W_i t} \left( 1 - e^{(\kappa_{1i} - W_i) (t_0 - t)} \right)
\]
\[
+ \frac{V_i}{\kappa_{1i}} \left( 1 - e^{\kappa_{1i} (t_0 - t)} \right)
\]
\[
\leq \frac{P_i}{\kappa_{1i} - W_i} e^{-W_i t} + \frac{V_i}{\kappa_{1i}}
\]
\[
\leq \frac{k_i}{\kappa_{1i} - W_i},
\]
which indicates that $|\epsilon_i| \leq \sqrt{\frac{\eta}{\kappa_{1i} - W_i}} k_i$ and $|\dot{\epsilon}_i| \leq \sqrt{\frac{\eta}{\kappa_{1i} - W_i}} k_i + k_i^2 k_i \leq \sqrt{\eta} + \sqrt{k_i^2} k_i$. Therefore, the prescribed constraint of synchronization tracking errors can be obtained by applying the constraint to sliding mode variable.
Next, we consider the convergence of tracking force error, which represents the transparency of the closed-loop teleoperation system. According to [40], the force tracking error is defined as \( e_f(t) = F_s^h - d_m \phi_m \dot{U}_m \). In the steady state \( \dot{x}_i \to 0 \), we have \( F_s^h + d_s \phi_s \dot{U}_s = \tilde{g}_s \in \mathbb{L}_\infty \), and

\[
d_m \phi_m \dot{U}_m + d_s \phi_s \dot{U}_s = \sum_{i=m,s} \sum_{r=1}^{q} m_{ir}(\ddot{x}_i)(F_r \tau_r(t) - \Omega_r^i)
\]

where \( \Omega_r^i = d_i \phi_i \dot{\tau}_r \). Since \( S_i, e_i, k_i, \) and \( x_i \) in the control input are bounded, one has \( \tau_r \in \mathbb{L}_\infty \), leading to the boundedness of \( (d_m \phi_m \dot{U}_m + d_s \phi_s \dot{U}_s) \). Due to the fact that \( e_f = (F_s^h + d_s \phi_s \dot{U}_s) - (d_m \phi_m \dot{U}_m + d_s \phi_s \dot{U}_s) \), it follows that \( e_f \in \mathbb{L}_\infty \). That completes the proof.

Remark 2: The gain function \( c^i/c \) plays a key role, not only in achieving varying-gain regulation, but also in prescribed-time convergence. As the settling time approaches \( T_\varsigma \to 0 \) implies that \( c^i/c \) may be singular. To solve this problem and guarantee the smoothness in the singularity regions, a dynamic damped reciprocal (DDR) approach can be employed based on damped reciprocal approach [41]. Then \( 1/c \) can be replaced as the damped reciprocal \( \varsigma/(\varsigma^2 + \lambda_2^2) \) with \( \lambda_2 = \lambda_2^e \exp(-c^i/c^2) \), where \( \lambda_2 \) and \( \epsilon_1 \) denote the nominal damped coefficient and user-defined singularity threshold, respectively. The damped reciprocal approach has the following properties:

\[
\frac{c_1}{c_2 + \lambda_2^2} \approx \frac{1}{c}, \text{ if } |c| \gg \lambda_d, \\
\frac{c_1}{c_2 + \lambda_2^2} \approx \frac{0}{c}, \text{ if } |c| \ll \lambda_d.
\]

which indicates that the damped reciprocal will be bounded if \( |c| \) is far less than \( \lambda_d \), and thus eliminate potential singularity phenomena. Furthermore, DDR method is also applicable to other terms with \( \varsigma \) as the denominator.

Remark 3: For \( \forall t \in [t_k, t_{k+1}) \), we have

\[
\frac{d}{dt}(\tau_{ir}^j(t) - U_{ir}^j(t)) = \frac{d}{dt}((\tau_{ir}^j(t) - U_{ir}^j(t)) \times (\tau_{ir}^j(t) - U_{ir}^j(t))) = \frac{1}{\varsigma(t)} \tau_{ir}^j(t).
\]

It follows from (22) that \( \tau_{ir}^j(t) \) is bounded and continuously differentiable such that \( \dot{\tau}_{ir}^j(t) \leq \lambda c_i^j \). In view of the fact that \( \tau_{ir}^j(t_k) - U_{ir}^j(t_k) = 0 \) and \( \lim_{t \to t_k^+} \tau_{ir}^j(t) - U_{ir}^j(t) = \varsigma_i^j \), there exists a positive scalar \( \tau^*_i \) such that \( \{t_k + t_k \geq \tau^*_i \geq \varsigma_i^j / \varsigma_i^j \} \). Hence Zeno behaviour can be effectively eliminated in the proposed scheme.

Remark 4: It is worth mentioning that, when \( k_i \to +\infty \), according to L'Hopital rule, the EBLF (30) will degenerate into a commonly-used quadratic form

\[
\lim_{k_i \to +\infty} V_i = \lim_{k_i \to +\infty} \frac{1}{2} \frac{k_i^2}{\tilde{S}_i^T \tilde{S}_i} = \frac{1}{2} \tilde{S}_i^T \tilde{S}_i.
\]

It shows that the EBLF (30) can be still used for stability analysis and control synthesis in the case of no constraint, which is essentially different from conventional BLF approaches [34]. Therefore, compared with log-type BLF [34] and tangent-type BLF [42], the exponential-type BLF (30) is a more generalized form used in a unified framework to handle nonlinear systems with and without constraint requirements simultaneously.

Remark 5: Adaptive fuzzy control can deal with nonlinear disturbances and uncertainties by using the universal approximation ability of fuzzy systems. However, the convergence domain depends on the upper bound of the disturbance so that the corresponding convergence accuracy is difficult to calculate, which is reflected in the asymptotic [43]–[45] and finite time stability analysis [46]. Compared with the above literature, the proposed method in this paper ensures that the convergence accuracy can be arbitrarily specified within the physically realizable range, which is more suitable for practical space teleoperation.

In addition to this, the settling time can be user-defined rather than dependent on initial values or control parameters in [46], relaxing the parameter constraint of control design.

Remark 6: Compared with T-S fuzzy model used in the developed control scheme, TP model transformation [47]–[51] provides a promising alternative to model nonlinear systems with less conservatism. Hence, TP model transformation can be incorporated into the proposed prescribed-time fuzzy control framework using CNO type membership functions, which will be conducted to reduce the conservatism of the stability condition in the future works.

IV. SIMULATION RESULTS

To validate the effectiveness of the developed adaptive prescribed-time event-triggered control strategy, two identical 2-DOF manipulators are set as the master and slave part, respectively. The body parameters of the space teleoperation system are given as \( m_{s1} = 1.5kg, m_{s2} = 0.5kg, m_{s3} = 1.5kg, m_{s4} = 0.5kg, l_{m1} = 1.0m, l_{m2} = 0.8m \), \( l_{s1} = 1.0m, l_{s2} = 0.8m \). The initial states are set as \( q_m(0) = [0.3\pi 0.2\pi]^T \) (rad), \( q_s(0) = [0.1\pi 0.3\pi]^T \) (rad), \( \dot{q}_m(0) = [0 0]^T \) (rad/s), \( \dot{q}_s(0) = [0 0]^T \) (rad/s), \( h_m \) and \( h_s \) are external disturbances with \( h_i = [h_{i1} h_{i2}]^T \), where \( h_{i1} \) and \( h_{i2} \) are random numbers in the range of \([-0.2, 0.2]\). The control parameters are chosen as \( \alpha = 0.5, \beta = 0.2, T = 10s, \eta = 0.1, \phi_m = \phi_s = \text{diag}(0.6, 0.6) \), \( E = 0.001, \varpi_m = \varpi_s = 0.2, \varpim = \varpis = 1, \epsilon = 0.01, \kappa_2m = \kappa_2s = 5, \kappa_3m = \kappa_3s = 12, \kappa_5m = \kappa_5s = 12, \delta_m = \delta_s = 1, \delta_1m = \delta_1s = 1, \delta_3m = \delta_3s = 0.5, d_m = d_s = 1, R_i = I \). The time-varying constraint function is designed as \( k_m = k_s = 1.8\exp(-0.4t) + 0.25 \). The time delays are composed of jittering delays and constant Earth-Space delay measured by DLR in the FORROST-ASTRA W3L mission [52], as shown in Fig. 2.

A 3-rule IT2 T-S fuzzy model is developed to describe the force interaction (4) with \( K_{i1} = K_{i2} = K_{i3} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, C^i_{11} = \begin{bmatrix} 0.1 & b_{min} \\ 0 & 0.2 \end{bmatrix}, C^i_{21} = \begin{bmatrix} 0.2 & b_{avg} \\ 0 & 0.3 \end{bmatrix}, C^i_{31} = \begin{bmatrix} 0.5 & b_{max} \\ 0 \end{bmatrix}, M^i_{11} = \begin{bmatrix} 0.1 + m_{min} \\ 0 \end{bmatrix}, M^i_{21} = \begin{bmatrix} 0.3 + m_{avg} \\ 0.5 + m_{max} \end{bmatrix}, M^i_{31} = \begin{bmatrix} 0.5 + m_{max} \\ 0 \end{bmatrix}, \) where the operating domain is characterized by \( x_i^j \in [-2, 2] \) for \( i \in \{m, s\} \).
The uncertainty interaction parameters \( b \) and \( m \) satisfy \( 0.1 = b_{\text{min}} \leq b(\hat{x}_i) \leq b_{\text{max}} = 0.3 \) and \( 0.1 = m_{\text{min}} \leq m(\hat{x}_i) \leq m_{\text{max}} = 0.5 \), respectively. \( b_{\text{avg}} = (b_{\text{min}} + b_{\text{max}})/2 \) and \( m_{\text{avg}} = (m_{\text{min}} + m_{\text{max}})/2 \). The membership functions of the IT2 T-S fuzzy model are defined as follows:

\[
\mu_{\tilde{Q}_1}^{0}(\hat{x}_i) = 0.8 - 0.8/\exp\left(-\frac{||\hat{x}_i|| + 3}{0.55}\right),
\]

\[
\mu_{\tilde{Q}_1}^1(\hat{x}_i) = 1 - \exp\left(-\frac{||\hat{x}_i|| + 3}{0.55}\right),
\]

\[
\mu_{\tilde{Q}_1}^{1}(\hat{x}_i) = 0.8/\exp\left(-\frac{||\hat{x}_i||}{0.55}\right),\mu_{\tilde{Q}_1}^{2}(\hat{x}_i) = 1/\exp\left(-\frac{||\hat{x}_i||}{0.55}\right),
\]

\[
\mu_{\tilde{Q}_1}^{2}(\hat{x}_i) = 1 - \mu_{\tilde{Q}_1}^{1}(\hat{x}_i) - \mu_{\tilde{Q}_1}^{2}(\hat{x}_i),
\]

\[
\bar{\theta}_{i1}(\hat{x}_i) = \bar{\theta}_{i1}(\hat{x}_i) = \frac{1}{2}, \quad \bar{\theta}_{i2}(\hat{x}_i) = \bar{\theta}_{i2}(\hat{x}_i) = \frac{1}{2}.
\]

A 2-rule IT2 fuzzy controller is designed with

\[
\mu_{\tilde{Q}_1}^0(\hat{x}_i) = 1 - \exp\left(-\frac{||\hat{x}_i||^2}{3.5}\right), \mu_{\tilde{Q}_1}^1(\hat{x}_i) = 1 - \exp\left(-\frac{||\hat{x}_i||^2}{0.8}\right),
\]

\[
\bar{\theta}_{i1}(\hat{x}_i) = \bar{\theta}_{i1}(\hat{x}_i) = \frac{1}{2}, \quad \bar{\theta}_{i2}(\hat{x}_i) = \bar{\theta}_{i2}(\hat{x}_i) = \frac{1}{2}.
\]

The tracking performance of synchronization positions driven by the event-triggered prescribed-time control scheme (22) with the assigned settling time \( T = 10s \) is illustrated in Fig. 3, where \( x_{m1} \) (\( x_{s1} \)) and \( x_{m2} \) (\( x_{s2} \)) represent the \( x \)-direction and \( y \)-direction of the end-effector position of the master (slave) robot. It can be seen from Fig. 3 that the convergence performance within the prescribed settling time is guaranteed despite the existence of time-varying delays, external disturbances, and actuator faults. Besides the prescribed-time property, the convergence accuracy meets predetermined requirements (i.e., \( ||e_m|| \leq 4.1 \times 10^{-3}(m), ||e_m|| \leq 4.8 \times 10^{-3}(m/s), t > 10s \)), as demonstrated in Tab. 1. The above two properties show that the proposed method achieves the practically prescribed-time stability of the closed-loop teleoperation system. The constraint control performance of the sliding mode variable is given in Fig. 4, in which the green and pink solid line stands for the upper and lower prescribed constraint function, respectively. In addition, \( S_1 \) and \( S_2 \) represent the components of the sliding mode variable along the \( x \) and \( y \) directions, respectively. Fig. 4
Fig. 6. Released intervals of the proposed event-triggered controller for the master manipulator with respect to Joint 1.

Fig. 7. Released intervals of the proposed event-triggered controller for the master manipulator with respect to Joint 2.

Fig. 8. Released intervals of the proposed event-triggered controller for the slave manipulator with respect to Joint 1.

Fig. 9. Released intervals of the proposed event-triggered controller for the slave manipulator with respect to Joint 2.

shows that, with the proposed controller, the sliding mode variable never exceeds the preassigned range, and thus the constraint requirements on the full states can be satisfied according to (55).

To further demonstrate the prescribed-time property, we consider the case in which the settling time is assigned as $T = 5s$ with the other conditions being exactly the same as the above case. Fig. 5 shows that the prescribed-time convergence performance can be ensured in the case of setting a different settling time. Furthermore, the performance of convergence to a predetermined residual set within a pre-specified settling time is guaranteed, as seen in Tab. 1. Figs. 6-9 illustrate that the proposed event-triggered control can effectively reduce the computing burden of communication channels, which is beneficial to overcome the adverse influence of time delay on system performance.

In summary, the numerical simulation results show that the proposed control strategy effectively tackles the uncertain human-machine-environment interaction in space teleoperation subject to various severe conditions, including jittering time delays, unknown control direction, and actuator faults.

It breaks through the long-standing contradiction between quantitative error and control performance with less local communication burden, thus guaranteeing the settling time and tracking accuracy available to be estimated and specified in a quantitative way.

<table>
<thead>
<tr>
<th>Prescribed Steady-state time (s)</th>
<th>$|\hat{e}_m|(m)$</th>
<th>$|\dot{\hat{e}}_m|(m/s)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T = 10$</td>
<td>$4.1 \times 10^{-3}$</td>
<td>$4.8 \times 10^{-3}$</td>
</tr>
<tr>
<td>$T = 5$</td>
<td>$4.6 \times 10^{-3}$</td>
<td>$6.8 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

V. Conclusion

In this paper, the event-triggered practically prescribed-time control problem of a class of nonlinear space teleoperation systems subject to multiple constraints and uncertainties is investigated. Together with the exponential-type Lyapunov function, uncertain interaction with human-operator/environments, uncertain control input, and full-state constraint with respect to operation space are addressed in the framework of interval
type-2 fuzzy control. Different from traditional finite-fixed-time control, the settling time can be used as an explicit tunable parameter in the designed piece-wise function and can be arbitrarily specified in the physically realizable range according to the actual task requirements. Based on that, a new adaptive event-triggered control scheme is proposed to ensure that the synchronization tracking errors will converge to a prescribed residual set within user-defined settling time. We further show that the computing burden of communication channels can be effectively reduced via the developed event-triggered mechanism, which is beneficial to overcome the adverse influence of time delay on system performance. In particular, the proposed control structure has demonstrated a great potential to obtain specified arbitrarily performance indicators, thereby reducing the complexity of parameter selection. The developed approach also provides an effective balance mechanism for the long-standing contradiction between the quantified effect of event-triggered control and the need for control performance, thus expanding the practical application of event-triggered control. Detailed simulation results have been presented to show the superior properties of the proposed method. The application to multi-master-multi-slave space teleoperation systems will be further studied in the future.

REFERENCES


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