Sliding Mode Reliable Control Under Redundant Channel: A Novel Censored Analog Fading Measurement

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Abstract—This paper considers the sliding mode control problem with redundant channels, in which the transmitted signals may be subject to the fading phenomenon. Firstly, the censoring strategy is utilized to judge the fading degree of the transmitted signals. If the measurements from the primary channel fail to pass the censorship, the ones from the redundant channel will be examined by the detector. A hold-input compensatory scheme will be adopted if the measurements from all channels are disqualifications. Furthermore, a channel-dependent state observer is designed to estimate the unavailable states by taking the redundant transmission and censored measurement into consideration. And then, the channel-dependent sliding mode controller is constructed. Both the stochastic stability of the closed-loop system and the reachability of the specified sliding surface are analyzed, and the corresponding sufficient conditions have been obtained. Finally, the numerical simulation results are provided.

Index Terms—redundant channels, fading networks, sliding mode control, censoring strategy.

I. INTRODUCTION

With the prevalence of network technology, more and more system signals including sensor/control signals are transmitted via communication networks [1]–[3]. In order to ensure the reliability of transmitted signals, the redundant channel transmission (RCT) has been widely used in industrial systems [4]. The key feature of the RCT is that, when the transmitted signals via the primary channel drop out or occur an anomaly, other redundant channels will automatically take actions. Some results concerning control and estimate/filtering problems under the RCT have been recently reported in [5]–[7]. Zhang et al [5] utilized two parallel channels to reduce the uncertain packet dropout rate in sensor networks, while Song et al [6] investigated the predictive control problem of polytopic discrete-time systems with N redundant channels. The above literatures concerning RCT mainly focused on reducing the packet loss rate. However, until now, how to enhance the communication quality of fading networks via RCT, that is, how to discard useless signal subject to great fading, has not been well studied.

Fading in channels, mainly caused by the shadowing and multiple-path propagation, is a notable feature, especially, when transmitting through wireless networks. It can be divided into two categories according to the type of transmitted signal: digital or analog fading channels. For digital channels, the received signal strength will affect the packet loss rate to some extent [8]. For analog channels, the amplitude attenuation of fading signals can be simply modeled as a multiplicative noise process [9], [10], which may be regarded as a broader sense than packet loss in mathematical viewpoint. Besides, there are other analog fading models, such as the finite-state Markov fading model [11], [12], the Rice fading model [13], etc. Up to now, some interesting results have been reported on analog fading channels in [14]–[16] and the references therein. In [14], Su and Chesi studied the robust stability of polytopic uncertain discrete-time systems via fading channels, and the independent fading channel was dedicated to each actuator. Xu et al. [15] investigated the mean square stabilization problem under the fading channels with additive white Gaussian noise, where the instantaneous fading gains were assumed to be known to the receiver side. It should be pointed out that the above assumption is more suitable for the slow-fading channels, not the fast-fading channels [16]. In practical applications, if the transmitted signals suffer from great fading, they will become almost useless even bring great negative effect on the system performance. Hence, the measurement censoring method in this paper is for avoiding the above unfavourable phenomena via getting rid of bad signals with greater fading. Actually, the kind of censoring method has been usually employed to examine the outliers in many engineering applications. For example, Jiang and Chen [17] investigated the fusion of censored decisions in the area of distributed detection. Besides, the Tobit Type model was also combined with the measurement censoring in [18], [19]. Particularly, if the transmitted signal is greater than the set threshold, the censored measurement can be obtained directly by the original signal, otherwise, it is set as the threshold value in the censored region [18]. In [19], a two-value variable, as an indicator variable, was utilized to reflect whether the measurement passes the threshold test.

As well known, sliding mode control (SMC) is an effective robust control method for the dynamic system with parameter uncertainties and external disturbances [20]–[25]. Its main feature is to drive the system states onto a specified sliding
surface and the state systems tend to zero along this sliding surface. More recently, the SMC technique has been utilized to deal with the network-induced phenomena, such as the packet dropout [26] and quantization [27]. However, the problem of SMC under fading measurement or redundant channels has only received few attentions [28].

As shown from the above discussion and the designing procedure later in this work, the redundant channels subject to fading will bring some characteristics different from the single channel with fading. A key issue for the redundant channels subject to fading is how to provide a judge standard for the channel selection. This will result in more difficulty in the design and analysis of controller, which motivates the present work on the SMC problem under the redundant channels subject to fading phenomenon. To this end, the measurement censoring method is employed to examine whether the transmitted signal is suffering from great decline. Then, the RCT update rule is designed via this censorship. That is, if the measurements from the primary channel fail to pass the censorship, the one from the redundant channel will be adopted. Moreover, the fading parameter-dependent state observer is utilized to estimate the unavailable states, and then, the corresponding SMC law are designed to ensure the reachability of the specified sliding surface and the stochastic stability of the closed-loop system. The main contributions are summarized below:

1) The redundant channel is utilized to improve the communication quality of analog fading networks via the measurement censoring strategy.
2) A compensatory update scheme is introduced to attenuate the effect of the simultaneous transmission failure of both the primary channel and the redundant channel.
3) An integral-like sliding function is constructed, and both the state observer and the controller are designed via the censored fading parameters. The corresponding sufficient conditions are obtained.

Notation: \( \| \cdot \| \) is the Euclidean norm. \( \mathcal{E}\{x|y\} \) is the conditional expectation of stochastic variable \( x \), while \( \mathcal{E}\{x|y\} \) is the expectation of stochastic variable \( x \), with \( \mathcal{E}\{x|y\} \) is the conditional expectation of \( x \) on \( y \). \( \text{Var}\{\cdot\} \) is the variance of stochastic variable. The symmetric matrix \( M > 0 \) \((M < 0)\) means that \( M \) is positive-definite (negative-definite). The symbol \( \otimes \) refers to the Kronecker product. \( \text{diag}\{\cdots\} \) is a block-diagonal matrix. \( \lambda(\cdot) \) represents the eigenvalues of the corresponding matrix.

II. PROBLEM FORMULATION

A. Discrete-time System with Fading Measurements

Consider the following discrete-time system:

\[
\begin{align*}
    x(k+1) &= Ax(k) + B(u(k) + d(x(k), k)) \\
    z(k) &= E x(k)
\end{align*}
\]

(1)

where \( x(k) \in \mathbb{R}^n \) is the state vector, \( u(k) \in \mathbb{R}^m \) is the control input, and \( z(k) \in \mathbb{R}^p \) is the controlled output. \( d(x(k), k) \) denotes the unknown external disturbance in the control channel satisfying \( \|d(x(k), k)\| \leq \alpha \|x(k)\| \) with \( \alpha \) a known positive constant. \( A, B \) and \( E \) are the known constant matrices, and the matrix \( B \) is assumed to be full column rank.

The networked control system (1) with one redundant channel is shown in Fig.1, in which the channel \( a \) is the primal channel and the channel \( b \) is the redundant channel for increasing the reliability of signal transmission. In this work, the system state \( x(k) \) is assumed to be unavailable, and the output signals \( y_i(k) \in \mathbb{R}^q, i \in \{0, 1\} \) are measured by the \( i \)-th group of sensors. Meanwhile, when the measurement output \( y_i(k) \) is transmitted to the controller via channel \( a \) or \( b \), the signal fading may occur according to the following model:

\[
y_i(k) = \xi_i(k) C_i x(k) + v_i(k)
\]

(2)

where \( \xi_i(k) \in [0, 1] \) is the mutually independent random variable. \( C_i \in \mathbb{R}^{q \times n} \) is the known output matrix of the \( i \)-th group of sensors. The unknown disturbance input \( v_i(k) \in \mathbb{R}^q \) belongs to \( L_2([0, \infty]) \) and its peak value satisfies \( \|v_i(k)\| \leq \bar{v}_i \).

According to the above analysis, both the fading output signals from the primary channel \( a \) and the redundant channel \( b \) are available to the controller/observer. Thus, our control target is to design a SMC law based on per-designed RCT rule such that the closed-loop system under the fading channels can achieve the stochastic stability with the prescribed \( H_{\infty} \) performance.

B. Measurement Censoring and RCT Rule

As discussed in Introduction, the fading measurements suffering from great declines are usually useless for computing control signals, even have a great negative effect on the system performance. Hence, it is vital to get rid of bad signals with great fading. In this work, the Tobit Type model as in [18], [19] is adopted to describe the censored measurement \( \hat{y}_i(k) \) of the \( i \)-th transmission channel as follows:

\[
\hat{y}_i(k) = \begin{cases} 
    y_i(k) & \text{if } \xi_i(k) \geq \hat{\xi}_i \\
    0 & \text{if } \xi_i(k) < \hat{\xi}_i
\end{cases}
\]

(3)

where \( \hat{\xi}_i \) is the constant threshold of fading gain. Here, it should be pointed out that in practical applications, it would be possible for us to utilize some prior statistical information and method as Monte Carlo simulations to estimate the probability distribution of fading gain \( g_i(\xi_i(k)) \). Especially, when the measurement signal is transmitted through slow varying analogy fading networks, the instantaneous value of the fading
gain $\xi_i(k)$ can be estimated as well, and similar assumptions are shown in [15], [29], [30].

Actually, the expression (3) may be taken as a bad data detector and can be utilized to examine whether the measurements $y_i(k)$ pass the censorship. Moreover, the censored measurement $\bar{y}_i(k)$ can be expressed as

$$\bar{y}_i(k) = \theta_i(k)(\xi_i(k)C\bar{z}(k) + v_i(k))$$

with the variable $\theta_i(k)$ given as:

$$\theta_i(k) = \begin{cases} 1 & \text{if } \xi_i(k) \geq \hat{\xi}_i \\ 0 & \text{if } \xi_i(k) < \hat{\xi}_i. \end{cases}$$

It is easily obtained that the mathematical expectation of $\theta_i(k)$ is $\bar{\theta}_i \triangleq \mathbb{E}\{\theta_i(k)\} = \int_{\hat{\xi}_i}^{\infty} g_i(\xi_i(k))d\xi_i(k)$. Defining $\bar{\theta}_i(k) \triangleq \bar{\theta}_i(\xi_i(k))$ as the censored fading gain, its mathematical expectation and variance are given as

$$\bar{\bar{\theta}}_i \triangleq \mathbb{E}\{\bar{\theta}_i(k)\} = \int_{\hat{\xi}_i}^{\infty} g_i(\xi_i(k))\xi_i(k)d\xi_i(k),$$

$$\sigma^2_\bar{\theta}_i \triangleq \text{Var}\{\bar{\theta}_i(k)\} = \int_{\hat{\xi}_i}^{\infty} g_i(\xi_i(k))\xi^2_i(k)d\xi_i(k) - \bar{\bar{\theta}}_i^2.$$

**Remark 1:** In some existing works such as [18], [19], if the measurement output exceeds the threshold (whether or not the threshold is constant), it would be taken as the original signal, otherwise, it would be replaced by the threshold. Different from the above method, this present work utilizes the fading gain $\xi_i(k)$ in (3) to examine the measurement output signal $y_i(k)$. If $\xi_i(k)$ is less than the threshold $\hat{\xi}_i$, the fading measurement $y_i(k)$ would be discarded proactively. Thus, the useless fading measurement that suffers great decline can be gotten rid of so that the adverse impact from fading channels may be attenuated. Besides, similar to the two-variable value $p_{i,k}$ in [19], the variable $\theta_i(k)$ in (5) is introduced to express whether the measurements $y_i(k)$ pass the censorship, which will be utilized in the following design of RCT rule.

According to the above discussion and the expressions (3)-(5), the actual received measurement output $\tilde{y}(k)$ will be updated according to the following scheme:

$$\tilde{y}(k) = \theta_a(k)y_a(k) + (1 - \theta_a(k))\theta_b(k)y_b(k) + (1 - \theta_a(k))(1 - \theta_b(k))\bar{y}(k - 1).$$

If the fading gain $\xi_a(k)$ of the primary channel $a$ is greater than the threshold $\hat{\xi}_a$, one has $\theta_a(k) = 1$ such that the received measurement output is given as $\tilde{y}(k) = y_a(k) = \xi_a(k)C\bar{z}(k) + v_a(k)$. Otherwise, the fading gain $\xi_b(k)$ of the redundant channel $b$ will be judged whether to satisfy the threshold condition. Moreover, a particularly awful situation is also taken into consideration, that is, the fading gains of both two channels fail to pass the threshold test. For this case, the controller will use the previous received signal for reducing the fading effect, i.e., $\tilde{y}(k) = \bar{y}(k - 1)$, when $\theta_a(k) = \theta_b(k) = 0$.

**C. Channel-dependent Observer**

From the expression (2), it can be seen that the two independent channels have their own group of sensors, and their quality may be different. With this consideration, the channel-dependent observer/controller will be designed in this work. To this end, we introduce the channel token $\rho(k)$ as:

$$\rho(k) = \theta_a(k)a + (1 - \theta_a(k))\theta_b(k)b + (1 - \theta_a(k))(1 - \theta_b(k))\rho(k - 1).$$

Notice that if the transmitted signal is from channel $a$, i.e., $\theta_a(k) = 1$, the channel token is $\rho(k) = \theta_a(k)a$; if the transmitted signal from channel $b$, i.e., $\theta_a(k) = 0$ and $\theta_b(k) = 1$, one has $\rho(k) = b$. Especially, if there is not new transmitted signal from channels $a$ and $b$, the channel token is chosen as the latest update transmission channel, i.e., if $\theta_a(k) = \theta_b(k) = 0$, $\rho(k) = \rho(k - 1)$.

Then, the following channel token-dependent state observer is constructed:

$$\hat{x}(k + 1) = A\hat{x}(k) + Bu(k) + L_{\rho(k)}(\bar{y}(k) - \hat{\theta}_{\rho(k)}C_{\rho(k)}\hat{x}(k))$$

where $\hat{x}(k)$ is the state estimate, and the design matrices $L_{\rho(k)} \in \mathbb{R}^{n \times q}$ are dependent on channel token $\rho(k)$. As $\theta_a(k) = \theta_b(k) = 0$, $\rho(k) = \rho(k - 1)$, one has $L_{\rho(k)} = L_{\rho(k - 1)}$, that is, the gain matrix $L_{\rho(k)}$ will remain unchanged. The variable $\hat{\theta}_{\rho(k)} \triangleq \hat{\theta}_{\rho(k)}/\theta_{\rho(k)}$ is the selection ratio of censoring coefficient with $0 < \mathbb{E}\{\xi_{\rho(k)}(k)\} \leq \hat{\theta}_{\rho(k)} < 1$. Here, it should be pointed out that it is unreasonable for the observer to utilize the original fading coefficient $\mathbb{E}\{\xi_{\rho(k)}(k)\}$, instead of $\hat{\theta}_{\rho(k)}$, due to the combined effect of both the redundant transmission and the censored measurement.

Denoting $e(k) \triangleq x(k) - \hat{x}(k)$, the estimation error dynamics is obtained from (1) and (8):

$$e(k + 1) = \hat{\theta}_{\rho(k)}L_{\rho(k)}C_{\rho(k)}x(k) - L_{\rho(k)}\bar{y}(k) + Bd(x(k),k) + (A - \hat{\theta}_{\rho(k)}L_{\rho(k)}C_{\rho(k)})e(k).$$

**III. Main results**

**A. Sliding mode controller design**

In this work, the integral-like sliding function is constructed as:

$$s(k) = G\hat{x}(k) - GA\hat{x}(k - 1)$$

where the matrix $G \in \mathbb{R}^{m \times n}$ should be designed to ensure the matrix $GB$ to be invertible. Actually, by designing $G = B^TP$ with $P > 0$, the invertibility of matrix $GB = B^TPB$ can be ensured.

Corresponding to the previous observer (8), we design the following channel-dependent SMC law:

$$u(k) = K_{\rho(k)}\hat{x}(k) - (GB)^{-1}GL_{\rho(k)} \times (\bar{y}(k) - \hat{\theta}_{\rho(k)}C_{\rho(k)}\hat{x}(k)) - \varphi(k)\text{sgn}(s(k))$$

with the channel-dependent gain matrix $K_{\rho(k)} \in \mathbb{R}^{m \times n}$ and $\varphi(k) \triangleq \alpha||\hat{x}(k)||$. It is worth noting that the designed observer (8) and controller (11) depend on the channel token $\rho(k)$, especially, on the communication quality of censored measurement $\bar{y}(k)$ via the coefficient $\hat{\theta}_{\rho(k)}$.

**Remark 2:** In discrete-time systems, the control input is held as a constant during the sampling period. The finite
sampling rate results in a quasi-sliding mode (QSM), that is, the state trajectories would move within the neighborhood of the specified surface \( s(k) = 0 \). Since the SMC law is updated only at the sampling instants, the bandwidth of QSM domain can be reduced with the increase of sampling frequency. This is just the aim of conventional ideal sliding motion along the sliding surface \( s(k) = 0 \).

Substituting (11) into (1), we obtain the following closed-loop system

\[
x(k+1) = (A + BK(\rho(k)) + \hat{\theta}(\rho(k))\tilde{G}L(\rho(k))C(\rho(k)))x(k) - (BK(\rho(k)) + \hat{\theta}(\rho(k))\tilde{G}L(\rho(k))C(\rho(k)))e(k)
\]

\[-\tilde{G}L(\rho(k))\bar{y}(k) + B\tilde{d}(k)
\]

(12)

with \( \tilde{G} \triangleq B(GB)^{-1}G \) and \( \tilde{d}(k) \triangleq d(x(k), k) - \varphi(k)sgn(s(k)) \). Note that \( sgn^T(s(k))sgn(s(k)) \leq m \), where \( m \) is the dimension of control input \( u(k) \), then we have

\[ ||\tilde{d}(k)|| \leq \alpha ||x(k)|| + \alpha \sqrt{m} ||e(k)|| \leq (1 + \sqrt{m}) ||x(k)|| + \alpha \sqrt{m} ||e(k)||. \]

(13)

It follows from (2) and (6) that the received measurement signal \( \bar{y}(k) \) can be rewritten as:

\[
\bar{y}(k) = (\tilde{\beta}_1 C_a + \tilde{\beta}_3 C_b)x(k) + \tilde{\beta}_3 \bar{y}(k-1) + \tilde{\beta}_4 v_a(k) + \tilde{\beta}_5 v_b(k)
\]

(14)

with

\[
\beta_1(k) \triangleq \theta_a, \quad \beta_1(k) \triangleq \bar{\theta}_a,
\]

\[
\beta_2(k) \triangleq (1 - \theta_a)\bar{\theta}_b, \quad \beta_3(k) \triangleq (1 - \theta_a)(1 - \theta_b),
\]

\[
\beta_4(k) \triangleq \theta_a \bar{\theta}_b, \quad \beta_5(k) \triangleq (1 - \theta_b)\bar{\theta}_a,
\]

\[
\beta_6(k) \triangleq \beta_6(k) - \bar{\beta}_7 (r = 1, \ldots, 5).
\]

Denote \( \eta(k) \triangleq [x^T(k) \quad e^T(k) \quad \bar{y}^T(k-1)]^T \) and \( v(k) \triangleq [v_a^T(k) \quad v_b^T(k)]^T \). Substituting (14) into (9) and (12), we obtain the following augmented closed-loop system:

\[
\eta(k+1) = \begin{bmatrix} A_1 & \bar{G}(A_1 - C_1(k)) & 0 \\ A_2 - C_1(k) & A_3 + C_2(k) \\ \tilde{G}(B_1 - D_1(k)) & B_1 - D_1(k) & B_2 + D_2(k) \\ B_3 + D_4(k) & B_4 - D_5(k) & 0 \\ B_5 + D_5(k) & B_6 - D_5(k) & 0 \\ B_7 + D_5(k) & B_8 - D_5(k) & 0 \\ \end{bmatrix} \eta(k) 
\]

(15)

\[ z(k) = \bar{E} \eta(k) \]

with

\[
A_1 \triangleq \begin{bmatrix} A + BK(\rho(k)) & -BK(\rho(k)) \\ \end{bmatrix},
\]

\[
A_2 \triangleq \begin{bmatrix} \hat{L}(\rho(k)) - \tilde{L}(\rho(k)) - \beta_3 L(\rho(k-1)) \\ \end{bmatrix},
\]

\[
A_3 \triangleq \begin{bmatrix} \beta_1 C_a + \beta_2 C_b & 0 & \beta_4 I_p \\ \beta_3 C_a & \beta_2 C_b & \beta_4 I_p \\ \end{bmatrix},
\]

\[
B_1 \triangleq \begin{bmatrix} -\beta_4 L_a & -\beta_5 L_b \\ \end{bmatrix},
\]

\[
B_2 \triangleq \begin{bmatrix} \beta_4 I_p & \beta_5 I_p \\ \end{bmatrix},
\]

\[
C_1(k) \triangleq \beta_1(k)C_a + \beta_2(k)C_b, \quad D_1(k) \triangleq \beta_3(k)D_1 + \beta_5(k)D_2 (j = 1, 2),
\]

\[
\bar{L}(\rho(k)) \triangleq \hat{L}(\rho(k))L(\rho(k))C(\rho(k)), \quad \bar{C} \triangleq \bar{\theta}_a L_a C_a + (1 - \bar{\theta}_a)\bar{\theta}_b L_b C_b, \quad \bar{E} \triangleq \begin{bmatrix} E & 00 \end{bmatrix}, \quad \bar{C}_{11} \triangleq \begin{bmatrix} L_a C_a & 00 \end{bmatrix}, \quad \bar{C}_{12} \triangleq \begin{bmatrix} L_b C_a & 00 \end{bmatrix}, \quad \bar{C}_{13} \triangleq \begin{bmatrix} 00 & L(\rho(k-1)) \end{bmatrix}, \quad \bar{C}_{21} \triangleq \begin{bmatrix} C_a & 0 \end{bmatrix}, \quad \bar{C}_{22} \triangleq \begin{bmatrix} C_b & 0 \end{bmatrix}, \quad \bar{C}_{23} \triangleq \begin{bmatrix} 00 & I_p \end{bmatrix}, \quad \bar{D}_{11} \triangleq \begin{bmatrix} L_a C_a & 0 \end{bmatrix}, \quad \bar{D}_{12} \triangleq \begin{bmatrix} 00 & I_p \end{bmatrix}, \quad \bar{D}_{21} \triangleq \begin{bmatrix} I_p & 0 \end{bmatrix}, \quad \bar{D}_{22} \triangleq \begin{bmatrix} 00 & I_p \end{bmatrix}.
\]

In the sequel, both the stochastic stability of the closed-loop system (15) with prescribed \( \bar{H}_u \) performance and the reachability of specified sliding surface \( s(k) = 0 \) will be analyzed. To this end, the definitions and lemma will be given.

**Definition 1:** The closed-loop system (15) with \( v(k) = 0 \) is said to be stochastically stable if, for the initial condition \( \eta(0) \neq 0 \), one has \( \mathbb{E}\{\sum_{k=0}^{\infty} ||\eta(k)||^2 \} < \infty \).

**Definition 2:** Given a scalar \( \gamma > 0 \), the closed-loop system (15) with \( v(k) \neq 0 \) is said to be stochastically stable with the disturbance attenuation level \( \gamma \), if the controlled output \( z(k) \) under the initial condition \( \eta(0) = 0 \) satisfies

\[
\mathbb{E}\{\sum_{k=0}^{\infty} ||z(k)||^2 \} < \gamma^2 \sum_{k=0}^{\infty} ||v(k)||^2.
\]

For all nonzero \( v(k) \).

From the above analysis, it can be found that the stochastic varying matrices \( C_j(k) \) and \( D_j(k) \) are involved in the closed-loop system (15). We further denote the stochastic varying matrices \( F_j(k) \triangleq C_j(k)\eta(k) + D_j(k)v(k), \) and give the following lemma before the analysis of stochastic stability.

**Lemma 1:** For the stochastic varying matrices \( F_j(k) \) (\( j = 1, 2 \)) and any symmetric matrix \( M > 0 \), one has:

\[
\mathbb{E}\{F_j(k)M F_j(k)^T\} = \mathbb{E}\{\eta^T(k)[(\sigma_a^2 + \sigma_b^2)\hat{C}_j M \hat{C}_j + \eta^T(k)^T[(1 - \bar{\theta}_a)\hat{G}\hat{C}_j \hat{C}_j^T M \hat{C}_j + \eta^T(k)^T \eta(k)] + \beta_4 (\hat{G} \hat{C}_j \eta(k) + \hat{D}_j v(k)) M (\hat{G} \hat{C}_j \eta(k) + \hat{D}_j v(k)) \} + \beta_5 (\hat{G} \hat{C}_j \eta(k) + \hat{D}_j v(k)) M (\hat{G} \hat{C}_j \eta(k) + \hat{D}_j v(k)) \} + \beta_6 (\hat{G} \hat{C}_j \eta(k) + \hat{D}_j v(k)) M (\hat{G} \hat{C}_j \eta(k) + \hat{D}_j v(k)) \} + \beta_7 (\hat{G} \hat{C}_j \eta(k) + \hat{D}_j v(k)) M (\hat{G} \hat{C}_j \eta(k) + \hat{D}_j v(k)) \} + \beta_8 (\hat{G} \hat{C}_j \eta(k) + \hat{D}_j v(k)) M (\hat{G} \hat{C}_j \eta(k) + \hat{D}_j v(k)) \}.
\]

(18)

**Proof:** It can be calculated that the expression (17) holds since \( \mathbb{E}\{\beta_1(k)^T \} = 0 \), \( (r = 1, \ldots, 5) \). Then, we have

\[
\mathbb{E}\{F_j(k)^T M F_j(k)\} = \mathbb{E}\{\eta^T(k)^T C_j^T(k) M C_j(k) \eta(k) + 2 \eta^T(k)^T C_j^T(k) M D_j(k) v(k) \} + \eta^T(k)^T D_j^T(k) M D_j(k) v(k) \}.
\]

(19)

Notice that

\[
\mathbb{E}\{C_j^T(k) M C_j(k) \} \}
\begin{align}
&\sigma^2 + \bar{d}_1^2 \int G_{i,j}^T M \hat{C}_{i,j} + (1 - \bar{d}_2)(\sigma^2 + \bar{d}_2^2) \int G_{i,j}^T M \hat{C}_{j,i} \\
&+ \beta_3 \int G_{i,j}^T M \hat{C}_{j,i} - (\beta_4 \hat{C}_{i,j} + \beta_2 \hat{C}_{j,i} + \beta_4 \bar{C}_{j,i})^T \\
&\times M(\beta_4 \hat{C}_{j,i} + \beta_4 \bar{C}_{j,i} + \beta_5 \bar{C}_{j,i})^T \\
&\leq \eta^T(k) A_{i}^T P A_{i} \eta(k) + (\hat{A}_i \eta(k) + B_i \nu(k))^T G \\
&\times (\hat{A}_i \eta(k) + B_i \nu(k)) + \tilde{d}^T(k) B^T P B \tilde{d}(k) \\
&+ 2\eta^T(k) A_{i}^T G (\hat{A}_i \eta(k) + B_i \nu(k)) + 2\eta^T(k) A_{i}^T P B \tilde{d}(k) \\
&+ 2(\hat{A}_i \eta(k) + B_i \nu(k))^T G B \tilde{d}(k) + F \tilde{d}^T(k) G F_1(k) \\
&+ (A_{2i} \eta(k) + B_i \nu(k))^T P (A_{2i} \eta(k) + B_i \nu(k)) \\
&+ \tilde{d}^T(x(k), k) B^T P B \tilde{d}(x(k), k) + \tilde{d}^T(x(k), k) P F_1(k) \\
&+ 2(\hat{A}_i \eta(k) + B_i \nu(k))^T P B \tilde{d}(x(k), k).
\end{align}

For the scalars \( \varsigma_k > 0 \ (h = 1, ..., 4) \), it can be calculated that
\begin{align}
&2\eta^T(k) A_{i}^T G (\hat{A}_i \eta(k) + B_i \nu(k)) \\
\leq &\varsigma_1 \eta^T(k) A_{i}^T P A_{i} \eta(k) + \varsigma_2^{-1} (\hat{A}_i \eta(k) + B_i \nu(k))^T \\
&\times G (\hat{A}_i \eta(k) + B_i \nu(k)), \\
&\varsigma_2 \eta^T(k) A_{i}^T P B \tilde{d}(k) \\
\leq &\varsigma_3 \eta^T(k) A_{i}^T P A_{i} \eta(k) + \varsigma_2^{-1} \hat{d}^T(k) B^T P B \tilde{d}(k), \\
&\varsigma_4 \eta^T(k) A_{i}^T (\hat{A}_i \eta(k) + B_i \nu(k)) \\
&+ \varsigma_2^{-1} \hat{d}^T(k) B^T P B \tilde{d}(k).
\end{align}

Following from (13) and (24), one has
\begin{align}
&\tilde{d}^T(x(k), k) B^T P B \tilde{d}(x(k), k) \\
\leq &\alpha^2 (1 + \sqrt{m})^2 \delta^2 T(x(k), k) \leq 2\alpha^2 m \delta e^T T(x(k), k)(e(k), k) \\
&\tilde{d}^T(x(k), k) d(x(k), k) \leq \alpha^2 \delta \tilde{d}^T(x(k), k)(x(k), k).
\end{align}

Then the closed-loop system (15) will be stochastically stable with the disturbance attenuation level \( \gamma \).

**Proof:** Choose the Lyapunov function candidate as
\begin{align}
&V(k) = V_1(k) + V_2(k) + V_3(k)
\end{align}
with
\begin{align}
V_1(k) &= x^T(k) P x(k), \\
V_2(k) &= e^T(k) P e(k), \\
V_3(k) &= \tilde{y}^T(k - 1) Q \tilde{y}(k - 1).
\end{align}
Along the closed-loop system (15), we have
\begin{align}
&\mathcal{E}\{V_1(k + 1)|\eta(k)\} + \mathcal{E}\{V_2(k + 1)|\eta(k)\} \\
&\leq \varsigma_1 \eta^T(k) A_{i}^T P A_{i} \eta(k) + (\hat{A}_i \eta(k) + B_i \nu(k))^T G \\
&\times (\hat{A}_i \eta(k) + B_i \nu(k)) + \tilde{d}^T(k) B^T P B \tilde{d}(k).
\end{align}

Besides, note that:
\begin{align}
(\hat{A}_i \eta(k) + B_i \nu(k))^T Q (\hat{A}_i \eta(k) + B_i \nu(k))
\end{align}
Thus, by utilizing Lemma 1 again, we obtain
\[
\mathcal{E}\{\gamma_T(k)Qy(k)|\eta(k)\} = (A_3\eta(k) + B_2v(k))^T Q (A_3\eta(k) + B_2v(k)) + F_2(k)QF_2(k) \\
\leq \eta_T(k)\{\beta_1\hat{C}_{21}Q\hat{C}_{21} + \bar{\beta}_2\hat{C}_{22}^TQ\hat{C}_{22} + \beta_3\hat{C}_{23}^TQ\hat{C}_{23}\}\eta(k) \\
+ \beta_4\{\hat{\psi}_k\hat{C}_{21}\eta(k) + \hat{D}_{21}(v(k))\}^T \hat{Q}\eta(k) + \hat{D}_{21}(v(k)) + \bar{\beta}_5\hat{D}_{22}(v(k)) + \hat{D}_{22}(v(k))^T \times \hat{Q}\eta(k) + \hat{D}_{22}(v(k)).
\]
(35)

Therefore, for \(v(k) = 0, \eta(k) \neq 0\), we have from (25), (33) and (35):
\[
\mathcal{E}\{V(k+1)|\eta(k)\} - V(k) \leq \eta_T(k)\hat{\Gamma}\eta(k)
\]
(36)

with
\[
\hat{\Gamma} = \text{diag}\{-P + \mu_4\delta I, -P + \mu_2\delta I, -Q\} + \chi_1A_T^TPA_1 + \chi_2A_T^T\hat{G}A_1 + \chi_3A_T^T\hat{P}A_2 + \beta_1(\hat{C}_{11}^T\hat{G}\hat{C}_{11} + \hat{C}_{11}^T\hat{P}\hat{C}_{11}) \\
+ \hat{C}_{21}^T\hat{G}\hat{C}_{21} + \bar{\beta}_2(\hat{G}_{12}^T\hat{C}_{12} + \hat{G}_{12}^T\hat{P}\hat{C}_{12} + \hat{C}_{22}^T\hat{Q}\hat{C}_{22}) \\
+ \bar{\beta}_3(\hat{C}_{13}^T\hat{G}\hat{C}_{13} + \hat{C}_{13}^T\hat{P}\hat{C}_{13} + \hat{C}_{23}^T\hat{Q}\hat{C}_{23}).
\]
(37)

It can be easily found that the inequality (23) implies that \(\hat{\Gamma} < 0\). Thus, it can be obtained from (36) that there exists a scalar \(\psi > 0\) satisfying \(\mathcal{E}\{V(k+1)|\eta(k)\} - V(k) < -\psi\eta(k)^2\). By taking mathematical expectation and summing up the above expression, we obtain \(\mathcal{E}\{V(k+1)\} - \mathcal{E}\{V(0)\} < -\psi\mathcal{E}\{\sum_{k=0}^{\infty} \eta(k)^2\}\), which implies \(\mathcal{E}\{\sum_{k=0}^{\infty} \eta(k)^2\} < \frac{1}{\psi}\mathcal{E}\{V(0)\} < \infty\). So, the closed-loop system (15) is stochastically stable according to Definition 1.

Next, we shall analyze the \(H_{\infty}\) performance of closed-loop system (15). To this end, considering the following index function with \(v(k) \neq 0\) under zero-initial condition
\[
J(k) = \mathcal{E}\{V(\eta(k+1))|\eta(k)\} - V(\eta(k)) \\
+ \mathcal{E}\{\|z(k)\|^2\} - \gamma^2\|v(k)\|^2.
\]
(38)

By applying Schur complement and the expressions (33) and (35), the condition \(J(k) < 0\) is equivalent to
\[
\left[ \begin{array}{c} \eta(k) \\ v(k) \end{array} \right]^T \Gamma \left[ \begin{array}{c} \eta(k) \\ v(k) \end{array} \right] < 0
\]
(39)

which can be ensured by the condition (23). Summing up (38) from 0 to \(\infty\), one gets
\[
\sum_{k=0}^{\infty} J(k) = \mathcal{E}\{V(\eta(\infty))\} - V(\eta(0)) \\
+ \mathcal{E}\{\sum_{k=0}^{\infty} \|z(k)\|^2\} - \gamma^2\sum_{k=0}^{\infty} \|v(k)\|^2 < 0.
\]
(40)

It is noted that \(V(0) = 0\) and \(\mathcal{E}\{V(\infty)\} \geq 0\), we have
\[
\mathcal{E}\{\sum_{k=0}^{\infty} \|z(k)\|^2\} < \gamma^2\sum_{k=0}^{\infty} \|v(k)\|^2,
\]
which complete the proof.

C. Analysis of reachability

The following theorem gives a sufficient condition on the reachability of the specified sliding surface (10). It will be shown that the disturbance \(v(k)\) has a great effect on the reaching domain \(\Omega\) of the specified sliding surface (10).

Theorem 2: Consider the system (1) with the redundant transmission protocol (6) and the channel-dependent SMC law (11). If there exist matrices \(P > 0, Q > 0\), matrices \(K_P\) and \(L_P\), and scalars \(\delta > 0, \epsilon_h > 0, h = 1, \ldots, 4\) satisfying (24) and the following matrix inequality:
\[
\Omega \triangleq \tilde{Z} + R + \sum_{j=1}^{3} \left(\kappa_jT_j\tilde{C}_{j1}\right) < 0
\]
(41)

with
\[
\tilde{Z} \triangleq \text{diag}\{-P + \mu_4\delta I, -P + \mu_2\delta I, -Q\},
\]
\[
R \triangleq \chi_1A_T^TPA_1 + \chi_2A_T^T\hat{G}A_1 + \chi_3A_T^T\hat{P}A_2 + 2A_T^TPA_4, \\
\kappa_1 \triangleq \sigma_a^2 + \gamma^2 + \beta_1^2, \quad \kappa_2 \triangleq (1 - \theta_a)(\sigma_a^2 + \gamma^2) + \beta_5^2, \\
\kappa_3 \triangleq \beta_3, \quad \mu_3 \triangleq 2\alpha_2m + 4\chi_3\alpha_2^2(1 + \sqrt{m})^2 + 2\chi_5\alpha_2^2, \\
\mu_4 \triangleq 4\chi_3\alpha_2^2m, \quad T_1 \triangleq G, \quad T_2 \triangleq P, \quad T_3 \triangleq Q, \\
A_4 \triangleq \left[ BK_{P}\right] - B K_{P}\right],
\]
and other parameters defined as in Theorem 1, \(\text{then the state trajectories of the closed-loop system (15) will be forced (in mean square) into the following domain} \Omega \text{ around the specified sliding surface} s(k) = 0:\)
\[
\Omega \triangleq \left\{s(k) \mid \|s(k)\| \leq \sqrt{\lambda_{\text{max}}(B_T^T PB)} \Delta \right\}
\]
(42)

with
\[
\Delta \triangleq \left(\begin{array}{ll} (2\chi_4\beta_4^2 + 2\beta_4) & L_a^TPL_a \\
L_a^TPL_a & \left(2\chi_4\beta_4^2 + 2\beta_4\|Q\|\right) \|\hat{v}_a\| \\
+ \left(2\chi_4\beta_4^2 + 2\beta_4\right) & L_a^T L_b^T \\
L_a^T L_b^T & \left(2\chi_4\beta_4^2 + 2\beta_4\|Q\|\right) \|\hat{v}_b\| \\
+ 4\beta_4^25 & \chi_4 L_a^T L_b^T \\
\chi_5 & \left(5L_a^T L_b^T \right)\|\hat{v}_b\| \end{array} \right)
\]

Proof: Construct the following Lyapunov function candidate as:
\[
U(k) = V(k) + s_T(k)(B_T^T PB)^{-1}s(k).
\]
(43)

By means of the state observer (8) and control law (11), we have from (10)
\[
s(k+1) = GB K_{P}\varphi(k) + \varphi(k)G B sgn(s(k)).
\]
(44)

Then, the second term in (43) can be calculated as
\[
s_T(k+1)(B_T^T PB)^{-1}s(k+1) \leq \left(2BK_{P}\varphi(k) - B K_{P}\varphi(k)\right)^T \\
\times P(BK_{P}\varphi(k) - BK_{P}\varphi(k)) + 2\text{sgn}(s(k))\varphi_T(k)\|B_T^T PB\varphi(k)\|\text{sgn}(s(k)).
\]
(45)
Noting that \( \text{sgn}^T(s(k)) \text{sgn}(s(k)) \leq m \), we obtain from (24)
\[
2\text{sgn}^T(s(k))\varphi^T(k)B^TPB\varphi(s(k)) \leq 2\alpha^2m\delta x^T(k)x(k).
\]
(46)

Besides, we have
\[
2\chi_4|B^TPB||\vec{d}(k)|^2 \\
\leq 4\chi_4\alpha^2(1 + \sqrt{m})^2\|x(k)\|^2 + 4\chi_4\alpha^2m\delta\|e(k)\|^2, \\
2\chi_5|B^TPB|\|dx(k),x(k)\|^2 \leq 2\chi_5\alpha^2\|x(k)\|^2.
\]
(47)

(48)

By adopting the similar derivation to the expressions (33) and (35) and utilizing (45)-(48), it yields from (43) that
\[
\mathcal{E}\{U(k + 1)|\eta(k)\} - U(k) \\
\leq \mathcal{E}^T(k)\begin{pmatrix} \dot{Z} & + 2\bar{\beta}_3\bar{\varphi}_T & + \bar{\beta}_3\bar{\varphi}_T & + \bar{\beta}_3\bar{\varphi}_T \end{pmatrix} \begin{pmatrix} \dot{X}_1 & \dot{X}_2 & \dot{X}_3 & \dot{X}_4 \end{pmatrix} + \chi_3\bar{\varphi}^T \begin{pmatrix} \dot{X}_4 \end{pmatrix}
\]

with prescribed \( H_{\infty} \) performance in Theorem 1 and the reachability of the specified sliding surface \( s(k) = 0 \) in Theorem 2 can be guaranteed simultaneously.

**Theorem 3:** Given the disturbance attenuation level \( \gamma > 0 \), if there exist symmetric matrices \( P > 0, Q > 0, T_{11} > 0, T_{22} \in \mathbb{R}^{(m-n)\times(n-m)}, \) and scalars \( \delta > 0, \varsigma_h > 0 (h = 1, \ldots, 4) \) satisfying (24) and the following matrix inequalities for all \( \rho(k) \in \Xi^* \)
\[
\begin{bmatrix}
\begin{array}{cccc}
\ddot{Z} & * & * & * \\
\ddot{\psi}_1 & \ddot{\phi}_1 & * & * \\
\ddot{\psi}_2 & 0 & \ddot{\phi}_2 & * \\
\ddot{\psi}_3 & 0 & 0 & \ddot{\phi}_3 \\
\ddot{\psi}_4 & 0 & 0 & 0
\end{array}
\end{bmatrix} < 0,
\]
(51)

\[
\begin{bmatrix}
\begin{array}{cccc}
\ddot{Z} & * & * & * \\
\ddot{\psi}_1 & \ddot{\phi}_1 & * & * \\
\ddot{\psi}_2 & 0 & \ddot{\phi}_2 & * \\
\ddot{\psi}_3 & 0 & 0 & \ddot{\phi}_3 \\
\ddot{\psi}_4 & 0 & 0 & 0
\end{array}
\end{bmatrix} < 0,
\]
(52)

Taking the mathematical expectation of (49) yields
\[
\mathcal{E}\{\Delta U(k)\} \triangleq \mathcal{E}\{U(k + 1)\} - \mathcal{E}\{U(k)\} \\
\leq \mathcal{E}(k)|\tilde{\eta}(k)| - s^T(k)(B^TPB)^{-1}s(k) + \Delta.
\]
(50)

If the state trajectories run out of the sliding domain \( \mathcal{O} \), i.e., \( \|s(k)\| \geq \sqrt{\max(B^TPB)\Delta} \), one has from (41) and (50) that \( \mathcal{E}(U(k + 1)) - U(k) < 0 \), which implies that the state trajectories will be forced back to the sliding domain \( \mathcal{O} \) again. Therefore, the reachability of specified sliding domain \( \mathcal{O} \) is attained in mean-square sense.

**Remark 3:** It is promised by Theorem 2 that the system state will be driven into the domain \( \mathcal{O} \) in (42) around the specified surface (so-called the sliding bound), but not staying on the sliding surface. This is just the characteristic of discrete-time SMC. It is noted that this domain \( \mathcal{O} \) involves in the parameters \( \bar{v}_i \) (i = a, b), i.e., the width of this domain is influenced by the disturbances \( v_i(k) \). Besides, the sliding domain \( \mathcal{O} \) can be further minimized through searching the sliding gain matrix \( G \) via some optimization algorithms as in [31].

**D. Solving algorithm**

It should be pointed out that the sufficient conditions (23) and (41) in Theorems 1 and 2 contain the nonlinear coupled terms \( P^T(B^TPB)^{-1}B^TP \), which are difficult to be solved. In the sequel, we shall give a solvable conditions such that both the stochastic stability of the closed-loop system (15) with
Then the stochastic stability of closed-loop system (15) with $H_\infty$ performance level $\gamma$ and the reachability of specified sliding domain $\mathcal{O}$ in (42) can be guaranteed simultaneously. Moreover, $K_{\rho(k)} = T_{11}^{-1} \hat{K}_{\rho(k)}$ and $L_{\rho(k)} = P^{-1} U_{\rho(k)}$.

Proof: First, for all $\rho(k) \in \mathbb{S}$, the matrices $K_{\rho(k)}$ are given as:

$$K_{\rho(k)} = T Y B K_{\rho(k)} = T \begin{bmatrix} I \\ 0 \end{bmatrix} = T_{11} K_{\rho(k)} = \begin{bmatrix} T_{11} K_{\rho(k)} \\ 0 \end{bmatrix}. \quad (53)$$

Due to,

$$T Y + (TY)^T - T Y P^{-1} (TY)^T - P = - (TY - P) P^{-1} (TY)^T - P < 0 \quad (54)$$

we can obtain

$$- T Y P^{-1} (TY)^T < P - TY - (TY)^T. \quad (55)$$

Denote $\hat{K}_{\rho(k)} = T_{11} K_{\rho(k)}$ and $U_{\rho(k)} = P L_{\rho(k)}$. By using Schur complement, it is easily shown that the expressions (23) and (41) can be guaranteed by (51) and (52), respectively. Moreover, these matrix inequalities (51) and (52) can be solved via linear matrix inequality method for given parameters $\varepsilon_b > 0$ ($h = 1, ..., 4$).

Remark 4: About the computational complexity of the above solving algorithm in the conditions (51)-(52) in Theorem 3, we need to solve 9 LMIs with $(n^2 + n + \frac{1}{2} q^2 + \frac{1}{2} q + 2 mn + 2 n q + 1)$ decision variables. Moreover, the free matrix $T$ with a special structure is utilized to overcome the difficulty from $P B K$ in Theorems 1 and 2. It should be pointed out that if the redundant model in this work is extended to $N$ redundant channels, both the dimensions of LMIs and the number of decision variables will greatly increase.

IV. NUMERICAL SIMULATIONS

Example 1: Consider the discrete-time system (1)-(2) with following parameters:

$$A = \begin{bmatrix} 1.1 & -0.1 \\ 0.5 & 0.4 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad E = \begin{bmatrix} 0.05 & 0 \\ 0.3 & 0.1 \end{bmatrix}, \quad C_a = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C_b = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}.$$

Suppose that the statistical information of fading channels is known, where the fading gains obey the following probability density functions:

$$g_a(\xi_a(k)) = 0.005(\xi_a^{0.89}(k) - 1), \quad 0 \leq \xi_a(k) \leq 1,$$

$$g_b(\xi_b(k)) = 0.003(\xi_b^{1.32}(k) - 1), \quad 0 \leq \xi_b(k) \leq 1.$$

Select the threshold $\tilde{\gamma}_a = 0.85$. Then, the mathematical expectations of censored fading gain $\bar{v}_i(k)$ ($i = a, b$) can be obtained as $0.7272, 0.6310,$ while the variances $\sigma_i^2$ can be calculated as $0.1582, 0.1958$. The mathematical expectations of variable $b_i(k)$ are 0.7712 and 0.6717. For the $H_\infty$ attenuation level $\gamma = 0.32$, solving the LMIs (24) and (51)-(52), yields

$$P = \begin{bmatrix} 3.957 & -1.448 \\ -1.448 & 1.153 \end{bmatrix}, \quad Q = \begin{bmatrix} 0.046 & -0.014 \\ -0.014 & 0.040 \end{bmatrix},$$

$$T = \begin{bmatrix} 0.179 & -4.836 \\ 0 & 2.577 \end{bmatrix}, \quad K_a = \begin{bmatrix} -0.426 & -0.079 \\ 0 & 0 \end{bmatrix},$$

$$K_b = \begin{bmatrix} -0.448 & -0.018 \\ 0 & 0 \end{bmatrix}, \quad U_a = \begin{bmatrix} 0.483 & 0.395 \\ -0.186 & -0.233 \end{bmatrix},$$

$$U_b = \begin{bmatrix} 0.597 & 0.278 \\ -0.230 & -0.183 \end{bmatrix}, \quad \delta = 0.0278.$$ 

Therefore, according to $K_{\rho(k)} = T_{11}^{-1} \hat{K}_{\rho(k)}$ and $L_{\rho(k)} = P^{-1} U_{\rho(k)}$, we obtain the desirable gains:

$$K_a = \begin{bmatrix} -2.386 & -0.441 \\ -0.117 & 0.048 \end{bmatrix}, \quad K_b = \begin{bmatrix} -2.507 & -0.103 \\ -0.015 & -0.142 \end{bmatrix}, \quad L_a = \begin{bmatrix} 0.144 & 0.022 \\ -0.019 & -0.131 \end{bmatrix}.$$

Under the initial condition $x(0) = [0.5 \ 0.7]^T$ and $\dot{z}(0) = [0.3 \ 0.9]^T$ with $v(k) = 0$, the simulation results are shown in Fig.2, which illustrates the stochastic stability of closed-loop system (15).

To verify the reliability of proposed method, some comparisons are provided. Firstly, suppose that there is only one channel $a$, i.e., not redundant channel $b$. Fig.3(a) shows the case that the controller directly utilizes the fading measurements $y_a(k)$ from the channel $a$ without using censored strategy, and Fig.3(b) shows the case that the proposed censored strategy is utilized in the channel $a$, i.e., $\hat{y}_a(k) = \theta_a(k) y_a(k) + (1 - \theta_a(k)) \bar{y}_a(k)$. That is, when the measurement signal from the channel $a$ does not pass the censoring threshold, the last signal $\bar{y}_a(k) - 1$ will be used for the controller. It can be seen from Fig.3(a) that the fading measurements will make the performance degradation or even instability. The censored fading measurements can partly improve the system performance seen from Fig.3(b). Then, the redundant channel case with the proposed censored strategy is shown in Fig.2(c), i.e., $\hat{y}(k) = \theta_a(k) y_a(k) + (1 - \theta_a(k)) \theta_b(k) y_b(k) + (1 - \theta_a(k))(1 - \theta_b(k)) \bar{y}(k) - 1,$ which attains a better convergence than the one in Fig.3(b).

In the sequel, we further verify the $H_\infty$ disturbance rejection attenuation level under the zero-initial condition $x(0) = 0$, and the disturbances $v_i(k)$ are selected as the stochastic bounded signals which form the uniform distribution with
The channel token \( \gamma \).

The observer state \( \rho(k) \).

The time response of \( \gamma \).

Fig. 2. Simulation results under nonzero initial condition with \( v(k) = 0 \).

(a) Case without censored strategy

(b) Case with censored strategy

Fig. 3. The comparison with or without censored strategy

\[ \bar{v}_a = \bar{v}_b = 0.0707 \text{ (partly shown in Fig. 4(a)).} \]

The evolution of the following function

\[ \gamma_d(k) = \sqrt{\frac{\sum_{n=0}^{k} \| z(n) \|^2}{\sum_{n=0}^{k} \| v(n) \|^2}} \]

is shown in Fig. 4(b), which illustrates the time response of \( \gamma_d(k) \) is much smaller than the given \( H_\infty \) attenuation level \( \gamma = 0.32 \).

Fig. 4. The verification of \( H_\infty \) performance

**Example 2:** Consider a three-order RLC circuit system shown in Fig. 5, which is composed by two resistances \( R_1, R_2 \), two capacitors \( C_1, C_2 \), and one inductor \( L \). The dynamic model of the circuit obtained via the famous Kirchoff Current and Voltage laws is given below:

\[
\begin{align*}
C_1 \dot{V}_{C_1}(t) &= -\frac{1}{R_1}(V_{C_1}(t) - V_{C_2}(t)) + \frac{1}{R_2}u(t) \\
C_2 \dot{V}_{C_2}(t) &= \frac{1}{R_2}(V_{C_1}(t) - V_{C_2}(t)) + i_L(t) \\
\dot{i}_L(t) &= V_{C_2}(t)
\end{align*}
\]

where \( V_{C_1}(t) \) and \( V_{C_2}(t) \) are the capacitance voltages, and \( i_L(t) \) is the current of the inductor \( L \). Let \( x_1(t) = V_{C_1}(t) \), \( x_2(t) = V_{C_2}(t) \) and \( x_3(t) = i_L(t) \), which can be rewritten as

\[
\dot{x}(t) = \begin{bmatrix}
-1 & \frac{1}{R_1C_1} & \frac{1}{R_1C_2} & 0 \\
\frac{1}{R_2C_1} & 0 & \frac{1}{R_2C_2} & 0 \\
\frac{1}{R_2C_1} & 0 & \frac{1}{R_2C_2} & 0 \\
0 & \frac{1}{R_1C_1} & 0 & 0
\end{bmatrix} x(t) + \begin{bmatrix}
\frac{1}{R_1C_1} \\
0 \\
0 \\
0
\end{bmatrix} u(t).
\]

Suppose that the circuit parameters are chosen as \( R_1 = 10 \Omega \), \( R_2 = 7 \Omega \), \( C_1 = 1 \text{F} \), \( C_2 = 1 \text{F} \) and \( L = 2 \text{H} \). Therefore, the above system matrices with the sampling time \( T = 0.5s \) are given as:

\[
A = \begin{bmatrix}
0.9334 & 0.0652 & -0.0169 \\
0.0652 & 0.8744 & -0.4730 \\
0.0848 & 0.2365 & 0.9396 \\
\end{bmatrix}, \quad B = \begin{bmatrix}
0.0483 \\
0.0017 \\
0.0001
\end{bmatrix}.
\]

The measurement matrices are given as \( C_a = \text{diag}\{1, 1, 1\}, \quad C_b = \text{diag}\{0.9, 0.8, 1.1\}, \) and \( E = \text{diag}\{0.2, 0.3, 0.5\} \). Besides, the external disturbance is chosen as \( d(x(k), k) = 0.1 \sqrt{x_1^2(k) + x_2^2(k) + x_3^2(k)} \).

The fading parameters and disturbances for both primary channel and redundant channel are the same with Example 1. For the \( H_\infty \) attenuation level \( \gamma = 0.6 \), solving the LMIs (24) and (51)-(52), we obtain the desirable control and observer gains:

\[
K_a = \begin{bmatrix}
-1.4116 & -2.2216 & 0.7006 \\
-3.5944 & -2.3303 & 1.4267 \\
-0.0083 & 0.196 & 0.0039 \\
0.0492 & 0.0434 & -0.0163 \\
0.0000 & 0.0035 & 0.0001 \\
0.0117 & 0.0249 & -0.0022 \\
0.1093 & 0.0576 & -0.0297 \\
0.0007 & 0.0064 & 0.0000
\end{bmatrix},
\]

\[
L_a = \begin{bmatrix}
-0.0000 \\
0.0000 \\
0.0000
\end{bmatrix}, \quad L_b = \begin{bmatrix}
0.0000 \\
0.0000 \\
0.0000
\end{bmatrix}.
\]
The simulation results are shown in Fig. 6 with the initial conditions $\mathbf{x}(0) = \begin{bmatrix} 1 & 0.7 & -0.5 \end{bmatrix}^T$ and $\hat{\mathbf{x}}(0) = \begin{bmatrix} 0.5 & 1 & -0.3 \end{bmatrix}^T$. Considering the stochastic characteristic of the channel fading and the external disturbances, we give the results of 50 individual experiments in this example. Fig. 6(c) shows the characteristic of discrete-time SMC, that is, the sliding variable can be forced into the domain and will not escape from it. All these results show the effectiveness of the desired SMC law with the redundant channel transmission under the analog fading networks.

![Fig. 6. Simulation results under 50 individual experiments](image)

V. CONCLUSION

This paper has investigated the SMC problem under redundant channel subject to fading measurements. The proposed censored measurement strategy can effectively attenuate the impact of some worse fading signals, by discarding the bad data via the measurement censoring strategy. Compared with some existing works on the redundant channel, the proposed redundant update scheme in this work has utilized the hold-input strategy to avoid the worst case that no information would be received by the controller/observer. It should be pointed out that the redundant channel might bring computational complexity on the sufficient conditions. Hence, the number of redundant channels requires compromise consideration in practical applications.

REFERENCES


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