Reduced-order Extended Dissipative Filtering for Nonlinear Systems with Sensor Saturation via Interval Type-2 Fuzzy Model

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Abstract—The system nonlinearity, sensor saturation and the uncertainty will hamper the analysis and affect the control performance. Filtering is a signal processing method which facilitates the system analysis and synthesis by signal estimation or noise suppression. To achieve generalized filtering problem for nonlinear systems with sensor saturations with lower computational burden, this paper addresses the reduced-order extended dissipative filter design for nonlinear sensor-saturated system which is modeled by interval type-2 (IT2) T-S fuzzy system. For IT2 T-S fuzzy systems, the main challenge exists in the acquisition of the information in IT2 membership functions (MFs) for analysis and design. A membership-function-dependent (MFD) method is applied to capture the information of the MFs for reducing the conservativeness introduced by MFs not involved in the analysis. An extended dissipative filtering method, under imperfect premise matching (IPM) concept that the membership functions of the filter are different from those of the model, is proposed for sensor-saturated IT2 fuzzy systems. The proposed method has a high flexibility in parameters adjustment of both the filter and the design condition, including extended dissipative matrices, approximation MFs and sensor saturation degree, one can freely choose the parameters according to the required performance of the fuzzy filter. A numerical example is given to demonstrate the effectiveness of the results.

Index Terms—Interval type-2 fuzzy system, reduced-order filter, sensor nonlinearity.

I. INTRODUCTION

Type-1 T-S fuzzy system has achieved great success in the analysis of nonlinear systems. It combines the linear subsystems by means of fuzzy set weighting, which makes it possible to use the analysis method similar to that of linear systems while modeling the nonlinearities. In recent years, some notable works of literature of FMB control system are presented in [1], [2]. In [1], on the basis of the fault-tolerant control research in [3], a fault-tolerant control design of FMB systems with additive and multiplicative actuator faults was proposed. [2] proposed a novel $\ell_2 - \ell_\infty / H_\infty$ control design for FMB system, in which an MF online learning policy was applied to increase the control performance. The IT2 fuzzy system improves the modeling of type-1 fuzzy system, in which the IT2 MFs can capture the uncertainty in nonlinear dynamics. Notable results for the control related problem of IT2 fuzzy systems have been obtained in the literature. [4] was the pioneering work proposed the IT2 fuzzy modeling concept and control method, and stability analysis technique under the fuzzy-model-based framework. On the stability analysis of IT2 fuzzy systems, [5] made a further work, in which the mismatched MFs between fuzzy model and fuzzy controller were considered to improve design flexibility and robustness property of the fuzzy controller. Based on the above work, [6]–[8] studied the control problem of IT2 fuzzy systems. Among them, [6], [8] mainly focused on IT2 fuzzy network control systems, while [7] gave controller design conditions satisfying extended dissipative performance. In addition, other control related issues such as tracking control [9], model reduction [10], [11] and fault detection [12] have also been discussed in the literature. In recent years, the IT2 fuzzy model has been further extended to the form of polynomial IT2 fuzzy model, and some notable related achievements can be found in [13]–[15].

Fuzzy filtering is a common solution for the state estimation of fuzzy systems. Some previous notable studies on filter design of fuzzy systems are demonstrated in [16]–[18]. Sensor saturation is a kind of nonlinearity that is crucial to system performance. For example, in [19]–[21], sensor saturation was considered in the modeling. [19] presented a distributed event-based filtering scheme for nonlinear systems. [20] and [21] considered $H_\infty$ filtering for multi-rate multisensor systems and Markov jump systems, respectively. For IT2 fuzzy systems, the problem of filter design was discussed in some previous literature such as [22]–[25]. Among them, [22] and [25] investigated the problem of extended dissipative filtering for continuous IT2 fuzzy system. In [24] and [23], based on IT2 fuzzy modeling, the $H_\infty$ filtering problem was studied for the continuous systems with nonlinear sensor and discrete sensor network systems, respectively. However, for different filtering performance requirements, some previous studies may provide guiding ideas, specific problems still need to be analyzed and designed for new applications. From the modeling point of view, the modeling capability of the IT2 fuzzy model is enhanced by capturing the uncertainty by the IT2 fuzzy sets. A more accurate fuzzy model is fundamentally significant to support the fuzzy-model-based (FMB) analysis and control design. However, due to immature analysis and
design techniques for IT2 FMB control systems, research on this class of nonlinear systems is still open with many challenges to overcome. To the best of the authors’ knowledge, the design of reduced-order extended dissipative filter for discrete-time IT2 fuzzy systems with sensor saturations has not been fully discussed. Besides, the utilization of the information of MFs is not much considered in the existing works but it plays import role to relax the results for system analysis and filter design when IPM concept and MFD technique are employed [26].

In this paper, the reduced-order extended dissipative filter design for discrete-time nonlinear system with sensor saturation via IT2 framework is studied. For an IT2 fuzzy system with sensor saturation, we aim to design a reduced-order filter with mismatched MFs to estimate the signal of the original system. Similar to [19], sensor saturation in IT2 fuzzy systems is decomposed into the linear and the nonlinear parts, in which the latter can be considered as a state to facilitate the formation of LMI-based conditions which describes the extended dissipative performance of the filtering error system. The discrete-time reduced-order extended dissipative IT2 FMB filter designed in the paper is a generalized filter, which can be converted into other filters by adjusting the parameters of the extended dissipative matrices, so that the application of the filter is extensive for nonlinear systems with uncertainty. In addition, the IPM IT2 FMB filter is flexible in the selection of MFs. The main contributions of the paper can be summarised in the following aspects: 1) A reduced-order extended dissipative fuzzy filter, a novel LMI-based condition is designed, 2) in the constraints for obtaining the reduced-order extended dissipative filter, a piecewise-linear MFD method which considers the error of the upper and lower bound MFs and their approximate MFs is embedded to relax the condition, and compared with other IT2 fuzzy filter design methods such as [22], [24], richer information from the MFs is utilized to support the analysis and design leading to superior performance of the extended dissipative filter. The rest of the paper is organized as follows. Section II presents the problem formulation and preliminaries. Section III presents the main results. Section IV gives an example to show the effectiveness. Section V concludes this paper.

II. Problem Formulation and Preliminaries

A. Original System Model

In this paper, we consider the following discrete-time sensor-saturated IT2 T-S fuzzy model (1) that represents a nonlinear system subject to uncertainty with $r$ rules:

\[
x(k+1) = \sum_{i=1}^{r} \tilde{w}_i(x(k))[A_i x(k) + B_i \omega(k)],
\]

\[
y(k) = \sum_{i=1}^{r} \tilde{w}_i(x(k))[\chi(C_i x(k)) + D_i \omega(k)],
\]

\[
z(k) = \sum_{i=1}^{r} \tilde{w}_i(x(k))[E_i x(k) + F_i \omega(k)],
\]

where $\tilde{w}_i(x(k))$ is the premise variable $i = 1, 2, \ldots, r$; $\tilde{w}_i(x(k)) = \frac{\varrho_i(x(k)) \pi_i(x(k)) + \pi_i(x(k)) \pi_i(x(k))}{\sum_{i=1}^{r} (\varrho_i(x(k)) \pi_i(x(k)) + \pi_i(x(k)) \pi_i(x(k)))}, \forall i$,

\[
\sum_{i=1}^{r} \tilde{w}_i(x(k)) \leq 1, \quad \varrho_i(x(k)) \text{ and } \pi_i(x(k)) \text{ are the lower and upper grades of membership, respectively.}
\]

\[
\tilde{w}_i(x(k)) \geq \tilde{w}_j(x(k)) \text{ for all } i.
\]

The nonlinear functions $\varrho_i(x(k))$ and $\pi_i(x(k))$ satisfy: 1) $0 \leq \varrho_i(x(k)) \leq 1$ and $0 \leq \pi_i(x(k)) \leq 1$. 2) $\varrho_i(x(k)) + \pi_i(x(k)) = 1$. $x(k) \in \mathbb{R}^{n_x}$, $y(k) \in \mathbb{R}^{n_y}$, $u(k) \in \mathbb{R}^{n_u}$, $\omega(k) \in \mathbb{R}^{n_w}$ and $z(k) \in \mathbb{R}^{n_z}$ are the system state vector, the measured output, the control input vector, the external input and the controlled output, respectively; $\chi(i)$ is the saturation function which satisfies $\chi(\phi_i) = \text{sign}(\phi_i) \text{min}(\phi_{i,max}, |\phi_i|)$. $A_i$, $B_i$, $C_i$, $D_i$, $E_i$, $F_i$ are matrices with appropriate dimensions, respectively; $k = 1, 2, \ldots$ is the discrete time instant.

Proposition 1: [19] Assuming that there exist diagonal matrices $T_1$ and $T_2$ such that $0 \leq T_1 < I \leq T_2$, then the saturation function $\chi(C_i x(k))$ in (1) can be decomposed into a linear and a nonlinear part as $\chi(C_i x(k)) = T_1 C_i x(k) + \Lambda(C_i x(k))$, where $\Lambda(C_i x(k))$ is a nonlinear vector-valued function satisfying sector condition which can be described as follows:

\[
\Phi_j = -\Lambda^T(C_j x(k)) (\Lambda(C_j x(k)) - T C_j x(k)) \geq 0
\]

where $T = T_2 - T_1$.

B. Fuzzy filter

We consider the following fuzzy filter to estimate the IT2 fuzzy system.

\[
\pi(k+1) = \sum_{j=1}^{q} \tilde{m}(\hat{z}(k))[\bar{A}_j \pi(k) + \bar{B}_j y(k)],
\]

\[
\tilde{z}(k) = \sum_{j=1}^{q} \tilde{m}(\hat{z}(k))[\bar{E}_j \pi(k)],
\]

where $\tilde{w}_i(x(k))$ is the premise variable $j = 1, 2, \ldots, q$; $\hat{z}(k) \in \mathbb{R}^{n \times n}$ is the system state vector of the filter; $\bar{A}$, $\bar{B}$ and $\bar{E}$ are matrices of the fuzzy filter with appropriate dimensions.

C. Filtering error system

The filtering error system is shown as follows.

\[
\zeta(k+1) = \sum_{i=1}^{r} \sum_{j=1}^{q} h_{ij}(\zeta(k))[\bar{A}_{ij} \zeta(k) + \bar{B}_{ij} \omega(k) + \bar{B}_{ij} \Lambda(k)],
\]

\[
e(k) = \sum_{i=1}^{r} \sum_{j=1}^{q} h_{ij}(\zeta(k))[\bar{E}_{ij} \zeta(k) + \bar{F}_{ij} \omega(k)],
\]
where \( \zeta(k) = [z^T(k), \pi^T(k)]^T \),
\[
\tilde{A}_{ij} = \begin{bmatrix} A_i & 0 \\ \mathcal{B}_i T_j C_i & \mathcal{A}_j \end{bmatrix}, \tilde{B}_{ij} = \begin{bmatrix} B_i & 0 \\ \mathcal{B}_i D_j & \mathcal{B}_j \end{bmatrix}, 
\tilde{E}_{ij} = \begin{bmatrix} E_i & -E_j \end{bmatrix}, \tilde{F}_i = \begin{bmatrix} F_i \end{bmatrix},
\]
where \( h_{ij}(\zeta(k)) = \sum_{i=1}^q \sum_{j=1}^q \tilde{w}_i(x(k)) \tilde{m}_j(\hat{x}(k)) = 1 \).

D. Definitions

Definition 1: [17] The closed-loop system (4) is said to be asymptotically stable under \( \omega(k) = 0 \) if the following is achieved:
\[
\lim_{k \to \infty} [\Theta(k)] = 0.
\]

For an asymptotically stable closed-loop system (4), we have \( e \equiv e(k) \in \ell_2(0, \infty) \) when \( \omega \equiv \omega(k) \in \ell_2(0, \infty) \).

Assumption 1: [27]-[29] Let \( \Theta \geq 0 \), \( \Omega_1 \leq 0 \), \( \Omega_2 \) and \( \Omega_3 = \Omega_3^T \) be matrices such that the following conditions are satisfied:
- \( \| \tilde{F}_i \| + \| \Theta \| = 0 \)
- \( \| [\Omega_1 + \Omega_2] \| + \| \Theta \| = 0 \)
- \( \tilde{F}_i^T \Omega_1 \tilde{F}_i + \tilde{F}_i^T \Omega_2 + \Omega_2 \tilde{F}_i + \Omega_3 \geq 0 \)

Definition 2: [28], [29] For given matrices \( \Theta, \Omega_1, \Omega_2 \) and \( \Omega_3 \) satisfying Assumption 1, the closed-loop system (4) is said to be extended dissipative if the following inequality holds for any \( k > 0 \) and all \( \omega(k) \in \ell_2(0, \infty) \) under zero initial condition:
\[
\sum_{k=0}^{T_j} J(k) \geq \sup_{0 \leq k \leq T_j} e(k)^T \Theta e(k)
\]
where \( J(k) = e(k)^T \Omega_1 e(k) + 2e(k)^T \Omega_2 \omega(k) + \omega(k)^T \Omega_3 \omega(k) \).

Remark 1: [28], [29] When utilizing the extended dissipative filter, the performance requirements of the filter should be determined first, and then the value of the extended dissipative matrices \( \Theta, \Omega_1, \Omega_2, \Omega_3 \) should be selected. Selections of extended dissipative matrices for several known filters are given as follows. If \( \Theta = 0 \), \( \Omega_1 = -I \), \( \Omega_2 = 0 \) and \( \Omega_3 = \gamma^2 I \), extended dissipative performance is transferred to the \( \mathcal{H}_\infty \) performance. If \( \Theta = I \), \( \Omega_1 = 0 \), \( \Omega_2 = 0 \) and \( \Omega_3 = \gamma^2 I \), extended dissipative performance is transferred to the \( \ell_2-\ell_\infty \) performance. If \( \Theta = 0 \), \( \Omega_1 = 0 \), \( \Omega_2 = I \) and \( \Omega_3 = \gamma I \), extended dissipative performance is transferred to the passivity performance. If \( \Theta = 0 \), \( \Omega_1 = Q \), \( \Omega_2 = S \) and \( \Omega_3 = R - \alpha I \), extended dissipative performance is transferred to the strict \( (Q, S, R) \)-dissipativity. Relevant studies on filtering or control design with the above specific performances can be found in [24], [30]-[32].

Proposition 2: [5], [33] Denote the lower and upper membership functions of \( h_{ij}(\theta(k)) \) as \( \tilde{h}_{ij}(\theta(k)) \) and \( 
\tilde{h}_{ij}(\theta(k)) \) respectively, which are chosen as the following piecewise linear forms:
\[
\tilde{h}_{ij}(\theta(k)) = \sum_{k=1}^q \sum_{i=1}^2 \sum_{j=1}^2 \sum_{n=1}^n \sum_{i=1}^n v_{i, k}(\theta_r) \bar{q}_{ij i_1 i_2 \ldots i_n k},
\]
where \( \bar{q}_{ij i_1 i_2 \ldots i_n k} \) is determined by the extended dissipative performance if there exist matrices \( P > 0 \), \( L > 0 \), \( \Pi_{ij} \), \( \Pi_{ij} \), \( X_{ij} < 0 \) and \( Y_{ij} > 0 \) with appropriate dimensions satisfying the following inequalities for \( i = 1, 2, \ldots, r; j = 1, 2, \ldots, q; \)
\[
\Xi_{ij} + \Pi_{ij} - \Pi_{ij} + \sum_{k=1}^q \sum_{n=1}^n \bigg[ -\bar{q}_{ij i_1 i_2 \ldots i_n k} \bar{q}_{ij i_1 i_2 \ldots i_n k}^T \bigg] \leq 0,
\]
where \( \bar{q}_{ij i_1 i_2 \ldots i_n k} \) and \( \bar{q}_{ij i_1 i_2 \ldots i_n k} \) are constant scalars to be determined; \( 0 \leq v_{i, k}(\theta_r) < 1 \) and \( v_{i, k}(\theta_r) + v_{i, k}(\theta_r) = 1 \) for \( r, s = 1, 2, \ldots, n; i = 1; \), and \( v_{i, k}(\theta_r) = 0 \) if otherwise. Then it is obtained that \( \sum_{k=1}^q \sum_{n=1}^n \sum_{i=1}^q \sum_{i=1}^q \Pi_{ij} v_{i, k}(\theta_r) = 1 \).

III. MAIN RESULTS

Theorem 1: Consider the closed-loop IT2 T-S fuzzy system in (4). Given scalar \( \mu > 0 \) and matrices \( \Omega_1, \Omega_2, \Omega_3 \) and \( \Theta \), it is asymptotically stable with an extended dissipative performance if there exist matrices \( P > 0, L > 0, \Pi_{ij}, \Pi_{ij}, X_{ij} < 0 \) and \( Y_{ij} > 0 \) with appropriate dimensions satisfying the following inequalities for \( i = 1, 2, \ldots, r; j = 1, 2, \ldots, q; \)
\[
\Xi_{ij} + \Pi_{ij} - \Pi_{ij} + \sum_{k=1}^q \sum_{n=1}^n \bigg[ -\bar{q}_{ij i_1 i_2 \ldots i_n k} \bar{q}_{ij i_1 i_2 \ldots i_n k}^T \bigg] \leq 0,
\]
where
\[
\Xi_{ij} = \begin{bmatrix} \tilde{A}_{ij}^T & \tilde{B}_{ij}^T \\ \tilde{B}_{ij}^T & \tilde{B}_{ij} \end{bmatrix} + \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix} - \mathcal{A}_j \mathcal{A}_j^T \leq 0,
\]
where \( \mathcal{A}_j \) stands for \( A + A^T \).

Proof 1: First, choose a Lyapunov functional candidates as
\[
V(k) = \zeta(k)^T P \zeta(k).
\]
Denote $\Omega_1 = -\Omega_1^T \Omega_1$, where $\Omega_1$ is a matrix with appropriate dimension and consider $\Delta V(k) = V(k+1) - V(k)$ and (2), then considering S-procedure [34].

$$\Delta V(k) - J(k) \leq \Delta V(k) - J(k) + \mu \sum_{j=1}^{q} \Phi_j$$

$$\leq \sum_{i=1}^{r} \sum_{j=1}^{q} h_{ij} \lambda^T(k) \Xi_{ij} \lambda(k)$$

$$\leq \sum_{i=1}^{r} \sum_{j=1}^{q} h_{ij} \lambda^T(k) \{ \Xi_{ij} + \Pi_{ij} - \Pi_{ij} \}$$

$$+ \sum_{k=1}^{r} \{ -h_{k} \Pi + \tilde{T}_{k} \Pi + (\alpha_{kl} - \beta_{kl}) \Xi_{k} - \alpha_{kl} \Pi_{kl} + (\Delta h_{k} - \alpha_{kl} \Pi_{kl} - X_{kl} + \tilde{T}_{k} \Pi + \tilde{T}_{k} \Pi_{kl}) \} \lambda(k)$$

$$= \sum_{i=1}^{r} \sum_{j=1}^{q} \sum_{k=1}^{r} \{ -\xi_{kl} \Pi_{kl} + \tilde{T}_{k} \Pi_{kl} + (\alpha_{kl} - \beta_{kl}) \Xi_{k} - \alpha_{kl} \Pi_{kl} + (\Delta h_{k} - \alpha_{kl} \Pi_{kl} - X_{kl} + \tilde{T}_{k} \Pi + \tilde{T}_{k} \Pi_{kl}) \} \lambda(k)$$

where $\lambda(k) = [\zeta^T(k), \omega^T(k), \Lambda^T(k)]$. From (11) to (13), we know that $\Delta V(k) - J(k) < 0$, which implies under zero initial condition for $\forall k \geq 0$ which will lead to

$$\sum_{l=0}^{k-1} J(l) \geq V(k). \quad (15)$$

From (14), it is obtained that

$$V(k) = \zeta^T(k) P \zeta(k) \geq \zeta^T(k) L \zeta(k) \geq 0. \quad (16)$$

Then we have:

$$\sum_{l=0}^{k-1} J(l) \geq \zeta^T(k) L \zeta(k). \quad (17)$$

Similar to [7], the following proof is divided into two cases: $\|\Theta\| = 0$ and $\|\Theta\| \neq 0$.

Case 1: $\|\Theta\| = 0$

From (17), we can easily know that

$$\sum_{k=0}^{T_f} J(k) \geq 0. \quad (18)$$

Be aware of $\epsilon(k)^T \Theta e(k) = 0$, then the inequality (5) holds.

Case 2: $\|\Theta\| \neq 0$

From Assumption 1, $\|\Theta\| \neq 0$ implies that $\|\hat{F}_{i}\| = 0$, $\Omega_1 = 0$, $\Omega_2 = 0$ and $\Omega_3 > 0$. Notice (14), then for $k \geq 0$ we can get

$$\sum_{k=0}^{T_f} J(k) - e^T(k) \Theta e(k) \geq \sum_{k=0}^{T_f} J(k) - \sum_{i=1}^{r} h_{ij} L \zeta(k)$$

$$\geq \sum_{k=0}^{T_f} J(k) - \sum_{i=1}^{r} h_{ij} L \zeta(k). \quad (19)$$

Notice $\Omega_1 < 0$, then we have $\Delta V(k) < 0$. Then the filtering error system (4) is asymptotically stable. The proof is completed.

Remark 2: $T_2$ can be chosen as $I$ and thus $T$ determines the size of the sector-shaped nonlinear region. The larger the value of $T$, the higher the admissibility of Theorem 1 for saturation. Nevertheless, when the conditions in Theorem 1 are satisfied for a large saturation region, the conservativeness of Theorem 1 will increase correspondingly, thus affecting the performance of the obtained reduced-order extended dissipative filter. Therefore, when selecting $T$, it is necessary to first estimate the degree of saturation, and on this basis to find the $T$ that satisfies both the saturation tolerance condition and the filter performance.

Theorem 2: Consider the closed-loop IT2 T-S fuzzy system in (4). Given scalar $\mu > 0$ and matrices $\Omega_1$, $\Omega_2$, $\Omega_3$ and $\Theta$, it is asymptotically stable with an extended dissipative performance index $\gamma$ if there exist matrices $\bar{P}$, $\bar{Q}$, $\bar{A}_f$, $\bar{B}_f$, $\bar{C}_f$, $\bar{L} \bar{P}_{ij}$, $\bar{P}_{ij}$, $\bar{X}_{ij} < 0$ and $\bar{Y}_{ij} > 0$ with appropriate dimensions satisfying the following inequalities for $i = 1, 2, \ldots, r$; $j = 1, 2, \ldots, c$:

$$\bar{E}_i \bar{F}_j + \bar{F}_j \bar{E}_i$$

$$\bar{P} = \begin{bmatrix} \bar{P} & \bar{Q} \\ \bar{Q} & \bar{P} \end{bmatrix}$$

$$\bar{H} = \begin{bmatrix} \bar{H}_{i,j} \\ \bar{H}_{j,i} \end{bmatrix}$$

$$\bar{A}_f = \begin{bmatrix} \bar{A}_f \\ \bar{A}_f \end{bmatrix}$$

$$\bar{B}_f = \begin{bmatrix} \bar{B}_f \\ \bar{B}_f \end{bmatrix}$$

The filter are given as

$$\begin{bmatrix} \bar{X}_{ij} \\ \bar{E}_j \end{bmatrix} = \begin{bmatrix} \bar{B}_{i,j}^{-1} & 0 \\ 0 & \bar{I} \end{bmatrix} \begin{bmatrix} \bar{A}_j \\ \bar{B}_j \end{bmatrix}.$$
Proof 2: Partition $P$ as

$$P = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix} > 0, \quad P_2 = \begin{bmatrix} P_4 \\ 0 \end{bmatrix}$$

where $P_4$ is a nonsingular matrix. Define

$$\Psi = \begin{bmatrix} I & 0 \\ P_3^{-1}P_4^T \end{bmatrix}, \quad \Xi = \begin{bmatrix} P_1 & 0 \\ 0 & I \end{bmatrix}$$

Denote $\Theta = \Theta^T\hat{\Theta}$, where $\Theta$ is a matrix with appropriate dimension. Then applying Schur complement, (11)-(14) indicate (31)-(34) by replacing $\hat{\Xi}_i$, $\Pi_i$, $\bar{\Xi}_i$, and $\bar{\Omega}_i$ with $\hat{\Xi}_i$, $\Pi_i$, $\bar{\Xi}_i$, and $\bar{\Omega}_i$.

$$\hat{\Xi}_i + \Pi_i - \bar{\Pi}_i + \sum_{k=1}^{r} \sum_{l=1}^{q} [ - \Sigma_{kl} \hat{\Xi}_i ] \Pi_{kl} + \Pi_{kl} + (\delta_{kl} - \beta_{kl}) \bar{\Xi}_i - \alpha_{kl} \Pi_{kl} + (\beta_{kl} - \alpha_{kl}) \bar{\Xi}_i - \alpha_{kl} \Pi_{kl} + (\beta_{kl} - \alpha_{kl}) \bar{\Xi}_i - \alpha_{kl} \Pi_{kl} < 0$$

$$L > 0, \hat{\Xi}_i > 0, \bar{\Omega}_i > 0, L - P < 0,$$

$$\begin{bmatrix} -L & \hat{V}_{ij} \hat{\Theta}^T \\ * & -I \end{bmatrix} < 0,$$

where

$$\hat{\Xi}_i = \begin{bmatrix} -P & \hat{E}_{ij}^T \Omega_2 \\ * & -\Omega_3 - S[F_{ij}^T \Omega_2] \end{bmatrix}$$

$$\hat{C}_i = [C_i, 0].$$

Performing congruence transformation to (31)-(34) by $\text{diag}(\Psi, I, I, I, \Xi), \text{diag}(\Psi, I, I, I, \Xi), \Xi$ and $\text{diag}(\Psi, I)$, respectively. Then (20)-(23) is obtained. Make $P_4^{-1}P_3 = I$, then (27) can be obtained by considering (30). The proof is completed.

IV. Simulation Example

Considering an IT2 fuzzy system with the following parameters:

$$A_1 = \begin{bmatrix} -0.1 & 0.1 & -0.2 \\ 0.2 & 0.5 & 0.1 \\ -0.3 & 0 & 0.3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.1 & 0.1 & -0.1 \\ 0.1 & 0.4 & 0.1 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.2 & 1.6 & 0.9 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.3 & 1.8 \\ 0.9 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 1.5 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1.4 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.8 & 0.1 & -0.1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.9 & 0.1 & -0.2 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.4 \end{bmatrix}, \quad F_2 = \begin{bmatrix} -0.2 \end{bmatrix}.$$
Original signal and its estimations

Estimation error

Membership Grade

Given \( \omega = \{0 \text{ for } 0 \leq k < 30; 0.1 \sin(0.2(k - 30))e^{-0.1(k - 30)} \text{ for } k \geq 30\} \), the results are shown in Fig. 2 and Fig. 3. Fig. 2 shows the original signal \( z(k) \) and its estimation signal by IT2 reduced-order filter.

Case 2: 1st-order reduced-order filter, \( \gamma_{\min} = 0.3860 \)

\[
\begin{align*}
\bar{A}_1 &= \begin{bmatrix} 0.0804 \end{bmatrix}, & \bar{A}_2 &= \begin{bmatrix} 0.1200 \end{bmatrix}, \\
\bar{B}_1 &= \begin{bmatrix} -0.2920 \end{bmatrix}, & \bar{B}_2 &= \begin{bmatrix} -0.3192 \end{bmatrix}, \\
E_1 &= \begin{bmatrix} -0.9942 \end{bmatrix}, & E_2 &= \begin{bmatrix} -1.0688 \end{bmatrix}.
\end{align*}
\]

From the results, under the sensor saturation, the transformed \( H_{\infty} \) reduced-order IT2 FMB filter can estimate signal \( z(k) \) efficiently. In addition, noticing \( 0.3789 < 0.3860 \), it indicates the higher the order of the reduced-order IT2 FMB filter, the better the performance of the estimation. As mentioned in Remark 2, before applying Theorem 2, it is necessary to estimate the degree of the saturation to determine the value of \( T \). A large \( T \) will affect the performance of the filter and even make the search for the filter fail. Therefore, one possible direction of optimization for the algorithm is the expression of saturation in LMI-based conditions. Besides, in spite of the acquisition of the information in upper and lower bounds of the uncertain MFs, there are limitations in approximating the original MFs with piecewise-linear MFs. One is that, for highly complex nonlinearity, the effect of piecewise-linear approximation on nonlinearity is not ideal. In addition, when the MFs are nonlinear and complex, it is necessary to use piecewise-linear MFs with a large number of segments to approximate them, which will make the algorithm suffer from a heavy calculation burden. MFs approximation methods based on polynomial function [35] or Taylor series [26], [36] can improve the above-mentioned, which can be one of the directions of future work.

V. Conclusion

In this paper, the problem of reduced-order extended dissipative filter design for discrete-time IT2 T-S fuzzy systems with sensor saturations is investigated. The sensor saturation is treated as sensor nonlinearity in a sector region. By means of convex linearization, the design method of reduced-order extended dissipative filter expressed in LMI condition is obtained. To facilitate the design of IPM filter and reduce the conservativeness of the condition, an MFD method based on piecewise linear membership functions is introduced. The proposed MFD method improves the utilization of the information in MFs leading to relaxed analysis and design results, and enhanced design flexibility in terms of freely choosing the filter’s membership functions. The related result will be helpful to the
analysis of other fuzzy-model-based problems. In addition, the proposed IT2 fuzzy extended dissipative filtering method has strong applicability and can be used to obtain other specific types of filters as required. The effectiveness of the analysis and design is verified by a simulation example.

REFERENCES


