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Novel Random Access Schemes for Small Data Transmission

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***Abstract*—Fifth Generation (5G) New Radio (NR) does not support data transmission during random access (RA) procedures, which results in unnecessary control signalling overhead, especially for small data transmission (SDT). Motivated by this, 3GPP has proposed 4/2-step SDT RA schemes based on the existing grant-based (4-step) and grant-free (2-step) RA schemes, with the aim to enable data transmission during RA procedures in Radio Resource Control (RRC) Inactive state. To compare the 4/2-step SDT RA schemes with the benchmark 4/2-step RA schemes, we provide a spatio-temporal analytical framework to evaluate the RA schemes, which jointly models the preamble detection, Physical Uplink Shared Channel (PUSCH) decoding, and data transmission procedures. Based on this analytical model, we derive the analytical expressions for the overall packet transmission success probability in each RACH attempt. Our results show that 2-step SDT RA scheme provides the highest overall packet transmission success probability, but performance gain decreases with the increase of device intensity.**

***Index Terms*—Grant-based, Grant-free, 4-step, 2-step, Small data.**

I. Introduction

As an emerging technology, a plethora of applications, including unmanned aerial vehicle (UAV), wearable devices, industrial wireless sensor networks (IWSN), and etc, are being revolutionized via Internet of Things (IoT), in which small data packets are the typical form of traffic generated by IoT devices (e.g., hundreds of bits) [1]. In view of this, minimizing control signalling overhead becomes a critical issue during small data transmission (SDT) since the control signalling is non-negligible compared to the small data packets.

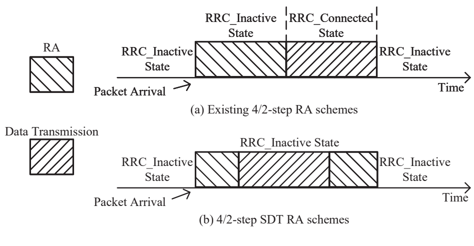

Fig. 1. Uplink transmission of (a) existing 4/2-step RA schemes; and (b) newly proposed 4/2-step SDT RA schemes.

The IoT device is triggered to perform random access (RA) when a new packet arrives, and transits to Radio Resource Control (RRC) Connected state for data transmission in existing 4-step and 2-step RA schemes as shown in Fig. 1(a). The 4-step RA scheme utilizes a scheduling-based transmission mechanism, namely grant-based (GB) access, in which the Physical Uplink Shared Channel (PUSCH) resource is allocated to the device only after the base station (BS) receives the preamble. The 2-step RA scheme is categorized as grant-free (GF) access, where the PUSCH resource is pre-allocated and transmitted along with the preamble [2].

RRC state transitions in the existing 4/2-step RA schemes result in unnecessary control signalling overhead, especially for SDT [3]. To optimize the support for SDT, 3GPP has identified the data transmission within 4/2-step RA schemes as the potential solutions in Release 17 [4], namely 4/2-step SDT RA schemes. Compared to the existing 4/2 step RA schemes, the 4/2-step SDT RA schemes, as shown in Fig. 1(b), have the potential capability to reduce the controlling overheads via enabling data transmission during RRC Inactive State.

Recent works in [5], [6] have evaluated the 4/2-step SDT RA schemes via system-level simulation. In [5], the authors evaluated the battery life and latency of the 4-step SDT scheme with various payload sizes and periods. In [6], the authors compared the performance of 4/2-step SDT RA schemes with the traditional 4/2-step RA schemes under different mobility patter, traffic pattern, and packet size. However, the general RA model and mathematical framework for the 4/2-step, and 4/2-step SDT RA schemes have never been fully established, and their comparative insights have not been investigated yet.

Stochastic geometry has been regarded as a powerful tool to capture the uncertainty of devices' locations in wireless networks [7], which has been utilized to analyse the 4/2-step RA schemes [8]–[10]. In [8], the authors analyzed the queue evolution of the 4-step RA scheme by developing a spatio-temporal mathematical framework. The work in [9] extended the preamble transmission probability analysis to the preamble collision in 4-step RA scheme. The work in [10] proposed a tractable approach to analyze the 2-step RA scheme under three different hybrid automatic repeat request (HARQ) mechanisms. However, to the best of our knowledge, existing works have either focused on studying preamble SINR outage [8] without considering collision, or assumed the collision happens during preamble transmission for simplicity [9], [10] and data transmission was ignored, whose model cannot capture accurate performance of each RA scheme.

Motivated by this, we present a tractable spatio-temporal mathematical framework to analyze 4/2-step, and 4/2-step SDT RA schemes based on stochastic geometry and probability theory, where preamble detection, PUSCH decoding, and data transmission procedures are jointly modelled. We then derive the overall packet transmission success probability in each RACH attempt. Finally, we develop a realistic simulation framework to verify the overall packet transmission success probability. Our results show that 2-step SDT RA scheme provides the highest overall packet transmission success probability. However, the performance gain compared with other RA schemes decreases with the increase of device intensity.

The rest of the paper is organized as follows. Section II presents the system model. Sections III derives the transmission success probability of each message, and overall packet transmission success probability in each RACH attempt. Section IV provides numerical results. Finally, Section V concludes the paper.

II. System Model

We consider a single BS and multiple IoT devices[1], where the devices are spatially distributed in $\mathbb{R}^2$ following independent homogeneous Poisson point process(PPP) Φ_{D} with intensity λ_{D}, and are assumed to be static all time once they are deployed.

A. Network and Traffic Model

We consider a flat Rayleigh fading channel, where the channel between two generic locations $x, y \in \mathbb{R}^2$ is assumed to follow $h(x,y) \sim \mathcal{CN}(0,1)$. We consider a standard power-law path-loss model with attenuation $r^{-\alpha}$, where r is the propagation distance from the device to BS, and α is the path-loss exponent. An ideal full path-loss inversion power control is assumed at all devices based on downlink transmission path-loss estimation, where each device compensates for its path-loss to keep the average received signal power at the BS equal to the same threshold ρ [8].

We model the new arrived packets N^m_{new} in the mth RACH attempt at each device using independent Poisson arrival process Λ^m_{new} with the intensity μ^m_{new}. First Come First Serve (FCFS) scheduling scheme is applied by placing the newly arrived packets at the end of the queue. Without loss of generality, we assume the buffer size of each device is large enough, and an infinite amount of RACH attempts is assumed, where no packet is dropped until the packet is successfully received by the BS.

B. Contention-Based Random Access Schemes

1) 4-step and 4-step SDT Random Access: The 4-step RA scheme is shown in Fig. 2(a). In step 1, each device randomly selects a preamble generated by Zadoff-chu (ZC) sequence, and transmits as Msg1 on the RA subframe. In step 2, the BS responds to the device with Msg2 containing timing advance, and PUSCH resource granted for Msg3 transmission under the condition that BS successfully detects the preamble. If not, the device reattempts in the next RACH opportunity. In step 3, the device transmits Msg3 including a device identity. If multiple devices select the same preamble and RA subframe in Msg1, they receive the same Msg2, and transmit their own Msg3 on the same PUSCH resource resulting in a collision. If the BS successfully decodes one specific Msg3 among colliding devices, in step 4, the BS sends Msg4 with an echo of the identity transmitted in Msg3 by the device. Those devices with matched identity succeed in the RA procedure and enter into the RRC Connected state for data transmission with HARQ, and all failed devices have to reattempt in the next RACH opportunity. After successful data transmission, the device receives the RRC release with suspend from the BS, and goes back to the RRC Inactive state.

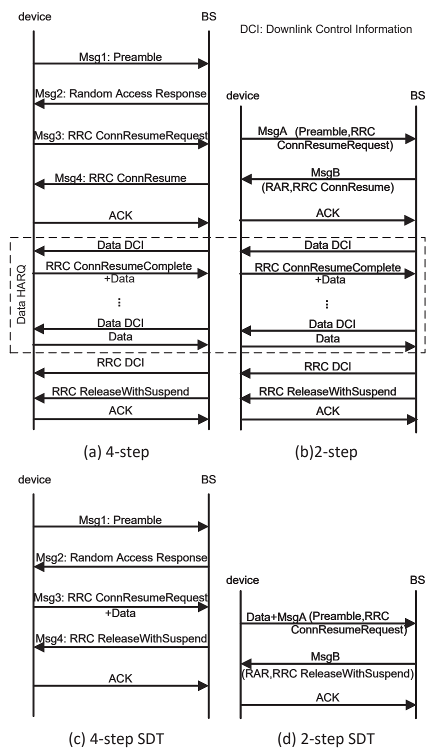

Fig. 2. Procedures of each scheme.

The 4-step SDT RA scheme is illustrated in Fig. 2(c), which enables devices to transmit data with Msg3 without entering into RRC Connected state [5]. It is noted that the BS would have no knowledge whether the device has a small packet to transmit or not. Even if devices have deterministic traffic and the traffic pattern are predictable, the device identity is not known to the BS until Msg3. Therefore, the device is required to indicate its wish to use 4-step SDT RA scheme to the BS in Msg1 by randomly selecting one of the special PRACH preambles, which have been dedicated to 4-step SDT by the BS in system information. Once the preamble is detected successfully, the BS grants a larger PUSCH resource for both Msg3 and data transmission. The device, whose Msg3

[1]The Physical Random Access Channel (PRACH) root sequence planning is applied to mitigate the false alarm ratio of preamble detection among neighbouring BSs, thus neighbouring BSs have different preamble sets [11].

and data are successfully decoded among colliding devices, receives Msg4.

2) 2-step and 2-step SDT Random Access: The 2-step and 2-step SDT RA schemes are illustrated in Fig. 2(b)(d), where data is either transmitted after MsgB in 2-step RA scheme in RRC Connected state, or together with MsgA in 2-step SDT RA scheme in RRC Inactive state, respectively. The MsgA consists of two parts, that is, preamble and PUSCH (corresponding to Msg3 in 4-step RA scheme), which are transmitted separately over time with independent channel. Unlike 4-step and 4-step SDT RA schemes, where the PUSCH resources are granted via Msg2, each preamble is mapped to a PUSCH in advance in 2-step and 2-step SDT RA schemes. Here, we consider the unique mapping relationship between preamble and PUSCH as the baseline. It is noted that if the preamble is detected but none of PUSCH transmission is decoded successfully among the colliding devices in 2-step and 2-step SDT RA schemes, the BS sends a fallback MsgB with an UL grant for Msg3 transmission, and all the colliding devices fallback to the 4-step RA scheme.

C. Random Access Model

To capture the characteristics of uplink transmission with different RA schemes, we jointly model the preamble detection, PUSCH decoding, and data transmission, which are based on ZC sequence characteristic, Power Delay Profile (PDP), SINR, and Block Error Rate (BLER), respectively.

1) Zadoff-chu Sequence Characteristic: The ZC sequence is defined as

$$z_r[k] \triangleq \exp\left[-j\pi r k(k+1)/N_{\rm ZC}\right], \quad (1)$$

where k is the sequence index, $N_{\rm ZC}$ denotes the sequence length, and r is the root number broadcasted to devices in the system information.

The ZC sequences have an ideal cyclic auto-correlation property, which means the magnitude of the cyclic correlation with a circularly shifted version of itself becomes a scaled delta function as

$$|c_{rr}[\tau]| = \left|\sum_{k=0}^{N_{\rm ZC}-1} z_r[k]\, z_r^*[k+\tau]\right| = N_{\rm ZC}\delta[\tau], \quad (2)$$

where $c_{rr}[\tau]$ is the discrete cyclic auto-correlation function of $z_r[k]$ at lag τ and $[\cdot]^*$ denotes the complex conjugate. From this property, we can observe how much the received sequences are shifted, compared to the reference ZC sequence.

Multiple preambles can be generated from a ZC sequence by cyclically shifting the sequence by a factor of cyclic shift value $\rm N_{CS}$. Thus, the ith preamble can be represented as

$$z_r^i[k] = z_r\left[(k + i{\rm N_{CS}}) \bmod N_{\rm ZC}\right]. \quad (3)$$

2) Power Delay Profile: PDP is utilized to model the preamble detection (i.e., Msg1 or MsgA) at the BS, which is the periodic correlation of the received preamble as a function of time. The BS detects preamble transmission via PDP, which indicates whether devices are requesting resources for uplink transmission. It is important to note that there is no interference and collision during preamble transmission. Without being limited to the hardware design of BS, we assume the BS can distinguish the devices based on their propagation delays in preamble detection. The received preamble from a typical device can be written as [12]

$$y_r^i[k] = \sqrt{\rho} h_0 z_r^i[k+\tau_0] + n_0, \quad (4)$$

where τ_0 is the sequence shift caused by the propagation delay from a typical device to the BS, ρ is the power control threshold, h_0 is the channel between the BS and a typical device, and n_0 denotes complex Gaussian noise with zero mean and variance σ_n^2.

The BS computes PDP of a typical device via time-domain correlation between the received preamble (4) and the local reference preamble sequence (3). Therefore, we formulate the PDP of a typical device in the mth RACH attempt as

$${\rm PDP}^m[\tau] = \left|\sum_{k=0}^{N_{\rm ZC-1}} \left(\sqrt{\rho} h_0 z_r^i[k+\tau_0] + n_0\right) z_r^i[k+\tau]^*\right|^2. \quad (5)$$

3) Signal to Noise plus Interference Ratio: SINR is utilized to model the PUSCH decoding (i.e., Msg3 or MsgA) when the preamble is successfully detected at the BS. As we mentioned earlier, each device transmits a randomly chosen preamble to the BS, and thus devices choosing same preamble in the same RA subframe cause the intra-cell interference in PUSCH transmission. We formulate SINR of the PUSCH transmission in mth RACH attempt as [8]

$${\rm SINR}^m = \rho |h_0|^2 / \left(\mathcal{I}_{\rm intra}^m + \sigma_n^2\right), \quad (6)$$

where

$$\mathcal{I}_{\rm intra}^m = \sum_{j\in\mathcal{Z}_{\rm in}} 1_{\{N_{{\rm New}_j}^m + N_{{\rm Cum}_j}^m > 0\}} \rho |h_j|^2. \quad (7)$$

In (6), h_0 is the channel from a typical device to the BS, and $\mathcal{I}_{\rm intra}$ is the aggregated intra-cell interference in the mth RACH attempt. In (7), $\mathcal{Z}_{\rm in}$ is the set of intra-cell interfering devices, $N_{{\rm New}_j}^m$ is the number of new arrived packets in the mth RACH attempt of jth interfering UE, $N_{{\rm Cum}_j}^m$ is the number of accumulated packets in the mth RACH attempt of jth interfering device. $1_{\{.\}}$is the indicator function that takes the value 1 if the statement $1_{\{.\}}$ is true, and zero otherwise.

4) Block Error Rate: For data transmission after RA procedure, we assume the BLER of data transmission is B. This is because the BS performs link adaption to adaptively change the modulation and coding scheme, which can guarantee the BLER target [13]. If the data is transmitted during the RA procedure, we do not consider data transmission separately for fairness.

III. Transmission Success Probability

We assume that the BS has an available preamble pool with the number of non-dedicated preambles ξ, known by the devices. Each preamble has an equal probability $1/\xi$ to be chosen by the device, hence the average number of the devices using the same preamble λ_{Dp} is

$$\lambda_{\mathrm{Dp}} = \lambda_{\mathrm{D}}/\xi, \tag{8}$$

where λ_{D} is the device intensity in the cell.

The device generates interference only when it has data to transmit in the mth RACH attempt, thus we define the non-empty buffer probability of a typical device as $\mathcal{T}^m = \mathbb{P}\left[N_{\mathrm{New}}^m + N_{\mathrm{Cum}}^m > 0\right]$, where N_{New}^m is the number of new arrived packets in the mth RACH attempt, and N_{Cum}^m is the number of accumulated packets in the mth RACH attempt.

Similar as [8], the intensity of accumulated packets $\mu_{\mathrm{cum}}^m (m > 1)$ in the mth RACH attempt is derived as

$$\mu_{\mathrm{Cum}}^m = \mu_{\mathrm{New}}^{m-1} + \mu_{\mathrm{Cum}}^{m-1} - \mathcal{P}^{m-1}\mathcal{T}^{m-1}, \tag{9}$$

where μ_{New}^{m-1} and μ_{Cum}^{m-1} are the intensity of new packets and accumulated packets in the $m-1$ RACH attempt. $\mathcal{P}^{m-1}$ and $\mathcal{T}^{m-1}$ are the overall transmission success probability and non-empty buffer probability in the Poisson approximation. Thus, the non-empty probability of each device in the mth RACH attempt is derived as

$$\mathcal{T}^m = 1 - e^{-\mu_{\mathrm{New}}^m - \mu_{\mathrm{Cum}}^m}. \tag{10}$$

Therefore, the intensity of interfering devices is $\lambda_{Dp}\mathcal{T}^m$, and the Probability Mass Function (PMF) of the number of interfering devices in the mth RACH attempt is derived as

$$\mathbb{P}\left[N = n\right] = e^{-\lambda_{\mathrm{Dp}}\mathcal{T}^m} \left(\lambda_{\mathrm{Dp}}\mathcal{T}^m\right)^n / n!, \tag{11}$$

where N is the number of interfering devices.

A. Preamble Detection in Msg1 and MsgA

The preamble detection is performed at step 1 of each RA scheme by calculating the PDP, where the peak values of devices choosing the same preamble are separate due to different propagation delays between each device and BS. For the brevity of exposition, we define the event with n interfering devices as $\mathcal{A} = \{N = n\}$. Considering that the preamble detection is successful when at least one PDP peak values among $n+1$ colliding devices is above the threshold λ_{th}, the preamble detection success probability of a typical device in mth RACH attempt conditioning on n interfering devices is defined as

$$\mathcal{P}_{\mathrm{pre}|\mathcal{A}}^m = 1 - \prod_{l=1}^{n+1} \mathbb{P}[\mathrm{PDP}_l^m\left[\tau_l\right] < \lambda_{\mathrm{th}}|\mathcal{A}], \tag{12}$$

which is characterized in the following lemma.

Lemma 1. *The preamble detection success probability of a typical device in the mth RACH attempt conditioning on n interfering devices is derived as*

$$\mathcal{P}_{\mathrm{pre}|\mathcal{A}}^m = 1 - \left[1 - \exp\left(-\frac{\lambda_{\mathrm{th}}}{\rho N_{\mathrm{ZC}}^2 + \sigma_n^2 N_{\mathrm{ZC}}}\right)\right]^{n+1}, \tag{13}$$

where N_{ZC} is the length of the preamble sequence, σ_{n}^2 is the noise power, ρ is the average received power at the BS, and λ_{th} is the threshold for preamble detection.

Proof. See Appendix A. □

B. PUSCH Decoding in Msg3 and MsgA

The PUSCH decoding is performed either in step 3 of 4-step and 4-step SDT RA schemes, or step 1 of the 2-step and 2-step SDT RA schemes. For the brevity of exposition, we define the preamble detection success of a typical device with n interfering devices as event $\mathcal{B} = \left\{\prod_{l=1}^{n+1} 1_{\{\mathrm{PDP}_l^m < \lambda_{\mathrm{th}}\}} = 0, \mathcal{A}\right\}$, where $1_{\{\mathrm{PDP}_l^m < \lambda_{\mathrm{th}}\}}$ takes value 1 if the PDP peak value of device l is below the threshold λ_{th}.

For an advanced receiver with capturing capability, a specific signal can still be decoded when the SINR of the strongest signal is larger than a specific threshold. Therefore, the PUSCH decoding success probability of a typical device in mth RACH attempt conditioning on n interfering devices and preamble detection success is defined as

$$\begin{aligned}\mathcal{P}_{\mathrm{pus}|\mathcal{B}}^m &= \mathbb{P}\left[\mathrm{SINR}_o > \gamma_{\mathrm{th}}, |h_o|^2 > |h_j|^2 |\mathcal{B}\right] \\ &= \mathbb{P}[\mathrm{SINR}_o > \gamma_{\mathrm{th}} | |h_o|^2 > |h_j|^2, \mathcal{B}]\mathbb{P}[|h_o|^2 > |h_j|^2 |\mathcal{B}],\end{aligned} \tag{14}$$

where $j \in \mathcal{Z}_{\mathrm{in}}$ represents the set of intra-cell interfering devices in (7), and the PUSCH decoding success probability is characterized in the following lemma.

Lemma 2. *The PUSCH decoding success probability conditioning on n interfering devices and preamble detection success is derived as*

$$\mathcal{P}_{\mathrm{pus}|\mathcal{B}}^m = \sum_{k=1}^{n+1} \binom{n+1}{k} (-1)^{k+1} \frac{\exp\left\{\dfrac{-k\gamma_{\mathrm{th}}\sigma_{\mathrm{n}}^2}{\rho}\right\}}{(n+1)\left(1+\gamma_{th}\right)^n}, \tag{15}$$

where σ_{n}^2 is the average noise power, ρ is the average received power at the BS, and γ_{th} is the SINR threshold for PUSCH decoding.

Proof. See Appendix B. □

C. Data Transmission after RACH

For the brevity of exposition, we define the preamble detection success, and PUSCH decoding success with n interfering devices as event $\mathcal{C} = \left\{\mathrm{SINR}_o > \gamma_{\mathrm{th}}, |h_o|^2 > |h_j|^2, \mathcal{B}\right\}$. We assume the BLER of each data transmission is B, hence the data transmission success probability of a typical device in mth RACH attempt conditioning on event $\mathcal{C}$ is derived as

$$\mathcal{P}_{\mathrm{data}|\mathcal{C}}^m = 1 - B^{\mathrm{K}}, \tag{16}$$

where K is the maximum data HARQ transmission times.

D. Overall Packet Transmission Success Probability

1) 4-step Random Access: The overall packet transmission success only occurs when preamble detection, PUSCH decoding, and data transmission are all successful. Hence, the overall packet transmission success probability of 4-step RA scheme in mth RACH attempt is derived as

$$\mathcal{P}_{4\mathrm{step}}^m = \sum_{n=0}^{\infty} \mathbb{P}\left[N = n\right] \mathcal{P}_{\mathrm{pre}|\mathcal{A}}^m \mathcal{P}_{\mathrm{pus}|\mathcal{B}}^m \mathcal{P}_{\mathrm{data}|\mathcal{C}}, \tag{17}$$

where $\mathbb{P}[N=n]$, $\mathcal{P}^m_{\mathrm{pre}|\mathcal{A}}$, $\mathcal{P}^m_{\mathrm{pus}|\mathcal{B}}$, and $\mathcal{P}_{\mathrm{data}|\mathcal{C}}$ are derived in (11), (13), (15), and (16).

2) 4-step SDT Random Access: The overall packet transmission success occurs when both preamble detection and PUSCH decoding are successful. Hence, the overall packet transmission success probability of 4-step SDT in mth RACH attempt is derived as

$$\mathcal{P}^m_{\mathrm{4stepSDT}} = \sum_{n=0}^{\infty} \mathbb{P}[N=n]\, \mathcal{P}^m_{\mathrm{pre}|\mathcal{A}} \mathcal{P}^m_{\mathrm{pus}|\mathcal{B}}, \tag{18}$$

where $\mathbb{P}[N=n]$, $\mathcal{P}^m_{\mathrm{pre}|\mathcal{A}}$, and $\mathcal{P}^m_{\mathrm{pus}|\mathcal{B}}$, are derived in (11), (13), and (15).

3) 2-step Random Access: Remind that if the BS detects the preamble but fails to decode any PUSCH signal among $n+1$ colliding devices in MsgA, the fallback mechanism in 2-step RA scheme allows the devices to transmit Msg3 following the 4-step RA scheme. Therefore, the packet can be successfully transmitted either after 2-step RA procedure with successful preamble detection and PUSCH decoding, or after 4-step RA procedure with successful fallback PUSCH decoding, the overall packet transmission success probability is derived as

$$\mathcal{P}^m_{\mathrm{2step}} = \sum_{n=0}^{\infty} \mathbb{P}[N=n]\, \mathcal{P}^m_{\mathrm{pre}|\mathcal{A}} \mathcal{P}^m_{\mathrm{pus}|\mathcal{B}} \mathcal{P}^m_{\mathrm{data}|\mathcal{C}} + \sum_{n=0}^{\infty} \mathbb{P}[N=n]\, \mathcal{P}^m_{\mathrm{pre}|\mathcal{A}} \left(1-(n+1)\mathcal{P}^m_{\mathrm{pus}|\mathcal{B}}\right) \mathcal{P}^m_{\mathrm{pus}|\mathcal{B}} \mathcal{P}_{\mathrm{data}|\mathcal{C}}, \tag{19}$$

where $\mathbb{P}[N=n]$, $\mathcal{P}^m_{\mathrm{pre}|\mathcal{A}}$, $\mathcal{P}^m_{\mathrm{pus}|\mathcal{B}}$, and $\mathcal{P}_{\mathrm{data}|\mathcal{C}}$ are derived in (11), (13), (15), and (16).

4) 2-step SDT Random Access: Similar as 2-step RA scheme, the overall packet transmission success probability of 2-step SDT RA scheme is derived as

$$\mathcal{P}^m_{\mathrm{2stepSDT}} = \sum_{n=0}^{\infty} \mathbb{P}[N=n]\, \mathcal{P}^m_{\mathrm{pre}|\mathcal{A}} \mathcal{P}^m_{\mathrm{pus}|\mathcal{B}} + \sum_{n=0}^{\infty} \mathbb{P}[N=n]\, \mathcal{P}^m_{\mathrm{pre}|\mathcal{A}} \left(1-(n+1)\mathcal{P}^m_{\mathrm{pus}|\mathcal{B}}\right) \mathcal{P}^m_{\mathrm{pus}|\mathcal{B}} \mathcal{P}_{\mathrm{data}|\mathcal{C}}, \tag{20}$$

where $\mathbb{P}[N=n]$, $\mathcal{P}^m_{\mathrm{pre}|\mathcal{A}}$, $\mathcal{P}^m_{\mathrm{pus}|\mathcal{B}}$, and $\mathcal{P}_{\mathrm{data}|\mathcal{C}}$ are derived in (11), (13), (15), and (16).

IV. Simulation and Discussion

We compare the results with the basic receiver in [9], where the BS cannot decode any PUSCH signal with multiple SINRs above the threshold. In all figures of this section, we use "Ana." and "Sim." to abbreviate "Analytical" and "Simulation", respectively. Unless otherwise stated, the devices are deployed in a 0.1 km^2 circle cell, $\mu^m_{\mathrm{New}} = 0.1$ packets/time-slot, $\rho = -90$ dBm, $\sigma^2_{\mathrm{n}} = -100.4$ dBm, $\gamma_{\mathrm{th}} = -10$ dB, $\alpha = 4$. $N_{\mathrm{ZC}} = 839$, $\lambda_{\mathrm{th}} = -51.5$ dBm, K = 1, $B = 0.1$.

Fig. 3 plots the overall packet transmission success probability with four RA schemes versus time slot with $\lambda_{\mathrm{Dp}} = 5$. The close match between the analytical curves and simulation points validates the accuracy of our developed spatio-temporal mathematical framework. We see that the overall

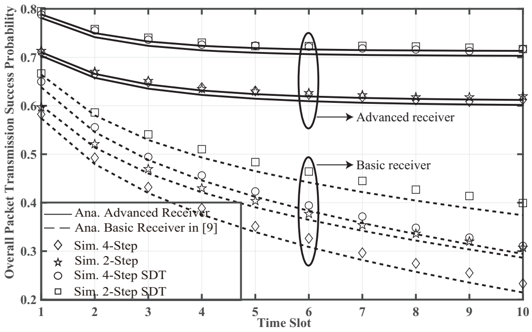

Fig. 3. Packet transmission success probability in each time slot. We set the device intensity λ_{Dp} =5 devices/preamble

packet transmission success probability of each scheme enters into the stable region in the advanced receiver. However, the overall packet transmission success probability of each scheme keeps decreasing as time evolves in the basic receiver. This is because the BS can not decode the PUSCH if multiple colliding devices' SINR is higher than the threshold in the basic receiver. Therefore, the new arrival packets can not be transmitted to the BS in time, and leads to the traffic congestion.

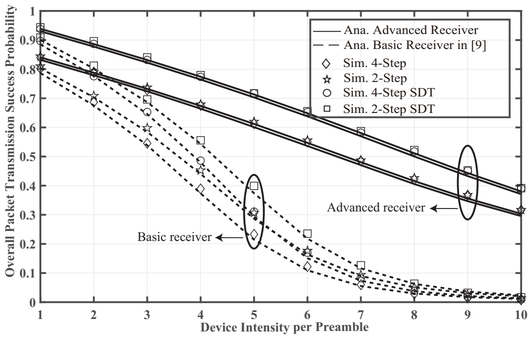

Fig. 4. Packet transmission success probability in 10th time slot with different device intensity

We also observe that the overall packet transmission success probabilities of 4/2-step SDT RA schemes are higher than that of 4/2-step RA schemes. This can be explained by the reason that the data is transmitted after RACH procedures in 4/2-step RA schemes, which decreases the overall packet transmission success probability. We also notice that the success probabilities of 2-step SDT RA scheme is higher than that of 4-step SDT RA scheme, and 2-step RA scheme is higher than that of 4-step RA scheme. This is because 2-step and 2-step SDT RA schemes integrate the fallback mechanism, which enables the packet to be successfully transmitted after fallback to 4-step RA scheme. This advantage is more obvious in the basic receiver due to higher fallback probability.

Fig. 4 plots the overall packet transmission success probability with four RA schemes versus device intensity in 10th time slot. We observe that the overall packet transmission success probability of all schemes decreases with the increasing device intensity, due to the increasing aggregate interference from more devices transmitting signals simultaneously. We also observe that the decreasing rate is almost linear in the advanced receiver, and more sharply in the basic receiver due to more serious traffic congestion.

V. Conclusion

In this paper, we analyzed the preamble detection probability, PUSCH decoding probability, data transmission success probability, and overall packet transmission success probability of a typical device with 4/2-step and 4/2-step SDT RA schemes by modelling the queue evolution over consecutive time slots. Our numerical results have shown that the 2-step SDT RA scheme achieves the highest packet transmission success probability, but its performance gain compared to the 4-step, 2-step, and 4-step SDT RA schemes decreases with the increase of device intensity.

Appendix A
A Proof of Lemma1

We can calculate the Cumulative Distribution Function (CDF) of PDP peak value of a typical device based on (5) as

$$\begin{aligned}&\mathbb{P}[\mathrm{PDP}_o^m\,[\tau_o] < \lambda_{\mathrm{th}}|\mathcal{A}]\\&= \mathbb{P}\left[\left|N_{\mathrm{ZC}}\sqrt{\rho}h_0 + \sum_{k=0}^{N_{\mathrm{ZC}}-1}\tilde{n_0}\right|^2 < \lambda_{\mathrm{th}}|\mathcal{A}\right].\end{aligned} \tag{21}$$

where the channel $h_0 \sim \mathcal{CN}(0,1)$, $\tilde{n_0} \sim \mathcal{CN}(0,\sigma_n^2)$, and $\left|N_{\mathrm{ZC}}\sqrt{\rho}h_0 + \sum_{k=0}^{N_{\mathrm{ZC}}-1}\tilde{n_0}\right|^2$ follows the exponential distribution with scale $\rho N_{\mathrm{ZC}}^2 + \sigma_n^2 N_{\mathrm{ZC}}$. Therefore, the probability that PDP peak value of a typical device is below the detection threshold is derived as

$$\mathbb{P}[\mathrm{PDP}_j^m\,[\tau_j] < \lambda_{\mathrm{th}}|\mathcal{A}] = 1 - \exp\left(-\frac{\lambda_{\mathrm{th}}}{\rho N_{\mathrm{ZC}}^2 + \sigma_n^2 N_{\mathrm{ZC}}}\right). \tag{22}$$

Substituting (22) into (12), we can obtain the preamble detection success probability as (13).

Appendix B
A Proof of Lemma2

As the channel gain $|h|^2$ follows the exponential distribution with unit mean, the Complementary Cumulative Distribution Function (CCDF) of the maximum channel gain between $n+1$ independent Rayleigh fading channel gains is derived as

$$F_{|h|^2_{max}}|_{N=n}\left(|h|^2\right) = 1 - \left(1 - \exp\left(-|h|^2\right)\right)^{n+1}. \tag{23}$$

Substituting (23) into (14) and noting that $\mathbb{P}[|h_o|^2 > |h_j|^2|\mathcal{B}] = 1/(n+1)$, we obtain

$$\begin{aligned}&\mathcal{P}^m_{\mathrm{pus}|\mathcal{B}}\\&= \frac{\mathbb{E}_{\mathcal{I}_{\mathrm{intra}}}\left\{1 - \left(1 - \exp\left\{\frac{\gamma_{\mathrm{th}}}{\rho}\left(\sigma_{\mathrm{n}}^2 + \mathcal{I}_{\mathrm{intra}}\right)\right\}\right)^{n+1}\right\}}{n+1}.\end{aligned} \tag{24}$$

Because of the independency of the PPP in different regions and after applying the binomial expansion for the numerator of (24), we obtain

$$\begin{aligned}&\mathcal{P}^m_{\mathrm{pus}|\mathcal{B}}\\&= \sum_{k=1}^{n+1}\binom{n+1}{k}(-1)^{k+1}\frac{\exp\left\{\frac{-k\gamma_{\mathrm{th}}\sigma_{\mathrm{n}}^2}{\rho}\right\}}{n+1}\mathcal{L}_{\mathcal{I}_{\mathrm{intra}}}\left(\frac{\gamma_{\mathrm{th}}}{\rho}\middle|\mathcal{B}\right),\end{aligned} \tag{25}$$

where $\mathcal{L}_{\mathcal{I}_{\mathrm{intra}}}(\cdot)$ denotes the Laplace Transform of the aggregate intra-cell interference $\mathcal{I}_{\mathrm{intra}}$. The Laplace Transform of the aggregate intra-cell interference $\mathcal{I}_{\mathrm{intra}}$ is given in [8] as

$$\mathcal{L}_{\mathcal{I}_{\mathrm{intra}}}\left(\frac{\gamma_{\mathrm{th}}}{\rho}\middle|\mathcal{B}\right) = 1/(1+\gamma_{th})^n. \tag{26}$$

Substituting the Laplace Transform of the aggregated intra-cell interference (26) into (25), we can obtain (15)

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