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Joint Design of Pilot Power and Pilot Pattern
for Sparse Cognitive Radio Systems

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Abstract—Existing works design the pilot pattern for sparse channel estimation, assuming that the power of all pilots is equal. However, equal power allocation is not optimal in cognitive radio (CR) systems. In this correspondence, we jointly design the pilot power and pilot pattern for sparse channel estimation in orthogonal-frequency-division-multiplexing-based CR systems, based on the rule of mutual incoherence property that minimizes the coherence of the measurement matrix used for the sparse channel estimation. Under the sum power constraint and peak power constraint, the pilot pattern is formulated as a second-order cone programming (SOCP). Then, we propose a joint design algorithm, which includes discrete optimization for pilot pattern and continuous optimization for pilot power. Simulation results show that the proposed algorithm can achieve better channel estimation performance in terms of mean square error and bit error rate and can further improve the spectrum efficiency by 2.4%. Compared with existing algorithms assuming equal pilot power, simulation results show that the proposed algorithm can achieve better channel estimation performance in terms of mean square error and bit error rate and can further improve the spectrum efficiency by 2.4%.

Index Terms—Cognitive radio (CR), compressed sensing (CS), orthogonal frequency-division multiplexing (OFDM), pilot design, sparse channel estimation.

I. INTRODUCTION

Radio spectrum is a precious and limited resource for wireless communications. In attempts to relieve the spectrum shortage, the concept of cognitive radio (CR) is proposed, which allows secondary users (SUs) to opportunistically access the spectrum that is originally allocated to primary users (PUs). SUs start communications with each other when the spectrum is not used by any PU. Therefore, CR can improve the usage of existing frequency bands without allocating a new spectrum resource [1]. On the other hand, with the great capability in combating frequency-selective fading and the high flexibility in allocating transmit resources, orthogonal frequency-division multiplexing (OFDM) has been suggested as a competitive candidate in CR systems [2]. In OFDM-based CR systems, the subcarriers are noncontiguous.

Hence, the efficient design of pilots including pilot pattern and pilot power is crucial to the performance of channel estimation and data detection. In [3], a scheme to design symbols for OFDM-based CR systems is proposed, where the pilot design assuming equal pilot power is formulated as an optimization problem that minimizes the upper bound related to the mean square error (MSE) of least squares (LS) channel estimation. In [4], a scheme that utilizes cross entropy (CE) optimization together with the analytical pilot power optimization is proposed to design pilot symbols to reduce the MSE of the LS channel estimation. In [5], parameter adaptation for wireless 48 multicarrier-based CR systems is investigated where the CE method is demonstrated to outperform the genetic algorithm (GA) and particle swarm optimization (PSO). However, all these literatures are based on the LS channel estimation.

Recently, sparse channel estimation that exploits the inherent sparse property of wireless multipath channels and applies the compressed sensing (CS) techniques for channel estimation has been proven to 55 improve the channel estimation performance and reduce the pilot overhead compared with the LS method [6, 7]. To further improve the performance of sparse channel estimation, one effective approach is to optimize the pilot design. In [8] and [9], it has been shown that the pilot pattern generated from the cyclic different set (CDS) is 60 optimal and have proposed a scheme to obtain a near-optimal pilot pattern when the CDS does not exist. In [10] and [11], two pilot design schemes based on CE optimization and stochastic approximation, respectively, are proposed to minimize the MSE of sparse channel estimation using the channel data. In [12], a pilot allocation method is proposed to obtain an optimized pilot pattern. In particular, it is shown in [14] that sparse channel estimation can achieve 11.5% improvement in spectrum efficiency with the same channel estimation performance compared with the LS channel estimation. However, all existing works design the pilot pattern for sparse channel estimation assuming that the sum of powers of pilots is equal.

In this correspondence, based on the work of Qi et al. in [14], we further consider the pilot power optimization for sparse channel estimation in OFDM-based CR systems. Note that the CR system employing sparse channel estimation is termed as the sparse CR system in this work. We jointly design the pilot power and pilot pattern based on the rule of mutual incoherence property (MIP) that minimizes the coherence of the measurement matrix used for the sparse channel estimation. Under the sum power constraint and peak power constraint, the pilot design is formulated as a joint optimization problem, which is then decoupled into tractable sequential formations. Given a pilot pattern, we formulate the design of pilot power as a second-order cone programming (SOCP). We propose a joint design algorithm, which includes discrete optimization for pilot pattern and continuous optimization for pilot power.

The notations used in this correspondence are defined as follows. Symbols for matrices (uppercase) and vectors (lowercase) are in boldface. \( (\cdot)^T, (\cdot)^H, \text{diag}\{\cdot\}, I_L, \mathbb{C}^{M \times N}, \mathbb{O}^{M \times N}, \mathbb{C}^N, \cdot, \|\cdot\|_1, \|\cdot\|_2, \|\cdot\|_\infty, \Re\{\cdot\}, \text{and } \Im\{\cdot\} \) denote the matrix transpose, conjugate transpose (Hermitian), the diagonal matrix, the identity matrix of size \( L \), the set exclusion, the set of \( M \times N \) complex matrices, the \( M \times N \) zero matrix, the complex Gaussian distribution, the absolute value of a scalar, the Frobenius norm of a matrix, the spectral norm of a matrix, the real part of a complex number, and the imaginary part of a complex number, respectively.
The relation between the transmit pilots and the receive pilots can be written in matrix notation as

\[ y = X F h + \eta \]

where

\[ X = \text{diag} \{ x(c_{p_1}), x(c_{p_2}), \ldots, x(c_{p_K}) \} \]

\[ \eta = [\eta(1), \eta(2), \ldots, \eta(K)]^T \sim CN(0, \sigma^2 I_K) \]

and \( F \) is a discrete Fourier transform submatrix given by

\[
F = \frac{1}{\sqrt{N}} \begin{bmatrix}
\omega^{0p_1} & \omega^{0p_2} & \cdots & \omega^{0p_{(L-1)}} \\
\omega^{1p_1} & \omega^{1p_2} & \cdots & \omega^{1p_{(L-1)}} \\
\cdots & \cdots & \cdots & \cdots \\
\omega^{(L-1)p_1} & \omega^{(L-1)p_2} & \cdots & \omega^{(L-1)p_{(L-1)}}
\end{bmatrix}
\]

where \( \omega = e^{-j 2 \pi / N} \). We further denote

\[ A \triangleq XF. \]

Then, (2) can be written as

\[ y = Ah + \eta. \]

If \( L \leq K \leq M \) and \( A \) has full column rank, (6) can be solved by 146 LS, which essentially employs the fast Fourier transform interpolations 147 with the estimated CIR given by 148

\[ \hat{h}_{LS} = (A^H A)^{-1} A^H y. \]

However, a large number of pilots is required. The CS theory shows 149 that we can reduce the number of pilots, i.e., \( 2S < K < L \), by exploiting 150 the sparse property of wireless channels. To identify the positions 151 of the \( S \) nonzero entries as well as estimating the coefficients of the 152 \( S \) nonzero entries, which results in totally \( 2S \) unknown parameters, 153 we have to use at least \( K > 2S \) pilots. With this condition, we can 154 apply CS algorithms, e.g., orthogonal matching pursuit (OMP), to 155 estimate \( h \). Existing works have already shown that the CS algorithms 156 outperform LS for channel estimation [15], [16].

The restrict isometry property (RIP) shows that \( h \) in (6) can be 158 recovered from the noiseless measurement \( y(\eta = 0) \) with a high 159 probability if the measurement matrix \( A \) satisfies the RIP [17]. It is 160 shown that \( A \in \mathbb{C}^{L \times L} \) in (6) satisfies the RIP if there exists a constant 161 \( \delta \) such that

\[
(1 - \delta) \|u\|_2^2 \leq \|Au\|_2^2 \leq (1 + \delta) \|u\|_2^2
\]

holds for all \( S \)-sparse vectors \( u \in \mathbb{C}^L \). However, it is computationally infeasible to check whether a given matrix \( A \) satisfies the RIP [16]. Alternatively, according to [18], we can minimize the coherence of \( A \), which is known as the MIP. The MIP condition is stronger than the 166 RIP in that the MIP implies the RIP but the converse is not true [18]. 167 Moreover, the MIP is more intuitive and practical than the RIP. Here, 168 we consider the pilot design including joint pilot power allocation and 169 pilot pattern optimization with respect to the MIP.

Given a pilot pattern, i.e.,

\[ p = \{ c_{p_1}, c_{p_2}, \ldots, c_{p_K} \} \]

and a pilot power vector, i.e.,

\[ v = \{ v_1, v_2, \ldots, v_K \} \]

\( ^1 u \in \mathbb{C}^L \) is said to be \( S \)-sparse (\( S \ll L \)) if the number of nonzero entries of \( u \) is equal to \( S \) or smaller than \( S \).
173 with \( v_i \) denoting the power of the \( i \)th pilot subcarrier, i.e.,
\[
v_i \triangleq |x(c_{p_i})|^2, \quad i = 1, 2, \ldots, K
\]
175 we define the coherence of \( A \) as the maximum absolute correlation
176 between any two different columns of \( A \), i.e.,
\[
g(p, v) \triangleq \max_{0 \leq m < n \leq L - 1} |\langle A(m), A(n) \rangle|
\]
\[
= \max_{0 \leq m < n \leq L - 1} \left| \sum_{i=1}^{K} v_i \omega^{c_{p_i}(n-m)} \right| / \sum_{i=1}^{K} v_i
\]
177 where \( \langle A(m), A(n) \rangle \) denotes the normalized inner product between
178 the \( m \)th column \( A(m) \) and the \( n \)th column \( A(n) \) of \( A \), i.e.,
\[
\langle A(m), A(n) \rangle \triangleq \frac{A^H(m)A(n)}{\|A(m)\|_2 \|A(n)\|_2},
\]
179 Let \( d = n - m \) and \( \Lambda = \{1, 2, \ldots, L - 1\} \). Then, (12) can be rewritten
180 ten as
\[
g(p, v) = \max_{d \in \Lambda} \left| \sum_{i=1}^{K} v_i \omega^{c_{p_i}d} \right| / \sum_{i=1}^{K} v_i.
\]
181 According to the MIP, the objective for the pilot design is to
182 minimize the coherence of \( A \), i.e., \( \min_{p, v} g(p, v) \). The constraint
183 for the integer vector \( p \) is \( p \subset \mathcal{P} \). Suppose the sum power of all pilot
184 symbols is
\[
\sum_{i=1}^{K} v_i = V_T.
\]
185 where \( V_T \) is the prespecified sum power constraint of SUs. Obviously,\n186 (15) is more general in practical OFDM-based CR systems than simply
187 assuming
\[
v_1 = v_2 = \cdots = v_K = \frac{V_T}{K}.
\]
188 in current literatures [8]–[10]. On the other hand, SUs should properly
189 control the peak power of pilot subcarriers regarding the linear region
190 of power amplifiers. The power of pilot subcarriers cannot be too
191 large or too small. As shown in Fig. 1, we denote \( V_L \) as the peak
192 power constraint related to the saturation power of the power amplifier.
193 Moreover, the pilot power should be greater than a threshold \( V_L \), which
194 is related to the cutoff power of the power amplifier as well as the
195 noise and interference level around SUs. In particular, \( V_L = 0 \) can be
196 regarded as a special case. Hence, we have
\[
V_L \leq v_i \leq V_H, \quad i = 1, 2, \ldots, K.
\]
197 With these constraints, the pilot design in OFDM-based CR systems
198 can be formulated as
\[
\min_{p, v} \quad g(p, v)
\]
\[
s.t. \quad p \subset \mathcal{P}
\]
\[
\sum_{i=1}^{K} v_i = V_T, \quad V_L \leq v_i \leq V_H
\]
199 which involves the joint optimization of the discrete integer vector \( p \)
200 and the continuous real-valued positive vector \( v \). Note that unlike most
201 literatures, investigating the optimal power allocation to maximize
202 the achievable rate of CR systems, in this paper, we focus on the
pilot design for sparse channel estimation, where the design of data
203 subcarriers is out of the scope of this work.

Apparently, it is analytically intractable to get a solution from (18). We now decouple this joint optimization problem with the following two kinds of sequential formulations.

1) Given a \( \tilde{p} \subset \mathcal{P} \), we first get
\[
g_\omega(\tilde{p}) \triangleq \min_v g(\tilde{p}, v)
\]
\[
s.t. \quad \sum_{i=1}^{K} v_i = V_T, \quad V_L \leq v_i \leq V_H
\]
then we solve
\[
\min_{\tilde{p} \subset \mathcal{P}} g_\omega(\tilde{p})
\]
to get an optimal \( \tilde{p} \). Meanwhile, the corresponding \( v \) is also obtained.

2) Given a feasible \( \tilde{v} \), we first get
\[
g_p(\tilde{v}) \triangleq \min_{v \subset \mathcal{P}} g(p, \tilde{v})
\]
then we solve
\[
\min_{\tilde{v} \subset \mathcal{P}} g_p(\tilde{v})
\]
to get an optimal \( \tilde{v} \). Meanwhile, the corresponding \( p \) is also obtained.

Comparing \( p \subset \mathcal{P} \) with the constraints in (15) and (17), it can be seen that \( p \) is less difficult to enumerate than \( v \). Therefore, it is better to decouple the joint optimization problem described by (18) with the 199 first kind of formulations described by (19) and (20).

III. PILOT POWER ALLOCATION

Regarding (19), with a given pilot pattern \( \tilde{p} \subset \mathcal{P} \), we first generate
a table, i.e.,
\[
G = \begin{bmatrix}
\omega^{c_{p_1}} & \omega^{c_{p_2}} & \cdots & \omega^{c_{p_M}} \\
\omega^{c_{p_1}+1} & \omega^{c_{p_2}+1} & \cdots & \omega^{c_{p_M}+1} \\
\vdots & \vdots & \ddots & \vdots \\
\omega^{(L-1)c_{p_1}} & \omega^{(L-1)c_{p_2}} & \cdots & \omega^{(L-1)c_{p_M}}
\end{bmatrix}
\]
where \( \omega = e^{-j2\pi/N} \). Once \( N \), \( L \) and \( C \) are given, \( G \) is determined.
We look up \( G \) and select the corresponding \( K \) columns indexed by \( \tilde{p} \) from \( G \), making up a \( L - 1 \) by \( K \) submatrix \( G(\tilde{p}) \). Then, from (14), (22) we have
\[
g(\tilde{p}, \tilde{v}) = \frac{1}{V_T} ||G(\tilde{p})\tilde{v}||_\infty.
\]
Therefore, (19) is equivalent to
\[
\min_{\tilde{v}} \quad ||G(\tilde{p})\tilde{v}||_\infty
\]
\[
s.t. \quad \sum_{i=1}^{K} v_i = V_T, \quad V_L \leq v_i \leq V_H
\]
where \( \mathbf{p} \) is given, and \( G(\mathbf{p}) \) is a complex-valued submatrix fast generated by looking up \( G \). Let \( b_i \) denote the \( i \)th row of \( G(\mathbf{p}) \), \( i = 1, 2, \ldots, L - 1 \). We further denote

\[
B_i = \begin{bmatrix}
\text{Re}(b_i) \\
\text{Im}(b_i)
\end{bmatrix}, \quad i = 1, 2, \ldots, L - 1
\tag{26}
\]

which is a real-valued matrix with two rows and \( K \) columns. Then, \( 253 (25) \) can be converted into a real-valued optimization problem as

\[
\begin{aligned}
\min_z \\
\text{s.t.} & \quad \| B_i v \|_2 \leq z, \quad i = 1, 2, \ldots, L - 1 \\
& \quad \sum_{i=1}^{K} v_i = V_T, \quad V_L \leq v_i \leq V_H
\end{aligned}
\tag{27}
\]

which is an SOCP optimization problem that contains \( L - 1 \) second-order conic constraints and some linear constraints. Typically, it can be solved by SOCP solvers, e.g., MOSEK. Then, we can obtain feasible solutions as \( \hat{z} \) and \( \hat{v} \) from (27), where \( g_v(\hat{p}) = \hat{z} \).

IV. JOINT PILOT DESIGN

If \( M \) and \( K \) are not small enough, \( \mathcal{P} \) can be a huge set. For example, if \( M = 512 \) and \( K = 16 \), \( \| \mathcal{P} \| = \binom{512}{16} = 8.4 \times 10^{29} \). It is impossible for SU to store \( \mathcal{P} \) into the memory and check them one by one until the best \( \mathbf{p} \in \mathcal{P} \) is found. Furthermore, it is very computationally inefficient to implement the exhaustive search from such huge space, particularly for SU equipped with power-constrained mobile devices.

The proposed joint design algorithm including discrete optimization for pilot pattern and continuous optimization for pilot power is described in Algorithm 1. At first, we input system parameters \( C, N, M, K, L, J, T_1, T_2 \), where \( T_1 \) and \( T_2 \) represent the number of outer-loop and inner-loop iterations, respectively. Each outer-loop iteration includes \( T_2 \) inner-loop iterations. Then, we initialize a zero-251 matrix \( \mathbf{D} \) to store the results of optimized pilot patterns after running 252 inner-loop iterations. Each row of \( \mathbf{D} \) stores a pilot pattern \( \mathbf{p} \), with the 253 corresponding objective value \( g_v(\mathbf{p}) \) stored in \( \mathbf{r} \), which is initialized 254 to be a zero vector. Then, we generate a table \( \mathbf{G} \) according to (23).

At each outer-loop iteration, indicated from step 4 to step 16, we 255 start by randomly generating a pilot pattern \( \mathbf{p} \in \mathcal{P} \). By introducing the 256 randomness to the algorithm so that it starts from different initial pilot 257 patterns, we can avoid that the algorithm falls into local optimums.

As \( T_1 \) increases to infinity, the algorithm will converge to the 260 global optimum. Then, we use the inner-loop iterations from step 6 to step 261 to obtain an optimized pilot pattern \( \mathbf{p} \), which is stored in each \( \mathbf{D} \) row of \( \mathbf{D} \) with the corresponding objective value \( g_v(\mathbf{p}) \) stored in \( \mathbf{r} \), 263 indicated by step 15. After we finish the outer-loop iterations, we select 264 the minimum from \( \mathbf{r} \) and output the corresponding row of \( \mathbf{D} \) as the 265 designed pilot pattern \( \mathbf{p}_o \), indicated by step 17 and step 18. We then 266 substitute \( \mathbf{p}_o \) into (27) to design the pilot power.

Algorithm 1 Joint Design of Pilot Power and Pilot Pattern

\[
1: \text{Input: } C, N, M, K, L, J, T_1, T_2. \\
2: \text{Initialization: } \mathbf{D} \leftarrow 0^{J \times K}, \mathbf{r} \leftarrow 0^J. \\
3: \text{Generate } \mathbf{G} \text{ according to (23).} \\
4: \text{for } l = 1, 2, \ldots, T_1 \\
5: \quad \text{randomly generate } \mathbf{p} \in \mathcal{P}, \mathbf{p}^* \leftarrow 0^K. \\
6: \quad \text{for } n = 1, 2, \ldots, T_2 \\
7: \quad \quad \text{if } \mathbf{p} = \mathbf{p}^* \\
8: \quad \quad \quad \text{break.} \\
9: \quad \text{end if} \\
10: \quad \text{for } k = 1, 2, \ldots, K/J \\
11: \quad \quad \text{Obtain } \mathbf{p}_{p,k} \text{ according to (31). } \mathbf{p} \leftarrow \mathbf{p}_{p,k}. \\
12: \quad \text{end for } [k] \\
13: \quad \text{end for } [n] \\
14: \quad \text{end for } (l) \\
15: \quad \text{Output } \mathbf{p}_o = D(l) \text{ as the designed pilot pattern.} \\
16: \text{Substitute } \mathbf{p}_o \text{ into (27) to design the pilot power.} \\
\]

Here, we use an auxiliary vector \( \mathbf{p}^* \), which always records the pilot 286 pattern obtained from the previous inner-loop iteration. If we find that 287 \( \mathbf{p} \) is exactly the same as \( \mathbf{p}^* \), which means we did not get a new pilot 288 pattern, there is no need to continue the inner-loop iterations because 289 the results thereafter will be exactly the same. Then, we break from 290 the inner-loop iterations. These procedures are indicated from step 7 291 to step 10. Additionally, we have to reset \( \mathbf{p} \) by \( \mathbf{p}^* = 0^K \) at the start 292 of each outer-loop iteration. This way, we can save the CPU running 293 time and therefore improve the efficiency by skipping the same routine. 294

The main contribution of Algorithm 1 is the group update of entries 295 of \( \mathbf{p} \), which is shown from step 11 to step 13. The update of \( \mathbf{p} \) 296 is implemented in a group of \( J \) entries each time, where \( J \) is divisible 297 by \( J \), i.e., \( K/J \) is a positive integer. For \( k = 1, 2, \ldots, K/J \), given 298 the latest \( \mathbf{p} \) from the last inner-loop iteration, we update the \( k \)th group of 299 entries of \( \mathbf{p} \) with the best group selected from 300

\[
\mathcal{W} = \{ \mathbf{w} | \mathbf{w} \subseteq \Psi, \| \mathbf{w} \|_0 = J \} \tag{28}
\]

where

\[
\Psi = \mathcal{C} \setminus \{ p(i) | i = 1, 2, \ldots, K, i \notin \Phi \} \tag{29}
\]

\[
\Phi = \{ kJ - J + 1, kJ - J + 2, \ldots, kJ \}. \tag{30}
\]

Mathematically, the resultant pilot pattern \( \mathbf{p}_{p,k} \) with the update of the 302 297th group of entries is given by

\[
\mathbf{p}_{p,k} = \text{arg} \min_{\mathbf{r}(i) \in \Psi, \mathbf{r}(i) \notin \Phi} g_v(\mathbf{p}) \tag{31}
\]

where the computation of \( g_v(\mathbf{p}) \) is provided in Section III. After we 304 obtain \( \mathbf{p}_{p,k} \) for given \( k \), we update \( \mathbf{p} \) by \( \mathbf{p} = \mathbf{p}_{p,k} \).

The complexity for pilot power allocation given a pilot pattern in 306 (27) is \( \mathcal{O}(L - 1)^{1.5}(K + 1)^3 \). Therefore, the computational com- 307 plexity for Algorithm 1 is

\[
\mathcal{O}(T_1 T_2 (L - 1)^{1.5}(K + 1)^3 K (M - K + J)) \tag{32}
\]

suppose that \( T_2 \) is small enough [19]. If \( T_2 \) is large, the inner-loop 309 iterations will terminate itself by procedures from step 7 to step 9, 310 leading to even lower complexity than (32).

Remark: If we set \( J = K \), Algorithm 1 degenerates to be the 312 exhaustive search, where \( \| \mathcal{P} \| = \binom{K}{K} \), and \( \| \Phi \| = K \). In this case, 313 no matter what \( T_1 \) and \( T_2 \) are, they are equivalent to \( T_1 = T_2 = 1, 314 \) resulting in \( \mathcal{O}((M - K + J)^3) \), which is the extraordinarily high com- 315 plexity of the exhaustive search. In practice, we usually set \( J = 1 \) or 316 \( J = 2 \) to reduce the complexity. For example, if \( J = 1 \), (32) reduces 317 to a polynomial complexity, i.e., \( \mathcal{O}(T_1 T_2 (L - 1)^{1.5}(K + 1)^3 K (M - 318 K + 1)) \).
To evaluate the performance of Algorithm 1, we first compare it with the CE method [14] assuming equal pilot power indicated by (16), where the joint pilot design is simplified to be the pilot pattern design. We set $V_F = 1$, which means that the sum power of pilot subcarriers is normalized. We set $V_L = 0.03$ and $V_M = 0.1$ so that the pilot power does not vary too much. The steps for optimal pilot power allocation in Algorithm 1 are skipped by directly substituting $v_1 = v_2 = \cdots = v_{16} = 0.0625$ into (24). The parameters of the CE method are selected to be the best in [14], where the maximum number of iterations, the number of random samples, the sample quantile, and the smoothing factor are set to be 50, 100,000, 0.001, and 0.3, respectively. It can be seen in Fig. 2 that Algorithm 1 is much faster convergent than the CE method. Here, we compare the convergence speed with respect to the running time instead of the number of iterations, because the running time of each iteration is different for different algorithms or parameters. Since we run the simulations under the exact same computer hardware and software, the running time is proportional to the computational complexity. We set $T_1 = 1000$ and $T_2 = 15$ for Algorithm 1. Once the running time, i.e., $T_3 = 342$ s, which is the running time for the CE method [14], is reached, we terminate Algorithm 1 so that we can compare Algorithm 1 with the CE method under the same computational complexity. As shown in Fig. 2, Algorithm 1 with $J = 1$ can achieve the best performance of the CE method in no more than 10 s, which indicates that Algorithm 1 is 359 times faster than the CE method and, therefore, is much more efficient and powerful. Since it has already been demonstrated in [5] that the CE method outperforms GA and PSO, Algorithm 1 is a remarkable candidate for integer optimization with its applications not being restricted to the pilot design. As shown in Fig. 2, although Algorithm 1 converges slower than that with $J = 1$, it can achieve 365 better performance than that with $J = 1$ if the running time is long enough, i.e., longer than 55 s. For those SUs equipped with powerful CPU and large capacity of battery, it is better to set $J = 2$ or even larger. The finally obtained pilot patterns $p$ with the corresponding objective $g(p, v)$ during 342 s of running time assuming equal pilot power are listed in Table I. Note that in [14], we suppose $v_1 = v_2 = \cdots = v_{16} = 1$, whereas in this paper, we suppose $v_1 = v_2 = \cdots = v_{16} = 0.0625$ satisfying $\sum_{i=1}^{16} v_i = 1$, the objective in [14] has to be divided by $K = 16$ when compared with this work.

We now evaluate the performance of joint design of pilot power [37] and pilot pattern and compare it with the pilot design assuming equal pilot power. As shown in Fig. 2, Algorithm 1 with $J = 1$ using joint [37] pilot design achieves better performance than Algorithm 1 with $J = 1$ [38] or $J = 2$, while its computational complexity is between $J = 1$ and $J = 2$. The obtained pilot pattern with the corresponding objective is also listed in Table I. The comparisons of MSE performance and the bit error rate (BER) performance for sparse channel estimation are shown in Figs. 3 and 4, respectively. Both the MSE and BER are averaged 383
Then, we have proposed a joint design algorithm. Simulation results have verified the effectiveness of the proposed algorithm and shown that the proposed algorithm can achieve better channel estimation than the pilot design assuming equal pilot power.  

The joint design can reduce the pilot overhead by 71 pilots and improve the spectrum efficiency by 13.9%, thus leading to additional 2.4% improvement compared with the pilot design assuming equal pilot power in [14].

VI. CONCLUSION

In this correspondence, we have investigated the joint design of pilot power and pilot pattern based on the rule of MIP. The pilot design has been formulated as a joint optimization problem, which is then decoupled into tractable sequential formations. Given a pilot pattern, we have formulated the design of pilot power as an SOCP problem. Then, we have proposed a joint design algorithm. Simulation results have verified the effectiveness of the proposed algorithm and shown that the proposed algorithm can achieve better channel estimation performance and further improve the spectrum efficiency by 2.4%, compared with existing algorithms assuming equal pilot power.

REFERENCES

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Index Terms—Cognitive radio (CR), compressed sensing (CS), orthogonal frequency-division multiplexing (OFDM), pilot design, sparse channel estimation.

I. INTRODUCTION
Radio spectrum is a precious and limited resource for wireless communications. In attempts to relieve the spectrum shortage, the concept of cognitive radio (CR) is proposed, which allows secondary users (SUs) to opportunistically access the spectrum that is originally allocated to primary users (PUs). SUs start communications with each other when the spectrum is not used by any PU. Therefore, CR can improve the usage of existing frequency bands without allocating a new spectrum resource [1]. On the other hand, with the great capability in combating frequency-selective fading and the high flexibility in allocating transmit resources, orthogonal frequency-division multiplexing (OFDM) has been suggested as a competitive candidate in CR systems [2]. In OFDM-based CR systems, the subcarriers are noncontiguous. Hence, the efficient design of pilots including pilot pattern and pilot power is crucial to the performance of channel estimation and data detection. In [3], a scheme to design pilot symbols for OFDM-based CR systems is proposed, where the pilot design assuming equal pilot power is formulated as an optimization problem that minimizes the upper bound related to the mean square error (MSE) of least 44 squares (LS) channel estimation. In [4], a scheme that utilizes cross 45 entropy (CE) optimization together with the analytical pilot power 46 optimization is proposed to design pilot symbols to reduce the MSE 47 of the LS channel estimation. In [5], parameter adaptation for wireless 48 multcarrier-based CR systems is investigated where the CE method is 49 demonstrated to outperform the genetic algorithm (GA) and particle 50 swarm optimization (PSO). However, all these literatures are based on 51 the LS channel estimation.

Recently, sparse channel estimation that exploits the inherent sparse property of wireless multipath channels and applies the compressed 53 sensing (CS) techniques for channel estimation has been proven to 55 improve the channel estimation performance and reduce the pilot overhead compared with the LS method [6], [7]. To further improve 57 the performance of sparse channel estimation, one effective approach is to optimize the pilot design. In [8] and [9], it has been shown 59 that the pilot pattern generated from the cyclic different set (CDS) is 60 optimal and have proposed a scheme to obtain a near-optimal pilot 61 pattern when the CDS does not exist. In [10] and [11], two pilot design 62 schemes based on CE optimization and stochastic approximation, 63 respectively, are proposed to minimize the MSE of sparse channel 64 estimation using the channel data. In [12], a pilot allocation method 65 based on the GA and a shifting mechanism is proposed for sparse 66 channel estimation in multiple-input–multiple-output OFDM systems. In [13], a pilot design scheme for OFDM transmission over two-way relay networks is presented. In [14], sparse channel estimation 69 is first introduced in OFDM-based CR systems. Based on the results of spectrum sensing, a scheme using constrained CE optimization is 71 proposed to obtain an optimized pilot pattern. In particular, it is shown in [14] that sparse channel estimation can achieve 11.5% improvement in spectrum efficiency with the same channel estimation performance 74 compared with the LS channel estimation. However, all existing works 75 design the pilot pattern for sparse channel estimation assuming that the 76 power of pilots is equal.

In this correspondence, based on the work of Qi et al. in [14], 77 we further consider the pilot power optimization for sparse channel estimation in OFDM-based CR systems. Note that the CR system 78 employing sparse channel estimation is termed as the sparse CR 81 system in this work. We jointly design the pilot power and pilot pattern based on the rule of mutual incoherence property (MIP) that minimizes the coherence of the measurement matrix used for the sparse 84 recovery. Under the sum power constraint and peak power constraint, 85 the pilot design is formulated as a joint optimization problem, which is then decoupled into tractable sequential formulations. Given a pilot pattern, we formulate the design of pilot power as a second-order cone programming (SOCP). Then, we propose a joint design algorithm, 89 which includes discrete optimization for pilot pattern and continuous 90 optimization for pilot power.

The notations used in this correspondence are defined as follows. Symbols for matrices (uppercase) and vectors (lowercase) are in boldface. (·)T, (·)H, diag{·}, I L, \( \mathbb{C}^{M \times N} \), \( 0^{M \times N} \), \( \mathbb{C}^{N} \), \( · \parallel \cdot \parallel_2 \), \( · \parallel \cdot \parallel_\infty \), Re{·}, and Im{·} denote the matrix transpose, conjugate transpose (Hermitian), the diagonal matrix, the identity matrix of size L, the set exclusion, the set of \( M \times N \) complex matrices, the \( M \times N \) zero matrix, the complex Gaussian distribution, the absolute 99
II. PROBLEM FORMULATION

We consider an OFDM-based CR system employing sparse channel estimation to exploit the inherent sparse property of wireless multipath channels. The channel is modeled as a finite impulse response filter with the channel impulse response (CIR) to be

$$y = X F h + \eta$$

where

$$X = \text{diag} \{ x(c_{p_1}), x(c_{p_2}), \ldots, x(c_{p_K}) \}$$

$$\eta = [\eta(1), \eta(2), \ldots, \eta(K)]^T \sim \mathcal{CN}(0, \sigma^2 I_K)$$

and $F$ is a discrete Fourier transform submatrix given by

$$F = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \omega^0 p_1 & \ldots & \omega^{L-1} p_1 \\ 1 & \omega^0 p_2 & \ldots & \omega^{L-1} p_2 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^0 p_K & \ldots & \omega^{L-1} p_K \end{bmatrix}$$

where $\omega = e^{-j 2\pi / N}$. We further denote

$$\hat{A} \triangleq XF.$$  

However, a large number of pilots is required. The CS theory shows that we can reduce the number of pilots, i.e., $2S < K < L$, by exploiting the sparse property of wireless channels. To identify the positions $149$ of the $S$ nonzero entries as well as estimating the coefficients of the $152$ $S$ nonzero entries, which results in totally $2S$ unknown parameters, $153$ we have to use at least $K > 2S$ pilots. With this condition, we can $154$ apply CS algorithms, e.g., orthogonal matching pursuit (OMP), to $155$ estimate $h$. Existing works have already shown that the CS algorithms $156$ outperform LS for channel estimation $157$.

The restrict isometry property (RIP) shows that $h$ in (6) can be $158$ recovered from the noiseless measurement $y(\eta = 0)$ with a high $159$ probability if the measurement matrix $A$ satisfies the RIP $17$. It is $160$ known that if $A \in \mathbb{C}^{K \times L}$ in (6) satisfies the RIP if there exists a constant $161$ $\delta(0 < \delta < 1)$ such that

$$\| \tilde{A} u \|_2^2 \leq (1 + \delta) \| u \|_2^2$$

holds for all $S$-sparse vectors $u \in \mathbb{C}^L$. However, it is computationally infeasible to check whether a given matrix $A$ satisfies the RIP $164$. Alternatively, according to $18$, we can minimize the coherence of $A$, which is known as the MIP. The MIP condition is stronger than the RIP in that the MIP implies the RIP but the converse is not true $18$. Moreover, the MIP is more intuitive and practical than the RIP. Here, we consider the pilot design including joint pilot power allocation and pilot pattern optimization with respect to the MIP.

Given a pilot pattern, i.e.,

$$p = \{ c_{p_1}, c_{p_2}, \ldots, c_{p_K} \}$$

and a pilot power vector, i.e.,

$$v = \{ v_1, v_2, \ldots, v_K \}$$

1$1 u \in \mathbb{C}^L$ is said to be $S$-sparse ($S \ll L$) if the number of nonzero entries of $u$ is equal to $S$ or smaller than $S$. 

Fig. 1. Joint design of pilot power and pilot pattern for SUs under peak power constraint.
173 with \( v_i \) denoting the power of the \( i \)th pilot subcarrier, i.e.,
\[
v_i \triangleq |x(c_{pi})|^2, \quad i = 1, 2, \ldots, K
\]
(11)
we define the coherence of \( A \) as the maximum absolute correlation
176 between any two different columns of \( A \), i.e.,
\[
g(p, v) \triangleq \max_{0 \leq m < n \leq L - 1} |\langle A(m), A(n) \rangle|
\]
(12)
where \( \langle A(m), A(n) \rangle \) denotes the normalized inner product between
178 the \( m \)th column \( A(m) \) and the \( n \)th column \( A(n) \) of \( A \), i.e.,
\[
\langle A(m), A(n) \rangle \triangleq \frac{A^H(m)A(n)}{\|A(m)\|_2 \|A(n)\|_2}.
\]
(13)
179 Let \( d = n - m \) and \( \Lambda = \{1, 2, \ldots, L - 1\} \). Then, (12) can be rewritten
180 as
\[
g(p, v) = \max_{d \in \Lambda} \left| \sum_{i=1}^{K} v_i \omega_{pi}^d \right| / \sum_{i=1}^{K} v_i.
\]
(14)
181 According to the MIP, the objective for the pilot design is to
182 minimize the coherence of \( A \), i.e., \( \min_{p,v} g(p,v) \). The constraint
183 for the integer vector \( p \) is \( p \subseteq P \). Suppose the sum power of all pilot
184 symbols is
\[
\sum_{i=1}^{K} v_i = V_T.
\]
(15)
185 where \( V_T \) is the prespecified sum power constraint of SUs. Obviously, \( (15) \) is more general in practical OFDM-based CR systems than simply
187 assuming
\[
v_1 = v_2 = \cdots = v_K = \frac{V_T}{K}.
\]
(16)
188 in current literatures [8]–[10]. On the other hand, SUs should properly
189 control the peak power of pilot subcarriers regarding the linear region
190 of power amplifiers. The power of pilot subcarriers cannot be too
191 large or too small. As shown in Fig. 1, we denote \( V_{\text{th}} \) as the peak
192 power constraint related to the saturation power of the power amplifier.
193 Moreover, the pilot power should be greater than a threshold \( V_L \), which
194 is related to the cutoff power of the power amplifier as well as the
195 noise and interference level around SUs. In particular, \( V_L = 0 \) can be
196 regarded as a special case. Hence, we have
\[
V_L \leq v_i \leq V_H, \quad i = 1, 2, \ldots, K.
\]
(17)
197 With these constraints, the pilot design in OFDM-based CR systems
198 can be formulated as
\[
\min_{p,v} g(p,v) \quad \text{s.t. } p \subseteq P \sum_{i=1}^{K} v_i = V_T, \quad V_L \leq v_i \leq V_H
\]
(18)
199 which involves the joint optimization of the discrete integer vector \( p \)
200 and the continuous real-valued positive vector \( v \). Note that unlike most
201 literatures, investigating the optimal power allocation to maximize
202 the achievable rate of CR systems, in this paper, we focus on the
pilot design for sparse channel estimation, where the design of data
203 subcarriers is out of the scope of this work.

Apparently, it is analytically intractable to get a solution from (18). We now decouple this joint optimization problem with the following two kinds of sequential formulations.

1) Given a \( \tilde{p} \subseteq P \), we first get
\[
g_v(\tilde{p}) \triangleq \min_{v} g(\tilde{p}, v)
\]
(20)
to get an optimal \( v \). Meanwhile, the corresponding \( v \) is also obtained.

2) Given a feasible \( \tilde{v} \), we first get
\[
g_p(\tilde{v}) \triangleq \min_{p \subseteq P} g(p, \tilde{v})
\]
(21)
then we solve
\[
\min_{p \subseteq P} g_p(\tilde{v})
\]
(22)
to get an optimal \( v \). Meanwhile, the corresponding \( p \) is also obtained.

Comparing \( p \subseteq P \) with the constraints in (15) and (17), it can be seen that \( p \) is less difficult to enumerate than \( v \). Therefore, it is better to decouple the joint optimization problem described by (18) with the first kind of formulations described by (19) and (20).

III. PILOT POWER ALLOCATION

Regarding (19), with a given pilot pattern \( \tilde{p} \subseteq P \), we first generate a table, i.e.,
\[
G = \begin{bmatrix}
\omega^{c_1} & \omega^{c_2} & \cdots & \omega^{c_M} \\
\omega^{l_1} & \omega^{l_2} & \cdots & \omega^{l_M} \\
\vdots & \vdots & \ddots & \vdots \\
\omega^{(L-1)c_1} & \omega^{(L-1)c_2} & \cdots & \omega^{(L-1)c_M}
\end{bmatrix}
\]
(23)
where \( \omega = e^{-j\pi/N} \). Once \( N \), \( L \) and \( C \) are given, \( G \) is determined.

We look up \( G \) and select the corresponding \( K \) columns indexed by \( \tilde{p} \) from \( G \), making up a \( L - 1 \) by \( K \) submatrix \( G(\tilde{p}) \). Then, from (14), (24) we have
\[
g(\tilde{p}, \tilde{v}) = \frac{1}{V_T} \|G(\tilde{p})\tilde{v}\|_\infty.
\]
(24)
Therefore, (19) is equivalent to
\[
\min_{\tilde{v}} \|G(\tilde{p})\tilde{v}\|_\infty \quad \text{s.t. } \sum_{i=1}^{K} v_i = V_T, \quad V_L \leq v_i \leq V_H
\]
(25)
where $\tilde{p}$ is given, and $G(\tilde{p})$ is a complex-valued submatrix fast generated by looking up $G$. Let $b_i$ denote the $i$th row of $G(\tilde{p})$, $i = 1, 2, \ldots, L - 1$. We further denote

$$B_i = \begin{bmatrix} \Re\{b_i\} \\ \Im\{b_i\} \end{bmatrix}, \quad i = 1, 2, \ldots, L - 1 \quad (26)$$

232 which is a real-valued matrix with two rows and $K$ columns. Then, 233 (25) can be converted into a real-valued optimization problem as

$$\min \ z$$

s.t. $\|B_i v\|_2 \leq z, \quad i = 1, 2, \ldots, L - 1$

$$\sum_{i=1}^{K} v_i = V_T, \quad V_L \leq v_i \leq V_H \quad (27)$$

234 which is an SOCP optimization problem that contains $L - 1$ second-235 order conic constraints and some linear constraints. Typically, it can be236 solved by SOCP solvers, e.g., MOSEK. Then, we can obtain feasible237 solutions as $\tilde{z}$ and $\tilde{v}$ from (27), where $g_i(\tilde{p}) = \tilde{z}$.

IV. JOINT PILOT DESIGN

239 If $M$ and $K$ are not small enough, $\mathcal{P}$ can be a huge set. For example, 240 if $M = 512$ and $K = 16$, $\|\mathcal{P}\| = \binom{512}{16} = 8.4 \times 10^{29}$. It is impossible 241 for SUUs to store $\mathcal{P}$ into the memory and check them one by one 242 until the best $p \in \mathcal{P}$ is found. Furthermore, it is very computationally 243 inefficient to implement the exhaustive search from such huge space, 244 particularly for SUUs equipped with power-constrained mobile devices. 245 The proposed joint design algorithm including discrete optimization 246 for pilot pattern and continuous optimization for pilot power is 247 described in Algorithm 1. At first, we input system parameters $C, N, 248 M, K, L, J, T_1,$ and $T_2$, where $T_1$ and $T_2$ represent the number 249 of outer-loop and inner-loop iterations, respectively. Each outer-loop 250 iteration includes $T_2$ inner-loop iterations. Then, we initialize a zero-251 matrix $D$ to store the results of optimized pilot patterns after running 252 inner-loop iterations. Each row of $D$ stores a pilot pattern $p$, with the 253 corresponding objective value $g_i(p)$ stored in $r_i$ which is initialized 254 to be a zero vector. Then, we generate a table $G$ according to (23). 255 At each outer-loop iteration, indicated from step 4 to step 16, we 256 start by randomly generating a pilot pattern $p \in \mathcal{P}$. By introducing the 257 randomness to the algorithm so that it starts from different initial pilot 258 patterns, we can avoid that the algorithm falls into local optimums. 259 As $T_1$ increases to infinity, the algorithm will converge to the global 260 optimum. Then, we use the inner-loop iterations from step 6 to step 261 to obtain an optimized pilot pattern $p$, which is stored in each 262 row of $D$ with the corresponding objective value $g_i(p)$ stored in $r_i$, 263 indicated by step 15. After we finish the outer-loop iterations, we select 264 the minimum from $r_i$ and output the corresponding row of $D$ as the 265 designed pilot pattern $p_o$, indicated by step 17 and step 18. We then 266 substitute $p_o$ into (27) to design the pilot power.

Algorithm 1 Joint Design of Pilot Power and Pilot Pattern

```plaintext
1: Input: C, N, M, K, L, J, T_1, T_2.
2: Initialization: D \leftarrow 0^{N \times K}, r \leftarrow 0^{T_1}.
3: Generate G according to (23).
4: for l = 1, 2, \ldots, T_1 do
5: randomly generate p \in \mathcal{P}, p^* \leftarrow 0^K.
6: for n = 1, 2, \ldots, T_2 do
7: if p = p^*
8: break.
9: end if
10: p^* \leftarrow p.
11: for k = 1, 2, \ldots, K/J do
12: Obtain p_{p,k} according to (31). p \leftarrow \hat{p}_{p,k}.
13: end for(k)
14: end for(n)
15: D(l) \leftarrow p. r(l) \leftarrow g_i(p).
16: end for(l)
17: t = arg min_{i=1,2,\ldots,T_1} r(i).
18: Output p_o = D(t) as the designed pilot pattern.
19: Substitute p_o into (27) to design the pilot power.
```

Here, we use an auxiliary vector $p^*$, which always records the pilot 286 pattern obtained from the previous inner-loop iteration. If we find that 287 $p$ is exactly the same as $p^*$, which means we did not get a new pilot 288 pattern, there is no need to continue the inner-loop iterations because 289 the results thereafter will be exactly the same. Then, we break from 290 the inner-loop iterations. These procedures are indicated from step 7 291 to step 10. Additionally, we have to reset $p^*$ by $p^* \leftarrow 0^K$ at the start 292 of each outer-loop iteration. This way, we can save the CPU running 293 time and therefore improve the efficiency by skipping the same routine. 294

The main contribution of Algorithm 1 is the group update of entries 295 of $p$, which is shown from step 11 to step 13. The update of 296 $K/J$ entries each time, where $J$ is a positive integer. For 297 $k = 1, 2, \ldots, K/J$, given the 298 latest $p$ from the last inner-loop iteration, we update the $k$th group of 299 entries of $p$ with the best group selected from 300

$$W = \{w | w \subseteq \Psi, \|w\|_0 = 1\} \quad (28)$$

where 301

$$\Psi = C \setminus \{p(i)|i = 1, 2, \ldots, K, i \notin \Phi\} \quad (29)$$

$$\Phi = \{k_J - J + 1, k_J - J + 2, \ldots, K_J\} \quad (30)$$

Mathematically, the resultant pilot pattern $p_{p,k}$ with the update of the 302 $k$th group of entries is given by

$$p_{p,k} = arg \min_{\{p|p(i) \neq p^*(i), i = 1, 2, \ldots, K/J\} \subseteq \Psi} g_i(p) \quad (31)$$

where the computation of $g_i(p)$ is provided in Section III. After we 304 obtain $p_{p,k}$ for given $p$ and $k$, we update $p$ by $p = \hat{p}_{p,k}$.

The complexity for pilot power allocation given a pilot pattern in 306 (27) is $O((L - 1)^{1.5}(K + 1)^3)$. Therefore, the computational com- 307 plexity for Algorithm 1 is

$$O(T_1T_2(L - 1)^{1.5}(K + 1)^3 \frac{K}{J} (M - K + J)) \quad (32)$$

suppose that $T_2$ is small enough [19]. If $T_2$ is large, the inner-loop 309 iterations will terminate itself by procedures from step 7 to step 9, 310 leading to even lower complexity than (32).

Remark: If we set $J = K$, Algorithm 1 degenerates to be the 311 exhaustive search, where $|W| = \binom{K}{K/J}$, and $|\Phi| = K$. In this case, 312 no matter what $T_1$ and $T_2$ are, they are equivalent to $T_1 = T_2 = 1$, 313 resulting in $O((M - K + J))$, which is the extraordinarily high com- 314 plexity of the exhaustive search. In practice, we usually set $J = 1$ or 315 $J = 2$ to reduce the complexity. For example, if $J = 1$, (32) reduces 316 to a polynomial complexity, i.e., $O(T_1T_2(L - 1)^{1.5}(K + 1)^3K(M - 318 K + 1))$. 319
To compare this work with [14], we set the same system parameters as [14]. We consider an OFDM-based CR system with $N = 1024$ subcarriers. After ideal spectrum sensing, there are $M = 512$ active subcarriers available for SUs, including three subcarrier blocks, i.e., $B_1 = \{1, 2, 3, \ldots, 256\}, B_2 = \{513, 514, \ldots, 640\}, B_3 = \{897, 898, \ldots, 1024\}$, with the number of contiguous subcarriers of each block being 256, 128, and 128, respectively. From $C = B_1 \cup B_2 \cup B_3$, which can also be regarded as the union of several active CR subbands for SUs, we want to select $K = 16$ pilot subcarriers for frequency-domain pilot-assisted channel estimation. A sparse multipath channel $h$ is generated with $L = 60$ taps, where $S = 5$ dominant nonzero channel taps are randomly placed among $L$ taps. The channel gain of each path is independent and identically distributed complex Gaussian distributed with unit variance, i.e., $CN(0, 1)$. Quadrature phase-shift keying modulation is employed in the simulations.

To evaluate the performance of Algorithm 1, we first compare it with the CE method [14] assuming equal pilot power indicated by (16), where the joint pilot design is simplified to be the pilot pattern design. We set $V_T = 1$, which means that the sum power of pilot subcarriers is normalized. We set $V_c = 0.03$ and $V_H = 0.1$ so that the pilot power does not vary too much. The steps for optimal pilot power allocation in Algorithm 1 are skipped by directly substituting $v_1 = v_2 = \cdots = v_{16} = 0.0625$ into (24). The parameters of the CE method are selected to be the best in [14], where the maximum number of iterations, the number of random samples, the sample quantile, and the smoothing factor are set to be 50, 100,000, 0.001, and 0.3, respectively. It can be seen in Fig. 2 that Algorithm 1 is much faster convergent than the CE method. Here, we compare the convergence speed with respect to the running time instead of the number of iterations, because the running time of each iteration is different for different algorithms or parameters. Since we run the simulations under the exact same computer hardware and software, the running time is proportional to the computational complexity. We set $T_I = 1000$ and $T_2 = 15$ for Algorithm 1. Once the running time, i.e., 342 s, which is the running time for the CE method [14], is reached, we terminate Algorithm 1 so that we can compare Algorithm 1 with the CE method under the same computational complexity. As shown in Fig. 2, Algorithm 1 with $J = 1$ can achieve the best performance of the CE method in no more than 10 s, which indicates that Algorithm 1 is 359 times faster than the CE method and, therefore, is much more efficient and powerful. Since it has already been demonstrated in [5] that the CE method outperforms GA and PSO, Algorithm 1 is a remarkable candidate for integer optimization with its applications not restricted to the pilot design. As shown in Fig. 2, although Algorithm 1 converges slower than that with $J = 1$, it can achieve better performance than that with $J = 1$ if the running time is long enough, i.e., longer than 55 s. For those SUs equipped with powerful CPU and large capacity of battery, it is better to set $J = 2$ or even 368 larger. The finally obtained pilot patterns $p$ with the corresponding objective $g(p, v)$ during 342 s of running time assuming equal pilot power are listed in Table I. Note that in [14], we suppose $v_1 = v_2 = \cdots = v_{16} = 1$, whereas in this paper, we suppose $v_1 = v_2 = \cdots = v_{16} = 0.0625$ satisfying $\sum_{i=1}^{16} v_i = 1$, the objective in [14] has to be divided by $K = 16$ when compared with this work.

We now evaluate the performance of joint design of pilot power and pilot pattern and compare it with the pilot design assuming equal pilot power. As shown in Fig. 2, Algorithm 1 with $J = 1$ using joint 377 pilot design achieves better performance than Algorithm 1 with $J = 1$ and $J = 2$, while its computational complexity is between $J = 1$ and $J = 2$. The obtained pilot pattern with the corresponding objective is also listed in Table I. The comparisons of MSE performance and the bit error rate (BER) performance for sparse channel estimation are shown in Figs. 3 and 4, respectively. Both the MSE and BER are averaged 383
Then, we have proposed a joint design algorithm. Simulation results have verified the effectiveness of the proposed algorithm and shown that the proposed algorithm can achieve better channel estimation performance and further improve the spectrum efficiency by 2.4%, compared with existing algorithms assuming equal pilot power.

We have formulated the design of pilot power as an SOCP problem. Given a pilot pattern, the joint design can reduce the pilot overhead by 71 pilots and improve almost the same performance as LS with the pilot interval being 7 and 6, respectively, is also provided. It is seen that Algorithm 1 with \( J = 1 \) using joint pilot design achieves 13.9\% improvement compared with the pilot design assuming equal pilot power in [14].

VI. CONCLUSION

In this correspondence, we have investigated the joint design of pilot power and pilot pattern based on the rule of MIP. The pilot design has been formulated as a joint optimization problem, which is then decoupled into tractable sequential formations. Given a pilot pattern, we have formulated the design of pilot power as an SOCP problem. Then, we have proposed a joint design algorithm. Simulation results have verified the effectiveness of the proposed algorithm and shown that the proposed algorithm can achieve better channel estimation performance and further improve the spectrum efficiency by 2.4%, compared with existing algorithms assuming equal pilot power.

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