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DOI:

[10.1109/TVT.2014.2374692](https://doi.org/10.1109/TVT.2014.2374692)

Document Version

Peer reviewed version

[Link to publication record in King's Research Portal](#)

Citation for published version (APA):

Qi, C., Wu, L., Huang, Y., & Nallanathan, A. (2015). Joint Design of Pilot Power and Pilot Pattern for Sparse Cognitive Radio Systems. *IEEE Transactions on Vehicular Technology*, 64(11), 5384-5390. [6966783].
<https://doi.org/10.1109/TVT.2014.2374692>

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Joint Design of Pilot Power and Pilot Pattern for Sparse Cognitive Radio Systems

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Abstract—Existing works design the pilot pattern for sparse channel estimation, assuming that the power of all pilots is equal. However, equal power allocation is not optimal in cognitive radio (CR) systems. In this correspondence, we jointly design the pilot power and pilot pattern for sparse channel estimation in orthogonal-frequency-division-multiplexing-based CR systems, based on the rule of mutual incoherence property that minimizes the coherence of the measurement matrix used for the sparse recovery. Under the sum power constraint and peak power constraint, the pilot design is formulated as a joint optimization problem, which is then decoupled into tractable sequential formations. Given a pilot pattern, we formulate the design of pilot power as a second-order cone programming. Then, we propose a joint design algorithm, which includes discrete optimization for pilot pattern and continuous optimization for pilot power. Simulation results show that the proposed algorithm can achieve better channel estimation performance in terms of mean square error and bit error rate and can further improve the spectrum efficiency by 2.4%, compared with existing algorithms assuming equal pilot power.

Index Terms—Cognitive radio (CR), compressed sensing (CS), orthogonal frequency-division multiplexing (OFDM), pilot design, sparse channel estimation.

I. INTRODUCTION

Radio spectrum is a precious and limited resource for wireless communications. In attempts to relieve the spectrum shortage, the concept of cognitive radio (CR) is proposed, which allows secondary users (SUs) to opportunistically access the spectrum that is originally allocated to primary users (PUs). SUs start communications with each other when the spectrum is not used by any PU. Therefore, CR can improve the usage of existing frequency bands without allocating a new spectrum resource [1]. On the other hand, with the great capability in combating frequency-selective fading and the high flexibility in allocating transmit resources, orthogonal frequency-division multiplexing (OFDM) has been suggested as a competitive candidate in CR systems [2]. In OFDM-based CR systems, the subcarriers are noncontiguous. Hence, the efficient design of pilots including pilot pattern and pilot power is crucial to the performance of channel estimation and data detection. In [3], a scheme to design pilot symbols for OFDM-based

CR systems is proposed, where the pilot design assuming equal pilot power is formulated as an optimization problem that minimizes the upper bound related to the mean square error (MSE) of least squares (LS) channel estimation. In [4], a scheme that utilizes cross entropy (CE) optimization together with the analytical pilot power optimization is proposed to design pilot symbols to reduce the MSE of the LS channel estimation. In [5], parameter adaptation for wireless multicarrier-based CR systems is investigated where the CE method is demonstrated to outperform the genetic algorithm (GA) and particle swarm optimization (PSO). However, all these literatures are based on the LS channel estimation.

Recently, sparse channel estimation that exploits the inherent sparse property of wireless multipath channels and applies the compressed sensing (CS) techniques for channel estimation has been proven to improve the channel estimation performance and reduce the pilot overhead compared with the LS method [6], [7]. To further improve the performance of sparse channel estimation, one effective approach is to optimize the pilot design. In [8] and [9], it has been shown that the pilot pattern generated from the cyclic different set (CDS) is optimal and have proposed a scheme to obtain a near-optimal pilot pattern when the CDS does not exist. In [10] and [11], two pilot design schemes based on CE optimization and stochastic approximation, respectively, are proposed to minimize the MSE of sparse channel estimation using the channel data. In [12], a pilot allocation method based on the GA and a shifting mechanism is proposed for sparse channel estimation in multiple-input-multiple-output OFDM systems. In [13], a pilot design scheme for OFDM transmission over two-way relay networks is presented. In [14], sparse channel estimation is first introduced in OFDM-based CR systems. Based on the results of spectrum sensing, a scheme using constrained CE optimization is proposed to obtain an optimized pilot pattern. In particular, it is shown in [14] that sparse channel estimation can achieve 11.5% improvement in spectrum efficiency with the same channel estimation performance compared with the LS channel estimation. However, all existing works design the pilot pattern for sparse channel estimation assuming that the power of pilots is equal.

In this correspondence, based on the work of Qi et al. in [14], we further consider the pilot power optimization for sparse channel estimation in OFDM-based CR systems. Note that the CR system employing sparse channel estimation is termed as the sparse CR system in this work. We jointly design the pilot power and pilot pattern based on the rule of mutual incoherence property (MIP) that minimizes the coherence of the measurement matrix used for the sparse recovery. Under the sum power constraint and peak power constraint, the pilot design is formulated as a joint optimization problem, which is then decoupled into tractable sequential formations. Given a pilot pattern, we formulate the design of pilot power as a second-order cone programming (SOCP). Then, we propose a joint design algorithm, which includes discrete optimization for pilot pattern and continuous optimization for pilot power.

The notations used in this correspondence are defined as follows. Symbols for matrices (uppercase) and vectors (lowercase) are in boldface. $(\cdot)^T$, $(\cdot)^H$, $\text{diag}\{\cdot\}$, $\mathbf{I}_L$, $\setminus$, $\mathbb{C}^{M \times N}$, $\mathbf{0}^{M \times N}$, $\mathcal{CN}$, $|\cdot|$, $\|\cdot\|_0$, $\|\cdot\|_2$, $\|\cdot\|_\infty$, $\text{Re}\{\cdot\}$, and $\text{Im}\{\cdot\}$ denote the matrix transpose, conjugate transpose (Hermitian), the diagonal matrix, the identity matrix of size L , the set exclusion, the set of $M \times N$ complex matrices, the $M \times N$ zero matrix, the complex Gaussian distribution, the absolute

Manuscript received February 27, 2014; revised July 4, 2014, and September 24, 2014; accepted November 16, 2014. This work was supported in part by the National Natural Science Foundation of China under Grants 61302097 and 61422105, by the Ph.D. Programs Foundation of the Ministry of Education of China under Grant 20120092120014, and by the Natural Science Foundation of Jiangsu Province under Grant BK20130019. The review of this paper was coordinated by Prof. X. Wang.

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Digital Object Identifier 10.1109/TVT.2014.2374692

Fig. 1. Joint design of pilot power and pilot pattern for SUs under peak power constraint.

99 value, the ℓ_0 norm, the ℓ_2 norm, the ℓ_∞ norm, real part, and imaginary
100 part, respectively. $\mathcal{O}(\cdot)$ denotes the order of complexity. $\hat{\phi}$ denotes the
101 estimate of the parameter of interest ϕ .

102 II. PROBLEM FORMULATION

103 We consider an OFDM-based CR system employing sparse channel
104 estimation to exploit the inherent sparse property of wireless multipath
105 channels. The channel is modeled as an finite impulse response filter
106 with the channel impulse response (CIR) to be

$$\mathbf{h} = [h(1), h(2), \dots, h(L)]^T. \quad (1)$$

107 $\mathbf{h}$ is sparse if the number of nonzero entries of $\mathbf{h}$, which is denoted as
108 S , is much smaller than the channel length L ($S \ll L$).

109 Similar to [3] and [14], we assume ideal spectrum sensing without
110 false alarm or missing detection. Based on the results of spectrum
111 sensing, the OFDM subcarriers occupied by PUs are first deactivated.
112 From the active subcarriers, we use some subcarriers to transmit pilot
113 symbols and the others to transmit data symbols for SUs. We assume
114 that the secondary transmitters of SUs broadcast the results of pilot
115 design to the secondary receivers of SUs through control signaling.

116 Suppose an OFDM-based CR system with N subcarriers. After
117 deactivating subcarriers occupied by PUs, there are M ($M \leq N$)
118 active subcarriers available for SUs. We denote these active subcarriers
119 as $\mathcal{C} = \{c_1, c_2, \dots, c_M\}$, where $\mathcal{C} \subseteq \mathcal{N}$ with $\mathcal{N} = \{1, 2, \dots, N\}$.
120 Without loss of generality, we suppose $1 \leq c_1 < c_2 < \dots < c_M \leq$
121 N . From $\mathcal{C}$, we select K ($K \leq M$) pilot subcarriers indexed by
122 $c_{p_1}, c_{p_2}, \dots, c_{p_K}$ ($1 \leq p_1 < p_2 < \dots < p_K \leq M$) to transmit pilot
123 symbols for frequency-domain pilot-assisted channel estimation, as
124 shown in Fig. 1. The indexes of pilot subcarriers make up a pilot
125 pattern $\mathbf{p} = \{c_{p_1}, c_{p_2}, \dots, c_{p_K}\}$, where $\mathbf{p} \subseteq \mathcal{C}$. We define the set of
126 all possible pilot patterns as $\mathcal{P} \triangleq \{\mathbf{w} | \mathbf{w} \subseteq \mathcal{C}, \|\mathbf{w}\|_0 = K\}$. Then,
127 we have $\mathbf{p} \in \mathcal{P}$. Note that all existing literatures of pilot design for
128 sparse channel estimation are all based on the equal pilot power
129 assumption. However, in OFDM-based CR systems, the pilot power
130 can be different, as shown in Fig. 1, where the equal pilot power is
131 not necessarily optimal. In this paper, we will study the joint design of
132 pilot power and pilot pattern.

133 We denote the transmit pilot symbols and the receive pilot sym-
134 bols as $\mathbf{x} = [x(c_{p_1}), x(c_{p_2}), \dots, x(c_{p_K})]^T$ and $\mathbf{y} = [y(c_{p_1}), y(c_{p_2}),$
135 $\dots, y(c_{p_K})]^T$, respectively. For LS channel estimation, we first
136 acquire channel frequency response (CFR) at pilot subcarriers by
137 $\{y(i)/x(i), i \in \mathbf{p}\}$ and then make interpolations for the rest of the
138 subcarriers. However, it usually demands a large number of pilots, i.e.,
139 $K > L$, so that the interpolations can approximate the true value of

CFR. The relation between the transmit pilots and the receive pilots
140 can be written in matrix notation as
141

$$\mathbf{y} = \mathbf{X}\mathbf{F}\mathbf{h} + \boldsymbol{\eta} \quad (2)$$

where

$$\mathbf{X} = \text{diag}\{x(c_{p_1}), x(c_{p_2}), \dots, x(c_{p_K})\} \quad (3)$$

$$\boldsymbol{\eta} = [\eta(1), \eta(2), \dots, \eta(K)]^T \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_K) \quad (4)$$

and $\mathbf{F}$ is a discrete Fourier transform submatrix given by

$$\mathbf{F} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \omega^{c_{p_1}} & \dots & \omega^{c_{p_1}(L-1)} \\ 1 & \omega^{c_{p_2}} & \dots & \omega^{c_{p_2}(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{c_{p_K}} & \dots & \omega^{c_{p_K}(L-1)} \end{bmatrix}$$

where $\omega = e^{-j2\pi/N}$. We further denote

$$\mathbf{A} \triangleq \mathbf{X}\mathbf{F}. \quad (5)$$

Then, (2) can be written as

$$\mathbf{y} = \mathbf{A}\mathbf{h} + \boldsymbol{\eta}. \quad (6)$$

If $L \leq K \leq M$ and $\mathbf{A}$ has full column rank, (6) can be solved by
146 LS, which essentially employs the fast Fourier transform interpolations
147 with the estimated CIR given by
148

$$\hat{\mathbf{h}}_{\text{LS}} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{y}. \quad (7)$$

However, a large number of pilots is required. The CS theory shows
149 that we can reduce the number of pilots, i.e., $2S < K < L$, by explor-
150 ing the sparse property of wireless channels. To identify the positions
151 of the S nonzero entries as well as estimating the coefficients of the
152 S nonzero entries, which results in totally $2S$ unknown parameters,
153 we have to use at least $K > 2S$ pilots. With this condition, we can
154 apply CS algorithms, e.g., orthogonal matching pursuit (OMP), to
155 estimate $\mathbf{h}$. Existing works have already shown that the CS algorithms
156 outperform LS for channel estimation [15], [16].
157

The restrict isometry property (RIP) shows that $\mathbf{h}$ in (6) can be
158 recovered from the noiseless measurement $\mathbf{y}(\boldsymbol{\eta} = \mathbf{0})$ with a high
159 probability if the measurement matrix $\mathbf{A}$ satisfies the RIP [17]. It is
160 said that $\mathbf{A} \in \mathbb{C}^{K \times L}$ in (6) satisfies the RIP if there exists a constant
161 δ ($0 < \delta < 1$) such that
162

$$(1 - \delta)\|\mathbf{u}\|_2^2 \leq \|\mathbf{A}\mathbf{u}\|_2^2 \leq (1 + \delta)\|\mathbf{u}\|_2^2 \quad (8)$$

holds for all S -sparse¹ vectors $\mathbf{u} \in \mathbb{C}^L$. However, it is computation-
163 ally infeasible to check whether a given matrix $\mathbf{A}$ satisfies the RIP.
164 Alternatively, according to [18], we can minimize the coherence of $\mathbf{A}$,
165 which is known as the MIP. The MIP condition is stronger than the
166 RIP in that the MIP implies the RIP but the converse is not true [18].
167 Moreover, the MIP is more intuitive and practical than the RIP. Here,
168 we consider the pilot design including joint pilot power allocation and
169 pilot pattern optimization with respect to the MIP.
170

Given a pilot pattern, i.e.,

$$\mathbf{p} = \{c_{p_1}, c_{p_2}, \dots, c_{p_K}\} \quad (9)$$

and a pilot power vector, i.e.,

$$\mathbf{v} = \{v_1, v_2, \dots, v_K\} \quad (10)$$

¹ $\mathbf{u} \in \mathbb{C}^L$ is said to be S -sparse ($S \ll L$) if the number of nonzero entries
of $\mathbf{u}$ is equal to S or smaller than S .

with v_i denoting the power of the i th pilot symbol transmitted by the c_{p_i} th subcarrier, i.e.,

$$v_i \triangleq |x(c_{p_i})|^2, \quad i = 1, 2, \dots, K \quad (11)$$

we define the coherence of $\mathbf{A}$ as the maximum absolute correlation between any two different columns of $\mathbf{A}$, i.e.,

$$\begin{aligned} g(\mathbf{p}, \mathbf{v}) &\triangleq \max_{0 \leq m < n \leq L-1} |\langle A(m), A(n) \rangle| \\ &= \max_{0 \leq m < n \leq L-1} \left| \sum_{i=1}^K v_i \omega^{c_{p_i}(n-m)} \right| / \sum_{i=1}^K v_i \end{aligned} \quad (12)$$

where $\langle A(m), A(n) \rangle$ denotes the normalized inner product between the m th column $A(m)$ and the n th column $A(n)$ of $\mathbf{A}$, i.e.,

$$\langle A(m), A(n) \rangle \triangleq \frac{A^H(m)A(n)}{\|A(m)\|_2 \|A(n)\|_2}. \quad (13)$$

Let $d = n - m$ and $\Lambda = \{1, 2, \dots, L-1\}$. Then, (12) can be rewritten as

$$g(\mathbf{p}, \mathbf{v}) = \max_{d \in \Lambda} \left| \sum_{i=1}^K v_i \omega^{c_{p_i} d} \right| / \sum_{i=1}^K v_i. \quad (14)$$

According to the MIP, the objective for the pilot design is to minimize the coherence of $\mathbf{A}$, i.e., $\min_{\mathbf{p}, \mathbf{v}} g(\mathbf{p}, \mathbf{v})$. The constraint for the integer vector $\mathbf{p}$ is $\mathbf{p} \in \mathcal{P}$. Suppose the sum power of all pilot symbols is

$$\sum_{i=1}^K v_i = V_T \quad (15)$$

where V_T is the prespecified sum power constraint of SUs. Obviously, (15) is more general in practical OFDM-based CR systems than simply assuming

$$v_1 = v_2 = \dots = v_K = \frac{V_T}{K} \quad (16)$$

in current literatures [8]–[10]. On the other hand, SUs should properly control the peak power of pilot subcarriers regarding the linear region of power amplifiers. The power of pilot subcarriers cannot be too large or too small. As shown in Fig. 1, we denote V_H as the peak power constraint related to the saturation power of the power amplifier. Moreover, the pilot power should be greater than a threshold V_L , which is related to the cutoff power of the power amplifier as well as the noise and interference level around SUs. In particular, $V_L = 0$ can be regarded as a special case. Hence, we have

$$V_L \leq v_i \leq V_H, \quad i = 1, 2, \dots, K. \quad (17)$$

With these constraints, the pilot design in OFDM-based CR systems can be formulated as

$$\begin{aligned} \min_{\mathbf{p}, \mathbf{v}} \quad & g(\mathbf{p}, \mathbf{v}) \\ \text{s.t.} \quad & \mathbf{p} \in \mathcal{P} \\ & \sum_{i=1}^K v_i = V_T, \quad V_L \leq v_i \leq V_H \end{aligned} \quad (18)$$

which involves the joint optimization of the discrete integer vector $\mathbf{p}$ and the continuous real-valued positive vector $\mathbf{v}$. Note that unlike most literatures, investigating the optimal power allocation to maximize the achievable rate of CR systems, in this paper, we focus on the

pilot design for sparse channel estimation, where the design of data subcarriers is out of the scope of this work.

Apparently, it is analytically intractable to get a solution from (18). We now decouple this joint optimization problem with the following two kinds of sequential formulations.

1) Given a $\tilde{\mathbf{p}} \in \mathcal{P}$, we first get

$$\begin{aligned} g_v(\tilde{\mathbf{p}}) &\triangleq \min_{\mathbf{v}} g(\tilde{\mathbf{p}}, \mathbf{v}) \\ \text{s.t.} \quad & \sum_{i=1}^K v_i = V_T, \quad V_L \leq v_i \leq V_H \end{aligned} \quad (19)$$

then we solve

$$\min_{\tilde{\mathbf{p}} \in \mathcal{P}} g_v(\tilde{\mathbf{p}}) \quad (20)$$

to get an optimal $\mathbf{p}$. Meanwhile, the corresponding $\mathbf{v}$ is also obtained.

2) Given a feasible $\tilde{\mathbf{v}}$, we first get

$$g_p(\tilde{\mathbf{v}}) \triangleq \min_{\mathbf{p} \in \mathcal{P}} g(\mathbf{p}, \tilde{\mathbf{v}}) \quad (21)$$

then we solve

$$\begin{aligned} \min_{\tilde{\mathbf{v}}} \quad & g_p(\tilde{\mathbf{v}}) \\ \text{s.t.} \quad & \sum_{i=1}^K \tilde{v}_i = V_T, \quad V_L \leq \tilde{v}_i \leq V_H \end{aligned} \quad (22)$$

to get an optimal $\mathbf{v}$. Meanwhile, the corresponding $\mathbf{p}$ is also obtained.

Comparing $\mathbf{p} \in \mathcal{P}$ with the constraints in (15) and (17), it can be seen that $\mathbf{p}$ is less difficult to enumerate than $\mathbf{v}$. Therefore, it is better to decouple the joint optimization problem described by (18) with the first kind of formulations described by (19) and (20).

III. PILOT POWER ALLOCATION

Regarding (19), with a given pilot pattern $\tilde{\mathbf{p}} \in \mathcal{P}$, we first generate a table, i.e.,

$$\mathbf{G} = \begin{bmatrix} \omega^{c_1} & \omega^{c_2} & \dots & \omega^{c_M} \\ \omega^{2c_1} & \omega^{2c_2} & \dots & \omega^{2c_M} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^{(L-1)c_1} & \omega^{(L-1)c_2} & \dots & \omega^{(L-1)c_M} \end{bmatrix} \quad (23)$$

where $\omega = e^{-j2\pi/N}$. Once N , L and C are given, $\mathbf{G}$ is determined.

We look up $\mathbf{G}$ and select the corresponding K columns indexed by $\tilde{\mathbf{p}}$ from $\mathbf{G}$, making up a $L-1$ by K submatrix $\mathbf{G}(\tilde{\mathbf{p}})$. Then, from (14), we have

$$g(\tilde{\mathbf{p}}, \mathbf{v}) = \frac{1}{V_T} \|\mathbf{G}(\tilde{\mathbf{p}})\mathbf{v}\|_{\infty}. \quad (24)$$

Therefore, (19) is equivalent to

$$\begin{aligned} \min_{\mathbf{v}} \quad & \|\mathbf{G}(\tilde{\mathbf{p}})\mathbf{v}\|_{\infty} \\ \text{s.t.} \quad & \sum_{i=1}^K v_i = V_T, \quad V_L \leq v_i \leq V_H \end{aligned} \quad (25)$$

229 where $\tilde{\mathbf{p}}$ is given, and $\mathbf{G}(\tilde{\mathbf{p}})$ is a complex-valued submatrix fast
 230 generated by looking up $\mathbf{G}$. Let $\mathbf{b}_i$ denote the i th row of $\mathbf{G}(\tilde{\mathbf{p}})$,
 231 $i = 1, 2, \dots, L - 1$. We further denote

$$\mathbf{B}_i = \begin{bmatrix} \text{Re}\{\mathbf{b}_i\} \\ \text{Im}\{\mathbf{b}_i\} \end{bmatrix}, \quad i = 1, 2, \dots, L - 1 \quad (26)$$

232 which is a real-valued matrix with two rows and K columns. Then,
 233 (25) can be converted into a real-valued optimization problem as

$$\begin{aligned} \min_z \quad & z \\ \text{s.t.} \quad & \|\mathbf{B}_i \mathbf{v}\|_2 \leq z, \quad i = 1, 2, \dots, L - 1 \\ & \sum_{i=1}^K v_i = V_T, \quad V_L \leq v_i \leq V_H \end{aligned} \quad (27)$$

234 which is an SOCP optimization problem that contains $L - 1$ second-
 235 order conic constraints and some linear constraints. Typically, it can be
 236 solved by SOCP solvers, e.g., MOSEK. Then, we can obtain feasible
 237 solutions as $\tilde{z}$ and $\tilde{\mathbf{v}}$ from (27), where $g_v(\tilde{\mathbf{p}}) = \tilde{z}$.

238 IV. JOINT PILOT DESIGN

239 If M and K are not small enough, $\mathcal{P}$ can be a huge set. For example,
 240 if $M = 512$ and $K = 16$, $\|\mathcal{P}\|_0 = \binom{512}{16} = 8.4 \times 10^{29}$. It is impossi-
 241 ble for SUs to store $\mathcal{P}$ into the memory and check them one by one
 242 until the best $\mathbf{p} \in \mathcal{P}$ is found. Furthermore, it is very computationally
 243 inefficient to implement the exhaustive search from such huge space,
 244 particularly for SUs equipped with power-constrained mobile devices.
 245 The proposed joint design algorithm including discrete optimiza-
 246 tion for pilot pattern and continuous optimization for pilot power is
 247 described in Algorithm 1. At first, we input system parameters $\mathcal{C}$, N ,
 248 M , K , L , J , T_1 , and T_2 , where T_1 and T_2 represent the number
 249 of outer-loop and inner-loop iterations, respectively. Each outer-loop
 250 iteration includes T_2 inner-loop iterations. Then, we initialize a zero-
 251 matrix $\mathbf{D}$ to store the results of optimized pilot patterns after running
 252 inner-loop iterations. Each row of $\mathbf{D}$ stores a pilot pattern $\mathbf{p}$, with the
 253 corresponding objective value $g_v(\mathbf{p})$ stored in $\mathbf{r}$, which is initialized
 254 to be a zero vector. Then, we generate a table $\mathbf{G}$ according to (23).
 255 At each outer-loop iteration, indicated from step 4 to step 16, we
 256 start by randomly generating a pilot pattern $\mathbf{p} \in \mathcal{P}$. By introducing the
 257 randomness to the algorithm so that it starts from different initial pilot
 258 patterns, we can avoid that the algorithm falls into local optimums.
 259 As T_1 increases to infinity, the algorithm will converge to the global
 260 optimum. Then, we use the inner-loop iterations from step 6 to step
 261 14 to obtain an optimized pilot pattern $\mathbf{p}$, which is stored in each
 262 row of $\mathbf{D}$ with the corresponding objective value $g_v(\mathbf{p})$ stored in $\mathbf{r}$,
 263 indicated by step 15. After we finish the outer-loop iterations, we select
 264 the minimum from $\mathbf{r}$ and output the corresponding row of $\mathbf{D}$ as the
 265 designed pilot pattern $\mathbf{p}_o$, indicated by step 17 and step 18. We then
 266 substitute $\mathbf{p}_o$ into (27) to design the pilot power.

Algorithm 1 Joint Design of Pilot Power and Pilot Pattern

267 1: Input: $\mathcal{C}$, N , M , K , L , J , T_1 , T_2 .
 268 2: Initializations: $\mathbf{D} \leftarrow \mathbf{0}^{T_1 \times K}$, $\mathbf{r} \leftarrow \mathbf{0}^{T_1}$.
 269 3: Generate $\mathbf{G}$ according to (23).
 270 4: **for** $l = 1, 2, \dots, T_1$
 271 5: randomly generate $\mathbf{p} \in \mathcal{P}$. $\mathbf{p}^* \leftarrow \mathbf{0}^K$.
 272 6: **for** $n = 1, 2, \dots, T_2$
 273 7: **if** $\mathbf{p} = \mathbf{p}^*$
 274 8: break.
 275 9: **end if**

10: $\mathbf{p}^* \leftarrow \mathbf{p}$.
 11: **for** $k = 1, 2, \dots, K/J$
 12: Obtain $\hat{\mathbf{p}}_{\mathbf{p},k}$ according to (31). $\mathbf{p} \leftarrow \hat{\mathbf{p}}_{\mathbf{p},k}$.
 13: **end for**(k)
 14: **end for**(n)
 15: $D(l) \leftarrow \mathbf{p}$. $r(l) \leftarrow g_v(\mathbf{p})$.
 16: **end for**(l)
 17: $t = \arg \min_{i=1,2,\dots,T_1} r(i)$.
 18: Output $\mathbf{p}_o = D(t)$ as the designed pilot pattern.
 19: Substitute $\mathbf{p}_o$ into (27) to design the pilot power.

Here, we use an auxiliary vector $\mathbf{p}^*$, which always records the pilot
 pattern obtained from the previous inner-loop iteration. If we find that
 $\mathbf{p}$ is exactly the same as $\mathbf{p}^*$, which means we did not get a new pilot
 pattern, there is no need to continue the inner-loop iterations because
 the results thereafter will be exactly the same. Then, we break from
 the inner-loop iterations. These procedures are indicated from step 7
 to step 10. Additionally, we have to reset $\mathbf{p}^*$ by $\mathbf{p}^* \leftarrow \mathbf{0}^K$ at the start
 of each outer-loop iteration. This way, we can save the CPU running
 time and therefore improve the efficiency by skipping the same routine.

The main contribution of Algorithm 1 is the group update of entries
 of $\mathbf{p}$, which is shown from step 11 to step 13. The update of $\mathbf{p}$ is
 implemented in a group of J entries each time, where K is divisible
 by J , i.e., K/J is a positive integer. For $k = 1, 2, \dots, K/J$, given the
 latest $\mathbf{p}$ from the last inner-loop iteration, we update the k th group of
 entries of $\mathbf{p}$ with the best group selected from

$$\mathcal{W} = \{\mathbf{w} | \mathbf{w} \subseteq \Psi, \|\mathbf{w}\|_0 = J\} \quad (28)$$

where

$$\Psi = \mathcal{C} \setminus \{p(i) | i = 1, 2, \dots, K, i \notin \Phi\} \quad (29)$$

$$\Phi = \{kJ - J + 1, kJ - J + 2, \dots, kJ\}. \quad (30)$$

Mathematically, the resultant pilot pattern $\hat{\mathbf{p}}_{\mathbf{p},k}$ with the update of the
 k th group of entries is given by

$$\hat{\mathbf{p}}_{\mathbf{p},k} = \arg \min_{\substack{\tilde{\mathbf{p}}(i)=p(i), \quad i=1,2,\dots,K, i \notin \Phi \\ \{\tilde{\mathbf{p}}(i), i \in \Phi\} \subset \mathcal{W}}} g_v(\tilde{\mathbf{p}}) \quad (31)$$

where the computation of $g_v(\tilde{\mathbf{p}})$ is provided in Section III. After we
 obtain $\hat{\mathbf{p}}_{\mathbf{p},k}$ for given $\mathbf{p}$ and k , we update $\mathbf{p}$ by $\mathbf{p} \leftarrow \hat{\mathbf{p}}_{\mathbf{p},k}$.

The complexity for pilot power allocation given a pilot pattern in
 (27) is $\mathcal{O}((L-1)^{1.5}(K+1)^3)$. Therefore, the computational com-
 plexity for Algorithm 1 is

$$\mathcal{O}\left(T_1 T_2 (L-1)^{1.5} (K+1)^3 \frac{K}{J} \binom{M-K+J}{J}\right) \quad (32)$$

suppose that T_2 is small enough [19]. If T_2 is large, the inner-loop
 iterations will terminate itself by procedures from step 7 to step 9,
 leading to even lower complexity than (32).

Remark: If we set $J = K$, Algorithm 1 degenerates to be the
 exhaustive search, where $\|\mathcal{W}\|_0 = \binom{M}{K}$, and $\|\Phi\|_0 = K$. In this case,
 no matter what T_1 and T_2 are, they are equivalent to $T_1 = T_2 = 1$,
 resulting in $\mathcal{O}(\binom{M-K+J}{J})$, which is the extraordinarily high com-
 plexity of the exhaustive search. In practice, we usually set $J = 1$ or
 $J = 2$ to reduce the complexity. For example, if $J = 1$, (32) reduces
 to a polynomial complexity, i.e., $\mathcal{O}(T_1 T_2 (L-1)^{1.5} (K+1)^3 K(M - K + 1))$.

Fig. 2. Comparisons of convergence and complexity for different algorithms or parameters.

V. SIMULATION RESULTS

To compare this work with [14], we set the same system parameters as [14]. We consider an OFDM-based CR system with $N = 1024$ subcarriers. After ideal spectrum sensing, there are $M = 512$ active subcarriers available for SUs, including three subcarrier blocks, i.e., $\mathcal{B}_1 = \{1, 2, 3, \dots, 256\}$, $\mathcal{B}_2 = \{513, 514, \dots, 640\}$, $\mathcal{B}_3 = \{897, 898, \dots, 1024\}$, with the number of contiguous subcarriers of each block being 256, 128, and 128, respectively. From $\mathcal{C} = \mathcal{B}_1 \cup \mathcal{B}_2 \cup \mathcal{B}_3$, which can also be regarded as the union of several active CR subbands for SUs, we want to select $K = 16$ pilot subcarriers for frequency-domain pilot-assisted channel estimation. A sparse multipath channel $\mathbf{h}$ is generated with $L = 60$ taps, where $S = 5$ dominant nonzero channel taps are randomly placed among L taps. The channel gain of each path is independent and identically distributed complex Gaussian distributed with unit variance, i.e., $\mathcal{CN}(0, 1)$. Quaternary phase-shift keying modulation is employed in the simulations.

To evaluate the performance of Algorithm 1, we first compare it with the CE method [14] assuming equal pilot power indicated by (16), where the joint pilot design is simplified to be the pilot pattern design. We set $V_T = 1$, which means that the sum power of pilot subcarriers is normalized. We set $V_L = 0.03$ and $V_H = 0.1$ so that the pilot power does not vary too much. The steps for optimal pilot power allocation in Algorithm 1 are skipped by directly substituting $v_1 = v_2 = \dots = v_{16} = 0.0625$ into (24). The parameters of the CE method are selected to be the best in [14], where the maximum number of iterations, the number of random samples, the sample quantile, and the smoothing factor are set to be 50, 100 000, 0.001, and 0.3, respectively. It can be seen in Fig. 2 that Algorithm 1 is much faster convergent than the CE method. Here, we compare the convergence speed with respect to the running time instead of the number of iterations, because the running time of each iteration is different for different algorithms or parameters. Since we run the simulations under the exact same computer hardware and software, the running time is proportional to the computational complexity. We set $T_1 = 1000$ and $T_2 = 15$ for Algorithm 1. Once the running time, i.e., 342 s, which is the running time for the CE method [14], is reached, we terminate Algorithm 1 so that we can compare Algorithm 1 with the CE method under the same computational complexity. As shown in Fig. 2, Algorithm 1 with $J = 1$ can achieve the best performance of the

TABLE I
COMPARISONS OF PILOT PATTERNS USING DIFFERENT ALGORITHMS OR PARAMETERS

Type	$g(\mathbf{p}, \mathbf{v})$	$\mathbf{p}$
CE method	0.3260	47, 95, 110, 162, 180, 193, 246, 513 524, 627, 640, 897, 910, 939, 976, 1019
Algorithm 1 with $J = 1$, equal	0.3141	18, 45, 57, 103, 141, 187, 242, 256 513, 593, 609, 640, 897, 909, 926, 977
Algorithm 1 with $J = 2$, equal	0.3077	30, 46, 77, 121, 182, 228, 247, 256 513, 529, 619, 633, 905, 932, 960, 1021
Algorithm 1 with $J = 1$, joint	0.2945	1, 34, 47, 98, 109, 181, 221, 236 252, 513, 529, 609, 635, 900, 934, 959

Fig. 3. Comparisons of MSE performance for different algorithms parameters.

CE method in no more than 10 s, which indicates that Algorithm 1 is 34 times faster than the CE method and, therefore, is much more efficient and powerful. Since it has already been demonstrated in [5] that the CE method outperforms GA and PSO, Algorithm 1 is a remarkable candidate for integer optimization with its applications not restricted to the pilot design. As shown in Fig. 2, although Algorithm 1 with $J = 2$ converges slower than that with $J = 1$, it can achieve better performance than that with $J = 1$ if the running time is long enough, i.e., longer than 55 s. For those SUs equipped with powerful CPU and large capacity of battery, it is better to set $J = 2$ or even larger. The finally obtained pilot patterns $\mathbf{p}$ with the corresponding objective $g(\mathbf{p}, \mathbf{v})$ during 342 s of running time assuming equal pilot power are listed in Table I. Note that in [14], we suppose $v_1 = v_2 = \dots = v_{16} = 1$, whereas in this paper, we suppose $v_1 = v_2 = \dots = v_{16} = 0.0625$ satisfying $\sum_{i=1}^{16} v_i = 1$, the objective in [14] has to be divided by $K = 16$ when compared with this work.

We now evaluate the performance of joint design of pilot power and pilot pattern and compare it with the pilot design assuming equal pilot power. As shown in Fig. 2, Algorithm 1 with $J = 1$ using joint pilot design achieves better performance than Algorithm 1 with $J = 1$ or $J = 2$, while its computational complexity is between $J = 1$ and $J = 2$. The obtained pilot pattern with the corresponding objective is also listed in Table I. The comparisons of MSE performance and the bit error rate (BER) performance for sparse channel estimation are shown in Figs. 3 and 4, respectively. Both the MSE and BER are averaged

Fig. 4. Comparisons of BER performance for different algorithms or parameters.

over 10 000 sparse channel realizations. The popular OMP algorithm is employed for sparse channel estimation given the designed pilot pattern and pilot power. For comparisons, the performance of LS channel estimation using $K = 75$ and $K = 87$ equally spaced pilots with the pilot interval being 7 and 6, respectively, is also provided. It is seen that Algorithm 1 with $J = 1$ using joint pilot design achieves almost the same performance as LS with $K = 87$. Therefore, the joint design can reduce the pilot overhead by 71 pilots and improve the spectrum efficiency by 13.9%, thus leading to additional 2.4% improvement compared with the pilot design assuming equal pilot power in [14].

VI. CONCLUSION

In this correspondence, we have investigated the joint design of pilot power and pilot pattern based on the rule of MIP. The pilot design has been formulated as a joint optimization problem, which is then decoupled into tractable sequential formations. Given a pilot pattern, we have formulated the design of pilot power as an SOCP problem. Then, we have proposed a joint design algorithm. Simulation results have verified the effectiveness of the proposed algorithm and shown that the proposed algorithm can achieve better channel estimation performance and further improve the spectrum efficiency by 2.4%, compared with existing algorithms assuming equal pilot power.

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Joint Design of Pilot Power and Pilot Pattern for Sparse Cognitive Radio Systems

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Abstract—Existing works design the pilot pattern for sparse channel estimation, assuming that the power of all pilots is equal. However, equal power allocation is not optimal in cognitive radio (CR) systems. In this correspondence, we jointly design the pilot power and pilot pattern for sparse channel estimation in orthogonal-frequency-division-multiplexing-based CR systems, based on the rule of mutual incoherence property that minimizes the coherence of the measurement matrix used for the sparse recovery. Under the sum power constraint and peak power constraint, the pilot design is formulated as a joint optimization problem, which is then decoupled into tractable sequential formations. Given a pilot pattern, we formulate the design of pilot power as a second-order cone programming. Then, we propose a joint design algorithm, which includes discrete optimization for pilot pattern and continuous optimization for pilot power. Simulation results show that the proposed algorithm can achieve better channel estimation performance in terms of mean square error and bit error rate and can further improve the spectrum efficiency by 2.4%, compared with existing algorithms assuming equal pilot power.

Index Terms—Cognitive radio (CR), compressed sensing (CS), orthogonal frequency-division multiplexing (OFDM), pilot design, sparse channel estimation.

I. INTRODUCTION

Radio spectrum is a precious and limited resource for wireless communications. In attempts to relieve the spectrum shortage, the concept of cognitive radio (CR) is proposed, which allows secondary users (SUs) to opportunistically access the spectrum that is originally allocated to primary users (PUs). SUs start communications with each other when the spectrum is not used by any PU. Therefore, CR can improve the usage of existing frequency bands without allocating a new spectrum resource [1]. On the other hand, with the great capability in combating frequency-selective fading and the high flexibility in allocating transmit resources, orthogonal frequency-division multiplexing (OFDM) has been suggested as a competitive candidate in CR systems [2]. In OFDM-based CR systems, the subcarriers are noncontiguous. Hence, the efficient design of pilots including pilot pattern and pilot power is crucial to the performance of channel estimation and data detection. In [3], a scheme to design pilot symbols for OFDM-based

CR systems is proposed, where the pilot design assuming equal pilot power is formulated as an optimization problem that minimizes the upper bound related to the mean square error (MSE) of least squares (LS) channel estimation. In [4], a scheme that utilizes cross entropy (CE) optimization together with the analytical pilot power optimization is proposed to design pilot symbols to reduce the MSE of the LS channel estimation. In [5], parameter adaptation for wireless multicarrier-based CR systems is investigated where the CE method is demonstrated to outperform the genetic algorithm (GA) and particle swarm optimization (PSO). However, all these literatures are based on the LS channel estimation.

Recently, sparse channel estimation that exploits the inherent sparse property of wireless multipath channels and applies the compressed sensing (CS) techniques for channel estimation has been proven to improve the channel estimation performance and reduce the pilot overhead compared with the LS method [6], [7]. To further improve the performance of sparse channel estimation, one effective approach is to optimize the pilot design. In [8] and [9], it has been shown that the pilot pattern generated from the cyclic different set (CDS) is optimal and have proposed a scheme to obtain a near-optimal pilot pattern when the CDS does not exist. In [10] and [11], two pilot design schemes based on CE optimization and stochastic approximation, respectively, are proposed to minimize the MSE of sparse channel estimation using the channel data. In [12], a pilot allocation method based on the GA and a shifting mechanism is proposed for sparse channel estimation in multiple-input-multiple-output OFDM systems. In [13], a pilot design scheme for OFDM transmission over two-way relay networks is presented. In [14], sparse channel estimation is first introduced in OFDM-based CR systems. Based on the results of spectrum sensing, a scheme using constrained CE optimization is proposed to obtain an optimized pilot pattern. In particular, it is shown in [14] that sparse channel estimation can achieve 11.5% improvement in spectrum efficiency with the same channel estimation performance compared with the LS channel estimation. However, all existing works design the pilot pattern for sparse channel estimation assuming that the power of pilots is equal.

In this correspondence, based on the work of Qi et al. in [14], we further consider the pilot power optimization for sparse channel estimation in OFDM-based CR systems. Note that the CR system employing sparse channel estimation is termed as the sparse CR system in this work. We jointly design the pilot power and pilot pattern based on the rule of mutual incoherence property (MIP) that minimizes the coherence of the measurement matrix used for the sparse recovery. Under the sum power constraint and peak power constraint, the pilot design is formulated as a joint optimization problem, which is then decoupled into tractable sequential formations. Given a pilot pattern, we formulate the design of pilot power as a second-order cone programming (SOCP). Then, we propose a joint design algorithm, which includes discrete optimization for pilot pattern and continuous optimization for pilot power.

The notations used in this correspondence are defined as follows. Symbols for matrices (uppercase) and vectors (lowercase) are in boldface. $(\cdot)^T$, $(\cdot)^H$, $\text{diag}\{\cdot\}$, $\mathbf{I}_L$, $\setminus$, $\mathbb{C}^{M \times N}$, $\mathbf{0}^{M \times N}$, $\mathcal{CN}$, $|\cdot|$, $\|\cdot\|_0$, $\|\cdot\|_2$, $\|\cdot\|_\infty$, $\text{Re}\{\cdot\}$, and $\text{Im}\{\cdot\}$ denote the matrix transpose, conjugate transpose (Hermitian), the diagonal matrix, the identity matrix of size L , the set exclusion, the set of $M \times N$ complex matrices, the $M \times N$ zero matrix, the complex Gaussian distribution, the absolute

Manuscript received February 27, 2014; revised July 4, 2014, and September 24, 2014; accepted November 16, 2014. This work was supported in part by the National Natural Science Foundation of China under Grants 61302097 and 61422105, by the Ph.D. Programs Foundation of the Ministry of Education of China under Grant 20120092120014, and by the Natural Science Foundation of Jiangsu Province under Grant BK20130019. The review of this paper was coordinated by Prof. X. Wang.

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Digital Object Identifier 10.1109/TVT.2014.2374692

Fig. 1. Joint design of pilot power and pilot pattern for SUs under peak power constraint.

99 value, the ℓ_0 norm, the ℓ_2 norm, the ℓ_∞ norm, real part, and imaginary
100 part, respectively. $\mathcal{O}(\cdot)$ denotes the order of complexity. $\hat{\phi}$ denotes the
101 estimate of the parameter of interest ϕ .

102 II. PROBLEM FORMULATION

103 We consider an OFDM-based CR system employing sparse channel
104 estimation to exploit the inherent sparse property of wireless multipath
105 channels. The channel is modeled as an finite impulse response filter
106 with the channel impulse response (CIR) to be

$$\mathbf{h} = [h(1), h(2), \dots, h(L)]^T. \quad (1)$$

107 $\mathbf{h}$ is sparse if the number of nonzero entries of $\mathbf{h}$, which is denoted as
108 S , is much smaller than the channel length L ($S \ll L$).

109 Similar to [3] and [14], we assume ideal spectrum sensing without
110 false alarm or missing detection. Based on the results of spectrum
111 sensing, the OFDM subcarriers occupied by PUs are first deactivated.
112 From the active subcarriers, we use some subcarriers to transmit pilot
113 symbols and the others to transmit data symbols for SUs. We assume
114 that the secondary transmitters of SUs broadcast the results of pilot
115 design to the secondary receivers of SUs through control signaling.

116 Suppose an OFDM-based CR system with N subcarriers. After
117 deactivating subcarriers occupied by PUs, there are M ($M \leq N$)
118 active subcarriers available for SUs. We denote these active subcarriers
119 as $\mathcal{C} = \{c_1, c_2, \dots, c_M\}$, where $\mathcal{C} \subseteq \mathcal{N}$ with $\mathcal{N} = \{1, 2, \dots, N\}$.
120 Without loss of generality, we suppose $1 \leq c_1 < c_2 < \dots < c_M \leq$
121 N . From $\mathcal{C}$, we select K ($K \leq M$) pilot subcarriers indexed by
122 $c_{p_1}, c_{p_2}, \dots, c_{p_K}$ ($1 \leq p_1 < p_2 < \dots < p_K \leq M$) to transmit pilot
123 symbols for frequency-domain pilot-assisted channel estimation, as
124 shown in Fig. 1. The indexes of pilot subcarriers make up a pilot
125 pattern $\mathbf{p} = \{c_{p_1}, c_{p_2}, \dots, c_{p_K}\}$, where $\mathbf{p} \subseteq \mathcal{C}$. We define the set of
126 all possible pilot patterns as $\mathcal{P} \triangleq \{\mathbf{w} | \mathbf{w} \subseteq \mathcal{C}, \|\mathbf{w}\|_0 = K\}$. Then,
127 we have $\mathbf{p} \in \mathcal{P}$. Note that all existing literatures of pilot design for
128 sparse channel estimation are all based on the equal pilot power
129 assumption. However, in OFDM-based CR systems, the pilot power
130 can be different, as shown in Fig. 1, where the equal pilot power is
131 not necessarily optimal. In this paper, we will study the joint design of
132 pilot power and pilot pattern.

133 We denote the transmit pilot symbols and the receive pilot sym-
134 bols as $\mathbf{x} = [x(c_{p_1}), x(c_{p_2}), \dots, x(c_{p_K})]^T$ and $\mathbf{y} = [y(c_{p_1}), y(c_{p_2}),$
135 $\dots, y(c_{p_K})]^T$, respectively. For LS channel estimation, we first
136 acquire channel frequency response (CFR) at pilot subcarriers by
137 $\{y(i)/x(i), i \in \mathbf{p}\}$ and then make interpolations for the rest of the
138 subcarriers. However, it usually demands a large number of pilots, i.e.,
139 $K > L$, so that the interpolations can approximate the true value of

CFR. The relation between the transmit pilots and the receive pilots
140 can be written in matrix notation as
141

$$\mathbf{y} = \mathbf{X}\mathbf{F}\mathbf{h} + \boldsymbol{\eta} \quad (2)$$

where

$$\mathbf{X} = \text{diag}\{x(c_{p_1}), x(c_{p_2}), \dots, x(c_{p_K})\} \quad (3)$$

$$\boldsymbol{\eta} = [\eta(1), \eta(2), \dots, \eta(K)]^T \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_K) \quad (4)$$

and $\mathbf{F}$ is a discrete Fourier transform submatrix given by

$$\mathbf{F} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & \omega^{c_{p_1}} & \dots & \omega^{c_{p_1}(L-1)} \\ 1 & \omega^{c_{p_2}} & \dots & \omega^{c_{p_2}(L-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega^{c_{p_K}} & \dots & \omega^{c_{p_K}(L-1)} \end{bmatrix}$$

where $\omega = e^{-j2\pi/N}$. We further denote

$$\mathbf{A} \triangleq \mathbf{X}\mathbf{F}. \quad (5)$$

Then, (2) can be written as

$$\mathbf{y} = \mathbf{A}\mathbf{h} + \boldsymbol{\eta}. \quad (6)$$

If $L \leq K \leq M$ and $\mathbf{A}$ has full column rank, (6) can be solved by
146 LS, which essentially employs the fast Fourier transform interpolations
147 with the estimated CIR given by
148

$$\hat{\mathbf{h}}_{\text{LS}} = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{y}. \quad (7)$$

However, a large number of pilots is required. The CS theory shows
149 that we can reduce the number of pilots, i.e., $2S < K < L$, by explor-
150 ing the sparse property of wireless channels. To identify the positions
151 of the S nonzero entries as well as estimating the coefficients of the
152 S nonzero entries, which results in totally $2S$ unknown parameters,
153 we have to use at least $K > 2S$ pilots. With this condition, we can
154 apply CS algorithms, e.g., orthogonal matching pursuit (OMP), to
155 estimate $\mathbf{h}$. Existing works have already shown that the CS algorithms
156 outperform LS for channel estimation [15], [16].
157

The restrict isometry property (RIP) shows that $\mathbf{h}$ in (6) can be
158 recovered from the noiseless measurement $\mathbf{y}(\boldsymbol{\eta} = \mathbf{0})$ with a high
159 probability if the measurement matrix $\mathbf{A}$ satisfies the RIP [17]. It is
160 said that $\mathbf{A} \in \mathbb{C}^{K \times L}$ in (6) satisfies the RIP if there exists a constant
161 δ ($0 < \delta < 1$) such that
162

$$(1 - \delta)\|\mathbf{u}\|_2^2 \leq \|\mathbf{A}\mathbf{u}\|_2^2 \leq (1 + \delta)\|\mathbf{u}\|_2^2 \quad (8)$$

holds for all S -sparse¹ vectors $\mathbf{u} \in \mathbb{C}^L$. However, it is computation-
163 ally infeasible to check whether a given matrix $\mathbf{A}$ satisfies the RIP.
164 Alternatively, according to [18], we can minimize the coherence of $\mathbf{A}$,
165 which is known as the MIP. The MIP condition is stronger than the
166 RIP in that the MIP implies the RIP but the converse is not true [18].
167 Moreover, the MIP is more intuitive and practical than the RIP. Here,
168 we consider the pilot design including joint pilot power allocation and
169 pilot pattern optimization with respect to the MIP.
170

Given a pilot pattern, i.e.,

$$\mathbf{p} = \{c_{p_1}, c_{p_2}, \dots, c_{p_K}\} \quad (9)$$

and a pilot power vector, i.e.,

$$\mathbf{v} = \{v_1, v_2, \dots, v_K\} \quad (10)$$

¹ $\mathbf{u} \in \mathbb{C}^L$ is said to be S -sparse ($S \ll L$) if the number of nonzero entries
of $\mathbf{u}$ is equal to S or smaller than S .

with v_i denoting the power of the i th pilot symbol transmitted by the c_{p_i} th subcarrier, i.e.,

$$v_i \triangleq |x(c_{p_i})|^2, \quad i = 1, 2, \dots, K \quad (11)$$

we define the coherence of $\mathbf{A}$ as the maximum absolute correlation between any two different columns of $\mathbf{A}$, i.e.,

$$g(\mathbf{p}, \mathbf{v}) \triangleq \max_{0 \leq m < n \leq L-1} |\langle A(m), A(n) \rangle|$$

$$= \max_{0 \leq m < n \leq L-1} \left| \frac{\sum_{i=1}^K v_i \omega^{c_{p_i}(n-m)}}{\sum_{i=1}^K v_i} \right| \quad (12)$$

where $\langle A(m), A(n) \rangle$ denotes the normalized inner product between the m th column $A(m)$ and the n th column $A(n)$ of $\mathbf{A}$, i.e.,

$$\langle A(m), A(n) \rangle \triangleq \frac{A^H(m)A(n)}{\|A(m)\|_2 \|A(n)\|_2}. \quad (13)$$

Let $d = n - m$ and $\Lambda = \{1, 2, \dots, L-1\}$. Then, (12) can be rewritten as

$$g(\mathbf{p}, \mathbf{v}) = \max_{d \in \Lambda} \left| \frac{\sum_{i=1}^K v_i \omega^{c_{p_i} d}}{\sum_{i=1}^K v_i} \right| \quad (14)$$

According to the MIP, the objective for the pilot design is to minimize the coherence of $\mathbf{A}$, i.e., $\min_{\mathbf{p}, \mathbf{v}} g(\mathbf{p}, \mathbf{v})$. The constraint for the integer vector $\mathbf{p}$ is $\mathbf{p} \in \mathcal{P}$. Suppose the sum power of all pilot symbols is

$$\sum_{i=1}^K v_i = V_T \quad (15)$$

where V_T is the prespecified sum power constraint of SUs. Obviously, (15) is more general in practical OFDM-based CR systems than simply assuming

$$v_1 = v_2 = \dots = v_K = \frac{V_T}{K} \quad (16)$$

in current literatures [8]–[10]. On the other hand, SUs should properly control the peak power of pilot subcarriers regarding the linear region of power amplifiers. The power of pilot subcarriers cannot be too large or too small. As shown in Fig. 1, we denote V_H as the peak power constraint related to the saturation power of the power amplifier. Moreover, the pilot power should be greater than a threshold V_L , which is related to the cutoff power of the power amplifier as well as the noise and interference level around SUs. In particular, $V_L = 0$ can be regarded as a special case. Hence, we have

$$V_L \leq v_i \leq V_H, \quad i = 1, 2, \dots, K. \quad (17)$$

With these constraints, the pilot design in OFDM-based CR systems can be formulated as

$$\min_{\mathbf{p}, \mathbf{v}} g(\mathbf{p}, \mathbf{v})$$

$$\text{s.t. } \mathbf{p} \in \mathcal{P}$$

$$\sum_{i=1}^K v_i = V_T, \quad V_L \leq v_i \leq V_H \quad (18)$$

which involves the joint optimization of the discrete integer vector $\mathbf{p}$ and the continuous real-valued positive vector $\mathbf{v}$. Note that unlike most literatures, investigating the optimal power allocation to maximize the achievable rate of CR systems, in this paper, we focus on the

pilot design for sparse channel estimation, where the design of data subcarriers is out of the scope of this work.

Apparently, it is analytically intractable to get a solution from (18). We now decouple this joint optimization problem with the following two kinds of sequential formulations.

1) Given a $\tilde{\mathbf{p}} \in \mathcal{P}$, we first get

$$g_v(\tilde{\mathbf{p}}) \triangleq \min_{\mathbf{v}} g(\tilde{\mathbf{p}}, \mathbf{v})$$

$$\text{s.t. } \sum_{i=1}^K v_i = V_T, \quad V_L \leq v_i \leq V_H \quad (19)$$

then we solve

$$\min_{\tilde{\mathbf{p}} \in \mathcal{P}} g_v(\tilde{\mathbf{p}}) \quad (20)$$

to get an optimal $\mathbf{p}$. Meanwhile, the corresponding $\mathbf{v}$ is also obtained.

2) Given a feasible $\tilde{\mathbf{v}}$, we first get

$$g_p(\tilde{\mathbf{v}}) \triangleq \min_{\mathbf{p} \in \mathcal{P}} g(\mathbf{p}, \tilde{\mathbf{v}}) \quad (21)$$

then we solve

$$\min_{\tilde{\mathbf{v}}} g_p(\tilde{\mathbf{v}})$$

$$\text{s.t. } \sum_{i=1}^K \tilde{v}_i = V_T, \quad V_L \leq \tilde{v}_i \leq V_H \quad (22)$$

to get an optimal $\mathbf{v}$. Meanwhile, the corresponding $\mathbf{p}$ is also obtained.

Comparing $\mathbf{p} \in \mathcal{P}$ with the constraints in (15) and (17), it can be seen that $\mathbf{p}$ is less difficult to enumerate than $\mathbf{v}$. Therefore, it is better to decouple the joint optimization problem described by (18) with the first kind of formulations described by (19) and (20).

III. PILOT POWER ALLOCATION

Regarding (19), with a given pilot pattern $\tilde{\mathbf{p}} \in \mathcal{P}$, we first generate a table, i.e.,

$$\mathbf{G} = \begin{bmatrix} \omega^{c_1} & \omega^{c_2} & \dots & \omega^{c_M} \\ \omega^{2c_1} & \omega^{2c_2} & \dots & \omega^{2c_M} \\ \vdots & \vdots & \ddots & \vdots \\ \omega^{(L-1)c_1} & \omega^{(L-1)c_2} & \dots & \omega^{(L-1)c_M} \end{bmatrix} \quad (23)$$

where $\omega = e^{-j2\pi/N}$. Once N , L and C are given, $\mathbf{G}$ is determined.

We look up $\mathbf{G}$ and select the corresponding K columns indexed by $\tilde{\mathbf{p}}$ from $\mathbf{G}$, making up a $L-1$ by K submatrix $\mathbf{G}(\tilde{\mathbf{p}})$. Then, from (14), we have

$$g(\tilde{\mathbf{p}}, \mathbf{v}) = \frac{1}{V_T} \|\mathbf{G}(\tilde{\mathbf{p}})\mathbf{v}\|_{\infty}. \quad (24)$$

Therefore, (19) is equivalent to

$$\min_{\mathbf{v}} \|\mathbf{G}(\tilde{\mathbf{p}})\mathbf{v}\|_{\infty}$$

$$\text{s.t. } \sum_{i=1}^K v_i = V_T, \quad V_L \leq v_i \leq V_H \quad (25)$$

229 where $\tilde{\mathbf{p}}$ is given, and $\mathbf{G}(\tilde{\mathbf{p}})$ is a complex-valued submatrix fast
 230 generated by looking up $\mathbf{G}$. Let $\mathbf{b}_i$ denote the i th row of $\mathbf{G}(\tilde{\mathbf{p}})$,
 231 $i = 1, 2, \dots, L - 1$. We further denote

$$\mathbf{B}_i = \begin{bmatrix} \text{Re}\{\mathbf{b}_i\} \\ \text{Im}\{\mathbf{b}_i\} \end{bmatrix}, \quad i = 1, 2, \dots, L - 1 \quad (26)$$

232 which is a real-valued matrix with two rows and K columns. Then,
 233 (25) can be converted into a real-valued optimization problem as

$$\begin{aligned} \min_z \quad & z \\ \text{s.t.} \quad & \|\mathbf{B}_i \mathbf{v}\|_2 \leq z, \quad i = 1, 2, \dots, L - 1 \\ & \sum_{i=1}^K v_i = V_T, \quad V_L \leq v_i \leq V_H \end{aligned} \quad (27)$$

234 which is an SOCP optimization problem that contains $L - 1$ second-
 235 order conic constraints and some linear constraints. Typically, it can be
 236 solved by SOCP solvers, e.g., MOSEK. Then, we can obtain feasible
 237 solutions as $\tilde{z}$ and $\tilde{\mathbf{v}}$ from (27), where $g_v(\tilde{\mathbf{p}}) = \tilde{z}$.

238 IV. JOINT PILOT DESIGN

239 If M and K are not small enough, $\mathcal{P}$ can be a huge set. For example,
 240 if $M = 512$ and $K = 16$, $\|\mathcal{P}\|_0 = \binom{512}{16} = 8.4 \times 10^{29}$. It is impossi-
 241 ble for SUs to store $\mathcal{P}$ into the memory and check them one by one
 242 until the best $\mathbf{p} \in \mathcal{P}$ is found. Furthermore, it is very computationally
 243 inefficient to implement the exhaustive search from such huge space,
 244 particularly for SUs equipped with power-constrained mobile devices.
 245 The proposed joint design algorithm including discrete optimiza-
 246 tion for pilot pattern and continuous optimization for pilot power is
 247 described in Algorithm 1. At first, we input system parameters $\mathcal{C}$, N ,
 248 M , K , L , J , T_1 , and T_2 , where T_1 and T_2 represent the number
 249 of outer-loop and inner-loop iterations, respectively. Each outer-loop
 250 iteration includes T_2 inner-loop iterations. Then, we initialize a zero-
 251 matrix $\mathbf{D}$ to store the results of optimized pilot patterns after running
 252 inner-loop iterations. Each row of $\mathbf{D}$ stores a pilot pattern $\mathbf{p}$, with the
 253 corresponding objective value $g_v(\mathbf{p})$ stored in $\mathbf{r}$, which is initialized
 254 to be a zero vector. Then, we generate a table $\mathbf{G}$ according to (23).
 255 At each outer-loop iteration, indicated from step 4 to step 16, we
 256 start by randomly generating a pilot pattern $\mathbf{p} \in \mathcal{P}$. By introducing the
 257 randomness to the algorithm so that it starts from different initial pilot
 258 patterns, we can avoid that the algorithm falls into local optimums.
 259 As T_1 increases to infinity, the algorithm will converge to the global
 260 optimum. Then, we use the inner-loop iterations from step 6 to step
 261 14 to obtain an optimized pilot pattern $\mathbf{p}$, which is stored in each
 262 row of $\mathbf{D}$ with the corresponding objective value $g_v(\mathbf{p})$ stored in $\mathbf{r}$,
 263 indicated by step 15. After we finish the outer-loop iterations, we select
 264 the minimum from $\mathbf{r}$ and output the corresponding row of $\mathbf{D}$ as the
 265 designed pilot pattern $\mathbf{p}_o$, indicated by step 17 and step 18. We then
 266 substitute $\mathbf{p}_o$ into (27) to design the pilot power.

Algorithm 1 Joint Design of Pilot Power and Pilot Pattern

267 1: Input: $\mathcal{C}$, N , M , K , L , J , T_1 , T_2 .
 268 2: Initializations: $\mathbf{D} \leftarrow \mathbf{0}^{T_1 \times K}$, $\mathbf{r} \leftarrow \mathbf{0}^{T_1}$.
 269 3: Generate $\mathbf{G}$ according to (23).
 270 4: **for** $l = 1, 2, \dots, T_1$
 271 5: randomly generate $\mathbf{p} \in \mathcal{P}$. $\mathbf{p}^* \leftarrow \mathbf{0}^K$.
 272 6: **for** $n = 1, 2, \dots, T_2$
 273 7: **if** $\mathbf{p} = \mathbf{p}^*$
 274 8: break.
 275 9: **end if**

10: $\mathbf{p}^* \leftarrow \mathbf{p}$. 276
 11: **for** $k = 1, 2, \dots, K/J$ 277
 12: Obtain $\hat{\mathbf{p}}_{\mathbf{p},k}$ according to (31). $\mathbf{p} \leftarrow \hat{\mathbf{p}}_{\mathbf{p},k}$. 278
 13: **end for**(k) 279
 14: **end for**(n) 280
 15: $D(l) \leftarrow \mathbf{p}$. $r(l) \leftarrow g_v(\mathbf{p})$. 281
 16: **end for**(l) 282
 17: $t = \arg \min_{i=1,2,\dots,T_1} r(i)$. 283
 18: Output $\mathbf{p}_o = D(t)$ as the designed pilot pattern. 284
 19: Substitute $\mathbf{p}_o$ into (27) to design the pilot power. 285

Here, we use an auxiliary vector $\mathbf{p}^*$, which always records the pilot 286
 pattern obtained from the previous inner-loop iteration. If we find that 287
 $\mathbf{p}$ is exactly the same as $\mathbf{p}^*$, which means we did not get a new pilot 288
 pattern, there is no need to continue the inner-loop iterations because 289
 the results thereafter will be exactly the same. Then, we break from 290
 the inner-loop iterations. These procedures are indicated from step 7 291
 to step 10. Additionally, we have to reset $\mathbf{p}^*$ by $\mathbf{p}^* \leftarrow \mathbf{0}^K$ at the start 292
 of each outer-loop iteration. This way, we can save the CPU running 293
 time and therefore improve the efficiency by skipping the same routine. 294

The main contribution of Algorithm 1 is the group update of entries 295
 of $\mathbf{p}$, which is shown from step 11 to step 13. The update of $\mathbf{p}$ is 296
 implemented in a group of J entries each time, where K is divisible 297
 by J , i.e., K/J is a positive integer. For $k = 1, 2, \dots, K/J$, given the 298
 latest $\mathbf{p}$ from the last inner-loop iteration, we update the k th group of 299
 entries of $\mathbf{p}$ with the best group selected from 300

$$\mathcal{W} = \{\mathbf{w} | \mathbf{w} \subseteq \Psi, \|\mathbf{w}\|_0 = J\} \quad (28)$$

where 301

$$\Psi = \mathcal{C} \setminus \{p(i) | i = 1, 2, \dots, K, i \notin \Phi\} \quad (29)$$

$$\Phi = \{kJ - J + 1, kJ - J + 2, \dots, kJ\}. \quad (30)$$

Mathematically, the resultant pilot pattern $\hat{\mathbf{p}}_{\mathbf{p},k}$ with the update of the 302
 k th group of entries is given by 303

$$\hat{\mathbf{p}}_{\mathbf{p},k} = \arg \min_{\substack{\tilde{\mathbf{p}}(i)=p(i), \quad i=1,2,\dots,K, i \notin \Phi \\ \{\tilde{\mathbf{p}}(i), i \in \Phi\} \subset \mathcal{W}}} g_v(\tilde{\mathbf{p}}) \quad (31)$$

where the computation of $g_v(\tilde{\mathbf{p}})$ is provided in Section III. After we 304
 obtain $\hat{\mathbf{p}}_{\mathbf{p},k}$ for given $\mathbf{p}$ and k , we update $\mathbf{p}$ by $\mathbf{p} \leftarrow \hat{\mathbf{p}}_{\mathbf{p},k}$. 305

The complexity for pilot power allocation given a pilot pattern in 306
 (27) is $\mathcal{O}((L-1)^{1.5}(K+1)^3)$. Therefore, the computational com- 307
 plexity for Algorithm 1 is 308

$$\mathcal{O}\left(T_1 T_2 (L-1)^{1.5} (K+1)^3 \frac{K}{J} \binom{M-K+J}{J}\right) \quad (32)$$

suppose that T_2 is small enough [19]. If T_2 is large, the inner-loop 309
 iterations will terminate itself by procedures from step 7 to step 9, 310
 leading to even lower complexity than (32). 311

Remark: If we set $J = K$, Algorithm 1 degenerates to be the 312
 exhaustive search, where $\|\mathcal{W}\|_0 = \binom{M}{K}$, and $\|\Phi\|_0 = K$. In this case, 313
 no matter what T_1 and T_2 are, they are equivalent to $T_1 = T_2 = 1$, 314
 resulting in $\mathcal{O}(\binom{M-K+J}{J})$, which is the extraordinarily high com- 315
 plexity of the exhaustive search. In practice, we usually set $J = 1$ or 316
 $J = 2$ to reduce the complexity. For example, if $J = 1$, (32) reduces 317
 to a polynomial complexity, i.e., $\mathcal{O}(T_1 T_2 (L-1)^{1.5} (K+1)^3 K (M - 318$
 $K + 1))$. 319

Fig. 2. Comparisons of convergence and complexity for different algorithms or parameters.

V. SIMULATION RESULTS

320

321 To compare this work with [14], we set the same system parameters
 322 as [14]. We consider an OFDM-based CR system with $N = 1024$ sub-
 323 carriers. After ideal spectrum sensing, there are $M = 512$ active sub-
 324 carriers available for SUs, including three subcarrier blocks, i.e., $\mathcal{B}_1 =$
 325 $\{1, 2, 3, \dots, 256\}$, $\mathcal{B}_2 = \{513, 514, \dots, 640\}$, $\mathcal{B}_3 = \{897, 898, \dots,$
 326 $1024\}$, with the number of contiguous subcarriers of each block being
 327 256, 128, and 128, respectively. From $\mathcal{C} = \mathcal{B}_1 \cup \mathcal{B}_2 \cup \mathcal{B}_3$, which
 328 can also be regarded as the union of several active CR subbands
 329 for SUs, we want to select $K = 16$ pilot subcarriers for frequency-
 330 domain pilot-assisted channel estimation. A sparse multipath channel
 331 $\mathbf{h}$ is generated with $L = 60$ taps, where $S = 5$ dominant nonzero
 332 channel taps are randomly placed among L taps. The channel gain of
 333 each path is independent and identically distributed complex Gaussian
 334 distributed with unit variance, i.e., $\mathcal{CN}(0, 1)$. Quaternary phase-shift
 335 keying modulation is employed in the simulations.

336 To evaluate the performance of Algorithm 1, we first compare it
 337 with the CE method [14] assuming equal pilot power indicated by
 338 (16), where the joint pilot design is simplified to be the pilot pattern
 339 design. We set $V_T = 1$, which means that the sum power of pilot
 340 subcarriers is normalized. We set $V_L = 0.03$ and $V_H = 0.1$ so that
 341 the pilot power does not vary too much. The steps for optimal pilot
 342 power allocation in Algorithm 1 are skipped by directly substituting
 343 $v_1 = v_2 = \dots = v_{16} = 0.0625$ into (24). The parameters of the CE
 344 method are selected to be the best in [14], where the maximum number
 345 of iterations, the number of random samples, the sample quantile,
 346 and the smoothing factor are set to be 50, 100 000, 0.001, and 0.3,
 347 respectively. It can be seen in Fig. 2 that Algorithm 1 is much faster
 348 convergent than the CE method. Here, we compare the convergence
 349 speed with respect to the running time instead of the number of
 350 iterations, because the running time of each iteration is different for
 351 different algorithms or parameters. Since we run the simulations under
 352 the exact same computer hardware and software, the running time
 353 is proportional to the computational complexity. We set $T_1 = 1000$
 354 and $T_2 = 15$ for Algorithm 1. Once the running time, i.e., 342 s,
 355 which is the running time for the CE method [14], is reached, we
 356 terminate Algorithm 1 so that we can compare Algorithm 1 with the
 357 CE method under the same computational complexity. As shown in
 358 Fig. 2, Algorithm 1 with $J = 1$ can achieve the best performance of the

TABLE I
COMPARISONS OF PILOT PATTERNS USING DIFFERENT ALGORITHMS OR PARAMETERS

Type	$g(\mathbf{p}, \mathbf{v})$	$\mathbf{p}$
CE method	0.3260	47, 95, 110, 162, 180, 193, 246, 513 524, 627, 640, 897, 910, 939, 976, 1019
Algorithm 1 with $J = 1$, equal	0.3141	18, 45, 57, 103, 141, 187, 242, 256 513, 593, 609, 640, 897, 909, 926, 977
Algorithm 1 with $J = 2$, equal	0.3077	30, 46, 77, 121, 182, 228, 247, 256 513, 529, 619, 633, 905, 932, 960, 1021
Algorithm 1 with $J = 1$, joint	0.2945	1, 34, 47, 98, 109, 181, 221, 236 252, 513, 529, 609, 635, 900, 934, 959

Fig. 3. Comparisons of MSE performance for different algorithms parameters.

CE method in no more than 10 s, which indicates that Algorithm 1 is 359
 34 times faster than the CE method and, therefore, is much more 360
 efficient and powerful. Since it has already been demonstrated in 361
 [5] that the CE method outperforms GA and PSO, Algorithm 1 is a 362
 remarkable candidate for integer optimization with its applications not 363
 restricted to the pilot design. As shown in Fig. 2, although Algorithm 1 364
 with $J = 2$ converges slower than that with $J = 1$, it can achieve 365
 better performance than that with $J = 1$ if the running time is long 366
 enough, i.e., longer than 55 s. For those SUs equipped with powerful 367
 CPU and large capacity of battery, it is better to set $J = 2$ or even 368
 larger. The finally obtained pilot patterns $\mathbf{p}$ with the corresponding 369
 objective $g(\mathbf{p}, \mathbf{v})$ during 342 s of running time assuming equal pilot 370
 power are listed in Table I. Note that in [14], we suppose $v_1 = v_2 =$ 371
 $\dots = v_{16} = 1$, whereas in this paper, we suppose $v_1 = v_2 = \dots =$ 372
 $v_{16} = 0.0625$ satisfying $\sum_{i=1}^{16} v_i = 1$, the objective in [14] has to be 373
 divided by $K = 16$ when compared with this work. 374

We now evaluate the performance of joint design of pilot power 375
 and pilot pattern and compare it with the pilot design assuming equal 376
 pilot power. As shown in Fig. 2, Algorithm 1 with $J = 1$ using joint 377
 pilot design achieves better performance than Algorithm 1 with $J = 1$ 378
 or $J = 2$, while its computational complexity is between $J = 1$ and 379
 $J = 2$. The obtained pilot pattern with the corresponding objective is 380
 also listed in Table I. The comparisons of MSE performance and the bit 381
 error rate (BER) performance for sparse channel estimation are shown 382
 in Figs. 3 and 4, respectively. Both the MSE and BER are averaged 383

Fig. 4. Comparisons of BER performance for different algorithms or parameters.

over 10 000 sparse channel realizations. The popular OMP algorithm is employed for sparse channel estimation given the designed pilot pattern and pilot power. For comparisons, the performance of LS channel estimation using $K = 75$ and $K = 87$ equally spaced pilots with the pilot interval being 7 and 6, respectively, is also provided. It is seen that Algorithm 1 with $J = 1$ using joint pilot design achieves almost the same performance as LS with $K = 87$. Therefore, the joint design can reduce the pilot overhead by 71 pilots and improve the spectrum efficiency by 13.9%, thus leading to additional 2.4% improvement compared with the pilot design assuming equal pilot power in [14].

VI. CONCLUSION

In this correspondence, we have investigated the joint design of pilot power and pilot pattern based on the rule of MIP. The pilot design has been formulated as a joint optimization problem, which is then decoupled into tractable sequential formations. Given a pilot pattern, we have formulated the design of pilot power as an SOCP problem. Then, we have proposed a joint design algorithm. Simulation results have verified the effectiveness of the proposed algorithm and shown that the proposed algorithm can achieve better channel estimation performance and further improve the spectrum efficiency by 2.4%, compared with existing algorithms assuming equal pilot power.

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