King's Research Portal

DOI:

[10.1109/TVT.2015.2404820](https://doi.org/10.1109/TVT.2015.2404820)

Document Version

Peer reviewed version

[Link to publication record in King's Research Portal](#)

Citation for published version (APA):

Li, M., Lin, M., Zhu, W-P., Huang, Y., Nallanathan, A., & Yu, Q. (2016). Performance Analysis of MIMO MRC Systems With Feedback Delay and Channel Estimation Error. *IEEE Transactions on Vehicular Technology*, 65(2), 707-717. <https://doi.org/10.1109/TVT.2015.2404820>

Citing this paper

Please note that where the full-text provided on King's Research Portal is the Author Accepted Manuscript or Post-Print version this may differ from the final Published version. If citing, it is advised that you check and use the publisher's definitive version for pagination, volume/issue, and date of publication details. And where the final published version is provided on the Research Portal, if citing you are again advised to check the publisher's website for any subsequent corrections.

General rights

Copyright and moral rights for the publications made accessible in the Research Portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognize and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the Research Portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the Research Portal

Take down policy

If you believe that this document breaches copyright please contact librarypure@kcl.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.

Performance Analysis of MIMO MRC Systems with Feedback Delay and Channel Estimation Error

Min Li, Min Lin, *Member, IEEE*, Wei-Ping Zhu, *Senior Member, IEEE*, Yongming Huang *Member, IEEE*, Arumugam Nallanathan, *Senior Member, IEEE*, and Quan Yu

Abstract—In this paper, we investigate the performance of a multiple-input multiple-output (MIMO) maximal ratio combining (MRC) system with feedback delay and channel estimation error. By taking these practical imperfect factors into account, we first formulate the system model and derive the moment generating function (MGF) of the output signal-to-noise ratio (SNR) which serves as the basis for further system performance analysis. Then, we compute the probability density function (PDF) and the cumulative distribution function (CDF) of the output SNR. Further, we derive the analytical expressions of the exact and approximate average symbol error rates (SERs) of the MIMO MRC system, which are used to investigate the performance loss in terms of the array gain and diversity order. Finally, computer simulations are conducted to show the efficacy of the analytical results, and the effect of feedback delay and channel estimation error on the system performance.

Index Terms—multiple-input multiple-output (MIMO), maximal ratio combining (MRC), feedback delay, channel estimation error, diversity order, performance analysis.

I. INTRODUCTION

THE multiple-input multiple-output (MIMO) technique employing antenna array on both the transmitter and receiver sides has received considerable attention in the past decades, due to its ability of increasing the capacity and improving the link quality for wireless networks [1]–[3]. Among various MIMO strategies, MIMO maximal ratio combining (MRC), which applies the principle of beamforming (BF) to maximizing the output signal-to-noise ratio (SNR), is of particular interest, and has been widely studied in open literatures [4]–[17].

When perfect channel state information (CSI) is available at both of the transmitter and receiver, closed-form expressions of

the probability density function (PDF) of the received SNR and average symbol error rate (SER) for the MIMO MRC systems have been obtained in [5] [6], and the outage probability and ergodic capacity have been derived in [7], where each entry of the channel matrix is assumed to be independent and identically distributed (i.i.d.). Performance analysis of double-correlated and arbitrarily correlated Rayleigh fading channels can be found in [8] and [9], respectively. Some statistical results of MIMO MRC systems over correlated Rician fading channels have been obtained in [10]. In practical MIMO systems, the receiver's combining vector is often calculated based on imperfect CSI due to the channel estimation error. Moreover, the transmit BF vector is chosen according to the outdated CSI because of the feedback delay. Under this situation, closed-form expressions for the moment generating function (MGF) and the PDF of the receive output SNR considering the effects of outdated and finite-rate feedback as well as further accurate analytical error rate expressions have been derived in [11]. In [12], the performance of transmit beamforming on multiple-antenna Rayleigh fading channels with imperfect channel feedback has been analyzed, where the feedback imperfections are characterized in terms of noisy channel estimation, feedback delay, and finite-rate channel quantization. The exact average SER, outage probability and ergodic capacity performance of beamforming in spatially correlated multiple-input single-output (MISO) systems with channel estimation error and feedback delay has been analyzed in [13]. However, the authors in [11]–[13] have limited their study to the MISO case, which can be regarded as a special case of MIMO MRC system with a single antenna at the receiver. More recently, many researchers have examined the effects of various errors on the MIMO MRC systems, where both the transmitter and receiver are equipped with multiple antennas. For example, the authors of [14] have investigated the effect of Gaussian estimation errors on the performance of maximal ratio transmission (MRT) in MIMO systems. In [15], the authors have studied the performance of MIMO wireless communications with transmit BF and receive MRC in the presence of channel estimation error and co-channel interferences. Furthermore, the joint impacts of channel estimation error, feedback delay and co-channel interference on the performance of MIMO systems employing MRT has been examined in [16]. It is worth-mentioning that the error term due to imperfect CSI is treated as noise or interference in [14]–[16]. This treatment is not highly efficient, since the error term can be demodulated via envelope detection in practical wireless systems, and thus can be used to enhance the output

This work is supported by the National Natural Science Foundation of China under Grants 61271255, 61231011 and 61422105, the Natural Science Foundation of Jiangsu Province under Grant BK20131068 and the Open research Fund of National Mobile Communications Research Laboratory in Southeast University under Grant 2012D15.

M. Li and Q. Yu are with the Institute of Communication Engineering, PLA University of Science and Technology, Nanjing 210007, China (e-mail: limin_sclsw@163.com).

M. Lin (Corresponding author) is with PLA University of Science and Technology, Nanjing 210007, and also with the National Mobile Communications Research Laboratory, Southeast University, Nanjing 210096, China (e-mail: linmin63@163.com).

W.-P. Zhu is with the Department of Electrical and Computer Engineering, Concordia University, Montreal, QC, Canada H3G 1M8 (e-mail: weiping@ece.concordia.ca).

Y. Huang is with the School of Information Science and Engineering, Southeast University, Nanjing 210096, China (e-mail: huangym@seu.edu.cn).

A. Nallanathan is with the Department of Informatics, King's College London, London WC2R 2LS, United Kingdom (e-mail: arumugam.nallanathan@kcl.ac.uk).

signal power [20]. Motivated by this observation, the authors of [17] have considered the error term as signal, and analyzed the bit error rate (BER) and outage probability (OP) of the singular value decomposition (SVD)-based MIMO systems with feedback delay and channel estimation error. However, the main drawback of the work in [17] is that only the channel gain of the signal term is regarded as random variable (RV). Actually, the channel gains of both the signal term and error term should be regarded as RVs in the performance analysis [11]. This observation motivates the work presented in this paper.

For reasons of clarity, we summarize the main contributions of this paper as follows:

- Based on the model of feedback delay and channel estimation error in [11], we present a closed-form expression of the output SNR for the MIMO MRC system, where a realistic scenario is considered by treating the error term due to imperfect CSI as signal, and assuming that the channel gains of both the signal and error terms are RVs.

- Analytical expressions are derived for the MGF, PDF and cumulative distribution function (CDF) of the output SNR to investigate the statistical properties of the MIMO MRC system with various errors, which is a general model containing the related work for example [11]-[13] as special cases. Meanwhile, these analytical results are different from those in [14]-[17] with accurate evaluation.

- New theoretical formulas are developed for the average symbol error rate (ASER) of the system, and a simple approximate ASER expression at high SNR is also derived to examine the asymptotic behavior of the MIMO MRC system conveniently.

The rest of the paper is organized as follows. In Section II, we formulate the model of the MIMO MRC system in the presence of the channel estimation error and feedback delay. In Section III, we give the derivation of the MGF, PDF and CDF for the output SNR. In Section IV, we work on closed-form expressions for the exact and approximate ASERs. Section V provides computer simulation results to validate our performance analysis, with respect to different errors, antenna configurations and modulation formats. Finally, Section VI draws the conclusion of our work in this paper.

Notations: Bold faced letters represent vectors or matrices, $(\cdot)^H$ the Hermitian transpose, $(\cdot)^*$ the complex conjugate, and $E[\cdot]$ the expectation. $\exp(\cdot)$ denotes the exponential function, $|\cdot|$ the absolute value, $\|\cdot\|_F$ the Frobenius norm, and $Q(\cdot)$ the Gaussian Q -function. χ_N^2 stands for a chi-square distributed random variable with N degrees of freedom, $\mathcal{N}_c(m, \sigma^2)$ for a complex Gaussian distribution with mean m and variance σ^2 , and $\min\{a, b\}$ for the minimum of a and b . $\text{diag}(a_1, a_2, \dots, a_N)$ represents an $N \times N$ diagonal matrix with $a_1, a_2, \dots, a_N$ as its diagonal elements, $\mathbf{I}_N$ an $N \times N$ identity matrix, and $o(x^n)$ the terms with order higher than n .

II. SYSTEM MODEL

As shown in Fig. 1, we consider a MIMO system with N_t antennas at the transmitter and N_r antennas at the receiver.

Fig. 1: System model of MIMO MRC with channel estimation error and feedback delay

The received signal can be written as

$$\mathbf{r}(t) = \mathbf{H}(t) \mathbf{w}(t) s(t) + \mathbf{n}(t), \quad (1)$$

where $s(t)$ is the transmitted data symbol with transmit power $P_s = E[|s(t)|^2]$, $\mathbf{w}(t)$ is the $N_t \times 1$ transmit BF weight vector normalized to $\|\mathbf{w}(t)\|_F = 1$. In addition, $\mathbf{H}(t)$ represents the $N_r \times N_t$ Rayleigh fading channel matrix with its components $[\mathbf{H}(t)]_{i,j}$ being i.i.d. complex Gaussian random variables of zero mean and unit variance, i.e., $[\mathbf{H}(t)]_{i,j} \sim \mathcal{N}_c(0, 1)$, and $\mathbf{n}(t)$ is an $N_r \times 1$ vector denoting the additive white Gaussian noise (AWGN) satisfying $\mathbf{n}(t) \sim \mathcal{N}_c(\mathbf{0}, N_0 \mathbf{I}_{N_r})$. After performing MRC with BF weight vector $\mathbf{v}(t)$ ($N_r \times 1$) satisfying $\|\mathbf{v}(t)\|_F = 1$, the output signal of the receiver is given by

$$y(t) = \mathbf{v}^H(t) \mathbf{r}(t) = \mathbf{v}^H(t) \mathbf{H}(t) \mathbf{w}(t) s(t) + \mathbf{v}^H(t) \mathbf{n}(t), \quad (2)$$

Thus, the output SNR can be expressed as

$$\gamma(t) = |\mathbf{v}^H(t) \mathbf{H}(t) \mathbf{w}(t)|^2 \gamma_s. \quad (3)$$

where $\gamma_s = E\{|s(t)|^2\}/N_0 = P_s/N_0$ is the transmit SNR. In what follows, we aim to derive a closed-form expression of $\gamma(t)$ in terms of the available CSI.

Let us consider first the perfect CSI case. By applying the spectral theorem, the matrix $\mathbf{H}^H(t) \mathbf{H}(t)$ can be decomposed as

$$\begin{aligned} \mathbf{H}^H(t) \mathbf{H}(t) &= [\mathbf{u}_1(t), \mathbf{u}_2(t), \dots, \mathbf{u}_{N_t}(t)] \\ &\times \text{diag}(\lambda_1(t), \lambda_2(t), \dots, \lambda_{N_t}(t)) \\ &\times [\mathbf{u}_1(t), \mathbf{u}_2(t), \dots, \mathbf{u}_{N_t}(t)]^H, \end{aligned} \quad (4)$$

where $\lambda_i(t)$'s are arranged in a non-increasing order. According to the principle of MIMO MRC [4], if perfect CSI is available at the transmitter, $\mathbf{w}(t)$ is chosen to be the eigenvector corresponding to the largest eigenvalue of $\mathbf{H}^H(t) \mathbf{H}(t)$, that is, $\mathbf{w}(t) = \mathbf{u}_1(t)$, and simultaneously, $\mathbf{v}(t)$ is given by $\mathbf{v}(t) = \mathbf{H}(t) \mathbf{u}_1(t) / \|\mathbf{H}(t) \mathbf{u}_1(t)\|_F$. Accordingly, the output SNR in (3) can be represented as

$$\gamma(t) = \lambda_1(t) \gamma_s. \quad (5)$$

In practice, since the CSI is estimated at the receiver and fed back to the transmitter through a delayed feedback channel as shown in Fig. 1, the transmit and receive BF vectors are

calculated based on the delayed CSI with a channel estimation error [18] [19]. As such, the channel model with the estimation error and feedback delay can be described as

$$\mathbf{H}(t) = \rho_d \hat{\mathbf{H}}(t - T_d) + \mathbf{E}_e + \mathbf{E}_d, \quad (6)$$

where T_d is the feedback delay, $\hat{\mathbf{H}}(t)$ is the estimated channel matrix at the receiver, satisfying $[\hat{\mathbf{H}}(t)]_{i,j} \sim \mathcal{N}_c(0, 1 - \sigma_e^2)$, $\mathbf{E}_e = \mathbf{H}(t) - \hat{\mathbf{H}}(t)$ is the channel estimation error matrix with $[\mathbf{E}_e]_{i,j} \sim \mathcal{N}_c(0, \sigma_e^2)$. In (6), $\mathbf{E}_d = \hat{\mathbf{H}}(t) - \rho_d \hat{\mathbf{H}}(t - T_d)$ is the additional channel estimation error matrix caused by the feedback delay, obeying $[\mathbf{E}_d]_{i,j} \sim \mathcal{N}_c(0, (1 - |\rho_d|^2)(1 - \sigma_e^2))$, where ρ_d denotes the normalized correlation coefficient between $[\hat{\mathbf{H}}(t)]_{i,j}$ and its delayed sample $[\hat{\mathbf{H}}(t - T_d)]_{i,j}$, namely,

$$\rho_d = \frac{E \left\{ [\hat{\mathbf{H}}(t)]_{i,j} [\hat{\mathbf{H}}(t - T_d)]_{i,j}^* \right\}}{1 - \sigma_e^2}, \quad (7)$$

Clearly, $\mathbf{E}_e$ is independent of $\hat{\mathbf{H}}(t)$ and $\mathbf{E}_d$. By letting $\tilde{\mathbf{H}}(t - T_d) = \sqrt{1 - \sigma_e^2} \hat{\mathbf{H}}(t - T_d)$ and the channel error matrix $\mathbf{E} = \mathbf{E}_e + \mathbf{E}_d$, the components of $\tilde{\mathbf{H}}(t - T_d)$ are i.i.d. with $\mathcal{N}_c(0, 1)$. Note that $\mathbf{E}$ is independent of $\tilde{\mathbf{H}}(t - T_d)$, whose entries are i.i.d. and obey $\mathcal{N}_c(0, 1 - |\rho_d|^2(1 - \sigma_e^2))$. The channel matrix can thus be rewritten as

$$\mathbf{H}(t) = \rho_d \sqrt{1 - \sigma_e^2} \tilde{\mathbf{H}}(t - T_d) + \mathbf{E}. \quad (8)$$

Since only the delayed version of the estimated CSI $\tilde{\mathbf{H}}(t - T_d) = \sqrt{1 - \sigma_e^2} \hat{\mathbf{H}}(t - T_d)$ is available at the transmitter, $\mathbf{w}(t)$ is calculated according to $\tilde{\mathbf{H}}(t - T_d)$. The eigen-decomposition of $\tilde{\mathbf{H}}^H(t) \tilde{\mathbf{H}}(t)$ can be denoted as

$$\tilde{\mathbf{H}}^H(t) \tilde{\mathbf{H}}(t) = \tilde{\mathbf{U}}(t) \tilde{\mathbf{D}}(t) \tilde{\mathbf{U}}^H(t), \quad (9)$$

where $\tilde{\mathbf{D}}(t) = \text{diag}(\tilde{\lambda}_1(t), \tilde{\lambda}_2(t), \dots, \tilde{\lambda}_{N_t}(t))$ with its diagonal elements arranged in a non-increasing order, and $\tilde{\mathbf{U}}(t) = [\tilde{\mathbf{u}}_1(t) \tilde{\mathbf{u}}_2(t) \dots \tilde{\mathbf{u}}_{N_t}(t)]$ with $\tilde{\mathbf{u}}_i(t)$ being the eigenvector corresponding to the eigenvalue $\tilde{\lambda}_i(t)$. As $\tilde{\mathbf{H}}^H(t - T_d) \tilde{\mathbf{H}}(t - T_d)$ is available at the transmitter, $\mathbf{w}(t)$ should be chosen as

$$\mathbf{w}(t) = \tilde{\mathbf{u}}_1(t - T_d), \quad (10)$$

Also, according to the principle of MRC at the receiver, $\mathbf{v}(t)$ is given by

$$\mathbf{v}(t) = \tilde{\mathbf{H}}(t - T_d) \tilde{\mathbf{u}}_1(t - T_d) / \left\| \tilde{\mathbf{H}}(t - T_d) \tilde{\mathbf{u}}_1(t - T_d) \right\|_F. \quad (11)$$

Furthermore, by considering the effect of various errors, and substituting (8), (10) and (11) into (2), the output signal of the

receiver can be rewritten as

$$\begin{aligned} y(t) = & \underbrace{\rho_d \sqrt{1 - \sigma_e^2} \left\| \tilde{\mathbf{H}}(t - T_d) \tilde{\mathbf{u}}_1(t - T_d) \right\|_F}_{\text{signal term}} s(t) \\ & + \underbrace{\frac{\tilde{\mathbf{u}}_1^H(t - T_d) \tilde{\mathbf{H}}^H(t - T_d)}{\left\| \tilde{\mathbf{H}}(t - T_d) \tilde{\mathbf{u}}_1(t - T_d) \right\|_F} \mathbf{E} \tilde{\mathbf{u}}_1(t - T_d)}_{\text{error term}} s(t) \\ & + \underbrace{\frac{\tilde{\mathbf{u}}_1^H(t - T_d) \tilde{\mathbf{H}}^H(t - T_d)}{\left\| \tilde{\mathbf{H}}(t - T_d) \tilde{\mathbf{u}}_1(t - T_d) \right\|_F} \mathbf{n}(t)}_{\text{noise term}}, \end{aligned} \quad (12)$$

Compared with (2), it can be found that the error term due to the imperfect CSI is added in the output signal. Since the error term can be demodulated with envelope detection [20], it should be treated as signal as in [17] rather than the noise or interference as in [14]-[16]. As a result, after the necessary computations, the output SNR at the receiver can be expressed as

$$\begin{aligned} \gamma(t) = & \left| \rho_d \sqrt{1 - \sigma_e^2} \left\| \tilde{\mathbf{H}}(t - T_d) \tilde{\mathbf{u}}_1(t - T_d) \right\|_F \right. \\ & \left. + \frac{\tilde{\mathbf{u}}_1^H(t - T_d) \tilde{\mathbf{H}}^H(t - T_d)}{\left\| \tilde{\mathbf{H}}(t - T_d) \tilde{\mathbf{u}}_1(t - T_d) \right\|_F} \mathbf{E} \tilde{\mathbf{u}}_1(t - T_d) \right|^2 \gamma_s. \end{aligned} \quad (13)$$

Note that conditioned on $\tilde{\mathbf{H}}(t - T_d)$ and $\tilde{\mathbf{u}}_1(t - T_d)$, $\gamma(t)$ is a non-central chi-square distributed RV following $\gamma(t) \sim \chi^2_2$, whose sum of mean square and variance of each freedom are respectively, given by [20]

$$\tau = |\rho_d|^2 (1 - \sigma_e^2) \tilde{\lambda}_1(t - T_d) \gamma_s, \quad (14)$$

$$\sigma^2 = \left[1 - |\rho_d|^2 (1 - \sigma_e^2) \right] \gamma_s / 2. \quad (15)$$

Remark 1: When perfect CSI is available, we have $\rho_d = 1$, $\sigma_e^2 = 0$, $\tilde{\mathbf{H}}(t - T_d) = \mathbf{H}(t)$, $\tilde{\mathbf{u}}_1(t - T_d) = \mathbf{u}_1(t)$, and $\mathbf{E}$ reduces to an $N_r \times N_t$ null matrix. Thus, (13) is simplified to (5), which can also be found in some related works, such as [6, Eq. (9)] and [7, Eq. (5)], implying that the MIMO MRC system with perfect CSI considered in prior literatures is a special case of our general model.

Remark 2: According to the idea of [14, Eq. (6)], [15, Eq. (5)] and [16, Eq. (6)], the output SNR is given by

$$\gamma(t) = \frac{|\rho_d|^2 (1 - \sigma_e^2) \tilde{\lambda}_1(t - T_d) \gamma_s}{\left[1 - |\rho_d|^2 (1 - \sigma_e^2) \right] \gamma_s + 1}, \quad (16)$$

where the error term due to the imperfect CSI is treated as noise or interference. Meanwhile, following the work of [17, Eq. (11)], the output SNR can be expressed as

$$\gamma(t) = \left[|\rho_d|^2 (1 - \sigma_e^2) \tilde{\lambda}_1(t - T_d) + 1 - |\rho_d|^2 (1 - \sigma_e^2) \right] \gamma_s. \quad (17)$$

where only the channel gain of the signal term is regarded as RV. Clearly, compared with these related works investigating MIMO MRC system with feedback delay and channel estimation error, our work has a rigorous system model and clear physical meaning.

III. OUTPUT SNR STATISTICS

In this section, we study the statistical properties of the MI-MO MRC system with feedback delay and channel estimation error, by deriving the closed-form expressions of MGF, PDF and CDF of the output SNR.

A. MGF of the output SNR

By using the conditional MGF $\psi_{(\gamma(t)|\tilde{\lambda}_1(t-T_d))}(s)$ of $\gamma(t)$ on $\tilde{\lambda}_1(t-T_d)$ along with the PDF $f_{\tilde{\lambda}_1(t-T_d)}(x)$ of $\tilde{\lambda}_1(t-T_d)$, the MGF of $\gamma(t)$ in (13) can be expressed as

$$\psi_{\gamma(t)}(s) = \int_0^\infty \psi_{(\gamma(t)|x)}(s) f_{\tilde{\lambda}_1(t-T_d)}(x) dx, \quad (18)$$

Before deriving an analytical expression of (18), we first introduce the following theorem, which has been proven in [20]. It states that if X satisfies a non-central χ_N^2 with the variance of each freedom being σ^2 and the sum of mean square being τ , its PDF is given by

$$f_X(x) = \frac{1}{2\sigma^2} \left(\frac{x}{\tau}\right)^{\frac{N-2}{4}} \exp\left(-\frac{\tau+x}{2\sigma^2}\right) I_{\frac{N}{2}-1}\left(\sqrt{x}\frac{\tau}{\sigma^2}\right), \quad (19)$$

and its MGF can be further calculated as

$$\begin{aligned} \psi_X(s) &= \int_0^\infty f_X(x) \exp(sx) dx = \frac{1}{(1-2\sigma^2s)^{N/2}} \exp\left(\frac{\tau s}{1-2\sigma^2s}\right), \end{aligned} \quad (20)$$

Due to the fact that $\gamma(t) \sim \chi_2^2$, whose sum of mean square and variance of each freedom are given by (14) and (15), it is straightforward to obtain

$$\begin{aligned} \psi_{(\gamma(t)|\tilde{\lambda}_1(t-T_d))}(s) &= \int_0^\infty f_{(\gamma(t)|\tilde{\lambda}_1(t-T_d))}(x) \exp(sx) dx \\ &= \frac{1}{1-[1-|\rho_d|^2(1-\sigma_e^2)]\gamma_s s} \exp\left\{\frac{|\rho_d|^2(1-\sigma_e^2)\tilde{\lambda}_1(t-T_d)\gamma_s s}{1-[1-|\rho_d|^2(1-\sigma_e^2)]\gamma_s s}\right\}, \end{aligned} \quad (21)$$

Using (21) into (18) yields

$$\begin{aligned} \psi_{\gamma(t)}(s) &= \frac{1}{1-[1-|\rho_d|^2(1-\sigma_e^2)]\gamma_s s} \\ &\times \int_0^\infty f_{\tilde{\lambda}_1(t-T_d)}(x) \exp\left\{\frac{|\rho_d|^2(1-\sigma_e^2)\gamma_s s}{1-[1-|\rho_d|^2(1-\sigma_e^2)]\gamma_s s} x\right\} dx \\ &= \frac{1}{1-[1-|\rho_d|^2(1-\sigma_e^2)]\gamma_s s} \psi_{\tilde{\lambda}_1(t-T_d)}\left\{\frac{|\rho_d|^2(1-\sigma_e^2)\gamma_s s}{1-[1-|\rho_d|^2(1-\sigma_e^2)]\gamma_s s}\right\} \\ &= \frac{1}{1-[1-|\rho_d|^2(1-\sigma_e^2)]\gamma_s s} \psi_{\tilde{\lambda}_1(t)}\left\{\frac{|\rho_d|^2(1-\sigma_e^2)\gamma_s s}{1-[1-|\rho_d|^2(1-\sigma_e^2)]\gamma_s s}\right\}. \end{aligned} \quad (22)$$

where $\psi_{\tilde{\lambda}_1(t-T_d)}(s)$ and $\psi_{\tilde{\lambda}_1(t)}(s)$ denote the MGF of $\tilde{\lambda}_1(t-T_d)$ and that of $\tilde{\lambda}_1(t)$, respectively. In (22), we have assumed that $\tilde{\lambda}_1(t-T_d)$ and $\tilde{\lambda}_1(t)$ have the same statistical property, due to the fact that the channel is ergodic.

Next, we consider the MGF of $\tilde{\lambda}_1(t)$. According to [6], the PDF of $\tilde{\lambda}_1(t)$ is given by

$$f_{\tilde{\lambda}_1(t)}(x) = \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{i^{m+1} x^m \exp(-ix)}{m!}, \quad (23)$$

where $N_{tr} = \min\{N_t, N_r\}$, and the coefficient $d_{i,m}$ dependent on the transmit-receive antenna combinations can be expressed as [6]

$$d_{i,m} = \frac{\Gamma(i+1) C_{i,m}}{m^{i+1} \left(\prod_{l=1}^{N_{tr}} \Gamma(N_r - l + 1) \Gamma(N_t - l + 1) \right)}, \quad (24)$$

where $\Gamma(\cdot)$ is the Gamma function, and $C_{i,m}$ is the coefficient term of $x^i \exp(-mx)$. Note that X is a central χ_N^2 RV with the variance of each freedom being σ^2 , its PDF and MGF are, respectively, given by [20]

$$f_X(x) = \frac{1}{2\sigma^2} \frac{1}{(N/2-1)!} x^{N/2-1} \exp\left(-\frac{x}{2\sigma^2}\right), \quad (25)$$

$$\psi_X(s) = \frac{1}{(1-2\sigma^2s)^{N/2}}, \quad (26)$$

For the PDF of $\tilde{\lambda}_1(t)$ in (23), it can be viewed as a linear combination of PDFs with $N = 2(m+1)$ and $\sigma^2 = 1/2i$ given in (25), and weighted by $d_{i,m}$. Thus, the MGF of $\tilde{\lambda}_1(t)$ can be written as

$$\psi_{\tilde{\lambda}_1(t)}(s) = \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \frac{d_{i,m}}{(1-s/i)^{m+1}}, \quad (27)$$

Using (27) into (22), one can finally obtain the MGF of $\gamma(t)$ as

$$\psi_{\gamma(t)}(s) = \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{(1-s/\alpha)^m}{(1-s/\beta)^{m+1}}. \quad (28)$$

where

$$\alpha = \frac{1}{[1-|\rho_d|^2(1-\sigma_e^2)]\gamma_s}, \quad (29)$$

$$\beta = \frac{1}{[1-|\rho_d|^2(1-\sigma_e^2)(1-\frac{1}{i})]\gamma_s}. \quad (30)$$

Remark 3: If only a single antenna is employed at the receiver, and simultaneously, the channel estimation error is ignored, we have $N_{tr} = N_r = 1$, $\sigma_e^2 = 0$, and $d_{1,N_t-1} = 1$. Thus $\psi_{\gamma(t)}(s) = \frac{(1-s/\alpha)^{N_t-1}}{(1-s/\beta)^{N_t}}$ with $\alpha = \frac{1}{(1-|\rho_d|^2)\gamma_s}$ and $\beta = \frac{1}{\gamma_s}$, which coincides with [11, Eq. (9)], indicating that the work in [11] is a special case of our result.

B. PDF of the output SNR

By applying the inverse Laplace transform and the MGF expression of $\gamma(t)$ in (28), the PDF of the output SNR can be obtained as

$$\begin{aligned} f_{\gamma(t)}(x) &= L^{-1}[\psi_{\gamma(t)}(s)] \\ &= \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} L^{-1}\left[\frac{(1-s/\alpha)^m}{(1-s/\beta)^{m+1}}\right] \\ &= \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^m d_{i,m} \frac{C_m^n}{(m-n)!} \frac{\beta^{m+1}}{\alpha^n} \\ &\times (1-\beta/\alpha)^{m-n} x^{m-n} \exp(-\beta x), \end{aligned} \quad (31)$$

where $L^{-1}[\cdot]$ denotes the inverse Laplace transform, which can be calculated with the help of the residue theorem as in [11].

Remark 4: For the MISO case as discussed in [11], we have $N_{tr} = N_r = 1$, $\sigma_e^2 = 0$, and $d_{1,N_t-1} = 1$, and thus (31) is simplified to

$$f_{\gamma(t)}(x) = \sum_{n=0}^m \frac{C_{N_t-1}^n}{(N_t-n-1)!} \frac{\beta^{N_t}}{\alpha^n} (1-\beta/\alpha)^{N_t-n-1} \times x^{N_t-n-1} \exp(-\beta x). \quad (32)$$

which is the same as [11, Eq. (13)].

C. CDF of the output SNR

The CDF of $\gamma(t)$ can be calculated from the integral of (31), that is,

$$F_{\gamma(t)}(x) = \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^m d_{i,m} \frac{C_m^n}{(m-n)!} \times \frac{\beta^{m+1}}{\alpha^n} (1-\beta/\alpha)^{m-n} \int_0^x t^{m-n} \exp(-\beta t) dt, \quad (33)$$

By applying the identity [21]

$$\int_0^x t^m \exp(-\alpha t) dt = \frac{m!}{\alpha^{m+1}} \left[1 - \exp(-\alpha x) \sum_{k=0}^m \frac{(\alpha x)^k}{k!} \right], \quad (34)$$

the CDF of the output SNR in (33) can be calculated as

$$\begin{aligned} F_{\gamma(t)}(x) &= \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \sum_{n=0}^m C_m^n \left(\frac{\beta}{\alpha} \right)^n \left(1 - \frac{\beta}{\alpha} \right)^{m-n} \\ &\times \left[1 - \exp(-\beta x) \sum_{k=0}^{m-n} \frac{(\beta x)^k}{k!} \right] \\ &= \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} - \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^m \sum_{k=0}^{m-n} d_{i,m} \\ &\times C_m^n \left(\frac{\beta}{\alpha} \right)^n \left(1 - \frac{\beta}{\alpha} \right)^{m-n} \frac{(\beta x)^k}{k!} \exp(-\beta x), \end{aligned} \quad (35)$$

With the help of the following equation [22]

$$\sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} = 1, \quad (36)$$

the CDF of $\gamma(t)$ can be further written in a simple closed-form expression as

$$F_{\gamma(t)}(x) = 1 - \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^m \sum_{k=0}^{m-n} d_{i,m} \times C_m^n \left(\frac{\beta}{\alpha} \right)^n \left(1 - \frac{\beta}{\alpha} \right)^{m-n} \frac{(\beta x)^k}{k!} \exp(-\beta x). \quad (37)$$

Remark 5: It is noted that the expression of (37) is reduced to

$$F_{\gamma(t)}(x) = 1 - \sum_{n=0}^{N_t-1} \sum_{k=0}^{N_t-n-1} C_{N_t-1}^n \left(\frac{\beta}{\alpha} \right)^n \times \left(1 - \frac{\beta}{\alpha} \right)^{N_t-n-1} \frac{(\beta x)^k}{k!} \exp(-\beta x). \quad (38)$$

with $N_{tr} = N_r = 1$, $\sigma_e^2 = 0$, and $d_{1,N_t-1} = 1$ for the MISO scenario without channel estimation error. Interestingly, (38) coincides with [23, Eq. (26)], meaning that we have indeed extended the prior CDF results on MISO systems to the MIMO MRC case.

IV. AVERAGE SER

In this section, we analyze the average SER (ASER) of the MIMO MRC system with feedback delay and channel estimation error. In particular, we derive closed-form expressions of the exact ASER with various modulation formats. Furthermore, we develop a novel approximate ASER at high SNR to investigate the array gain and diversity order of the system in the presence of various errors.

A. Exact ASER

It is well-known that the SER of a wireless system with various modulation formats over AWGN channel is given by [20]

$$P_s(\gamma(t)|a, b) = aQ\left(\sqrt{2b\gamma(t)}\right), \quad (39)$$

where a and b are the parameters specified by the modulations. For example, for M -ary pulse amplitude modulation (M -PAM), we have $a = 2(M-1)/M$ and $b = 3/(M^2-1)$; a good approximation for M -ary phase shift keying (M -PSK) is given by $a = 2$ and $b = \sin^2(\pi/M)$; and a tight upper bound for M -ary quadrature amplitude modulation (M -QAM) is specified by $a = 4$ and $b = 3/2(M-3)$. Furthermore, the ASER over fading channels can be written as

$$\begin{aligned} P_s(a, b) &= E_{\gamma(t)}\{P_s(\gamma(t)|a, b)\} \\ &= \int_0^\infty aQ\left(\sqrt{2bz}\right) f_{\gamma(t)}(z) dz, \end{aligned} \quad (40)$$

Substituting (31) into (40) yields

$$\begin{aligned} P_s(a, b) &= a \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^m d_{i,m} \frac{C_m^n}{(m-n)!} \frac{\beta^{m+1}}{\alpha^n} \\ &\times (1-\beta/\alpha)^{m-n} \int_0^\infty z^{m-n} \exp(-\beta z) Q\left(\sqrt{2bz}\right) dz, \end{aligned} \quad (41)$$

By utilizing the following equality [21]

$$\begin{aligned} \frac{a^m}{\Gamma(m)} \int_0^\infty \exp(-at) t^{m-1} Q\left(\sqrt{bt}\right) dt \\ = \frac{1}{2} \left[1 - \mu \sum_{k=0}^{m-1} C_{2k}^k \left(\frac{1-\mu^2}{4} \right)^k \right], \end{aligned} \quad (42)$$

where $\mu = \sqrt{\frac{b}{2a+b}}$, the ASER is thus given by

$$\begin{aligned} P_s(a, b) &= \frac{a}{2} \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} \sum_{n=0}^m d_{i,m} C_m^n \left(\frac{\beta}{\alpha} \right)^n \\ &\times \left(1 - \frac{\beta}{\alpha} \right)^{m-n} \left[1 - \mu \sum_{k=0}^{m-n} C_{2k}^k \left(\frac{1-\mu^2}{4} \right)^k \right]. \end{aligned} \quad (43)$$

where $\mu = \sqrt{\frac{b}{\beta+b}}$. Therefore, (43) generalizes the ASER expression of [6] with perfect CSI to the case of considering the channel estimation error and feedback delay.

B. Approximate ASER at high SNR

According to [22], an alternative calculation of the ASER can be expressed as

$$P_s(a, b) = \frac{a}{2} \sqrt{\frac{b}{\pi}} \int_0^\infty \frac{\exp(-bu)}{\sqrt{u}} F_{\gamma(t)}(u) du, \quad (44)$$

In order to obtain an accurate approximation for (44), we first present the following theorem. (see Appendix A for proof)

Theorem 1: The first-order expansion of the CDF of $\gamma(t)$ given by (37) can be expressed as

$$F_{\gamma(t)}(x) = \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{\beta^{m+1}}{\alpha^m} x + \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{\beta^m}{\alpha^m} o(\beta x), \quad (45)$$

Substituting (45) into (44), we have

$$P_s(a, b) = \underbrace{\frac{a}{2} \sqrt{\frac{b}{\pi}} \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{\beta^{m+1}}{\alpha^m} \int_0^\infty \exp(-bu) \sqrt{u} du}_{I_1} + \underbrace{\frac{a}{2} \sqrt{\frac{b}{\pi}} \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{\beta^m}{\alpha^m} \int_0^\infty \frac{\exp(-bu)}{\sqrt{u}} o(\beta u) du}_{I_2}, \quad (46)$$

I_1 can be calculated with the integral formula [24, Eq. (3.381.4)] as

$$I_1 = \frac{a}{4b} \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \times \frac{[1-|\rho_d|^2(1-\sigma_e^2)]^m}{[1-|\rho_d|^2(1-\sigma_e^2)(1-\frac{1}{i})]^{m+1}} \frac{1}{\gamma_s}, \quad (47)$$

Meanwhile, the integral in I_2 can be denoted with the help of [24, Eq. (3.381.4)] as

$$\begin{aligned} & \int_0^\infty \frac{\exp(-bu)}{\sqrt{u}} o(\beta u) du \\ &= \sum_{i=2}^\infty k_i \beta^i \int_0^\infty u^{i-1/2} \exp(-bu) du \\ &= \sum_{i=2}^\infty k_i \frac{\Gamma(i+1/2)}{b^{i+1/2}} \beta^i \\ &= o(\beta), \end{aligned} \quad (48)$$

From (29), (30) and (48), and after some mathematical manipulations, I_2 in (46) can be derived as

$$I_2 = \frac{a}{2} \sqrt{\frac{b}{\pi}} \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{[1-|\rho_d|^2(1-\sigma_e^2)]^m}{[1-|\rho_d|^2(1-\sigma_e^2)(1-\frac{1}{i})]^{m+1}} \times o\left(\frac{1}{1-|\rho_d|^2(1-\sigma_e^2)(1-\frac{1}{i})} \frac{1}{\gamma_s}\right), \quad (49)$$

Noting that (49) can be regarded as a linear combination of $o(\frac{1}{\gamma_s})$, I_2 can be written as $I_2 = o(\frac{1}{\gamma_s})$. Therefore, with the expressions of I_1 and I_2 , the ASER can be expressed as

$$P_s(a, b) = (G_a \gamma_s)^{-G_d} + o(\gamma_s^{-G_d}). \quad (50)$$

where G_a and G_b represent the array gain and the diversity order of the MIMO MRC system with various errors, which

Fig. 2: PDF of the MIMO MRC system with different antenna combinations and various SNR expressions

are, respectively, given by

$$G_a = \frac{4b}{a \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)i-2i^2} d_{i,m} \frac{[1-|\rho_d|^2(1-\sigma_e^2)]^m}{[1-|\rho_d|^2(1-\sigma_e^2)(1-\frac{1}{i})]^{m+1}}}, \quad (51)$$

$$G_d = 1, \quad (52)$$

It is to be noted that when SNR γ_s is very large, we have $o(\gamma_s^{-G_d}) \approx 0$. Thus, the ASER at high SNR can be approximated as

$$P_s(a, b) = (G_a \gamma_s)^{-G_d}. \quad (53)$$

It has already been proven in [25] that the diversity order of MIMO MRC system equals $N_t N_r$, when perfect CSI is available at both sides. However, from (52), we can find that the diversity order reduces to one due to the existence of channel estimation error and feedback delay, no matter how many antennas are employed. This scenario implies that the quality of CSI significantly affects the performance of the MIMO MRC system. The same conclusion has also been drawn in [11], where the diversity order of the transmit beamforming in MISO system impaired by feedback delay is also reduced to one while the diversity order of the transmit BF with perfect CSI is N_t .

V. COMPUTER SIMULATION

In this section, we carry out computer simulations to validate the theoretical results obtained in previous sections, and to investigate the system performance in various scenarios with different channel estimation errors, antenna configurations and modulation formats. In our simulation, we apply the Jake's fading to model the error, namely, $\rho_d = J_0(2\pi f_m \tau)$, with $f_m \tau$ being the Doppler spread rate ratio and $J_0(\cdot)$ the zero order Bessel function of the first kind, and set the transmit SNR as $\gamma_s = 10dB$ in Fig.2~Fig.7. In all plots, the label $N_t \times N_r$ means the combination of antennas at the transmitter and the receiver.

Fig. 3: PDF of the 6×1 MIMO MRC system with various parameters

Fig. 4: PDF of the 2×2 MIMO MRC system with various parameters

First of all, we investigate the PDF of the proposed method by comparing with the previous works such as [14] and [17]. Supposing that the values of feedback delay and channel estimation error are chosen as $f_m \tau = 0.1$ and $\sigma_e^2 = 0.01$, respectively, we consider the following three antenna combinations: 2×2 , 4×2 and 4×4 , and plot the PDF curves of the output SNR in Fig.2. Here, the results of [17] are obtained without taking the co-channel interference into account. As we expect, the PDF curves obtained from our method and [17] shift to higher SNR values than those from [14], which attributes to the different treatment of the error term due to imperfect CSI. Meanwhile, it can also be observed that due to the different treatment of the channel gain owing to the error term, the PDF peaks of our method are different from those of [17]. Furthermore, we find that a better performance can be obtained as the number of antennas increases, which clearly demonstrates the advantage of employing multiple antennas.

Secondly, we study the impact of various errors on the PDF of the MIMO MRC system. The validity of the theoretical results of our paper is verified by the comparison between eq. (31) and eq. (15) in [11]. The PDF simulation results are given in Fig.3 and the antenna configuration is assumed to be 6×1 . As we can see that the theoretical formulas in [11] match

Fig. 5: PDF of the MIMO MRC system with different antenna combinations and various parameters

our analytical results perfectly for different values of $f_m \tau$ and σ_e^2 , which demonstrate that the PDF formula of [11] is a special case of our general result. Fig. 4 depicts the theoretical and simulated PDFs of a 2×2 system with various feedback delays and different channel estimation errors. It is seen that the PDF curves from the Monte Carlo experiments are highly consistent with those given by theoretical formulas regardless of different values of $f_m \tau$ and σ_e^2 . This implies that the derived exact expression in (31) can accurately evaluate the PDF of the output SNR. As for different parameters, four different cases are considered in our simulation, that is, (i) MIMO MRC system without channel estimation error and feedback delay, namely, $f_m \tau = 0, \sigma_e^2 = 0$; (ii) only with feedback delay, namely, $f_m \tau = 0.3, \sigma_e^2 = 0$; (iii) only with channel estimation error, namely, $f_m \tau = 0, \sigma_e^2 = 0.1$; and (iv) with both feedback delay and channel estimation error. Here, case (iv) is further divided into three scenarios: short feedback delay and small variance of channel estimation error, namely, $f_m \tau = 0.1, \sigma_e^2 = 0.01$; moderate feedback delay and variance of channel estimation error, namely, $f_m \tau = 0.2, \sigma_e^2 = 0.05$; and long feedback delay and large variance of channel estimation error, namely, $f_m \tau = 0.3, \sigma_e^2 = 0.1$. It is found that the perfect CSI case corresponds to the best performance for its curve shifting towards the higher value of SNR. For fixed feedback delay, the performance becomes worse with the increase of channel estimation error. For fixed channel estimation error, the performance gets better with shorter feedback delay. In addition, Fig. 5 gives the PDF of the MIMO MRC system with different antenna combinations and various parameters. For the cases with the same parameters, the performance becomes better as the number of transmit antennas increases, such as from 2×2 to 3×2 , or the number of receive antennas increases, e.g. 4×2 vs 4×3 . This is because more array gain is provided by increasing the antenna number. For the same antenna combination, it is obvious that the cases with short feedback delay and small variance of channel estimation error have better performance.

Thirdly, we present the CDF of the output SNR in Fig. 6 for a 3×2 antenna combination and in Fig. 7 for different antenna combinations, both with various parameters. We can see the theoretical curves match very well with the Monte

Fig. 6: CDF of the 3×2 MIMO MRC system with various parameters

Fig. 7: CDF of the MIMO MRC system with different antenna combinations and various parameters

Carlo simulation results. It is known that for the same SNR, the performance upgrades with the smaller CDF value. Thus, a similar conclusion as in Fig. 4 can be drawn, namely, a better performance can be obtained with shorter feedback delay and smaller variance of channel estimation error. In Fig. 7, it is shown that a better performance is achieved by setting short feedback delay and small variance of channel estimation error for the same antenna combination which coincides with the result in Fig. 5.

Finally, we examine the validity of the derived exact ASER expression and its approximation at high SNR. In Fig. 8, we plot the average SER with various parameters and modulation formats for a 4×2 antenna combination. Two modulations, BPSK and 4PAM, are included in our simulation. Three cases, $f_m \tau = 0, \sigma_e^2 = 0$, $f_m \tau = 0.1, \sigma_e^2 = 0.01$ and $f_m \tau = 0.3, \sigma_e^2 = 0.1$ are considered. We can see a good match between the Monte Carlo simulations and the exact analytical results. As well, the approximate analytical results approach closely the exact results at high SNR for various parameters and modulation formats. Compared with the perfect CSI case, the case with feedback delay and channel estimation error has a large performance loss as the diversity order has been reduced from $N_t N_r$ to 1. It is obvious that the ASER of BPSK

Fig. 8: Average SER of the 4×2 MIMO MRC system with various parameters and modulations

Fig. 9: Average SER of the 8×4 MIMO MRC system with various parameters and modulations

is better than that of 4PAM for the same parameters. Also, the case with $f_m \tau = 0.1, \sigma_e^2 = 0.01$ has a better ASER than the one with $f_m \tau = 0.3, \sigma_e^2 = 0.1$ due to a shorter feedback delay and smaller variance of channel estimation error. The average SER with various parameters and modulation formats for a 8×4 antenna combination is also given in Fig. 9, where 8PAM and 16PSK are investigated for $f_m \tau = 0.2, \sigma_e^2 = 0.05$ and $f_m \tau = 0.3, \sigma_e^2 = 0.1$. A good match between the Monte Carlo simulations and the exact analytical results as well as the approximate analytical results approaching closely the exact results at high SNR show the validity of our ASER formula and the asymptotic ASER results. In Fig. 10, we present the exact ASER and the approximate ASER at high SNR for different antenna combinations, where the parameters are set to be $f_m \tau = 0.2, \sigma_e^2 = 0.05$, and the modulation format is BPSK. The approximate ASER approaches very closely the exact ASER at high SNRs for various antenna combinations, showing a good robustness of our analytical results. Furthermore, although the antenna combinations with more transmit or receive antennas give a better performance, they have the same diversity order as shown by the respective curves which have the same slope at high SNRs.

Fig. 10: Average SER of the MIMO MRC system with different antenna combinations and various parameters in the case of BPSK modulation

VI. CONCLUSION

In this paper, we have analyzed the MIMO MRC systems with feedback delay and channel estimation error. Closed-form expressions for the PDF and CDF of the output SNR have been obtained. The exact and approximate average SERs of the system have also been derived. It is shown that the channel estimation error and the feedback delay have reduced the diversity order from $N_t N_r$ to 1 and thus caused a significant performance loss. Computer simulations have confirmed the validity of the performance analysis and shown the performance difference among the systems with different channel estimation errors, feedback delays, antenna configurations and modulation formats.

APPENDIX A PROOF OF THEOREM 1

The first-order expansion of $1 - \exp(-\eta x) \sum_{k=0}^m \frac{(\eta x)^k}{k!}$ can be expressed as

$$1 - \exp(-\eta x) \sum_{k=0}^m \frac{(\eta x)^k}{k!} = \frac{(\eta x)^{m+1}}{(m+1)!} + o((\eta x)^{m+1}), \quad (54)$$

Substituting (54) into the first line of (35), we have

$$F_{\gamma(t)}(x) = \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)-2i^2} d_{i,m} \sum_{n=0}^m C_m^n \left(\frac{\beta}{\alpha}\right)^n \times \left(1 - \frac{\beta}{\alpha}\right)^{m-n} \left[\frac{(\beta x)^{m-n+1}}{(m-n+1)!} + o((\beta x)^{m-n+1}) \right], \quad (55)$$

In order to derive the first-order expansion of $F_{\gamma(t)}(x)$, we need to find the first nonzero coefficient in the Maclaurin expansion. Letting $n = m$, one can obtain the first-order expansion of $\sum_{n=0}^m C_m^n \left(\frac{\beta}{\alpha}\right)^n \left(1 - \frac{\beta}{\alpha}\right)^{m-n} \left[\frac{(\beta x)^{m-n+1}}{(m-n+1)!} + o((\beta x)^{m-n+1}) \right]$ as $\frac{\beta^{m+1}}{\alpha^m} x + \frac{\beta^m}{\alpha^m} o(\beta x)$, and hence the first-

order expansion of the CDF is finally given by

$$F_{\gamma(t)}(x) = \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)-2i^2} d_{i,m} \frac{\beta^{m+1}}{\alpha^m} x + \sum_{i=1}^{N_{tr}} \sum_{m=|N_t-N_r|}^{(N_t+N_r)-2i^2} d_{i,m} \frac{\beta^m}{\alpha^m} o(\beta x). \quad (56)$$

REFERENCES

- [1] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, "Capacity limits of MIMO channels," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 5, pp. 684-702, June. 2003.
- [2] C.-B. Chae, A. Forenza, R. W. Heath Jr., M. R. McKay, and I. B. Collings, "Adaptive MIMO transmission techniques for broadband wireless communication systems," *IEEE Commun. Mag.*, vol. 48, no. 5, pp. 112-118, May. 2010.
- [3] A. Paulraj, R. Nabar, and D. Gore, *Introduction to Space-Time Wireless Communications*. Cambridge, U. K.: Cambridge Univ. Press, 2003.
- [4] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, pp. 1458-1461, Oct. 1999.
- [5] A. J. Grant, "Performance analysis of transmit beamforming," *IEEE Trans. Commun.*, vol. 53, no. 4, pp. 738-744, Apr. 2005.
- [6] P. A. Dighe, R. K. Mallik, and S. S. Jamuar, "Analysis of transmit-receive diversity in Rayleigh fading," *IEEE Trans. Commun.*, vol. 51, no. 4, pp. 694-703, Apr. 2003.
- [7] A. Maaref, and S. Aissa, "Closed-form expressions for the outage and ergodic Shannon capacity of MIMO MRC systems," *IEEE Trans. Commun.*, vol. 56, no. 7, pp. 1092-1095, July. 2005.
- [8] M. McKay, A. Grant, and I. Collings, "Performance analysis of MIMO MRC in double-correlated Rayleigh environments," *IEEE Trans. Commun.*, vol. 55, no. 3, pp. 497-507, Mar. 2007.
- [9] A. Maaref, and S. Aissa, "Eigenvalue distributions of Wishart-type random matrices with application to the performance analysis of MIMO MRC systems," *IEEE Trans. Wireless Commun.*, vol. 6, no. 7, pp. 2678-2689, July. 2007.
- [10] M. Kang, and M.-S. Alouini, "Largest eigenvalue of complex Wishart matrices and performance analysis of MIMO MRC systems," *IEEE J. Sel. Areas Commun.*, vol. 21, no. 3, pp. 418-426, Apr. 2003.
- [11] Y. Ma, D. Zhang, A. Leith, and Z. Wang, "Error performance of transmit beamforming with delayed and limited feedback," *IEEE Trans. Wireless Commun.*, vol. 8, no. 3, pp. 1164-1170, Mar. 2009.
- [12] Y. Isukapalli, R. Annamalai, and B. D. Rao, "Performance analysis of transmit beamforming for MISO systems with imperfect feedback," *IEEE Trans. Commun.*, vol. 57, no. 1, pp. 222-231, Jan. 2009.
- [13] A. Firag, P. J. Smith, H. A. Suraweera, and A. Nallanathan, "Performance of beamforming in correlated MISO systems with estimation error and feedback delay," *IEEE Trans. Wireless Commun.*, vol. 10, no. 8, pp. 2592-2602, Aug. 2011.
- [14] Y. Chen, and C. Tellambura, "Performance analysis of maximum ratio transmission with imperfect channel estimation," *IEEE Commun. Lett.*, vol. 9, no. 4, pp. 322-324, Apr. 2005.
- [15] K. S. Ahn, "Performance analysis of MIMO-MRC systems with channel estimation error in the presence of cochannel interferences," *IEEE Signal Process. Lett.*, vol. 15, no. , pp. 445-448, May. 2008.
- [16] T. M. C. Chu, T. Q. Duong, and H.-J. Zepernick, "Performance analysis for multiple-input multiple-output-maximum ratio transmission systems with channel estimation error, feedback delay and co-channel interference," *IET Commun.*, vol. 7, no. 4, pp. 279-285, Apr. 2013.
- [17] E. K. S. Au, S. Jin, M. R. McKay, W. H. Mow, and X.-Q. Gao, "Analytical performance of MIMO-SVD systems in Ricean fading channels with channel estimation error and feedback delay," *IEEE Trans. Wireless Commun.*, vol. 7, no. 4, pp. 1315-1325, Apr. 2008.
- [18] T. Yoo, and A. Goldsmith, "Capacity and power allocation for fading MIMO channels with channel estimation error," *IEEE Trans. Inform Theory.*, vol. 52, no. 5, pp. 2203-2214, May. 2006.
- [19] S. Zhou, and G. Giannakis, "Adaptive modulation for multiantenna transmissions with channel mean feedback," *IEEE Trans. Wireless Commun.*, vol. 3, no. 5, pp. 1626-1636, Sep. 2004.
- [20] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1995.
- [21] M. K. Simon and M.-S. Alouini, *Digital Communication over Fading Channels*. New York: Wiley, 2000.
- [22] M. Li, M. Lin, Q. Yu, W.-P. Zhu, and L. Dong, "Optimal beamformer design for dual-hop MIMO AF relay networks over Rayleigh fading channels," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 8, pp. 1402-1414, Sep. 2012.

- [23] H. A. Suraweera, T. A. Tsiftsis, G. K. Karagiannidis, and A. Nallanathan, "Effect of feedback delay on amplify-and-forward relay networks with beamforming," *IEEE Trans. Veh. Technol.*, vol. 60, no. 3, pp. 1265-1271, Mar. 2011.
- [24] I. S. Gradshteyn, I. M. Ryzhik, and A. Jeffrey, *Table of integrals, series, and products*, 6th ed. Academic press, 2000.
- [25] S. Jin, M. R. McKay, X.-Q. Gao, and I. B. Collings, "Asymptotic SER and outage probability of MIMO MRC in correlated fading," *IEEE Signal Process. Lett.*, vol. 14, no. 1, pp. 9-12, Jan. 2007.

Min Li Min Li received the B.S. and M.S. degrees from the Institute of Communication Engineering, PLA University of Science and Technology, Nanjing, China, in 2004 and 2007, respectively, all in electrical engineering. He is currently working towards the Ph.D. degree in electronic engineering, from the PLA University of Science and Technology. His research interests are MIMO beamforming, space-time coding, multiple-terminal synchronization in virtual MIMO, wireless relay, distributed beamforming and associated performance analysis.

He won the outstanding M. S. degree paper award from the Jiangsu Province.

Min Lin (M'13) received the B.S. degree from National University of Defense Technology, the M.S. degree from Nanjing Institute of Communication Engineering, and Ph.D. degree from Southeast University in 1993, 2000, and 2008, respectively, all in electrical engineering. He is presently an Associate Professor of Southeast University, and also an Adjunct Professor of PLA University of Science and Technology, Nanjing, China. His current research interests include wireless communications and array signal processing. He is the author or coauthor of

more than 80 papers, and holds 4 patents. Dr. Lin is a Senior Member of Chinese Institute of Electronics (CIE).

Wei-Ping Zhu (SM'97) received the B.E. and M.E. degrees from Nanjing University of Posts and Telecommunications, and the Ph.D. degree from Southeast University, Nanjing, China, in 1982, 1985, and 1991, respectively, all in electrical engineering. He was a Postdoctoral Fellow from 1991 to 1992 and a Research Associate from 1996 to 1998 with the Department of Electrical and Computer Engineering, Concordia University, Montreal, Canada. During 1993-1996, he was an Associate Professor with the Department of Information Engineering,

Nanjing University of Posts and Telecommunications. From 1998 to 2001, he worked with hi-tech companies in Ottawa, Canada, including Nortel Networks and SR Telecom Inc. Since July 2001, he has been with Concordia's Electrical and Computer Engineering Department as a full-time faculty member, where he is presently a Full Professor. His research interests include digital signal processing fundamentals, speech and audio processing, and signal processing for wireless communication with a particular focus on MIMO systems and cooperative relay networks.

Dr. Zhu was an Associate Editor of the IEEE Transactions on Circuits and Systems Part I: Fundamental Theory and Applications from 2001 to 2003, and an Associate Editor of Circuits, Systems and Signal Processing from 2006 to 2009. He was also a Guest Editor for the IEEE Journal on Selected Areas in Communications for the special issues of: Broadband Wireless Communications for High Speed Vehicles, and Virtual MIMO during 2011-2013. Since 2011, he has served as an Associate Editor for the IEEE Transactions on Circuits and Systems Part II: Express Briefs. Dr. Zhu was the Secretary of Digital Signal Processing Technical Committee (DSPTC) of the IEEE Circuits and System Society during 2012-2014, where he is presently the Chair of the DSPTC.

Yongming Huang (M'10) received the B.S. and M.S. degrees from Nanjing University, China, in 2000 and 2003, respectively. In 2007 he received the Ph.D. degree in electrical engineering from Southeast University, China. Since March 2007 he has been a faculty in the School of Information Science and Engineering, Southeast University, China. During 2008/2009, Dr. Huang was visiting the Signal Processing Lab, Electrical Engineering, Royal Institute of Technology (KTH), Stockholm, Sweden.

His current research interests include space time wireless communications, cooperative wireless communications, energy efficient wireless communications and optimization theory. He serves as an Associate Editor for the IEEE Transactions on Signal Processing, EURASIP Journal on Advances in Signal Processing, and EURASIP Journal on Wireless Communications and Networking.

Arumugam Nallanathan (S'97-M'00-SM'05) is a Professor of Wireless Communications in the Department of Informatics at King's College London (University of London). He served as the Head of Graduate Studies in the School of Natural and Mathematical Sciences at King's College London, 2011/12. He was an Assistant Professor in the Department of Electrical and Computer Engineering, National University of Singapore from August 2000 to December 2007. His research interests include 5G Technologies, Millimeter wave communications,

Cognitive Radio and Relay Networks. In these areas, he co-authored nearly 250 papers. He is a co-recipient of the Best Paper Award presented at the 2007 IEEE International Conference on Ultra-Wideband (ICUWB2007). He is a Distinguished Lecturer of IEEE Vehicular Technology Society.

He is an Editor for IEEE Transactions on Communications and IEEE Transactions on Vehicular Technology. He was an Editor for IEEE Transactions on Wireless Communications (2006-2011), IEEE Wireless Communications Letters and IEEE Signal Processing Letters. He served as the Chair for the Signal Processing and Communication Electronics Technical Committee of IEEE Communications Society, Technical Program Co-Chair (MAC track) for IEEE WCNC 2014, Co-Chair for the IEEE GLOBECOM 2013 (Communications Theory Symposium), Co-Chair for the IEEE ICC 2012 (Signal Processing for Communications Symposium), Co-Chair for the IEEE GLOBECOM 2011 (Signal Processing for Communications Symposium), Technical Program Co-Chair for the IEEE International Conference on UWB 2011 (IEEE ICUWB 2011), Co-Chair for the IEEE ICC 2009 (Wireless Communications Symposium), Co-chair for the IEEE GLOBECOM 2008 (Signal Processing for Communications Symposium) and General Track Chair for IEEE VTC 2008. He received the IEEE Communications Society SPCE outstanding service award 2012 and IEEE Communications Society RCC outstanding service award 2014.

Quan Yu received the B.S. degree in information physics from Nanjing University in 1986, the M.S. degree in radio wave propagation from Xidian University in 1988, and the Ph.D. degree in fiber optics from the University of Limoges in 1992. Since 1992, he joined the faculty of the Institute of China Electronic System Engineering Corporation, where he has been a Senior Engineer, and currently a Research Fellow. His main areas of research interest are the architecture of wireless networks, optimization of protocols and cognitive radios. Dr. Yu is a member

of Chinese Academy of Engineering (CAE).