John Callanan

Mendelssohn and Kant on Mathematics and Metaphysics

Abstract: The difference between the method of metaphysics and the method of mathematics was an issue of central concern for Kant in both the Pre-Critical and Critical periods. I will argue that when Kant speaks of the 'philosophical method' in the Doctrine of Method in the *Critique of Pure Reason* (*CPR*), he frequently has in mind not his own methodology but rather the method of conceptual analysis associated with rationalism. The particular target is Moses Mendelssohn's picture of analysis contained in his submission for the 1763 Prize Essay competition. By the time of the first Critique, I argue, Kant wants to maintain his own longstanding commitment to the distinctness of the methods of metaphysics and mathematics. However, Kant wants to use this same analysis of the source of the distinction to diagnose the origins of the dogmatism that is engendered by the method of the rationalists.

1 Introduction

It is frequently held that a central claim of Kant's Critical philosophy was that the method of mathematics is essentially different from that of metaphysics. A typical recent explanation of Kant's central claim puts it as follows:

> According to Kant's Transcendental Doctrine of Method, philosophy cannot be developed along the lines of the definitions-axioms-proofs scheme that is known from mathematics, and this is for the following reason: mathematics is based on pure intuition, while philosophy is not...None of this can be done in philosophy, or so Kant argues, simply because our abstract philosophical concepts do not exhibit the same kind of intuitive content. Hence, philosophy cannot be done—even in parts—in the style of mathematics.[1]

This is a familiar characterization of Kant's approach.[2] Yet taken in its plainest sense it must be incorrect. This can be seen from the simple point that for Kant metaphysical concepts *do* of course contain intuitional content. The entire strategic thrust of the Transcendental Analytic is just to show that synthetic a

1 Leitgeb (2013, 270).

2 See Carson (1999), De Jong (1995), Kemp Smith (2003), Kitcher (1975), Hintikka (1992), Shabel (2006).

priori knowledge in metaphysics is possible just because the concepts that we deploy are ones that require a schematization relating to intuition in order to manifest their *Sinn und Bedeutung* – by inquiry from concepts alone we are not "in the least being able to show whence they could have their application and their object, thus how in pure understanding without sensibility they could have any significance and objective validity" (*Critique of Pure Reason* (*CPR*) A 242/B 299–300).[3]

In the first section of the first chapter of Transcendental Doctrine of Method ('the discipline of pure reason in dogmatic use'),[4] where Kant unpacks the meaning of the mathematical method, the account cannot connote the relating of a concept to an intuition *simpliciter*, since one of Kant's explicit claims in the *Critique* is that in metaphysics too contentful cognition occurs only if there are intuitions present corresponding to the concepts deployed in judgment.[5] In this regard, Kant is clear that metaphysics is akin to mathematics:

> But if we consider these principles of pure understanding in themselves as to their origin, then they *are anything but* cognitions from concepts. For they would not even be possible *a priori* if we did not bring in pure intuition (in mathematics) or the conditions of a possible experience in general. (*CPR* A 301/B 357 – emphasis added)

There is then a puzzle with regard to the claims of the dogmatic use section. On the one hand, Kant clearly maintains the distinctness of the methods of metaphysics and mathematics. On the other hand, it seems like drawing that distinction in terms of a criterion concerning concepts involving intuitional content runs precisely against Kant's own vision of metaphysics, and indeed places his inquiries on the wrong side of the metaphysics/mathematics distinction. This puzzle can be resolved though by considering the Pre-Critical origins of the dogmatic use section. Specifically, I claim that the section ought to be read as a continuation of Kant's engagement with Mendelssohn on just this question of the distinctness of the methods of mathematics and metaphysics that began with their submissions to the 1763 Berlin Prize Essay competition.

3 See *CPR* A 50/B 74, A 79/B 104–105, A 139–140, B 178–179, A 155–156/B 194–195, A 258/B 314. All English translations of Kant are from The Cambridge Edition of the Works of Immanuel Kant. All references to Kant's works are to the volume and page number of the Akadamie-Ausgabe (AA), except the Critique of Pure Reason (*CPR*), which uses the standard A/B-edition pagination.

4 Henceforth the 'dogmatic use section'.

5 E.g. *CPR* A 51/B 75. I'll follow Allison (2004) in referring to Kant's 'discursivity thesis' as the claim that concepts and intuitions are individually necessary and jointly sufficient for knowledge.

Firstly, I will argue that when Kant speaks of the 'philosophical method' in the dogmatic use section, he means to critically engage with rationalism's traditional reliance upon conceptual analysis as the central means of inquiry. Kant's own transcendental methodology is better understood as sharing with the method of mathematics the claim that the veridical deployment of concepts requires the informing of those abstract discursive representations with possible intuitional content. Kant nevertheless also maintains the distinctness of the methods, and attempts to retain a sense whereby philosophy is still properly understood as reasoning from concepts, albeit not in the unchecked rationalist sense. As such, the dogmatic use section's ambiguity stems from the double duty that it is intended to satisfy: on the one hand, Kant wants to maintain his own longstanding commitment to the distinctness of the methods of metaphysics and mathematics; on the other hand, Kant wants to use this same examination of the source of the distinction to diagnose the origins of the dogmatism that is engendered by the method of the rationalists.

Kant's desire to set metaphysics on the "secure course of a science" demands that the metaphysician attend to the methodology that she employs in her task, and it is this aspect of metaphysical inquiry that she has to date neglected (*CPR* B *xiv*). This contrasts, he thinks, with the case of other sciences, where progress has been achieved just because the scientist has paid heed to the methodological presuppositions in play when initially conceiving of the targets of those inquiries. Kant provides some detail with regard to the initial example, that of the first unidentified geometer:

> A new light broke upon the first person who demonstrated the isosceles triangle... For he found that what he had to do was not trace what he saw in this figure, or even trace its mere concept, and read off, as it were, from the properties of the figure, but rather that he had to produce the latter from what he himself had thought into the object and presented (through construction) according to *a priori* concepts, and that in order to know something securely *a priori* he had to ascribe to the thing nothing except what followed necessarily from what he himself had put into it in accordance with its concept. (*CPR* B *xi-xii*)[6]

The "revolution in thinking" that Kant later advocates in metaphysics is epitomized by the particular self-understanding of this first geometer's own method, and to this extent Kant clearly thinks there is something to be gained by imitating the methodology of geometry.

However, this prescription contrasts notably with Kant's own more explicit reservations expressed later in the *Critique* regarding the methods of the meta-

6 Kant has Proposition I.5 of Euclid's *Elements* in mind here – see Heath (1908, 251).

physician and the mathematician. In the dogmatic use section Kant returns to the theme of the difference between mathematical and "philosophical" cognition. Here the methods are contrasted as essentially different, and the difference concerns the manner in which each operates with its concepts: whereas "[**p**]**hilosophical** cognition is **rational cognition** from **concepts**, mathematical cognition that from the **construction** of concepts. But to construct a concept means to exhibit *a priori* the intuition corresponding to it" (*CPR* A 713/B 741). Here Kant claims that the philosopher cannot mimic the method of the mathematician, since the latter "constructs" concepts and in so doing gains an "intuitive" use of reason, while the philosopher's use of reason is merely "discursive", i.e. involves the manipulation of conceptual representations alone (*CPR* A 719/B 747). His conclusion is that "[t]here are thus two uses of reason, which regardless of the universality of cognition and its *a priori* generation, which they have in common, they are nevertheless very different in procedure" (*CPR* A 723/B 751).

Kant can be seen both as firstly demanding some shared methodology and latterly denying that very possibility. Furthermore, Kant's claim regarding the difference between the philosophical and mathematical methods in the dogmatic use section doesn't seem appropriate with regard to his own methodology. The philosophical method is "**rational cognition** from concepts" and contrasts with the mathematical method which performs rational cognition from the "**construction** of concepts" (*CPR* A 713/B 741). Kant claims that "to **construct** a concept means to exhibit *a priori* the intuition corresponding to it" (*ibid.*). Construction requires the presentation of an a priori intuition, which Kant holds to be an example of "a **non-empirical** intuition" (*ibid.*). In expressing the difference between an a priori intuition and an empirical one, Kant gives the example of a triangle drawn in "mere imagination", in contrast to an empirical one drawn on paper (*ibid.*). The connotation attached to "non-empirical" seems to be that of geometrical procedures pursued in what I'll refer to as *imaginational space*, in contrast to the empirical representations of physical space. Yet the contrast here is still peculiar, since the implied connotation with regard to philosophical cognition is that it proceeds without appeal to intuition generally, whether empirical or non-empirical. Kant continues in the dogmatic use section by characterizing a certain class of necessarily true propositions generated solely from consideration of the relations between concepts as "dogmata" and claims that in metaphysics properly conceived, there are no dogmata at all (*CPR* A 737/B 765). Again, the idea of philosophical cognition as "rational cognition from concepts" if taken to mean 'cognition from concepts *alone*' would seem to preclude Kant's own metaphysical method.

In an attempt to explicate the difference between the mathematical and the metaphysical methods, Kant makes a well-known appeal to proposition I.32 of

Euclid's *Elements*, that the sum of the internal angles of all triangles are necessarily equal to the sum of two right angles.[7] However, the example here serves to confuse rather than clarify the question. Here Kant considers two inquirers, a metaphysician and a geometer, asked to investigate the nature of the concept <*triangle*> in an attempt to derive a hitherto ungrasped truth about triangles generally. On Kant's account, the metaphysician fails miserably – he returns with clear and distinct concepts such as <*straight line*>, <*three*>, etc., but these are just the sub-concepts contained in the concept of <*triangle*> given to him in the first place. Therefore, the metaphysician "may reflect on this concept as long as he wants, yet he will never produce anything new" (*CPR* A 716/B 745).

The geometer by contrast, by engaging in a process of diagrammatic reasoning, performs the proof procedure for proposition I.32, and discovers the proof. In using this example though, it looks like Kant is again making just the wrong kind of distinction for his purposes, since here it looks as if all the philosopher can do is *analyse* the abstract content of the concept <*triangle*> (i.e. decompose it into its constituent conceptual components) whereas the mathematician's access to the proof of proposition I.32 is secured by fact that she engages in "a chain of inferences that is always guided by intuition" (*CPR* A 717/B 745). The point of the example again seems to hinge on the characterization of the philosopher's method as restricted to conceptual analysis. That precisely isn't Kant's own methodology though, so the example seems particularly ill-chosen if meant to illuminate the proper procedure of metaphysics.

Consideration of both Kant's and Mendelssohn's Prize Essay submissions can provide a context for the explanation of the source of this puzzle. In his submission Mendelssohn claimed that metaphysics could and should imitate geometry in the latter's method, which he claimed is that of analysis. Kant's account in the *Critique of Pure Reason* is motivated as a reaction against this fundamental mischaracterization of the methodology of *geometry* that Mendelssohn put forward. The discussion in the 'dogmatic use' section is complicated by Kant's pursuing three distinct claims: firstly, he is expressing his longstanding commitment to the claim that there is some important difference to be noted between metaphysics and mathematics such that metaphysics cannot mimic mathematics; secondly, that metaphysics in the rationalist tradition had sought to follow an incorrect picture of mathematical practice; thirdly, that once the correct picture of geometry is in place, both the differences and the elements in common between metaphysics and mathematics can be ascertained. The circumstances of

7 Heath (1908, 316).

Kant's resolution of these issues, spurred by Mendelssohn's submission, account for the complex nature of Kant's attack.

The remainder of the paper is as follows. In § 2 I examine Kant's account of the difference between mathematical and metaphysical cognition in the *Inquiry*, focusing on Kant's early identification of the necessary role of the representation of particulars for mathematics. In § 3 I examine Mendelssohn's submission, which claimed that mathematical knowledge could be secured through analysis. In § 4 I turn to how Kant uses the example of I.32 to show how his new Critical model of cognition allows him to explain the nature of the contribution of the representation of particulars in a priori knowledge, a model that generalizes to metaphysical knowledge. Once the proper mathematical method is shown, Kant claims that we can see the paucity of the rationalist model of analysis. In § 5 I conclude by examining the remaining senses in which Kant nevertheless held that metaphysics is properly understood as cognition 'from concepts' and that metaphysics and mathematics have distinct methods.

2 Mathematics and Metaphysics in the *Inquiry*

By 1763, when Kant was composing the *Inquiry*, he had already come to hold that metaphysics had laboured under a misapprehension, namely that it could imitate the methodology of mathematics. That this was impossible would have been clear had metaphysicians paid sufficient attention to how mathematical practice actually takes place, and specifically with regard to the crucial issue of the conditions under which we come to possess mathematical concepts. In mathematics we acquire the relevant concepts through a voluntary and creative act of defining them, by bringing together sub-concepts into a synthetic whole:

> There are two ways in which one can arrive at a general concept: either by the arbitrary combination of concepts, or by separating out that cognition which has been rendered distinct by means of analysis. Mathematics only ever draws up its definitions in the first way. For example, think arbitrarily of four straight lines bounding a plane surface so that the opposite sides are not parallel to each other. Let this figure be called a *trapezium*. The concept which I am defining is not given prior to the definition itself; on the contrary, it only comes into existence as a result of that definition. Whatever the concept of a cone may ordinarily signify, in mathematics the concept is the product of the arbitrary representation of a right-angled triangle which is rotated on one of its sides. In this and in all other cases the definition obviously comes into being as a result of *synthesis*. (*Inquiry*, AA 2:276)

Kant's account depends on a distinction between concepts that are created and those that are "given".[8] That a concept is given for Kant does not entail that it is non-acquired, but rather that it has been acquired in some non-arbitrary manner – presumably as being indispensable to the course of ordinary experience.[9] By contrast, a voluntarily created concept is marked by the 'arbitrariness' of this act of creation. An arbitrary combination should not be taken to signify that the propositional content expressible with the concept is contingent – it rather merely marks the fact of the concept's possession as contingent, since it has taken place through a self-conscious decision to form that concept (presumably without being prompted by the pragmatic needs that stimulate the acquisition of given concepts). At this Pre-Critical stage, Kant held that the method of metaphysics was that of *analysis*, which proceeded towards definitions through decomposition of a concept into its fundamental sub-concepts. This presents the most important point of contrast with the method of geometry – "geometers acquire their concepts by means of *synthesis*, whereas philosophers can only acquire their concepts by means of *analysis* – and that completely changes the method of thought" (AA 2:289).[10]

Kant holds that a singular difficulty for metaphysics stems from the fact that the *signs* with which it performs its analysis are those of natural language. Language is encumbered with a range of inherent vagaries, such as that "in metaphysics in particular, words acquire their meaning as a result of linguistic usage…it frequently happens that the same words are employed for concepts which, while very similar, nonetheless conceal within themselves considerable differences" (AA 2:284). However, even if the indeterminacy of natural language use weren't as difficult a problem as it is, the very type of sign that it employs would still hamper metaphysics. This can be seen best by contrast with the signs deployed in geometry. Kant's account of geometrical knowledge hinges upon the epistemic role of "individual signs" (AA 2:279). Individual signs are representations whose explicit intensional content is the presentation of a particular: they present the universal "*in concreto*". Linguistically expressed concepts

8 See *Jäsche Logic* §§ 4–5, AA 9:93–94.

9 Kant is not explicit however with regard to what in fact determines the parameters for givenness in this sense.

10 This confidence in the method of analysis as the sole one for metaphysics is abandoned by the Critical period, and Kant claims that the method of the first *Critique* is synthetic (see *Prolegomena*, AA 4:274). For discussion of Kant's pre-Critical endorsement of conceptual analysis see Schönfeld (2000); for the meaning of the synthetic method in the first *Critique*, see de Jong (1995) and Merritt (2006).

"represent the universal *in abstracto*" (AA 2:279), i.e. they are representations whose explicit intensional purport is to express generality.[11]

The distinction between the two types of sign corresponds to the characteristic representational function performed. Each sign-type offers a distinct *way* of expressing a universal, e.g. triangularity can be expressed both through an image of a token triangle or through the tokening of the word 'triangularity'. That the signs deployed in metaphysics are invariably signs *in abstracto* presents an ironic impediment for the task of expressing universal and necessary truths. In Kant's view, signs *in abstracto* are clearly inferior to individual signs with regard to their power to express universals. For example, an individual sign in geometry, such as an image of a triangle, provides a concrete example of the universal of triangularity. It is in the nature of triangularity itself that all triangles consist of three straight lines laid end to end. The structure of the universal can be expressed in terms of these other universals ('straight', 'line', 'three', etc.) and their relations to each other. These very elements and their relations are also expressed by the individual sign deployed: the drawing of the triangle is also simply nothing but a unified image of three straight lines laid end to end. Just as the universal can be decomposed into other constituent universals, so too can the individual sign be decomposed into representational sub-components that are themselves individual signs of just those corresponding constituent universals.

The individual sign of a triangle is thus structurally isomorphic with the universal it expresses – as Kant puts it, "in geometry the signs are similar to the things signified" (AA 2:292). As such, it is expressively transparent with regard to the structure of the universal it represents – one can just see in the features of the image the properties of the universal that it represents. This contrasts with the signs *in abstracto* that we deploy to signify the universal of triangularity – the word 'triangularity' merely decomposes into a series of letters or phonemes. Words cannot "show in their composition the constituent concepts of which the whole idea, indicated by the word, consists" (AA 2:278–279).

This feature manifests itself not merely with regard to revealing the internal structure of single individual signs but also with regard to their combinatorial properties in propositional knowledge. Individual signs are distinct with regard to the epistemic function of expressing the necessary truth of the propositions concerning the relations between universals. Abstract signs, unlike individual

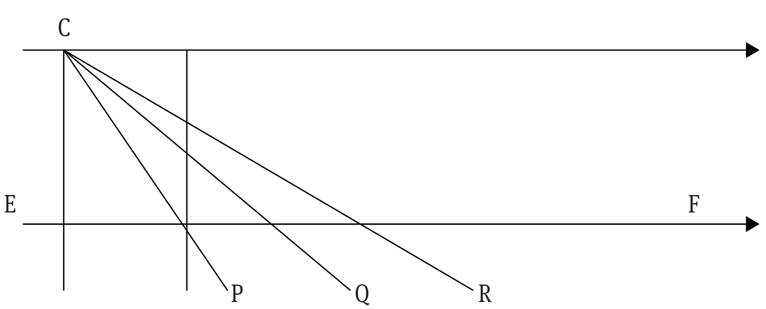
11 My view of the epistemic role of signs *in concreto* (which is performed by *intuitions* in the Critical system) is broadly in accord with Parson's (1983) account. For differing accounts, see Hintikka (1969), Howell (1973) and Thompson (1972).

signs, are not "capable of indicating in their combinations the relations of the philosophical thoughts to each other" (AA 2:279). Just as a given word is not structurally isomorphic with the universal it seeks to express, neither are combinations of words in sentences expressively transparent with regard to the propositional characterization of the relevant relations between universals. But this *is* a feature of propositions expressed through the deployment of individual signs – such a deployment can make evident the necessary connection between two or more universals in a way that the activity of combining words in a sentence cannot. The example given is that of a geometrical diagrammatic proof deployed to express the infinite divisibility of space:

> Suppose for example, that the geometer wishes to demonstrate that space is infinitely divisible. He will take, for example, from a straight line standing vertically between two parallel lines; from a point on one of these parallel lines he will draw lines to intersect the other two lines. By means of this symbol he recognises with the greatest certainty that the division can be carried on *ad infinitum*. (*Inquiry*, AA 2:279)

The example is one Kant used before in the *Physical Monadology* (AA 1:478 – see Figure 1):

Figure 1:



Here the suasive force of mathematical proof is held to make essential use of individual signs – we can just see, in the visual and epistemic sense, that we can continue to draw lines from point C intersecting further and further out along the line EF *ad infinitum*. The proof procedures of metaphysics are hampered once again just by their use of natural language. Kant runs through an argument for the "claim that all bodies consist of simple substances" – he starts from the two premises that bodies are composite wholes of substances, and that composition is an accidental property. Kant then suggests a thought experiment whereby "all composition in a body could be suspended in imagination, but in such a way that the substances, of which the body consists, would continue to exist" (AA 2:279). The non-compositional features of substances only obtain insofar as they are simple, Kant claims, and so he concludes that all bodies are composed of simple substances. Here however "neither figures nor visible signs are capable of expressing either the thoughts or the relations which hold between them… [t]he universal must rather be considered *in abstracto*" (AA 2:278–279).[12]

Although Kant maintained the security of mathematics, he was far more doubtful regarding the state of metaphysics, claiming that "[m]etaphysics is without doubt the most difficult of all the things into which man has insight. But so far no metaphysics has ever been written" (AA 2:283). Kant's dramatic pessimism here is explicitly linked to his claims regarding the processes of signification and concept acquisition peculiar to mathematics and metaphysics. His insistence that no metaphysics has yet been written is connected to his claim that metaphysics had imitated the method of mathematics. The geometer justifiably creates concepts in acts of definition and the metaphysician assumes that he can do so also. The synthetic formation of metaphysical concepts however is unchecked by the deployment of accompanying individual signs – the latter being that which guarantees reference – and is thus simply an exercise in fiction. What is validly *defined* in mathematics is merely *invented* in metaphysics.[13]

For this reason, Kant thinks, "nothing has been more damaging to philosophy than mathematics, and in particular the *imitation* of its method in contexts where it cannot possibly be employed" (AA 2:283). Yet this conclusion obscures the crucial insight that Kant brings from the *Inquiry* to the *Critique* and which motivates the discursivity thesis. In the Pre-Critical period, Kant had already held that individual signs are the indispensable means for grasping necessary truths about mathematical universals; according to the Critical model of cognition, *all* grasping of necessary truth must occur via the presentation of particulars through intuition. The *Inquiry* thus marks a crucial step in Kant's rejection of the rationalist model of discursive cognition, since it is there that he first recognizes the importance of resisting the thought that the appropriate manner of expressing truths about abstract entities, such as universals, is through the exclusive use of abstract signs, such as concepts.

12 Kant's choice of example here is not arbitrary, since it concerns a conclusion that is directly at odds with the conclusion of the preceding mathematical proof. This tension between the conclusions of metaphysics and mathematics was one that concerned Kant throughout the pre-Critical and Critical period, as Emily Carson has explored (see Carson 1999 and 2004).

13 As examples of invented concepts Kant suggests Leibniz's notion of a "slumbering monad" (AA 2:277) – for discussion, see Carson (1999).

3 Mendelssohn's *On Evidence*

There are striking similarities between Kant and Mendelssohn's submissions, such as their shared opinion that metaphysics' reliance upon natural language as a source of its inferiority to mathematics. Mendelssohn also claims that the "abstract signs" deployed in metaphysics guarantee that its proofs can never attain the same "perspicuity" as found in mathematics.[14] He holds that mathematical proofs, by contrast, use "essential signs" – signs that represent things "*in concreto*" (265). Metaphysical proofs have to date all lacked perspicuity just because of their "type of notation" – "[t]hey lacked the aid of essential signs. Everything in the language of philosophers remains arbitrary. The words and the connections among them contain nothing that would essentially agree with the nature of thoughts and the connections among them" (272).

The differences between the two submissions are also clear. The central difference is that, despite Mendelssohn's claim regarding the inferior *perspicuity* of metaphysical proof, he holds that there is no concern to be raised at all with regard to the *certainty* of the properly performed results of metaphysical inquiry. Metaphysical certainty is just the same as the certainty of geometry, simply because, the matter of sign usage aside, the inquiries employ exactly the same method, that of *analysis*. Mendelssohn thus claims "it is possible, through interlocking inferences, to trace the most eminent truths of metaphysics back to such principles which, as far as their nature is concerned, are as undeniable as the first principles and postulates of geometry" (260).

For Mendelssohn, the "entire force" of geometrical certainty is acquired through "the necessary connection of concepts". This is due to the paired theories of inference and concepts to which he is committed. Mendelssohn considers concepts as a kind of abstract sign with an infinitely dense substructure. He does not concern himself with the account of where our concepts come from; rather, he assumes that the concept possession conditions are irrelevant to the knowledge acquisition process: since we *do* possess these concepts, and given that we are in a position to ascertain necessary truths related to them, this achievement must be explained by the fact that the possessed concepts contain *within* them a potentially infinite store of necessary truths. He compares concepts to "seeds of grain" which might have a mediocre outward appearance that belies the amount that can be garnered from them, since every concept "is linked to endless truths and can be reduced by analysis to other concepts and truths" (271).

14 *On Evidence in Metaphysical Sciences*, in Mendelssohn (1997).

All inquiry is then just the analyzing of the core concepts relevant to a discipline. Geometry is a science that can be characterized as the single activity of the analysis of the concept <*extension*>. We are capable of inferring from the bare concept <*extension*> a necessary truth regarding other concepts, e. g., <*triangle*>, etc. and this phenomenon could only be explained if "this truth [can be found] originally and implicitly in the initial concept of extension":

> Thus, this truth also lay tangled up, as one might say, in the original concept of extension, but it escaped our attention and could not be distinctly known and distinguished until, through analysis, we unpacked all the parts of this concept and separated them from one another. The analysis of concepts is for the understanding nothing more than what the magnifying glass is for sight. It does not produce anything that was not to be found in the object. But it spreads out the parts of the object and makes it possible for our senses to distinguish much that they would otherwise not have noticed. The analysis of concepts does nothing different from this; it makes the parts and members of these concepts, which were previously obscure and unnoticed, distinct and recognizable but it does not introduce anything into the concepts that was not already to be found in them. (258)

We are in possession of necessary truths, such as those of geometry, and since our rational capacities do not "introduce" but rather merely recognize representational content, it must be the case that these truths are realized by virtue of unpacking given contents. The claim regarding the structure of concepts follows from the epistemological claim as to what can be achieved with nothing but a bare concept and the magnifying glass of analysis:

> [T]here is no doubt that all geometric truths that geometry teaches us to *unpack* or *untangle* from the concept of extension must be encountered all *tangled up* in it. For what else can the profoundest inferences do but analyze a concept and make distinct what was obscure? Such inferences cannot bring in what is not to be found in the concepts, and it is easy to see that it is also not possible, by means of the principle of contradiction, to derive from the concept what is not to be found in it. (257)[15]

For Mendelssohn, we can acquire knowledge regarding what might have seemed like newly formed concepts (such as <*triangle*>) just through analysis of <*extension*>. In both the quoted passages above, Mendelssohn's language clearly anticipates not just Kant's characterization of analytic judgment,[16] but also his characterization of the first geometer in preface to the *Critique*. It cannot be,

15 See Shabel (2006) for discussion.

16 In terms of predicates "already thought" in the concept in the subject position of a judgment (e. g. *CPR* A 7/B 11, B 15) and also of predicates determinable through the principle of non-contradiction alone (e. g. *CPR* A 7–8/B 11–12, A 151/B 190).

Mendelssohn says, that the inferences a geometer performs "introduces anything into the concepts" or that the inferences performed might themselves "bring in" representational content, since our inferential capacity is essentially a recognitional capacity, thus one that we direct towards pre-existing contents located within an initial concept's implicit discursive structure. By contrast, even by this Pre-Critical point Kant was already claiming that the performance of inferences might themselves "bring in" representational content and thereby constitute the possession conditions for mathematical concepts.

Like Kant though, Mendelssohn claims that the power of geometry's individual signs is that they "agree in their nature and connection with the nature and connection of the thoughts" and that for example "lines are placed together in figures in the same manner as the concepts are placed together in our soul" (264). Although not as perspicuous in its procedure as geometry, in metaphysics "the same certainty reigns". Mendelssohn acknowledges though that metaphysical conclusions, since formed through analysis, can be understood as just claims regarding the connections between concepts, and as such merely express "possibilities". The task of the metaphysician is in a sense more demanding than that of the mathematician, since although the latter can establish her conclusions without direct appeal to the existence of things to which those conclusions apply, metaphysical propositions often aspire to express claims *just about* the existence of things (such as the self or God), and so now "the important step into the realm of actuality must take place" (274). Despite the metaphysician's method being identical to that of the mathematician, and despite the fact the mathematician cannot step into the realm of actuality, Mendelssohn claims that the movements are in fact easily achieved by the metaphysician. Mendelssohn cites as examples the *cogito* and the ontological argument and claims that "[w]e have Descartes to thank for these two transitions from the possible to the actual" (275–276). For Mendelssohn these two claims are as well founded as the propositions of geometry. The conclusions of both essays are then radically at odds. In the *Inquiry*, Kant diagnosed the failures of metaphysics as based on the assumption of a shared methodology with mathematics, only to discover that the prize-winning submission was one that argued that, just by virtue of the shared methodology of the analysis of concepts, our acquisition of knowledge of God's existence was as straightforward a matter as that of acquiring knowledge of the properties of triangles.

4 The Critical Account

By the time Kant presented his account of mathematical cognition in the first *Critique*, nearly two decades later, his view of knowledge *per se* had changed entirely. The change could be expressed by contrasting it with the rationalist model of knowledge that Mendelssohn's essay epitomized, namely that of successful analysis. The achievement of veridical representation was modelled as achieved through the purification of acts of *thinking*. The transition from the state of lacking knowledge to the state of possessing it is to be understood as the Leibnizian transition from a state of indistinct and obscure thought to clear and distinct thought (*CPR* A 270–271/B 326–327). The confusion of obscure thought is due to the impurities of sensory representations infecting the capacity of thought itself. As well as our discursive capacities, such as the understanding and reason, Kant claims we also possess a range of non-discursive capacities. Amongst those non-discursive representational capacities are the familiar ones of the sensory modalities, memory and imagination. The *Critique* though marks the recognition of the requirement of a further, distinct type of non-discursive representational capacity, that of intuition.[17] In a broader sense Kant's project is motivated by the denial of the rationalist picture, and by the thought of the necessary co-deployment of both discursive and non-discursive capacities as the jointly sufficient conditions for knowledge acquisition.

By 1781, Kant had discovered that individual signs, in the form of intuitions, were required for the expression of propositional knowledge relating to both mathematical *and* metaphysical concepts. Kant's endorsement of the discursivity thesis holds that veridical representation, rather than being an achievement of pure thought, is instead a distinct epistemic achievement involving both conceptualization and the input of non-conceptual individual signs (*qua* intuition) in a single cognitive act, i. e. cognition. A crucial aspect of this model is that for Kant our intuitional capacity can also be activated imaginationally, whereby we can access the intuitional outputs without directly activating any of our sensory modalities. When this activity of the interaction of our imaginational and intuitional capacities occurs, the representations produced in imaginational space are "pure" (*CPR* B 3).

In the 'dogmatic use' section Kant repeats his claim from the *Inquiry* that there is an essential difference between the methods of metaphysics and mathematics. This difference is now expressed though in terms of a procedure that is

17 For the role of the intuitional capacity as one contributing its own representational content see Warren (1998) and Waxman (2005).

peculiar to mathematics, that of construction, and explains why definitions can only be provided in mathematics:

> Thus there remain no other concepts that are fit for being defined than those containing an arbitrary synthesis which can be constructed *a priori*; and thus only mathematics has definitions. For the object that it thinks it also exhibits *a priori* in intuition, and this can surely contain neither more nor less than the concept, since through the explanation of the concept the object is originally given, i.e., without the explanation being derived from anywhere else. (*CPR* A 729–730/B 757–758).

Kant's account of construction involves the amalgamation of several of the key claims seen before: construction itself is nothing but the acquisition of concepts through acts of definition, where the latter is understood as manifested within the inferences performed upon individual signs. Only through this procedure can certain objects even be known, since those individual signs are constitutive of the proof-procedures that must be performed to acquire geometrical knowledge. Whereas before the deployment of individual signs had been a mere "aid to thought", Kant now views the representation of particulars as necessary for the very capacity to represent the objects that fall under our concepts.

Kant uses a different geometrical proof to express this point, proposition I.32 of the *Elements*. As he had done in the *Inquiry*, Kant contrasts the practice of conceptual analysis with the practice of the mathematician:

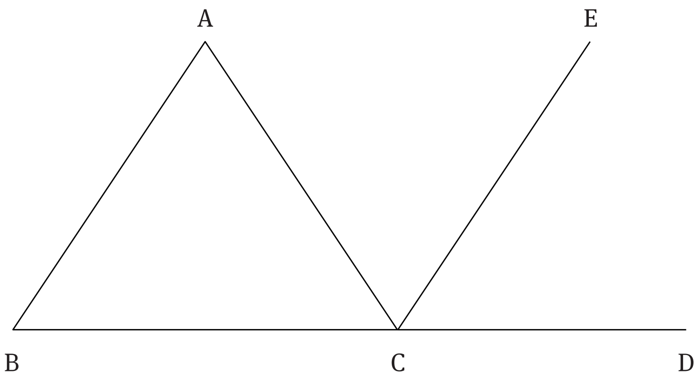
> Give a philosopher the concept of a triangle, and let him try to find out in his way how the sum of its angles must be related to a right angle. He has nothing but the concept of a figure enclosed by three straight lines, and in it the concept of equally many angles. Now he may reflect on this concept as long as he wants, yet he will never produce anything new. He can analyse and make distinct the concept of a straight line, or of an angle, or of the number three, but he will not come upon any other properties that do not already lie in these concepts. But not let the geometer take up this question. He begins at once to construct a triangle. Since he knows that two right angles together are exactly equal to all the adjacent angles that can be drawn at one point on a straight line, he extend one side of his triangle, and obtains two adjacent angles that together are equal to two right ones. Now he divides the external one of these angles by drawing a line parallel to the opposite side of the triangle, and sees that here there arises an external adjacent angle which is equal to an internal one, etc. In such a way, through a chain of inferences that is always guided by intuition, he arrives at a fully illuminating and at the same time general solution of the question. (*CPR* A 716–717/B 745).[18]

---

18 The use of this example was not uncommon, though I'd suggest that Kant was inspired here by Leibniz's use of I.32 in the *New Essays* (Book IV, Ch. ii, § 2: 367). Leibniz describes the demonstrative reasoning employed in proving Proposition I.32 as a "chain of items of intuitive knowledge [*enchaînement des connaissances intuitives*]" just as Kant describes the reasoning

Here Kant describes the proof performed in the *Elements* (see Figure 2).

Figure 2:



Briefly, the proof proceeds as follows: we construct a triangle ABC, then extend BC to point D, and draw a line CE that is parallel to BA. We see that the angle ∡abc is identical to that at ∡ecd. Since AC is a transversal of two parallel lines, the opposing angles at ∡bac and ∡ace are equal. We see then that the internal angles of ABC are equal to the sum of ∡ecd, ∡ace and ∡acb. Furthermore, that latter set of angles are together equal to the sum of two right angles, since we can see that those angles together rest upon the straight line BD. Thus the internal angles of ABC must be equal to the sum of two right angles.

The claim is that this proof procedure for establishing proposition I.32 is not performed through conceptual analysis alone.[19] There are two senses in which the proof is "always guided by intuition" however. Firstly, there is the simple point that the drawn diagram is a deployment of an individual sign. Secondly, though, there is the epistemic suasive sense in which certain steps of the proof take place (to repeat Kant's claim from the *Inquiry*) "with the degree of assurance characteristic of seeing something with one's own eyes". At crucial points in the diagram we are supposed just to *see*, in both the visual and epistemic senses, that the angle ∡abc is the same as ∡ecd; similarly, we are supposed just to see that ∡acb is shared in common between the triangle and the angles

with regard to the same proposition as proceeding through a "chain of inferences [*eine Kette von Schlüssen*] that is always guided by intuition".

19 E.g. Friedman (1985), Shabel (2003; 2006).

that go to make the straight line BD. The epistemic intuitiveness of the proof is for Kant a product of the use of the representation of spatial particulars.

This allows for an understanding of the original import of Kant's use of the example of the first geometer in the B preface. There Kant praised the first geometer for recognizing that the nature of the task at hand involved neither an empiricist "reading off" of properties from a drawn figure – all that would do would be to present a series of images of three straight lines laid end-to-end – nor a *rationalist* reading off from the unpacked intension of a given concept, but instead drawing out from the proof procedure what the geometer had himself put in. Far from our being able to untangle the concept <*extension*> into all the truths of geometry, Kant is simply challenging Mendelssohn (and Wolffians in general) to untangle the conceptual content <*sum of interior angles necessarily equal to the sum of two right angles*> from the initial concept <*figure enclosed by three straight lines laid end-to-end*>. No matter how far we decompose that concept, Kant thinks, the former constituent just won't reveal itself. Applying the magnifying glass of analysis to the latter and "spreading out all its parts" reveals nothing more than the spread-out parts: <*figure*>, <*three*>, <*straight*>, <*line*>, etc. and not the propositional knowledge that intuition can reveal.[20]

5 The Distinction Between Mathematics and Metaphysics

The 'philosopher' targeted in the Euclidean example section is Mendelssohn, and the practice of transcendental inquiry ought not to be thought of in terms of his rationalist conception of the philosophical method. Nevertheless, Kant maintains (i) that transcendental philosophy is properly understood as discursive cognition and (ii) that there are some genuine differences between transcendental philosophy and mathematics. There is the difference firstly that the mathematical method constructs its concept through acts of definition, whereas the philosophical method does not. But it is clear that there are two distinct elements to the construction procedure that Kant appeals to in the 'dogmatic use' section. Firstly, there is the discursivity thesis, and Kant's claim from the *Inquiry* that in mathematical cognition we think "the universal in the particular" (*CPR* A 714/B 742). A second feature however is that in mathematics the concept can be acquired through the provision of an intuition that one has *literally produced* and brought into being through the cooperation of one's own imagination-

20 See *CPR* A 718–719/B 746–747. Cf. *Metaphysik Vigilantius* (K3), AA 29:969.

al and intuitional cognitive capacities – this is what it is for an intuition to be the product of an a priori *exhibition* (*CPR* A 713/B 741).[21] This second feature is not shared with metaphysics, since there the opportunities for knowledge depend on occasions of being given objects whose existence is always accounted for by some distinct ontological ground. We cannot summon up in the imagination genuine tokens of the extension of the concept <*substance*>, for example, though we *can* summon up genuine tokens of geometrical figures in imagination. In extending the demand of intuitions for concept-application to metaphysical concepts, Kant maintains a distinct similarity between metaphysical and mathematical inquiries; in distinguishing intuitions that can be received from those that can be exhibited *a priori*, he retains for himself grounds for the claim that there is a strict distinction between the two.

Inattentiveness to both these features of the mathematician's practice that Kant thinks has had a deleterious effect on metaphysics. While both mathematics and metaphysics deploy concepts that require the use of intuitions, and to that extent share a methodology, the former can deploy intuitions by acts of will and imagination. It is due to this latter feature that we can define mathematical concepts, since we can stipulate the things to which they refer by bringing their referents into being. To presume that we could do this with regard to metaphysical concepts however would be to abandon the claim that our experience of empirical reality is essentially restricted by the receptive nature of human cognition. Transcendental philosophy concerns the conditions of a possible rule-like synthesis of intuitions, but those intuitions must be given, i.e. they can only be exhibited in experience (*CPR* A 567/B 595, A 714–715/B 742–743). In this sense then transcendental philosophy is still cognition merely "from concepts", since it must stipulate its claims only with regard to discursive conditions on possible intuition rather than conceptualized instances of actual token intuitions.

Kant's construction procedure therefore essentially involves the cognitive achievement of producing an individual sign just out of one's own representational resources. The notion of providing an intuition a priori must be understood literally – with the mathematical use of reason "we can determine our concepts *a priori* in intuition, for we create the objects themselves in space and time" (*CPR* A 723/B 751). Through the act of construction we define a concept and thereby bring that concept into being for the first time, just as Kant had held in the *Inquiry*; in the *Critique*, though, Kant maintains that a further feature

21 Cf. *CPR* B 65–66, where Kant states that in geometry "[y]ou must therefore give your object *a priori* in intuition, and ground your synthetic proposition on this." See also *CPR* A 234/B 387.

of the creation of the concept is that it can occur through a presentation of an individual sign that the agent herself has also created and that is both necessary and sufficient as a genuine member of the extension of that concept. Although I can draw a triangle on paper, I may also do so in the pure imagination. When I perform the latter task, I am not dependent on being given any *empirical* objects in order to engage in the construction procedure and yet that imaginational triangle nevertheless counts as a genuine member of concept's extension.[22]

When Kant expresses the difference between the metaphysical and mathematical methods, it is invariably with appeal to this feature of construction. In his metaphysics lectures he characterizes concept construction as occurring when an a priori intuition is produced and where the latter is understood as "that which everyone can give to himself".[23] Mathematical inquiry has a distinct method for Kant, though only because mathematical concepts can be acquired via imaginational intuition, without the prompting of sensory stimuli from given empirical objects. Were metaphysics to share the exact same method of mathematics, this would entail our having the representational resources to produce empirical intuitions in our imagination corresponding to the Categories. To assume this though would be to ascribe to human representation a capacity akin to the intellectual intuition of a divine being, for whom thought of *any* object, empirical objects included, is coterminous with an act of that object's being brought into existence (*CPR* A 252/B 308).

Kant's aims in the dogmatic use section, stemming from the *Inquiry*, are not just to contrast his own metaphysical method with that of mathematics but also to point to aspects of similarity, by attending to the actual reasoning practices of the mathematician. His claim is that the metaphysical tradition has mischaracterized the method that it seeks to emulate. When the mathematical method is correctly characterized, both the differences and similarities between the proper methods of metaphysics can be more clearly appreciated. Kant attempts to demonstrate this by showing how the paradigmatic epistemic achievement of geometrical knowledge is properly characterized not along rationalist lines, as an achievement of clear and distinct abstract thought, but rather as that of the conceptually informed cognition of spatiotemporally represented particulars.[24]

---

22 *CPR* A 722/B 750, note.

23 *Metaphysics L2*, AA 28:532. See also the *Metaphysical Foundations of Natural Science*, AA 4:470, 4:486–488, *On a Discovery*, AA 8:192, note.

24 For comments on earlier talks from which this paper is drawn, I would like to thank audiences at King's College London, Cambridge, University of Amsterdam, Humboldt University and University of California at Berkeley, as well as an anonymous referee.

Bibliography

Allison, Henry E. (2004): Kant's Transcendental Idealism: An Interpretation and Defense (rev. and enl. ed.), New Haven and London.

Carson, Emily (1999): Kant on the Method of Mathematics, in: Journal of the History of Philosophy 37, pp. 629–52.

Carson, Emily (2004): Metaphysics, Mathematics and the Distinction Between the Sensible and the Intelligible in Kant's Inaugural Dissertation, in: Journal of the History of Philosophy 42, pp. 165–194.

De Jong, Willem R. (1995): How is Metaphysics as a Science Possible? Kant on the Distinction between Philosophical and Mathematical Method, in: Review of Metaphysics 49, pp. 235–274.

Friedman, Michael (1985): Kant's Theory of Geometry, in: The Philosophical Review 94, pp. 455–506.

Heath, Thomas L. (ed. and trans.) (1908): The Thirteen Books Of Euclid's Elements, Vol. I, Cambridge.

Hintikka, Jaakko (1969): On Kant's Notion of Intuition (Anschauung), in Terence Penelhum and J.J. Macintosh (eds.): Kant's First Critique, Belmont, pp. 38–53.

Hintikka, Jaakko (1992): Kant on the Mathematical Method, in: Carl J. Posy (ed.): Kant's Philosophy of Mathematics, Dordrecht, pp. 21–42.

Howell, Robert (1973): Intuition, Synthesis and Individuation in the Critique of Pure Reason, in: *Noûs 7*, pp. 207–232.

Kant, Immanuel (1992a): Lectures on Logic, J. Michael Young (trans. and ed.), Cambridge.

Kant, Immanuel (1992b): Theoretical Philosophy 1755–1770, David E. Walford, with collaboration by Ralf Meerbote (trans. and eds.), Cambridge.

Kant, Immanuel (1997): Lectures on Metaphysics, Karl Ameriks and Steve Naragon (trans. and eds.), Cambridge.

Kant, Immanuel (1998): The Critique of Pure Reason, P. Guyer and A. Wood (trans. and eds.), Cambridge.

Kant, Immanuel (2002): Theoretical Philosophy After 1781, Henry Allison and Peter Heath (eds. and trans.), Cambridge.

Kemp Smith, Norman (2003): A Commentary to Kant's Critique of Pure Reason, New York.

Kitcher, Philip (1975): Kant and the Foundations of Mathematics, in: The Philosophical Review 84, pp. 23–50.

Leibniz, Gottfried Wilhelm (1996): New Essays on Human Understanding, Peter Remnant and Jonathan Bennett (trans. and eds.), Cambridge.

Leitgeb, Hannes (2013): Scientific Philosophy, Mathematical Philosophy, and All That, in: Metaphilosophy 44, pp. 267–275.

Mendelssohn, Moses (1997): Philosophical Writings, Daniel Dahlstrom (trans. and ed.), Cambridge.

Merritt, Melissa M. (2006): Science and the Synthetic Method of the Critique of Pure Reason, in: The Review of Metaphysics 59, pp. 517–539.

Parsons, Charles (1983): Kant's Philosophy of Arithmetic, in: idem: Mathematics and Philosophy: Selected Essays, Ithaca, NY, pp. 110–49.

Schönfeld, Martin (2000): The Philosophy of the Young Kant, New York.

Shabel, Lisa (2003): Mathematics in Kant's Critical Philosophy, London & New York.

Shabel, Lisa (2006): Kant's Philosophy of Mathematics, in: Paul Guyer (ed.), The Cambridge Companion to Kant and Modern Philosophy, Cambridge, pp. 94–128.
Thompson, Manley (1972): Singular Terms and Intuitions in Kant's Epistemology, in: The Review of Metaphysics 26, pp. 314–343.
Warren, Daniel (1998): Kant and the A Priority of Space, in: The Philosophical Review 107, pp. 179–224.
Waxman, Wayne (2005): Kant and the Empiricists: Understanding Understanding, Oxford.