Delay-dependent stabilization condition for T-S fuzzy neutral systems

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Abstract—In this paper, the stabilization problems for a class of Takagi-Sugeno (T-S) fuzzy neutral systems are explored. Utilizing Pólya’s theorem and some homogeneous polynomials techniques, the delay-dependent stabilization condition for T-S fuzzy neutral systems are proposed in terms of a linear matrix inequality (LMI) to guarantee the asymptotic stabilization of T-S fuzzy neutral systems. Lastly, an example is illustrated to demonstrate the effectiveness and applicability of the proposed method.

Keywords—Fuzzy control, Takagi-Sugeno (T-S) fuzzy model, linear matrix inequality (LMI), Pólya’s theorem.

I. INTRODUCTION

During the past decades, fuzzy logic control has been widely adopted to analyze nonlinear systems [1], [2]. In addition to fuzzy logic control, a great deal of effort has been devoted to describe a nonlinear model using Takagi-Sugeno (T-S) fuzzy model. Moreover, many fuzzy modelling approaches [3], [4] which are provided to represent nonlinear models as T-S fuzzy model. Through the modeling procedure, T-S fuzzy model can be described by fuzzy IF-THEN rules, which represents local linear input-output relations of a nonlinear model. In addition to fuzzy modeling scheme, the parallel distributed compensation (PDC) technique [5] is adopted to stabilize the overall T-S fuzzy model. For the stabilization condition analysis, Lyapunov direct method [6], [7] is mainly investigated to yield the stabilization conditions of T-S fuzzy model. Furthermore, many studies utilize linear matrix inequalities (LMIs) to find a feasible solution, if available. Furthermore, the solutions can be found via linear matrix inequalities (LMIs) techniques.

Time-delay phenomenon exists in many practical systems, such as chemical engineering systems, robotic arm systems and network systems [8], [9]. In general, the practical systems with time-delay are more complicated than those systems without time-delays. Besides, time-delay may cause instability and reduce the system performance under some situations; therefore, there has been an increasing interest in the stabilization problem for time-delay systems, and a lot of results on these topics have been explored in the literature [10], [11].

In addition, time-delay phenomenon exists in both the state and the derivative of the state in neutral systems. For this reason, stabilization problem for neutral systems has been explored in many studies [12], [13]. For example, in [14], a state matrix decomposition is adopted and a delay-dependent stability condition for fuzzy neutral systems was proposed. A descriptor system approach is adopted for uncertain fuzzy neutral system in [15]. Recently, a polynomial technique has been adopted to reduce the conservatism of the stabilization condition [16]–[18]. Inspired by these works and reference therein, we will explore the delay-dependent stabilization condition for T-S fuzzy neutral systems via polynomial technique.

Following the introduction, the paper is organized as follows. In Section II, a general description of T-S fuzzy neutral system is introduced and the state feedback fuzzy controller is also designed. In Section III, based on homogeneous polynomial technique and Pólya’s theorem, a delay-dependent stabilization conditions for T-S fuzzy neutral system is formulated in terms of LMIs in this section. In Section IV, a numerical example is given to demonstrate the feasibility and effectiveness of the proposed approach. Finally, the conclusions are given in Section V.

Notation: The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote, respectively, the $n$-dimensional Euclidean space and the set of all $n \times m$ real matrices. $A^T$ denotes the transpose of matrix $A$. $X \preceq Y$ or $X < Y$, respectively, where $X$ and $Y$ are symmetric matrices, means that $X - Y$ is negative semi-definite or negative definite, respectively. $I$ is the identity matrix with a compatible dimension (without confusion). $\text{diag}(q_1, \cdots, q_n)$ represents a block-diagonal, $q_1 \cdots q_n$ as the diagonal elements. Denotes a symmetric matrix, where * represents the entries implied by symmetry. The matrices, if not explicitly stated, are assumed to have compatible dimensions. ! denotes factorial for combinatoric expression. Let $K(h)$ be the set of $r$-tuples defined as [19]:

$$K(h) = \{ (k_1 k_2 \cdots k_r) : k_1 + k_2 + \cdots + k_r = h, \ \forall k_i \in I^+ (\text{positive integers}), i = 1, 2, \cdots, r \}$$

where $h$ is the total polynomial degree. Since the number of
The state feedback fuzzy controller for T-S fuzzy neutral system (1) is represented as follows.

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(t) [A_i x(t) + A_{ci} x(t-h(t)) + A_{di} \dot{x}(t-h(t))] \\
+ B_i u(t)
\]

where \( r \) is the number of fuzzy rules. The state of system \( x(t) \in \mathbb{R}^{n \times 1} \) and the input \( u(t) \in \mathbb{R}^{m \times 1} \). The matrices \( A_i, A_{ci}, A_{di}, B_i \in \mathbb{R}^{n \times n} \) are system matrices, and initial vector \( \phi(t) \) belongs to the set of continuous functions. The time-varying delay \( h(t) \) satisfies \( 0 \leq h(t) \leq h_M \) and \( 0 \leq h(t) \leq h_D \). \( \mu_i(t) = \omega_i(t) \prod_{i=1}^{r} M_{ij}(\xi(t)). \)

The state feedback fuzzy controller for T-S fuzzy neutral system (1) is represented as follows.

\[
u(t) = \sum_{k \in K(r-1), r \geq 2} \mu_k F_k x(t)
\]

By substituting (2) into (1), the closed-loop system can be obtained as (3).

\[
\dot{x}(t) = (A(t) + B(t) F(t)) x(t) + A_{c}(t) x(t-h(t)) \\
+ A_{d}(t) \dot{x}(t-h(t))
\]

III. MAIN RESULTS

Before discussing the proof of the theorems, here are some lemmas which are used in the proof.

**Lemma 1:** [20] For any positive symmetric constant matrix \( R_1 \in \mathbb{R}^{n \times n} \) and a scalar \( h_M > 0 \), if there exists a vector function \( \dot{x}(s) : [0, h_M] \rightarrow \mathbb{R}^{n} \) such that the integrals \( \int_{t-h(t)}^{t} \dot{x}(s) R_1 \dot{x}(s) ds \) and \( \int_{t-h(t)}^{t} \dot{x}(s) R_2 \dot{x}(s) ds \) are well defined, then the following inequality holds:

\[
-h_M \int_{t-h(t)}^{t} \dot{x}(s) R_1 \dot{x}(s) ds \leq \int_{t-h(t)}^{t} \dot{x}(s) R_2 \dot{x}(s) ds.
\]

**Lemma 2:** [21] (Pólya’s theorem) For a positive integer \( r \), \( \{ \Delta_r: (\mu_1, \ldots, \mu_r) \mid \mu_i \geq 0, \sum_{i=1}^{r} \mu_i = 1 \} \). If a real homogeneous polynomial \( F(\mu_1, \ldots, \mu_r) \) is positive definite, then for a sufficiently large \( d \), all the coefficients of \( (\mu_1 + \cdots + \mu_r)^d F(\mu_1, \ldots, \mu_r) \) are positive.

**Lemma 3:** [16] Consider the T-S fuzzy system,

\[
\dot{x}(t) = \sum_{i=1}^{r} \mu_i(t) [A_i x(t) + B_i u(t)]
\]

where \( x(t) \) is the state of system, \( u(t) \) is the control input, \( A(t) = \sum_{i=1}^{r} \mu_i(t) A_i \quad B(t) = \sum_{i=1}^{r} \mu_i(t) B_i \). The T-S fuzzy system (4) is asymptotically stabilizable via the state feedback controller \( G_k = \bar{G}_k X^{-1} \) if and only if there exist a symmetric positive definite matrix \( X > 0 \) and \( d \in \mathbb{N} \) such that:

\[
\sum_{k' \in K(d)} \sum_{i=1}^{r} \frac{d_{k,i}}{\pi(k') \pi(k-k')} \left( A_i X + \Omega \right) + (B_i G_{k-k'-e_i} + \Omega) < 0
\]

where \( k \in K(d+r), k > k' \), symbol \( \prec \) is the componentwise.

Before discussing the following stability conditions, we first define the following function:

\[
P_1(t) = \sum_{i=1}^{r} \mu_i(t) P_{1i}, \quad P_2(t) = \sum_{i=1}^{r} \mu_i(t) P_{2i}
\]

\[
P_3(t) = \sum_{i=1}^{r} \mu_i(t) P_{3i}, \quad S_1(t) = \sum_{i=1}^{r} \mu_i(t) S_{1i}
\]

\[
P_4(t) = \sum_{i=1}^{r} \mu_i(t) P_{4i}
\]

The main result on the asymptotic stability of the T-S fuzzy neutral system (1) is propound in the following theorem.

**Theorem 1:** If there exist integers \( d_a > 0, d_b > 0, \) some positive matrices \( P_1, P_{1i}, P_{2i}, P_{3i}, P_{4i} \in \mathbb{R}^{n \times n} \), and matrices \( S_{1i} = [S_{11i}, S_{12i}, S_{13i}, S_{14i}], \) \( S_{11i}, \ldots, S_{14i} \in \mathbb{R}^{n \times n} \) such that the following inequalities (6) and (7) are satisfied for some positive scalar \( g_1, g_2, g_3 \) and \( 0 \leq h(t) \leq h_M, \leq \frac{h(t)}{h_D} \) then the fuzzy neutral system (1) is asymptotically stabilizable via the state feedback controller \( F_k = F_k X^{-1}, X \in \mathbb{R}^{n \times n} \).

\[
\Omega < 0
\]

\[
P_{3i} < P_{4i}
\]

where

\[
\Omega = diag \left[ \Omega_{1a}(1), \Omega_{2a}(1), \ldots, \Omega_{a}(J(d+2), \Omega_{b}(J(d+1)) \right]
\]

\[
\Omega_{ka, kb} = \sum_{k_i \in K(d_a)} \frac{d_{a}}{\pi(k_a)} \sum_{k_j \in K(d_b)} \frac{d_{b}}{\pi(k_b)}\Omega
\]

\( k_a \in K(d_a + 2), \ k_b \in K(d_b + 1), k_a > k_a, k_b > k_b' \)
\[
\Omega = \begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\
\ast & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\
\ast & \ast & \Omega_{33} & \Omega_{34} & \Omega_{35} \\
\ast & \ast & \ast & \Omega_{44} & \Omega_{45} \\
\ast & \ast & \ast & \ast & \Omega_{55}
\end{bmatrix}
\]

\[
\begin{align*}
\Omega_{11} &= \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} \left( (S_{11i} + \ast) + P_{i1} \right) \\
+ (A_{i}X + \ast) + \sum_{i=1}^{r} \frac{1!}{\pi(k_{b} - k_{b}')} [(B_{i} \bar{F}_{k_{a} - k_{a}' - e_{i} + \ast})]
\end{align*}
\]

\[
\Omega_{12} = \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} [g_{1}X^{T}A_{i}^{T}]
+ \sum_{i=1}^{r} \frac{1!}{\pi(k_{b} - k_{b}')} [g_{1}\bar{F}_{k_{a} - k_{a}' - e_{i} + \ast}B_{i}^{T}]
\]

\[
\begin{align*}
\Omega_{13} &= \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} [g_{2}X^{T}A_{i}^{T}]
+ \sum_{i=1}^{r} \frac{1!}{\pi(k_{b} - k_{b}')} [g_{2}\bar{F}_{k_{a} - k_{a}' - e_{i} + \ast}B_{i}^{T}]
+ \sum_{i=1}^{r} \frac{1!}{\pi(k_{b} - k_{b}')} [\bar{P}_{0}^{T} - X]
\end{align*}
\]

\[
\begin{align*}
\Omega_{14} &= \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} [A_{di}X + S_{14i}^{T}]
+ g_{3}X^{T}A_{i}^{T}]
+ \sum_{i=1}^{r} \frac{1!}{\pi(k_{b} - k_{b}')} [g_{3}\bar{F}_{k_{a} - k_{a}' - e_{i} + \ast}B_{i}^{T}]
\end{align*}
\]

\[
\begin{align*}
\Omega_{15} &= \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} \left( (S_{11i} + \ast) \\
+ \sum_{j=1}^{r} \frac{1!}{\pi(k_{b} - k_{b}')} [\bar{P}_{0}^{T} - X]
\end{align*}
\]

\[
\begin{align*}
\Omega_{22} &= \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} [g_{1}A_{ci}X + \ast]
- (S_{12i} + \ast)
\end{align*}
\]

\[
\begin{align*}
\Omega_{23} &= \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} [g_{2}X^{T}A_{i}^{T}]
+ \sum_{i=1}^{r} \frac{1!}{\pi(k_{b} - k_{b}')} [g_{1}\bar{F}_{k_{a} - k_{a}' + \ast}B_{i}^{T}]
- (S_{12i} + \ast)
\end{align*}
\]

\[
\begin{align*}
\Omega_{24} &= \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} [g_{1}X^{T}A_{i}^{T}]
+ g_{1}A_{di}X - S_{14i}^{T}
\end{align*}
\]

\[
\begin{align*}
\Omega_{25} &= \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} [\bar{P}_{2i} + h_{M}\bar{P}_{3i}]
\end{align*}
\]

\[
\begin{align*}
\Omega_{33} &= \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} [\bar{P}_{2i} + h_{M}\bar{P}_{3i}]
\end{align*}
\]

\[
\begin{align*}
\Omega_{34} &= \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} \pi(k_{b} - k_{b}')[g_{2}A_{di}X]
+ \sum_{i=1}^{r} \frac{1!}{\pi(k_{a} - k_{a}')} \pi(k_{b} - k_{b})[-g_{3}X]
\end{align*}
\]

\[
\Omega_{35} = \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} \pi(k_{b} - k_{b})[S_{13i}]
\]

\[
\Omega_{44} = \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} \pi(k_{b} - k_{b})[(g_{3}A_{di}X + \ast)]
+ \sum_{j=1}^{r} \frac{1!}{\pi(k_{a} - k_{a})}[-(1 - h_{D})\bar{P}_{2j}]
\]

\[
\Omega_{45} = \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} \pi(k_{b} - k_{b})[S_{14i}]
\]

\[
\Omega_{55} = \sum_{i=1}^{r} \frac{(r-1)![(k_{ai} - k_{ai}')]!}{\pi(k_{a} - k_{a}')} \pi(k_{b} - k_{b})[-h_{3}\bar{P}_{3i}]
\]

**Proof:** Firstly, let us consider the Lyapunov-Krasovskii function

\[
V(t) = V_{1}(t) + V_{2}(t) + V_{3}(t) + V_{4}(t)
\]

where

\[
V_{1}(t) = x^{T}(t)P_{0}x(t)
\]

\[
V_{2}(t) = \int_{t-h(t)}^{t} x^{T}(s)P_{1}(s)x(s)ds
\]

\[
V_{3}(t) = \int_{t-h(t)}^{t} \dot{x}^{T}(s)P_{2}(s)\dot{x}(s)ds
\]

\[
V_{4}(t) = \int_{t-h_{M}}^{t} (s - (t - h_{M}))\dot{x}^{T}(s)P_{3}(s)\dot{x}(s)ds
\]

and \(P_{0}, P_{1}(s), P_{2}(s), P_{3}(s)\) are symmetric positive definite matrices.

The time derivative of Lyapunov-Krasovskii function is

\[
\dot{V}(t) = \dot{V}_{1}(t) + \dot{V}_{2}(t) + \dot{V}_{3}(t) + \dot{V}_{4}(t)
\]

where

\[
\begin{align*}
\dot{V}_{1}(t) &= \dot{x}^{T}(t)P_{0}x(t) + x^{T}(t)P_{0}\dot{x}(t)
\end{align*}
\]

\[
\begin{align*}
\dot{V}_{2}(t) &= \int_{t-h(t)}^{t} \dot{x}^{T}(s)P_{1}(s)x(s)ds
\end{align*}
\]

\[
\begin{align*}
\dot{V}_{3}(t) &= \dot{x}^{T}(t)P_{2}(t)\dot{x}(t)
\end{align*}
\]

\[
\begin{align*}
\dot{V}_{4}(t) &= \int_{t-h_{M}}^{t} \dot{x}^{T}(s)P_{3}(s)\dot{x}(s)ds.
\end{align*}
\]

According to \(0 \leq \dot{h}(t) \leq h_{D}\), we can obtain

\[
\dot{V}_{2}(t) = x^{T}(t)P_{1}(t)x(t) - (1 - h_{D})
\times x^{T}(t - (t - h(t)))P_{1}(t - h(t))x(t - h(t))
\]

\[
\dot{V}_{3}(t) = \dot{x}^{T}(t)P_{2}(t)\dot{x}(t) - (1 - h_{D})
\times \dot{x}^{T}(t - h(t))P_{2}(t - h(t))\dot{x}(t - h(t)).
\]
From Newton-Leibniz formula and the T-S fuzzy neutral system in (1), the following equalities are always hold:

$$\Pi_1 = 2\xi^T(t)S_1(t)[x(t) - x(t - h(t))] - \int_{t-h(t)}^t \dot{x}(s)ds = 0$$  \hspace{1cm} (12)

$$\Pi_2 = 2\xi^T(t)S_2(t)[A(t)x(t) + A_c(t)x(t - h(t)) + A_d(t)\dot{x}(t - h(t)) + B(t)K(t)x(t) - \dot{x}(t)] + B(t)K(t)x(t) - \dot{x}(t)] = 0$$  \hspace{1cm} (13)

$$\Pi_3 = h_M\xi^T(t)S_1(t)P_{4}^{-1}(t)S_{T}^T(t)\xi(t) - \int_{t-h(t)}^t \xi^T(t)S_1(t)P_{4}^{-1}(t)S_{T}^T(t)\xi(t)ds = 0$$  \hspace{1cm} (14)

where \( \xi^T(t) = [x^T(t) \ x^T(t - h(t)) \ \dot{x}^T(t) \ \dot{x}^T(t - h(t))]. \)

From the condition \( 0 \leq h(t) \leq h_M, \) we can obtain the following result

$$\Pi_3 = h_M\xi^T(t)S_1(t)P_{4}^{-1}(t)S_{T}^T(t)\xi(t) - \int_{t-h(t)}^t \xi^T(t)S_1(t)P_{4}^{-1}(t)S_{T}^T(t)\xi(t)ds \leq h_M\xi^T(t)S_1(t)P_{4}^{-1}(t)S_{T}^T(t)\xi(t)$$  \hspace{1cm} (15)

By (9)-(15) with \( P_3(s) \geq P_4(t) \) we can obtain

$$\dot{V}(t) \leq \Lambda(t) + h_M\xi^T(t)S_1(t)P_{4}^{-1}(t)S_{T}^T(t)\xi(t) - \int_{t-h(t)}^t \xi^T(t)S_1(t)P_{4}^{-1}(t)S_{T}^T(t)\xi(t)ds \leq \Lambda(t) + h_M\xi^T(t)S_1(t)P_{4}^{-1}(t)S_{T}^T(t)\xi(t)$$  \hspace{1cm} (16)

where

$$\Lambda(t) = \dot{x}^T(t)P_0x(t) + x^T(t)P_0\dot{x}(t) + \dot{x}^T(t)P_1x(t) - (1 - h_P)x^T(t - h(t))P_1(t - h(t))x(t - h(t)) - (1 - h_P)x^T(t - h(t))P_1(t - h(t))x(t - h(t)) + \dot{x}^T(t)P_2(t)\dot{x}(t) + h_M\xi^T(t)P_3(t)\dot{x}(t) + 2\xi^T(t)S_1(t)[x(t) - x(h(t))] + 2\xi^T(t)S_2(t)[A(t)x(t) + A_c(t)x(t - h(t)) + A_d(t)\dot{x}(t - h(t)) + B(t)K(t)x(t) - \dot{x}(t)].$$

In order for \( \dot{V}(x(t)) < 0 \) for all \( x(t) \neq 0, \) (16) should be negative. By Schur complement and pre- and post-multiplying both sides with \( \text{diag}[X, X, X, X, X] \) and define \( X = S_{21}^T, \ S_{22} = g_1S_{21}, \ S_{23} = g_2S_{21}, \ S_{24} = g_3S_{21}, \) we can get the following result from (16)

$$\xi^T(t)\Xi(t)\xi(t) < 0$$  \hspace{1cm} (17)

where

$$\Xi(t) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} \\ * & * & \Xi_{33} & \Xi_{34} & \Xi_{35} \\ * & * & * & \Xi_{44} & \Xi_{45} \\ * & * & * & * & \Xi_{55} \end{bmatrix}$$

$$\Xi_{11} = \left[ (S_{11}(t) + \ast) + P_1(t) + (A(t)X + \ast) + (B(t)\bar{F}(t) + \ast) \right]$$  \hspace{1cm} (18)

$$\Xi_{12} = \left[ A_c(t)X + g_1X^T A(t)^T + S_{12}^T(t) - \bar{S}_{11}^T(t) + g_1F(t)^TB(t) \right]$$  \hspace{1cm} (19)

$$\Xi_{13} = \left[ g_2X^T A(t)^T + S_{13}^T(t) + \bar{P}_{0}^T - X + g_3F(t)^TB(t) \right]$$  \hspace{1cm} (20)

$$\Xi_{14} = \left[ A_d(t)X + g_3X^T A(t)^T + S_{14}^T(t) + g_3F(t)^TB(t) \right]$$  \hspace{1cm} (21)

$$\Xi_{15} = \left[ S_{15}(t) \right]$$  \hspace{1cm} (22)

$$\Xi_{22} = \left[ (g_1A_c(t)X + \ast) - (S_{12}(t) + \ast) - (1 - h_D)P_1(t - h(t)) \right]$$  \hspace{1cm} (23)

$$\Xi_{23} = \left[ g_2X^T A(t)^T - \bar{S}_{13}^T(t) - g_1X \right]$$  \hspace{1cm} (24)

$$\Xi_{24} = \left[ g_1A_d(t)X + g_3X^T A(t)^T - \bar{S}_{14}^T(t) \right]$$  \hspace{1cm} (25)

$$\Xi_{25} = \left[ S_{12}(t) \right]$$  \hspace{1cm} (26)

$$\Xi_{33} = \left[ P_2(t) + h_M\bar{P}_3(t) - (g_2X + \ast) \right]$$  \hspace{1cm} (27)

$$\Xi_{34} = \left[ g_2A_d(t)X - g_3X \right]$$  \hspace{1cm} (28)

$$\Xi_{35} = \left[ S_{13}(t) \right]$$  \hspace{1cm} (29)

$$\Xi_{44} = \left[ (g_3A_2^T(t)X + \ast) - (1 - h_D)\bar{P}_2(t - h(t)) \right]$$  \hspace{1cm} (30)

$$\Xi_{45} = \left[ S_{14}(t) \right]$$  \hspace{1cm} (31)

$$\Xi_{55} = \left[ (1 - h_M)\bar{P}_3(t) \right]$$  \hspace{1cm} (32)

$$\bar{S}_{11}(t) = X_S S_{11}(t)X, \ S_{12}(t) = X_S S_{12}(t)X, \ P_0(t) = X_P X_0(t)X, \ S_{13}(t) = X_S S_{13}(t)X, \ S_{14}(t) = X_S S_{14}(t)X, \ P_1(t) = X_P X_1(t)X, \ P_2(t) = X_P X_2(t)X, \ P_1(t - h(t)) = X_P X_1(t - h(t))X$$

$$\bar{P}_3(t) = X_P X_3(t)X, \ P_2(t - h(t)) = X_P X_2(t - h(t))X. \hspace{1cm} (33)$$

Clearly, (17) is equivalent to (18)

$$\Phi(t) < 0$$  \hspace{1cm} (18)
\[
\Phi_{15} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [\tilde{S}_{11}(t)]
\]
\[
\Phi_{22} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [(g_1 A_c(t)X + \ast) - (\tilde{S}_{12}(t) + \ast)]
\]
\[
- \left( \sum_{i=1}^{r} \mu_i \right) \left( (1 - h_D) \tilde{P}_1(t - h(t)) \right)
\]
\[
\Phi_{23} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [g_2 X^T A_c(t)^T - \tilde{S}_{15}^T(t)]
\]
\[
- \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [g_1 X]
\]
\[
\Phi_{24} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [g_1 A_d(t)X + g_3 X^T A_c(t)^T - \tilde{S}_{14}^T(t)]
\]
\[
\Phi_{25} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [\tilde{S}_{12}(t)]
\]
\[
\Phi_{33} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [\tilde{P}_2(t) + h_M \tilde{P}_3(t)]
\]
\[
- \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [(g_2 X + \ast)]
\]
\[
\Phi_{34} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [g_2 A_d(t)X]
\]
\[
- \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [g_3 X]
\]
\[
\Phi_{35} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [\tilde{S}_{13}(t)]
\]
\[
\Phi_{44} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [(g_3 A_d(t)X + \ast)]
\]
\[
+ \left( \sum_{i=1}^{r} \mu_i \right) \left( (1 - h_D) \tilde{P}_2(t - h(t)) \right)
\]
\[
\Phi_{45} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [\tilde{S}_{14}(t)]
\]
\[
\Phi_{55} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{-1} [-h_M^{-1} \tilde{P}_3(t)].
\]

By applying Lemma 2 and Lemma 3 to (18), yields

\[
(\sum_{i=1}^{r} \mu_i(t))^{d_a} \left( \sum_{j=1}^{r} \mu_j(t - h(t)) \right)^{d_b} \Phi(t) = \sum_{k' \in K(d_a)} \frac{d_a!}{\pi(k_a')} \sum_{k'' \in K(d_b)} \frac{d_b!}{\pi(k_b')} \Omega < 0
\]
\[
= \sum_{k'_a \in K(d_a)} \sum_{k'_b \in K(d_b)} \Omega^{k_a,k_b} < 0.
\]

Therefore, if (19) is satisfied which implies that \(\Omega < 0\), and the closed-loop T-S fuzzy neutral system is asymptotically stable. This completes the proof of the theorem.

\[\text{IV. NUMERICAL EXAMPLE}\]

In this section, a numerical example is provided to demonstrate the validity and feasibility of the proposed result.

\textbf{Example 1:} Consider the following T-S fuzzy neutral system:

\[
x(t) = \sum_{i=1}^{2} \mu_i(t)[A_i x(t) + A_{ci} x(t-h(t)) + A_{di} \dot{x}(t-h(t))]
\]
\[
+ B_i u(t)
\]
\[
(20)
\]

where

\[
A_1 = \begin{bmatrix} 0.3 & 0.6 \\ 0.8 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0.3 \\ 1 & 0.6 \end{bmatrix},
\]
\[
A_{c1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, A_{c2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix},
\]
\[
A_{d1} = \begin{bmatrix} -0.5 & 1 \\ 0.4 & 0.3 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.3 & 0 \\ 0.7 & -0.2 \end{bmatrix},
\]
\[
B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, h(t) = 0.3 + 0.2cos(t).
\]

By applying the convex optimization problem in Theorem 1 with \(g_1 = 0.32\), \(g_2 = 2.55\), \(g_3 = 0.1\), \(h_M = 0.5\), and \(h_D = 0.2\), the following matrices and controller gain can be obtained:

\[
\begin{align*}
\text{Fig. 1.} & \quad \text{The state response for closed-loop T-S fuzzy neutral systems with initial condition} \ x(0) = [0.5, -0.4].
\end{align*}
\]
Fig. 2. The variation of \(\dot{h}(t)\) and \(h_D\).

\[
P_0 = \begin{bmatrix} 14.1414 & 13.7123 \\ 13.7123 & 13.5008 \end{bmatrix}, \quad X = \begin{bmatrix} 0.4453 & 0.2166 \\ 0.2166 & 0.2209 \end{bmatrix},
\]

\[
P_{11} = \begin{bmatrix} 8.9149 & 8.3762 \\ 8.3762 & 8.1529 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 7.4549 & 7.0032 \\ 7.0032 & 6.9467 \end{bmatrix},
\]

\[
P_{21} = \begin{bmatrix} 0.8856 & 0.3300 \\ 0.3300 & 0.2413 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 1.4665 & 0.7449 \\ 0.7449 & 0.5225 \end{bmatrix},
\]

\[
P_{31} = \begin{bmatrix} 0.7179 & 0.1753 \\ 0.1753 & 0.1872 \end{bmatrix}, \quad P_{32} = \begin{bmatrix} 1.1204 & 0.1160 \\ 0.1160 & 0.0456 \end{bmatrix},
\]

\[
F_{10} = [-1.3569 - 23.2738], \quad F_{01} = [-2.2770 - 22.3444].
\]

The state responses for closed-loop T-S fuzzy neutral system with delay time \(0.3 + 0.2\cos(t)\) and \(x(0) = [0.5, -0.4]\) is shown in Fig. 1. From the simulation results, it can be seen the designed fuzzy controller ensures the asymptotic stability of the closed-loop T-S fuzzy neutral system. One can observe that the states converge to the equilibrium states after some transient times. Fig. 2 shows the variation of \(\dot{h}(t)\) and \(h_D\).

V. CONCLUSIONS

In this paper, a stabilization problem for T-S fuzzy neutral system is investigated. Based on the polynomial technique and some variable transformation, a delay-dependent stabilization condition is proposed for T-S fuzzy neutral system. Furthermore, the results can be formulated in terms of LMI forms. A numerical example is given to illustrate the effectiveness of the proposed methods.

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