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Delay-dependent stabilization condition for T-S fuzzy neutral systems

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Abstract—In this paper, the stabilization problems for a class of Takagi-Sugeno (T-S) fuzzy neutral systems are explored. Utilizing Pólya’s theorem and some homogeneous polynomials techniques, the delay-dependent stabilization condition for T-S fuzzy neutral systems are proposed in terms of a linear matrix inequality (LMI) to guarantee the asymptotic stabilization of T-S fuzzy neutral systems. Lastly, an example is illustrated to demonstrate the effectiveness and applicability of the proposed method.

Keywords—Fuzzy control, Takagi-Sugeno (T-S) fuzzy model, linear matrix inequality (LMI), Pólya’s theorem.

I. INTRODUCTION

During the past decades, fuzzy logic control has been widely adopted to analyze nonlinear systems \cite{1, 2}. In addition to fuzzy logic control, a great deal of effort has been devoted to describe a nonlinear model using Takagi-Sugeno (T-S) fuzzy model. Moreover, many fuzzy modelling approaches \cite{3, 4} which are provided to represent nonlinear models as T-S fuzzy model. Through the modeling procedure, T-S fuzzy model can be described by fuzzy IF-THEN rules, which represents local linear input-output relations of a nonlinear model. In addition to fuzzy modeling scheme, the parallel distributed compensation (PDC) technique \cite{5} is adopted to stabilize the overall T-S fuzzy model. For the stabilization condition analysis, Lyapunov direct method \cite{6, 7} is mainly investigated to yield the stabilization conditions of T-S fuzzy model. Furthermore, many studies utilize linear matrix inequalities (LMIs) to find a feasible solution, if available. Furthermore, the solutions can be found via linear matrix inequalities (LMIs) techniques.

Time-delay phenomenon exists in many practical systems, such as chemical engineering systems, robotic arm systems and network systems \cite{8, 9}. In general, the practical systems with time-delay are more complicated than those systems without time-delays. Besides, time-delay may cause instability and reduce the system performance under some situations; therefore, there has been an increasing interest in the stabilization problem for time-delay systems, and a lot of results on these topics have been explored in the literature \cite{10, 11}.

In addition, time-delay phenomenon exists in both the state and the derivative of the state in neutral systems. For this reason, stabilization problem for neutral systems has been explored in many studies \cite{12, 13}. For example, in \cite{14}, a state matrix decomposition is adopted and a delay-dependent stability condition for fuzzy neutral systems was proposed. A descriptor system approach is adopted for uncertain fuzzy neutral system in \cite{15}. Recently, a polynomial technique has been adopted to reduce the conservatism of the stabilization condition \cite{16}–\cite{18}. Inspired by these works and reference therein, we will explore the delay-dependent stabilization condition for T-S fuzzy neutral systems via polynomial technique.

Following the introduction, the paper is organized as follows. In Section II, a general description of T-S fuzzy neutral system is introduced and the state feedback fuzzy controller is also designed. In Section III, based on homogeneous polynomial technique and Pólya’s theorem, a delay-dependent stabilization conditions for T-S fuzzy neutral system is formulated in terms of LMIs in this section. In Section IV, a numerical example is given to demonstrate the feasibility and effectiveness of the proposed approach. Finally, the conclusions are given in Section V.

Notation: The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote, respectively, the $n$-dimensional Euclidean space and the set of all $n \times m$ real matrices. $A^T$ denotes the transpose of matrix $A$. $X \preceq Y$ or $X < Y$, respectively, where $X$ and $Y$ are symmetric matrices, means that $X - Y$ is negative semi-definite or negative definite, respectively. $I$ is the identity matrix with a compatible dimension (without confusion). $\text{diag}(q_1, \cdots, q_n)$ represents a block-diagonal, $q_1 \cdots q_n$ as the diagonal elements. Denotes a symmetric matrix, where $*$ represents the entries implied by symmetry. The matrices, if not explicitly stated, are assumed to have compatible dimensions. ! denotes factorial for combinatoric expression. Let $K(h)$ be the set of $r$-tuples defined as \cite{19}:

$$K(h) = \{ (k_1, k_2, \cdots, k_r) : k_1 + k_2 + \cdots + k_r = h, \forall k_i \in I^+ (\text{positive integers}), i = 1, 2, \cdots, r \}$$

where $h$ is the total polynomial degree. Since the number of
fuzzy base is $r$, the number of elements in $K(h)$ is expressed by $J(h) = (r + h - 1)! / (h!/(r - 1)!)$. For example, $r = 2$, $h = 3$

$$J(3) = (2 + 3 - 1)! / (3!/(2 - 1)!) = 4$$

$$K(3) = \{ (30), (21), (12), (03) \} = \{ (t(1), t(2), t(3), t(4)) \}$$

For clarity, the following notations are adopted:

$$k = k_1 k_2 \cdots k_r$$

$$\mu_k = \mu_{k_1} \mu_{k_2} \cdots \mu_{k_r}$$

$$e_i = 0 \cdots 1 \cdots 0$$

$$k - e_i = k_1 k_2 \cdots (k_i - 1) \cdots k_r$$

$$\pi(k) = (k_1)! (k_2)! \cdots (k_r)!.$$  

II. PRELIMINARIES

To begin, consider the following T-S fuzzy neutral system:

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(t)[A_i x(t) + A_{ci} x(t - h(t)) + A_{di} \dot{x}(t - h(t))$$

$$+ B_i u(t)]$$

$$= A(t)x(t) + A_c(t)x(t - h(t)) + A_d(t)\dot{x}(t - h(t))$$

$$+ B(t)u(t)$$

$$x(t) = \phi(t), \ \forall t \in [-\max(\Delta t), 0], \ i = 1, \ldots, r$$  \hspace{1cm} (1)

where $r$ is the number of fuzzy rules. The state of system $x(t) \in \mathbb{R}^{n \times 1}$ and the input $u(t) \in \mathbb{R}^{m \times 1}$. The matrices $A_i$, $A_{ci}$, $A_{di} \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$ are system matrices, and initial vector $\phi(t)$ belongs to the set of continuous functions. The time-varying delay $h(t)$ satisfies $0 \leq h(t) \leq h_M$ and $0 \leq h(t) \leq h_D$. $\mu_i(t) = \omega_i(t)/\sum_{j=1}^{r} \omega_i(t) = \prod_{i=1}^{r} M_{ij}(\xi(t))$. $M_{ij}$ is the membership degree of $\xi(t)$, and $\omega_i(t) \geq 0$ for all $t$, $i = 1, \ldots, r$. It is clear that $\mu_i(t) \geq 0$, and $\sum_{i=1}^{r} \mu_i(t) = 1$.

The state feedback fuzzy controller for T-S fuzzy neutral system (1) is represented as follows.

$$u(t) = \sum_{k \in K(r-1), r \geq 2} \mu_k F_k x(t)$$

$$= F(t)x(t)$$  \hspace{1cm} (2)

By substituting (2) into (1), the closed-loop system can be obtained as (3).

$$\dot{x}(t) = (A(t) + B(t)F(t))x(t) + A_c(t)x(t - h(t))$$

$$+ A_d(t)\dot{x}(t - h(t))$$

$$+ B(t)u(t)$$

$$x(t) = \phi(t), \ \forall t \in [-\max(\Delta t), 0], \ i = 1, \ldots, r$$  \hspace{1cm} (3)

III. MAIN RESULTS

Before discussing the proof of the theorems, here are some lemmas which are used in the proof.

Lemma 1: [20] For any positive symmetric constant matrix $R_1 \in \mathbb{R}^{n \times n}$ and a scalar $\mu > 0$, if there exists a vector function $\dot{x}(s) : [0, h_M] \rightarrow \mathbb{R}^{n}$ such that the integrals $\int_{t-h_M}^{t} \dot{x}(s)R_1 x'(s)ds$ and $\int_{t-h_M}^{t} \dot{x}(s)x'(s)ds$ are well defined, then the following inequality holds:

$$-h_M \int_{t-h_M}^{t} \dot{x}^T(s)R_1 \dot{x}(s)ds \leq -h(t) \int_{t-h(t)}^{t} \dot{x}^T(s)R_1 x'(s)ds$$

$$\leq - \left( \int_{t-h(t)}^{t} \dot{x}(s)ds \right)^T R_1 \left( \int_{t-h(t)}^{t} \dot{x}(s)ds \right).$$

Lemma 2: [21] (Pólya’s theorem) For a positive integer $r$, $\{ \Delta_r; (\mu_1, \ldots, \mu_r) \mid \sum_{i=1}^{r} \mu_i = 1 \}$. If a real homogeneous polynomial $F(\mu_1, \ldots, \mu_r)$ is positive definite, then for a sufficiently large $d$, all the coefficients of $F(\mu_1 + \cdots + \mu_r, d)$ are positive.

Lemma 3: [16] Consider the T-S fuzzy system,

$$\dot{x}(t) = \sum_{i=1}^{r} \mu_i(t)[A_i x(t) + B_i u(t)]$$

$$+ (B(t)G_k x(t) - e_i)$$

where $x(t)$ is the state of system $u(t)$ is the control input, $A(t) = \sum_{i=1}^{r} \mu_i(t)\xi_i(t)A_i, B(t) = \sum_{i=1}^{r} \mu_i(t)\xi_i(t)B_i$. The T-S fuzzy system (4) is asymptotically stabilizable via the state feedback controller $G_k = G_k X^{-1}$ if and only if there exist a symmetric positive definite matrix $X > 0$ and $d \in \mathbb{N}$ such that:

$$\sum_{k' \in K(d)} \sum_{i=1}^{r} \frac{d_i}{\pi(k')} \left( \frac{k_i - k_i'}{\pi(k - k')^2} (A_i X + \star) + (B(t)G_k - e_i, \star) \right) < 0$$

where $k \in K(d + r), k > k'$, symbol $\star$ is the componentwise.

Before discussing the following stability conditions, we first define the following function:

$$P_{1i}(t) = \sum_{i=1}^{r} \mu_i(t)P_{1i}, \ P_{2i}(t) = \sum_{i=1}^{r} \mu_i(t)P_{2i}$$

$$P_{3i}(t) = \sum_{i=1}^{r} \mu_i(t)P_{3i}, \ S_1(t) = \sum_{i=1}^{r} \mu_i(t)S_1i$$

$$P_{4i}(t) = \sum_{j=1}^{r} \mu_j(t)P_{4j}.$$

The main result on the asymptotic stability of the T-S fuzzy neutral system (1) is propounded in the following theorem.

Theorem 1: If there exist integers $d_a > 0$, $d_b > 0$, some positive matrices $P_{ia}, P_{ib}, P_{ia}, P_{4j} \in \mathbb{R}^{n \times n}$, and matrix $S_{ij} = \left[ S_{ij}^{(1)}, S_{ij}^{(2)}, S_{ij}^{(3)}, S_{ij}^{(4)} \right], S_{ij}^{(1)}, \ldots, S_{ij}^{(4)} \in \mathbb{R}^{n \times n}$ such that the following inequalities (6) and (7) are satisfied for some positive scalar $g_i, g_2, g_3$ and $\Omega(t) = h(t) \leq h_M, 0 \leq h(t) \leq h_D$ then the fuzzy neutral system (1) is asymptotically stabilizable via the state feedback controller $F_k = F_k X^{-1} \in \mathbb{R}^{n \times n}$.

$$\Omega < 0$$

$$P_{3i} < P_{a_k}$$

where

$$\Omega = diag \left[ \Omega_{1a}, \Omega_{2a}, \ldots, \Omega_{a_t(J+d+2)}, \Omega_{a_t(J+d+1)} \right]$$

$$\Omega_{k_a, k_b} = \sum_{k_a \in K(d_a)} \sum_{k_b \in K(d_b)} \frac{d_{a_k}}{\pi(k_a)} \frac{d_{b_k}}{\pi(k_b)}$$

$k_a \in K(d_a + 2), k_b \in K(d_b + 1), k_a > k_a^{'}, k_b > k_b^{'}$.
\[
\hat{\Omega} = \begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\
\Omega_{21} & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} \\
\ast & \ast & \Omega_{33} & \Omega_{34} & \Omega_{35} \\
\ast & \ast & \ast & \Omega_{44} & \Omega_{45} \\
\ast & \ast & \ast & \ast & \Omega_{55}
\end{bmatrix}
\]

\[
\Omega_{11} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [(S_{11i} + \ast) + P_{1i}]
+ (A_1X + \ast)] + \sum_{i=1}^{r} \frac{1}{\pi(k_b - k'_b)} [(B_i \bar{F} \kappa_{a' \rightarrow e_i} + \ast)]
\]

\[
\Omega_{12} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [g_1X^T A_i^T
+ \bar{S}_{12i} + A_{ci} X - \bar{S}^T_{11i}]
+ \sum_{i=1}^{r} \frac{1}{\pi(k_b - k'_b)} [g_1 \bar{F} \kappa_{a' \rightarrow e_i} B_i^T]
\]

\[
\Omega_{13} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [g_2X^T A_i^T
+ \bar{S}^T_{13i}] + \sum_{i=1}^{r} \frac{1}{\pi(k_b - k'_b)} [g_2 \bar{F} \kappa_{a' \rightarrow e_i} B_i^T]
+ \sum_{i=1}^{r} \frac{1}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [\bar{P}_0^T - X]
\]

\[
\Omega_{14} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [A_{di} X + \bar{S}^T_{14i}]
+ g_3X^T A_i^T] + \sum_{i=1}^{r} \frac{1}{\pi(k_b - k'_b)} [g_3 \bar{F} \kappa_{a' \rightarrow e_i} B_i^T]
\]

\[
\Omega_{15} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [(g_1 A_{ci} X + \ast)
- (\bar{S}_{12i} + \ast)]
+ \sum_{j=1}^{r} \frac{1}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [(-1 + h_D) \bar{P}_{ij}]]
\]

\[
\Omega_{23} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [g_2X^T A_i^T
- \bar{S}^T_{13i}] + \sum_{i=1}^{r} \frac{1}{\pi(k_b - k'_b)} \frac{1}{\pi(k_a - k'_a)} [\bar{S}^T_{12i}]
\]

\[
\Omega_{25} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [P_{2i} + h_M \bar{P}_{3i}]
+ \sum_{i=1}^{r} \frac{1}{\pi(k_b - k'_b)} \frac{1}{\pi(k_a - k'_a)} [(-g_2X + \ast)]
\]

\[
\Omega_{34} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [g_2 A_{di} X]
+ \sum_{i=1}^{r} \frac{1}{\pi(k_b - k'_b)} \frac{1}{\pi(k_a - k'_a)} [-g_3X]
\]

\[
\Omega_{35} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [g_3 A_{di} X + \ast]
+ \sum_{j=1}^{r} \frac{1}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [-1 + h_D \bar{P}_{2j}]
\]

\[
\Omega_{44} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [g_3 A_{di} X + \ast]
+ \sum_{j=1}^{r} \frac{1}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [-h_M \bar{P}_{3j}]
\]

\[
\Omega_{55} = \sum_{i=1}^{r} \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1}{\pi(k_b - k'_b)} [-h_M \bar{P}_{3j}]
\]

**Proof:** Firstly, let us consider the Lyapunov-Krasovskii function

\[
V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t)
\]

where

\[
V_1(t) = x^T(t) P_0 x(t)
\]

\[
V_2(t) = \int_{t-h(t)}^{t} x^T(s) P_1(s) x(s) ds
\]

\[
V_3(t) = \int_{t-h(t)}^{t} \dot{x}^T(s) P_2(s) \dot{x}(s) ds
\]

\[
V_4(t) = \int_{t-h_M}^{t} (s - (t - h_M)) \dot{x}^T(s) P_3(s) \dot{x}(s) ds
\]

and \(P_0, P_1, P_2, P_3\) are symmetric positive definite matrices.

The time derivative of Lyapunov-Krasovskii function is

\[
\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t)
\]

where

\[
\dot{V}_1(t) = \dot{x}^T(t) P_0 x(t) + x^T(t) P_0 \dot{x}(t)
\]

\[
\dot{V}_2(t) = \dot{x}^T(t) P_1(t) x(t)
- (1 - \hat{h}(t)) x^T(t - h(t)) P_1(t - h(t)) x(t - h(t))
\]

\[
\dot{V}_3(t) = \dot{x}^T(t) P_2(t) \dot{x}(t)
- (1 - \hat{h}(t)) \dot{x}^T(t - h(t)) P_2(t - h(t)) \dot{x}(t - h(t))
\]

\[
\dot{V}_4(t) = h_M \dot{x}^T(t) P_3(t) \dot{x}(t) - \int_{t-h_M}^{t} \dot{x}^T(s) P_3(s) \dot{x}(s) ds.
\]

According to \(0 \leq \hat{h}(t) \leq h_D\), we can obtain

\[
\dot{V}_2(t) = x^T(t) P_1(t) x(t) - (1 - h_D)
\times x^T(t - h(t)) P_1(t - h(t)) x(t - h(t))
\]

\[
\dot{V}_3(t) = \dot{x}^T(t) P_2(t) \dot{x}(t) - (1 - h_D)
\times \dot{x}^T(t - h(t)) P_2(t - h(t)) \dot{x}(t - h(t)).
\]
From Newton-Leibniz formula and the T-S fuzzy neutral system in (1), the following equalities are always hold:

$$\Pi_1 = 2\xi^T(t)S_1(t)[x(t) - x(t - h(t))] - \int_{t-h(t)}^t \dot{x}(s)ds = 0$$  \hspace{1cm} (12)

$$\Pi_2 = 2\xi^T(t)S_2(t)[A(t)x(t) + A_c(t)x(t - h(t))] + A_d(t)\dot{x}(t - h(t)) + B(t)K(t)x(t) - \dot{x}(t) = 0$$  \hspace{1cm} (13)

$$\Pi_3 = h_M\xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)$$

$$- \int_{t-h(t)}^t \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)ds = 0$$  \hspace{1cm} (14)

where \(\xi^T(t) = [x^T(t) \ x^T(t-h(t)) \ \dot{x}^T(t) \ \dot{x}^T(t-h(t))]\).

From the condition \(0 \leq h(t) \leq h_M\), we can obtain the following result

$$\Pi_3 = h_M\xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)$$

$$- \int_{t-h(t)}^t \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)ds \leq h_M\xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)$$

$$- \int_{t-h(t)}^t \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)ds$$

$$\leq \Delta(t) + h_M\xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)$$  \hspace{1cm} (16)

where

$$\Delta(t) = \dot{x}^T(t)P_0x(t) + \dot{x}^T(t)P_0\dot{x}(t) + x^T(t)P_1(t)x(t)$$

$$+ \int_{t-h(t)}^t [\dot{x}^T(t)P_3(s)x(s) + x^T(t)S_1^T(t)\xi(t)]ds$$

By (9)-(15) with \(P_3(s) \geq P_2(t)\) we can obtain

$$\tilde{V}(t) \leq \Delta(t) + h_M\xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)$$

$$- \int_{t-h(t)}^t [\dot{x}^T(s)P_3(s) + \xi^T(t)S_1(t)]x(t)$$

$$\dot{\xi}(t) = \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)$$

In order for \(\tilde{V}(x(t)) < 0\) for all \(x(t) \neq 0\), (16) should be negative. By Schur complement and pre- and post-multiplying both sides with \(\text{diag}[X, X, X, X, X]\) and define \(X = S_{21}^T\), \(S_{22} = g_1S_{21}, S_{23} = g_2S_{21}, S_{24} = g_3S_{21}\), we can get the following result from (16)

$$\xi^T(t)\Xi(t)\xi(t) < 0$$  \hspace{1cm} (17)

where

$$\Xi(t) = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\
* & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} \\
* & * & \Xi_{33} & \Xi_{34} & \Xi_{35} \\
* & * & * & \Xi_{44} & \Xi_{45} \\
* & * & * & * & \Xi_{55}
\end{bmatrix}$$

$$\Xi_{11} = [(S_{11}(t) + \alpha) + \dot{P}_1(t) + (A(t)X + \alpha) + (B(t)\dot{F}(t) + \alpha)]$$

$$\Xi_{12} = [A_c(t)X + g_1X^TA(t)^T + S_{12}(t) - \dot{S}^T_{11}(t)]$$

$$\Xi_{13} = [g_2X^TA(t)^T + S_{13}(t) + \dot{P}_6(t) - X + g_3F^T(t)B(t)^T]$$

$$\Xi_{14} = [A_d(t)X + g_3X^TA(t)^T + S_{14}(t) + g_3F^T(t)B(t)^T]$$

$$\Xi_{15} = [\dot{S}_{12}(t)]$$

$$\Xi_{22} = [(g_1A_c(t)X + \alpha) - (S_{12}(t) + \alpha) - (1 - h_D)\dot{P}_1(t) - h_D(t)]$$

$$\Xi_{23} = [g_2X^TA_c(t)^T - \dot{S}_{13}(t) - g_3X]$$

$$\Xi_{24} = [g_1A_d(t)X + g_3X^TA_c(t)^T - \dot{S}_{14}(t)]$$

$$\Xi_{25} = [\dot{S}_{12}(t)]$$

$$\Xi_{33} = [\dot{P}_2(t) + h_M\dot{P}_3(t) - (g_2X + \alpha)]$$

$$\Xi_{34} = [g_2A_d(t)X - g_3X]$$

$$\Xi_{35} = [\dot{S}_{15}(t)]$$

$$\Xi_{44} = [(g_3A_c(t)X + \alpha) - (1 - h_D)\dot{P}_2(t) - h_D(t)]$$

$$\Xi_{45} = [\dot{S}_{14}(t)]$$

$$\Xi_{55} = [-h_M\dot{P}_3(t)]$$

Clearly, (17) is equivalent to (18)

$$\Phi(t) < 0$$  \hspace{1cm} (18)

where

$$\Phi(t) = \begin{bmatrix}
\phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} \\
* & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} \\
* & * & \phi_{33} & \phi_{34} & \phi_{35} \\
* & * & * & \phi_{44} & \phi_{45} \\
* & * & * & * & \phi_{55}
\end{bmatrix}$$

$$\phi_{11} = \sum_{j=1}^{r} \mu_j [(B(t)\dot{F}(t) + \alpha)] + \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)\tau^{-1} \dot{P}_1(t)$$

$$+ (S_{11}(t) + \alpha) + (A(t)X + \alpha)$$

$$\phi_{12} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)\tau^{-1} [A_c(t)X + g_1X^TA(t)^T]$$

$$+ \dot{S}_{12}(t) - \dot{S}_{12}(t) + \sum_{j=1}^{r} \mu_j [g_1F^T(t)B(t)^T]$$

$$\phi_{13} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)\tau^{-1} [g_2X^TA(t)^T + \dot{S}_{13}(t)]$$

$$+ \sum_{j=1}^{r} \mu_j [g_2F^T(t)B(t)^T] + \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)\tau^{-1} \dot{P}_6(t)$$

$$\phi_{14} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)\tau^{-1} [A_d(t)X + g_3X^TA(t)^T + \dot{S}_{14}(t)]$$

$$+ \sum_{j=1}^{r} \mu_j [g_3F^T(t)B(t)^T]$$
\[
\Phi_{15} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{r-1} [\tilde{S}_{11}(t)] \\
\Phi_{22} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{r-1} \left[ (g_1 A_c(t)X + \ast) - (\tilde{S}_{12}(t) + \ast) \right] \\
- \left( \sum_{i=1}^{r} \mu_i \right)^r [(1 - h_D) \tilde{P}_{1}(t - h(t))] \\
\Phi_{23} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{r-1} [g_2 X^T A_c(t)^T - \tilde{S}_{14}^T(t)] \\
- \left( \sum_{i=1}^{r} \mu_i \right)^r [g_1 X] \\
\Phi_{24} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{r-1} [g_1 A_d(t)X + g_3 X^T A_c(t)^T - \tilde{S}_{14}^T(t)] \\
\Phi_{25} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{r-1} [\tilde{S}_{12}(t)] \\
\Phi_{33} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{r-1} [\tilde{P}_{2}(t) + h_M \tilde{P}_3(t)] \\
- \left( \sum_{i=1}^{r} \mu_i \right)^r [g_2 X + \ast] \\
\Phi_{34} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{r-1} [g_2 A_d(t)X] \\
- \left( \sum_{i=1}^{r} \mu_i \right)^r [g_3 X] \\
\Phi_{35} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{r-1} [\tilde{S}_{13}(t)] \\
\Phi_{44} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{r-1} [(g_3 A_d(t)X + \ast)] \\
+ \left( \sum_{i=1}^{r} \mu_i \right)^r [(1 - h_D) \tilde{P}_{2}(t - h(t))] \\
\Phi_{45} = \sum_{j=1}^{r} \mu_j \left( \sum_{i=1}^{r} \mu_i \right)^{r-1} [-h_M^{-1} \tilde{P}_3(t)].
\]

By applying Lemma 2 and Lemma 3 to (18), yields
\[
(\sum_{i=1}^{r} \mu_i(t))^{d_a} (\sum_{j=1}^{r} \mu_j(t - h(t)))^{d_b} \Phi(t) \\
= \sum_{k' \in K(d_a)} \frac{d_a!}{\pi(k'_a)} \sum_{k'' \in K(d_b)} \frac{d_b!}{\pi(k''_b)} \Omega < 0 \\
= \sum_{k'_a \in K(d_a)} \sum_{k''_b \in K(d_b)} \Omega^{k_a, k_b} < 0.
\]

Therefore, if (19) is satisfied which implies that $\Omega < 0$, and the closed-loop T-S fuzzy neutral system is asymptotically stable. This completes the proof of the theorem.

**IV. Numerical Example**

In this section, a numerical example is provided to demonstrate the validity and feasibility of the proposed result.

**Example 1:** Consider the following T-S fuzzy neutral system:
\[
\dot{x}(t) = \sum_{i=1}^{2} \mu_i(t)[A_i x(t) + A_{ci} x(t - h(t)) + A_{di} \dot{x}(t - h(t)) + B_i u(t)]
\]

where
\[
A_1 = \begin{bmatrix} 0.3 & 0.6 \\ 0.8 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0.3 \\ 1 & 0.6 \end{bmatrix},
\]
\[
A_{c1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, \quad A_{c2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix},
\]
\[
A_{d1} = \begin{bmatrix} -0.5 & 1 \\ 0.4 & 0.3 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.3 & 0 \\ 0.7 & -0.2 \end{bmatrix},
\]
\[
B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad h(t) = 0.3 + 0.2 \cos(t).
\]

By applying the convex optimization problem in Theorem 1 with $g_1 = 0.32$, $g_2 = 2.55$, $g_3 = 0.1$, $h_M = 0.5$, and $h_D = 0.2$, the following matrices and controller gain can be obtained:

![Fig. 1. The state response for closed-loop T-S fuzzy neutral systems with initial condition $x(0) = [0.5, -0.4]$.](image)
The state responses for closed-loop T-S fuzzy neutral system is investigated. Based on the polynomial technique and some variable transformation, a delay-dependent stabilization condition is proposed for T-S fuzzy neutral system. Further-
more, the results can be formulated in terms of LMI forms.

![Graph of h(t) and h_D](image)

The state responses for closed-loop T-S fuzzy neutral system with delay time $0.3 + 0.2 \cos(t)$ and $x(0) = [0.5, -0.4]$ is shown in Fig. 1. From the simulation results, it can be seen that the designed fuzzy controller ensures the asymptotic stability of the closed-loop T-S fuzzy neutral system. One can observe that the states converge to the equilibrium states after some transient times. Fig. 2 shows the variation of $h(t)$ and $h_D$.

V. Conclusions

In this paper, a stabilization problem for T-S fuzzy neutral system is investigated. Based on the polynomial technique and some variable transformation, a delay-dependent stabilization condition is proposed for T-S fuzzy neutral system. Furthermore, the results can be formulated in terms of LMI forms. A numerical example is given to illustrate the effectiveness of the proposed methods.

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