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Delay-dependent stabilization condition for T-S fuzzy neutral systems

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Abstract—In this paper, the stabilization problems for a class of Takagi-Sugeno (T-S) fuzzy neutral systems are explored. Utilizing Pólya's theorem and some homogeneous polynomials techniques, the delay-dependent stabilization condition for T-S fuzzy neutral systems are proposed in terms of a linear matrix inequality (LMI) to guarantee the asymptotic stabilization of T-S fuzzy neutral systems. Lastly, an example is illustrated to demonstrate the effectiveness and applicability of the proposed method.

Keywords—Fuzzy control, Takagi-Sugeno (T-S) fuzzy model, linear matrix inequality (LMI), Pólya's theorem.

I. INTRODUCTION

During the past decades, fuzzy logic control has been widely adopted to analyze nonlinear systems [1], [2]. In addition to fuzzy logic control, a great deal of effort has been devoted to describe a nonlinear model using Takagi-Sugeno (T-S) fuzzy model. Moreover, many fuzzy modelling approaches [3], [4] which are provided to represent nonlinear models as T-S fuzzy model. Through the modeling procedure, T-S fuzzy model can be described by fuzzy IF-THEN rules, which represents local linear input-output relations of a nonlinear model. In addition to fuzzy modeling scheme, the parallel distributed compensation (PDC) technique [5] is adopted to stabilize the overall T-S fuzzy model. For the stabilization condition analysis, Lyapunov direct method [6], [7] is mainly investigated to yield the stabilization conditions of T-S fuzzy model. Furthermore, many studies utilize linear matrix inequalities (LMIs) to find a feasible solution, if available. Furthermore, the solutions can be found via linear matrix inequalities (LMIs) techniques.

Time-delay phenomenon exists in many practical systems, such as chemical engineering systems, robotic arm systems and network systems [8], [9]. In general, the practical systems with time-delay are more complicated than those systems without time-delays. Besides, time-delay may cause instability and reduce the system performance under some situations; therefore, there has been an increasing interest in the stabilization problem for time-delay systems, and a lot of results on these topics have been explored in the literature [10], [11].

In addition, time-delay phenomenon exists in both the state and the derivative of the state in neutral systems. For this reason, stabilization problem for neutral systems has been explored in many studies [12], [13]. For example, in [14], a state matrix decomposition is adopted and a delay-dependent stability condition for fuzzy neutral systems was proposed. A descriptor system approach is adopted for uncertain fuzzy neutral system in [15]. Recently, a polynomial technique has been adopted to reduce the conservatism of the stabilization condition [16]–[18]. Inspired by these works and reference therein, we will explore the delay-dependent stabilization condition for T-S fuzzy neutral systems via polynomial technique.

Following the introduction, the paper is organized as follows. In Section II, a general description of T-S fuzzy neutral system is introduced and the state feedback fuzzy controller is also designed. In Section III, based on homogeneous polynomial technique and Pólya's theorem, a delay-dependent stabilization conditions for T-S fuzzy neutral system is formulated in terms of LMIs in this section. In Section IV, a numerical example is given to demonstrate the feasibility and effectiveness of the proposed approach. Finally, the conclusions are given in Section V.

Notation: The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ denote, respectively, the n -dimensional Euclidean space and the set of all $n \times m$ real matrices. A^T denotes the transpose of matrix A . $X \leq Y$ or $X < Y$, respectively, where X and Y are symmetric matrices, means that $X - Y$ is negative semi-definite or negative definite, respectively. I is the identity matrix with a compatible dimension (without confusion). $\text{diag}[q_1, \dots, q_n]$ represents a block-diagonal, $q_1 \cdots q_n$ as the diagonal elements. Denotes a symmetric matrix, where $\star$ represents the entries implied by symmetry. The matrices, if not explicitly stated, are assumed to have compatible dimensions. $!$ denotes factorial for combinatoric expression. Let $K(h)$ be the set of r -tuples defined as [19]:

$$K(h) = \{ (k_1 k_2 \cdots k_r) : k_1 + k_2 + \cdots + k_r = h, \\ \forall k_i \in I^+ (\text{positive integers}), i = 1, 2, \dots, r \}$$

where h is the total polynomial degree. Since the number of

fuzzy base is r , the number of elements in $K(h)$ is expressed by $J(h) = (r + h - 1)! / (h!(r - 1)!)$. For example, $r = 2$, $h = 3$

$$\begin{aligned} J(3) &= (2 + 3 - 1)! / (3!(2 - 1)!) = 4 \\ K(3) &= \{ (30), (21), (12), (03) \} \\ &= \{ t(1), t(2), t(3), t(4) \} \end{aligned}$$

For clarity, the following notations are adopted:

$$\begin{aligned} k &= k_1 k_2 \cdots k_r \\ \mu^k &= \mu_1^{k_1} \mu_2^{k_2} \cdots \mu_r^{k_r} \\ e_i &= 0 \cdots \underbrace{1}_{i^{th}} \cdots 0 \\ k - e_i &= k_1 k_2 \cdots (k_i - 1) \cdots k_r \\ \pi(k) &= (k_1!) (k_2!) \cdots (k_r!). \end{aligned}$$

II. PRELIMINARIES

To begin, consider the following T-S fuzzy neutral system:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \mu_i(t) [A_i x(t) + A_{ci} x(t - h(t)) + A_{di} \dot{x}(t - h(t)) \\ &\quad + B_i u(t)] \\ &= A(t)x(t) + A_c(t)x(t - h(t)) + A_d(t)\dot{x}(t - h(t)) \\ &\quad + B(t)u(t) \\ x(t) &= \phi(t), \forall t \in [-\max\{h_M\}, 0], i = 1, \dots, r \end{aligned} \quad (1)$$

where r is the number of fuzzy rules. The state of system $x(t) \in \mathbb{R}^{n \times 1}$ and the input $u(t) \in \mathbb{R}^{m \times 1}$. The matrices A_i , A_{ci} , $A_{di} \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$ are system matrices, and initial vector $\phi(t)$ belongs to the set of continuous functions. The time-varying delay $h(t)$ satisfies $0 \leq h(t) \leq h_M$ and $0 \leq \dot{h}(t) \leq h_D$. $\mu_i(t) = \omega_i(t) / \sum_{i=1}^r \omega_i(t)$, $\omega_i(t) = \prod_{j=1}^p M_{ij}(\xi(t))$. M_{ij} is the membership degree of $\xi(t)$, and $\omega_i(t) \geq 0$ for all t , $i = 1, \dots, r$. It is clear that $\mu_i(t) \geq 0$, and $\sum_{i=1}^r \mu_i(t) = 1$.

The state feedback fuzzy controller for T-S fuzzy neutral system (1) is represented as follows.

$$\begin{aligned} u(t) &= \sum_{k \in K(r-1), r \geq 2} \mu^k F_k x(t) \\ &= F(t)x(t) \end{aligned} \quad (2)$$

By substituting (2) into (1), the closed-loop system can be obtained as (3).

$$\begin{aligned} \dot{x}(t) &= (A(t) + B(t)F(t))x(t) + A_c(t)x(t - h(t)) \\ &\quad + A_d(t)\dot{x}(t - h(t)) \end{aligned} \quad (3)$$

III. MAIN RESULTS

Before discussing the proof of the theorems, here are some lemmas which are used in the proof.

Lemma 1: [20] For any positive symmetric constant matrix $R_1 \in \mathbb{R}^{n \times n}$ and a scalar $h_M > 0$, if there exists a vector function $\dot{x}(s) : [0, h_M] \rightarrow \mathbb{R}^n$ such that the integrals $\int_{t-h(t)}^t \dot{x}(s) R_1 \dot{x}(s) ds$ and $\int_{t-h(t)}^t \dot{x}(s) ds$ are well defined, then the following inequality holds:

$$-h_M \int_{t-h_M}^t \dot{x}^T(s) R_1 \dot{x}(s) ds \leq -h(t) \int_{t-h(t)}^t \dot{x}^T(s) R_1 \dot{x}(s) ds$$

$$\leq - \left(\int_{t-h(t)}^t \dot{x}(s) ds \right)^T R_1 \left(\int_{t-h(t)}^t \dot{x}(s) ds \right).$$

Lemma 2: [21] (Pólya's theorem) For a positive integer r , $\{\Delta_r: (\mu_1, \dots, \mu_r) \mid \mu_i \geq 0, \sum_{i=1}^r \mu_i = 1\}$. If a real homogeneous polynomial $F(\mu_1, \dots, \mu_r)$ is positive definite, then for a sufficiently large d , all the coefficients of

$$(\mu_1 + \dots + \mu_r)^d F(\mu_1, \dots, \mu_r)$$

are positive.

Lemma 3: [16] Consider the T-S fuzzy system,

$$\dot{x}(t) = \sum_{i=1}^r \mu_i(t) [A_i x(t) + B_i u(t)] \quad (4)$$

where $x(t)$ is the state of system, $u(t)$ is the control input, $A(t) = \sum_{i=1}^r \mu_i(\xi(t)) A_i$, $B(t) = \sum_{i=1}^r \mu_i(\xi(t)) B_i$. The T-S fuzzy system (4) is asymptotically stabilizable via the state feedback controller $G_k = \bar{G}_k X^{-1}$ if and only if there exist a symmetric positive definite matrix $X > 0$ and $d \in \mathbb{N}$ such that:

$$\begin{aligned} \sum_{k' \in K(d)} \sum_{i=1}^r \frac{d!}{\pi(k')} \left(\frac{(k_i - k'_i)}{\pi(k - k')} (A_i X + \star) \right. \\ \left. + (B_i G_{k-k'-e_i} + \star) \right) < 0 \end{aligned} \quad (5)$$

where $k \in K(d + r)$, $k \succ k'$, symbol $\succ$ is the componentwise.

Before discussing the following stability conditions, we first define the following function:

$$\begin{aligned} P_1(t) &= \sum_{i=1}^r \mu_i(t) P_{1i}, \quad P_2(t) = \sum_{i=1}^r \mu_i(t) P_{2i} \\ P_3(t) &= \sum_{i=1}^r \mu_i(t) P_{3i}, \quad S_1(t) = \sum_{i=1}^r \mu_i(t) S_{1i} \\ P_4(t) &= \sum_{j=1}^r \mu_j(t) P_{4j}. \end{aligned}$$

The main result on the asymptotic stability of the T-S fuzzy neutral system (1) is propounded in the following theorem.

Theorem 1: If there exist integers $d_a > 0$, $d_b > 0$, some positive matrices $\bar{P}_0, \bar{P}_{1i}, \bar{P}_{2i}, \bar{P}_{3i}, \bar{P}_{4j} \in \mathbb{R}^{n \times n}$, and matrix $\bar{S}_{1i}^T = [\bar{S}_{11i}^T, \bar{S}_{12i}^T, \bar{S}_{13i}^T, \bar{S}_{14i}^T], \bar{S}_{11i}^T, \dots, \bar{S}_{14i}^T \in \mathbb{R}^{n \times n}$ such that the following inequalities (6) and (7) are satisfied for some positive scalar g_1, g_2, g_3 and $0 \leq h(t) \leq h_M$, $0 \leq \dot{h}(t) \leq h_D$ then the fuzzy neutral system (1) is asymptotically stabilizable via the state feedback controller $F_k = \bar{F}_k X^{-1}$, $X \in \mathbb{R}^{n \times n}$.

$$\Omega < 0 \quad (6)$$

$$P_{3i} < P_{4j} \quad (7)$$

where

$$\Omega = \text{diag} \left[\Omega^{t_a(1), t_b(1)}, \dots, \Omega^{t_a(J(d+2)), t_b(J(d+1))} \right]$$

$$\Omega^{k_a, k_b} = \sum_{k'_a \in K(d_a)} \frac{d_a!}{\pi(k'_a)} \sum_{k'_b \in K(d_b)} \frac{d_b!}{\pi(k'_b)} \bar{\Omega}$$

$$k_a \in K(d_a + 2), k_b \in K(d_b + 1), k_a \succ k'_a, k_b \succ k'_b$$

$$\begin{aligned}
\bar{\Omega} &= \begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13} & \bar{\Omega}_{14} & \bar{\Omega}_{15} \\ \star & \bar{\Omega}_{22} & \bar{\Omega}_{23} & \bar{\Omega}_{24} & \bar{\Omega}_{25} \\ \star & \star & \bar{\Omega}_{33} & \bar{\Omega}_{34} & \bar{\Omega}_{35} \\ \star & \star & \star & \bar{\Omega}_{44} & \bar{\Omega}_{45} \\ \star & \star & \star & \star & \bar{\Omega}_{55} \end{bmatrix} \\
\bar{\Omega}_{11} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [(\bar{S}_{11i} + \star) + \bar{P}_{1i} \\
&\quad + (A_i X + \star)] + \sum_{i=1}^r \frac{1!}{\pi(k_b - k'_b)} [(B_i \bar{F}_{k_a - k'_a - e_i} + \star)] \\
\bar{\Omega}_{12} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [g_1 X^T A_i^T \\
&\quad + \bar{S}_{12i}^T + A_{ci} X - \bar{S}_{11i}^T] \\
&\quad + \sum_{i=1}^r \frac{1!}{\pi(k_b - k'_b)} [g_1 \bar{F}_{k_a - k'_a - e_i}^T B_i^T] \\
\bar{\Omega}_{13} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [g_2 X^T A_i^T \\
&\quad + \bar{S}_{13i}^T] + \sum_{i=1}^r \frac{1!}{\pi(k_b - k'_b)} [g_2 \bar{F}_{k_a - k'_a - e_i}^T B_i^T] \\
&\quad + \sum_{i=1}^r \frac{r!}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [\bar{P}_0^T - X] \\
\bar{\Omega}_{14} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [A_{di} X + \bar{S}_{14i}^T \\
&\quad + g_3 X^T A_i^T] + \sum_{i=1}^r \frac{1!}{\pi(k_b - k'_b)} [g_3 \bar{F}_{k_a - k'_a - e_i}^T B_i^T] \\
\bar{\Omega}_{15} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [\bar{S}_{11i}] \\
\bar{\Omega}_{22} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [(g_1 A_{ci} X + \star) \\
&\quad - (\bar{S}_{12i} + \star)] \\
&\quad + \sum_{j=1}^r \frac{r!}{\pi(k_a - k'_a)} [-(1 - h_D) \bar{P}_{1j}] \\
\bar{\Omega}_{23} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [g_2 X^T A_{ci}^T \\
&\quad - \bar{S}_{13i}^T] + \sum_{i=1}^r \frac{r!}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [-g_1 X] \\
\bar{\Omega}_{24} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [g_3 X^T A_{ci}^T \\
&\quad + g_1 A_{di} X - \bar{S}_{14i}^T] \\
\bar{\Omega}_{25} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [\bar{S}_{12i}] \\
\bar{\Omega}_{33} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [\bar{P}_{2i} + h_M \bar{P}_{3i}] \\
&\quad + \sum_{i=1}^r \frac{r!}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [(-g_2 X + \star)]
\end{aligned}$$

$$\begin{aligned}
\bar{\Omega}_{34} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [g_2 A_{di} X] \\
&\quad + \sum_{i=1}^r \frac{r!}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [-g_3 X] \\
\bar{\Omega}_{35} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [\bar{S}_{13i}] \\
\bar{\Omega}_{44} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [(g_3 A_{di} X + \star)] \\
&\quad + \sum_{j=1}^r \frac{r!}{\pi(k_a - k'_a)} [-(1 - h_D) \bar{P}_{2j}] \\
\bar{\Omega}_{45} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [\bar{S}_{14i}] \\
\bar{\Omega}_{55} &= \sum_{i=1}^r \frac{(r-1)!(k_{ai} - k'_{ai})}{\pi(k_a - k'_a)} \frac{1!}{\pi(k_b - k'_b)} [-h_M^{-1} \bar{P}_{3i}].
\end{aligned}$$

Proof: Firstly, let us consider the Lyapunov-Krasoviskii function

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \quad (8)$$

where

$$\begin{aligned}
V_1(t) &= x^T(t) P_0 x(t) \\
V_2(t) &= \int_{t-h(t)}^t x^T(s) P_1(s) x(s) ds \\
V_3(t) &= \int_{t-h(t)}^t \dot{x}^T(s) P_2(s) \dot{x}(s) ds \\
V_4(t) &= \int_{t-h_M}^t (s - (t - h_M)) \dot{x}^T(s) P_3(s) \dot{x}(s) ds
\end{aligned}$$

and $P_0, P_1(s), P_2(s), P_3(s)$ are symmetric positive definite matrices.

The time derivative of Lyapunov-Krasoviskii function is

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) \quad (9)$$

where

$$\begin{aligned}
\dot{V}_1(t) &= \dot{x}^T(t) P_0 x(t) + x^T(t) P_0 \dot{x}(t) \\
\dot{V}_2(t) &= x^T(t) P_1(t) x(t) \\
&\quad - (1 - \dot{h}(t)) x^T(t - h(t)) P_1(t - h(t)) x(t - h(t)) \\
\dot{V}_3(t) &= \dot{x}^T(t) P_2(t) \dot{x}(t) \\
&\quad - (1 - \dot{h}(t)) \dot{x}^T(t - h(t)) P_2(t - h(t)) \dot{x}(t - h(t)) \\
\dot{V}_4(t) &= h_M \dot{x}^T(t) P_3(t) \dot{x}(t) - \int_{t-h_M}^t \dot{x}^T(s) P_3(s) \dot{x}(s) ds.
\end{aligned}$$

According to $0 \leq \dot{h}(t) \leq h_D$, we can obtain

$$\dot{V}_2(t) = x^T(t) P_1(t) x(t) - (1 - h_D) \times x^T(t - h(t)) P_1(t - h(t)) x(t - h(t)) \quad (10)$$

$$\dot{V}_3(t) = \dot{x}^T(t) P_2(t) \dot{x}(t) - (1 - h_D) \times \dot{x}^T(t - h(t)) P_2(t - h(t)) \dot{x}(t - h(t)). \quad (11)$$

From Newton-Leibniz formula and the T-S fuzzy neutral system in (1), the following equalities are always hold:

$$\Pi_1 = 2\xi^T(t)S_1(t)[x(t) - x(t-h(t)) - \int_{t-h(t)}^t \dot{x}(s)ds] = 0 \quad (12)$$

$$\Pi_2 = 2\xi^T(t)S_2(t)[A(t)x(t) + A_c(t)x(t-h(t)) + A_d(t)\dot{x}(t-h(t)) + B(t)K(t)x(t) - \dot{x}(t)] = 0 \quad (13)$$

$$\Pi_3 = h_M \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t) - \int_{t-h_M}^t \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)ds = 0 \quad (14)$$

where $\xi^T(t) = [x^T(t) \quad x^T(t-h(t)) \quad \dot{x}^T(t) \quad \dot{x}^T(t-h(t))]$.

From the condition $0 \leq h(t) \leq h_M$, we can obtain the following result

$$\begin{aligned} \Pi_3 &= h_M \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t) \\ &\quad - \int_{t-h_M}^t \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)ds \\ &\leq h_M \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t) \\ &\quad - \int_{t-h(t)}^t \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)ds. \end{aligned} \quad (15)$$

By (9)-(15) with $P_3(s) \geq P_4(t)$ we can obtain

$$\begin{aligned} \dot{V}(t) &\leq \Lambda(t) + h_M \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t) \\ &\quad - \int_{t-h(t)}^t [(\dot{x}^T(s)P_3(s) + \xi^T(t)S_1(t)) \\ &\quad \times P_3^{-1}(s)(P_3(s)\dot{x}(s) + S_1^T(t)\xi(t))]ds \\ &\leq \Lambda(t) + h_M \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t) \end{aligned} \quad (16)$$

where

$$\begin{aligned} \Lambda(t) &= \dot{x}^T(t)P_0x(t) + x^T(t)P_0\dot{x}(t) + x^T(t)P_1(t)x(t) \\ &\quad - (1-h_D)x^T(t-h(t))P_1(t-h(t))x(t-h(t)) \\ &\quad - (1-h_D)\dot{x}^T(t-h(t))P_2(t-h(t))\dot{x}(t-h(t)) \\ &\quad + \dot{x}^T(t)P_2(t)\dot{x}(t) + h_M\dot{x}^T(t)P_3(t)\dot{x}(t) \\ &\quad + 2\xi^T(t)S_1(t)[x(t) - x(t-h(t))] \\ &\quad + 2\xi^T(t)S_2(t)[A(t)x(t) + A_c(t)x(t-h(t)) \\ &\quad + A_d(t)\dot{x}(t-h(t)) + B(t)K(t)x(t) - \dot{x}(t)]. \end{aligned}$$

In order for $\dot{V}(x(t)) < 0$ for all $x(t) \neq 0$, (16) should be negative. By Schur complement and pre- and post-multiplying both sides with $\text{diag}[X, X, X, X, X]$ and define $X = S_{21}^{-1}$, $S_{22} = g_1 S_{21}$, $S_{23} = g_2 S_{21}$, $S_{24} = g_3 S_{21}$, we can get the following result from (16)

$$\xi^T(t)\Xi(t)\xi(t) < 0 \quad (17)$$

where

$$\Xi(t) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\ \star & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} \\ \star & \star & \Xi_{33} & \Xi_{34} & \Xi_{35} \\ \star & \star & \star & \Xi_{44} & \Xi_{45} \\ \star & \star & \star & \star & \Xi_{55} \end{bmatrix}$$

$$\begin{aligned} \Xi_{11} &= [(\bar{S}_{11}(t) + \star) + \bar{P}_1(t) + (A(t)X + \star) + (B(t)\bar{F}(t) + \star)] \\ \Xi_{12} &= [A_c(t)X + g_1 X^T A(t)^T + \bar{S}_{12}^T(t) - \bar{S}_{11}^T(t) \\ &\quad + g_1 \bar{F}^T(t)B^T(t)] \\ \Xi_{13} &= [g_2 X^T A(t)^T + \bar{S}_{13}^T(t) + \bar{P}_0^T - X + g_2 \bar{F}^T(t)B^T(t)] \\ \Xi_{14} &= [A_d(t)X + g_3 X^T A(t)^T + \bar{S}_{14}^T(t) + g_3 \bar{F}^T(t)B^T(t)] \\ \Xi_{15} &= [\bar{S}_{11}(t)] \\ \Xi_{22} &= [(g_1 A_c(t)X + \star) - (\bar{S}_{12}(t) + \star) - (1-h_D)\bar{P}_1(t-h(t))] \\ \Xi_{23} &= [g_2 X^T A_c(t)^T - \bar{S}_{13}^T(t) - g_1 X] \\ \Xi_{24} &= [g_1 A_d(t)X + g_3 X^T A_c(t)^T - \bar{S}_{14}^T(t)] \\ \Xi_{25} &= [\bar{S}_{12}(t)] \\ \Xi_{33} &= [\bar{P}_2(t) + h_M \bar{P}_3(t) - (g_2 X + \star)] \\ \Xi_{34} &= [g_2 A_d(t)X - g_3 X] \\ \Xi_{35} &= [\bar{S}_{13}(t)] \\ \Xi_{44} &= [(g_3 A_d^T(t)X + \star) - (1-h_D)\bar{P}_2(t-h(t))] \\ \Xi_{45} &= [\bar{S}_{14}(t)] \\ \Xi_{55} &= [-h_M^{-1}\bar{P}_3(t)] \\ \bar{S}_{11}(t) &= X S_{11}(t)X, \quad \bar{S}_{12}(t) = X S_{12}(t)X, \quad \bar{P}_0(t) = X P_0(t)X, \\ \bar{S}_{13}(t) &= X S_{13}(t)X, \quad \bar{S}_{14}(t) = X S_{14}(t)X, \quad \bar{P}_1(t) = X P_1(t)X, \\ \bar{P}_2(t) &= X P_2(t)X, \quad \bar{P}_1(t-h(t)) = X P_1(t-h(t))X \\ \bar{P}_3(t) &= X P_3(t)X, \quad \bar{P}_2(t-h(t)) = X P_2(t-h(t))X. \end{aligned}$$

Clearly, (17) is equivalent to (18)

$$\Phi(t) < 0 \quad (18)$$

where

$$\Phi(t) = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} \\ \star & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} \\ \star & \star & \Phi_{33} & \Phi_{34} & \Phi_{35} \\ \star & \star & \star & \Phi_{44} & \Phi_{45} \\ \star & \star & \star & \star & \Phi_{55} \end{bmatrix}$$

$$\begin{aligned} \Phi_{11} &= \sum_{j=1}^r \mu_j [(B(t)\bar{F}(t) + \star)] + \sum_{j=1}^r \mu_j (\sum_{i=1}^r \mu_i)^{r-1} [\bar{P}_1(t) \\ &\quad + (\bar{S}_{11}(t) + \star) + (A(t)X + \star)] \end{aligned}$$

$$\begin{aligned} \Phi_{12} &= \sum_{j=1}^r \mu_j (\sum_{i=1}^r \mu_i)^{r-1} [A_c(t)X + g_1 X^T A(t)^T \\ &\quad + \bar{S}_{12}^T(t) - \bar{S}_{11}^T(t)] + \sum_{j=1}^r \mu_j [g_1 \bar{F}^T(t)B^T(t)] \end{aligned}$$

$$\begin{aligned} \Phi_{13} &= \sum_{j=1}^r \mu_j (\sum_{i=1}^r \mu_i)^{r-1} [g_2 X^T A(t)^T + \bar{S}_{13}^T(t)] \\ &\quad + \sum_{j=1}^r \mu_j [g_2 \bar{F}^T(t)B^T(t)] + \sum_{j=1}^r \mu_j (\sum_{i=1}^r \mu_i)^r [\bar{P}_0^T - X] \end{aligned}$$

$$\begin{aligned} \Phi_{14} &= \sum_{j=1}^r \mu_j (\sum_{i=1}^r \mu_i)^{r-1} [A_d(t)X + g_3 X^T A(t)^T + \bar{S}_{14}^T(t)] \\ &\quad + \sum_{j=1}^r \mu_j [g_3 \bar{F}^T(t)B^T(t)] \end{aligned}$$

$$\begin{aligned}
\Phi_{15} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [\bar{S}_{11}(t)] \\
\Phi_{22} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [(g_1 A_c(t) X + \star) - (\bar{S}_{12}(t) + \star)] \\
&\quad - \left(\sum_{i=1}^r \mu_i \right)^r [(1 - h_D) \bar{P}_1(t - h(t))] \\
\Phi_{23} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [g_2 X^T A_c(t)^T - \bar{S}_{13}^T(t)] \\
&\quad - \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^r [g_1 X] \\
\Phi_{24} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [g_1 A_d(t) X + g_3 X^T A_c(t)^T - \bar{S}_{14}^T(t)] \\
\Phi_{25} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [\bar{S}_{12}(t)] \\
\Phi_{33} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [\bar{P}_2(t) + h_M \bar{P}_3(t)] \\
&\quad - \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^r [(g_2 X + \star)] \\
\Phi_{34} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [g_2 A_d(t) X] \\
&\quad - \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^r [g_3 X] \\
\Phi_{35} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [\bar{S}_{13}(t)] \\
\Phi_{44} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [(g_3 A_d(t) X + \star)] \\
&\quad + \left(\sum_{i=1}^r \mu_i \right)^r [(1 - h_D) \bar{P}_2(t - h(t))] \\
\Phi_{45} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [\bar{S}_{14}(t)] \\
\Phi_{55} &= \sum_{j=1}^r \mu_j \left(\sum_{i=1}^r \mu_i \right)^{r-1} [-h_M^{-1} \bar{P}_3(t)].
\end{aligned}$$

By applying Lemma 2 and Lemma 3 to (18), yields

$$\begin{aligned}
&\left(\sum_{i=1}^r \mu_i(t) \right)^{d_a} \left(\sum_{j=1}^r \mu_j(t - h(t)) \right)^{d_b} \Phi(t) \\
&= \sum_{k'_a \in K(d_a)} \frac{d_a!}{\pi(k'_a)} \sum_{k'_b \in K(d_b)} \frac{d_b!}{\pi(k'_b)} \bar{\Omega} < 0 \\
&= \sum_{k'_a \in K(d_a)} \sum_{k'_b \in K(d_b)} \Omega^{k_a, k_b} < 0.
\end{aligned} \tag{19}$$

Therefore, if (19) is satisfied which implies that $\Omega < 0$, and the closed-loop T-S fuzzy neutral system is asymptotically stable. This completes the proof of the theorem. ■

IV. NUMERICAL EXAMPLE

In this section, a numerical example is provided to demonstrate the validity and feasibility of the proposed result.

Example 1: Consider the following T-S fuzzy neutral system:

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^2 \mu_i(t) [A_i x(t) + A_{ci} x(t - h(t)) + A_{di} \dot{x}(t - h(t)) \\
&\quad + B_i u(t)]
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0.3 & 0.6 \\ 0.8 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0.3 \\ 1 & 0.6 \end{bmatrix}, \\
A_{c1} &= \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, A_{c2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}, \\
A_{d1} &= \begin{bmatrix} -0.5 & 1 \\ 0.4 & 0.3 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.3 & 0 \\ 0.7 & -0.2 \end{bmatrix}, \\
B_1 = B_2 &= \begin{bmatrix} 1 \\ 1 \end{bmatrix}, h(t) = 0.3 + 0.2 \cos(t).
\end{aligned}$$

By applying the convex optimization problem in Theorem 1 with $g_1 = 0.32$, $g_2 = 2.55$, $g_3 = 0.1$, $h_M = 0.5$, and $h_D = 0.2$, the following matrices and controller gain can be obtained:

Fig. 1. The state response for closed-loop T-S fuzzy neutral systems with initial condition $x(0) = [0.5, -0.4]$.

Fig. 2. The variation of $\dot{h}(t)$ and h_D .

$$\begin{aligned}
 P_0 &= \begin{bmatrix} 14.1414 & 13.7123 \\ 13.7123 & 13.5008 \end{bmatrix}, \quad X = \begin{bmatrix} 0.4453 & 0.2166 \\ 0.2166 & 0.2209 \end{bmatrix}, \\
 P_{11} &= \begin{bmatrix} 8.9149 & 8.3762 \\ 8.3762 & 8.1529 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 7.4549 & 7.0032 \\ 7.0032 & 6.9467 \end{bmatrix}, \\
 P_{21} &= \begin{bmatrix} 0.8856 & 0.3300 \\ 0.3300 & 0.2413 \end{bmatrix}, \quad P_{22} = \begin{bmatrix} 1.4665 & 0.7449 \\ 0.7449 & 0.5225 \end{bmatrix}, \\
 P_{31} &= \begin{bmatrix} 0.7179 & 0.1753 \\ 0.1753 & 0.1872 \end{bmatrix}, \quad P_{32} = \begin{bmatrix} 1.1204 & 0.1160 \\ 0.1160 & 0.0456 \end{bmatrix}, \\
 F_{10} &= [-1.3569 \quad -23.2738], \quad F_{01} = [-2.2770 \quad -22.3444].
 \end{aligned}$$

The state responses for closed-loop T-S fuzzy neutral system with delay time $0.3 + 0.2\cos(t)$ and $x(0) = [0.5, -0.4]$ is shown in Fig. 1. From the simulation results, it can be seen the designed fuzzy controller ensures the asymptotic stability of the closed-loop T-S fuzzy neutral system. One can observe that the states converge to the equilibrium states after some transient times. Fig. 2 shows the variation of $\dot{h}(t)$ and h_D .

V. CONCLUSIONS

In this paper, a stabilization problem for T-S fuzzy neutral system is investigated. Based on the polynomial technique and some variable transformation, a delay-dependent stabilization condition is proposed for T-S fuzzy neutral system. Furthermore, the results can be formulated in terms of LMI forms. A numerical example is given to illustrate the effectiveness of the proposed methods.

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