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An Abstract Theory and Ontology of Motion Based on the Regions Connection Calculus

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Abstract. In this paper, we present a framework abstracting motion by creating a qualitative representation of classes describing motion, and use the continuity constraints implicitly embedded in the semantics of these classes to create a framework that enables plausible reasoning about them. In particular, we propose a topology-based calculus of motion whose primitive is a *motion class*. We subsequently construct a set of primitive motion classes that exhaustively describes the change in topology between two moving objects, and show how compound motion classes are formed from these primitive motion classes using continuity constraints we make explicit. We use composition tables to define queries in the spatio-temporal domain and enable the extension of the classes to reason about the change in topology among three objects as they move.

1 Introduction

Qualitative formalisms have been accepted as suitable methods for high-level reasoning about spatial and spatio-temporal knowledge [2]. This is due to the fact that the epistemic nature of spatio-temporal information renders it vague, highly-dynamic or unknown, making it difficult to obtain the numerical values necessary for a quantitative representation. Because of this, numerical approaches to reasoning about the spatio-temporal domain are limited [2], and qualitative formalisms that provide an abstraction of such domains have prevailed [3].

The general principle of qualitative spatio-temporal representations is to capture one or more aspects of space, e.g. topology [1] [4] [5] [6], orientation [7] or distance [9] and formulate a set of relations that represents the possible interactions among two spatial objects with respect to the chosen aspect of space [3]. The resulting set of relations must satisfy the property of being *Jointly Exhaustive and Pairwise Disjoint* (JEPD), which means that only one relation may hold between any two spatial objects at a time, and that together, the relations make up all forms of interaction among spatial objects [3]. Reasoning is performed by means of a composition table [3], which uses known relations between certain objects to predict unknown relations that may hold between other objects.

Because of the relational nature of the qualitative representations of space, opposed to the absolute Newtonian numerical approaches, it seemed natural to use to describe the elements of this calculi, and furthermore, use the elements of the calculi provide natural language descriptions of motion with respect to the calculi. For example [11] uses the topology-based qualitative theory of space of [1] to construct a set of six motion classes consisting of *leave*, *reach*, *cross*, *hit*, *internal* and *external*. [8] does the same but embeds vagueness in the representation of the regions under study to come up with a set of motion classes specific to vague regions.

What is common among the above sets of classes of motion is that their formulations were rather ad-hoc. More specifically, because they were directed towards capturing some semantics of motion and not to use it as a construct for a calculus of change, they did not necessitate that the set of motion classes formulated possess the properties necessary for a calculus. More specifically, they are all non-exhaustive, i.e. they do not represent all the forms that motion can take, which renders them unusable for a calculus of motion.

In this paper, we present a qualitative calculus of motion. The paper aims at formulating an abstract representation at motion by using a qualitative representation of space to create its primitive, a *motion class*, presents a high-level view of motion by ridding its of its quantitative aspects. We begin by defining the necessary conditions for a set of classes to become the primitive of such calculus, then we present a methodology for deriving the elements of the set of classes constituting the calculus from a jointly-exhaustive and pairwise-disjoint set of spatial relations. We follow by introducing reasoning mechanisms that enables the prediction of the possible topologies between moving objects other than the ones we have knowledge about.

Although the formalism we present is a general one and can be used to derive motion classes from a qualitative spatial calculus that represents any aspect of space, for readability purposes, we choose introduce it by deriving its constructs from the well known spatial theory of the *Region Connection Calculus* (RCC8) [12]. The result is an abstract theory of motion where topology determines the spatial interactions among the objects under study.

The paper is organized as follows. Section 2 reviews the RCC8 calculus concentrating on the continuity properties it possesses, and how the RCC8 conceptual neighborhood captures spatio-temporal relationships. Section 3 presents the spatio-temporal representation based on motion classes and Section 4 shows how to combine two motion classes to form a compound motion. Section 5 constructs reasoning mechanisms through the use of composition tables to formulate spatio-temporal queries about motion. Finally Section 6 present a summary, some conclusions and future directions.

2 The Region Connection Calculus: RCC8 Set

Although many qualitative theories for dealing with space exist in the literature, the RCC8 calculus [12] has attracted more attention within the QSR community

Fig. 1. The RCC8 Relations

than others. RCC8 uses the notion of 'connectedness' to formulate a set of JEPD topological relations that may hold between two regions at any time. For any two regions x and y , the topological relation that may exist between x and y is represented by $r_i(x,y) \in \text{RCC8}$ and can be one of the following: $\text{DC}(x,y)$ (x is disconnected from y), $\text{EC}(x,y)$ (x is externally connected to y), $\text{PO}(x,y)$ (x partially overlaps y), $\text{EQ}(x,y)$ (x is equal to y), $\text{TPP}(x,y)$ (x is a tangential proper part of y) and its inverse $\text{TPPI}(x,y)$ (y is a tangential proper part of x), $\text{NTPP}(x,y)$ (x is a non-tangential proper part of y) and its inverse $\text{NTPPI}(x,y)$ (y is a non-tangential proper part of x). Figure 1 shows the complete set.

Reasoning is performed by means of a composition table [12], which generally takes the form of answering the following query: for three spatial regions x , y and z , and RCC8 relations rcc_i and rcc_j , if $rcc_i(x,y)$ and $rcc_j(y,z)$ hold, what are the possible RCC8 relations that may hold between x and z ?

In addition to its high expressive power and flexibility, the RCC8 relations possess an inherit continuity property in which each relation can be mapped to another from the set via a continuous transition. This property has been modeled by what is known as the RCC8's conceptual neighborhood graph (CNG) [12] shown in figure 2.

Figure 2 shows the continuity of the process of transforming one RCC8 relation to another. Transforming some $rcc_i \in \text{RCC8}$ to some $rcc_j \in \text{RCC8}$ can be done via one direct transition (e.g. DC to PO), making rcc_i and rcc_j immediate conceptual neighbors, or indirectly (e.g. PO and DC). In the indirect case, there must be some sequence S of RCC8 relations that connect rcc_i and rcc_j . The fact that S cannot be empty ensures the continuity of the transitions among members of RCC8.

Because of the continuity properties explained above, the RCC8 relations can be interpreted to model the topological relations between the spatio-temporal evolutions of spatial regions in addition to topological relations between static spatial regions. An example is shown in figure 3 where the relation PO (partial overlap) between two regions x and y is shown to hold during the temporal

Fig. 2. The RCC8 Conceptual Neighborhood

Fig. 3. Spatio-temporal Interpretation of PO

interval $[t_1, t_2]$. In the figure, the horizontal and vertical axes represent the evolution of space and time respectively. Hence, the figure shows not the static regions x and y , but the evolution of their spatial extents through time. Hence, the relation $PO(x, y)$ holds when the evolution of region x overlaps that of region y during some interval, e.g. $[t_1, t_2]$.

Because of its spatio-temporal nature, it is possible to construct from the RCC8 set a set of motion class descriptions. In [11], six mutually exclusive classes have been formulated. They are *leave*(z, x, y), *reach*(z, x, y), *hit*(z, x, y), *cross*(z, x, y), *internal*(z, x, y) and *external*(z, x, y) representing in each case the motion class that holds between two spatio-temporal moving regions x and y during interval z .

Fig. 4. Muller's Motion Classes

Figure 4 shows the six classes and is read as follows. The horizontal axis represents the spatial extent of the region and the vertical axis represents the temporal evolution. Hence, the beginning of interval z is located in its lowest t value and its end is located at its highest t value. The spatial extents of regions x and y evolve as time increases forming a motion class. We do not give the formal definition of these classes. The interested reader can refer to [11].

3 A Qualitative Set of Motion Classes

In this section, we present the building blocks for a calculus which abstracts the motion of one spatial region with respect to another by describing it via a set of classes which it can take. More specifically, given two regions x and y moving during the interval $[t, t + \Delta]$, we study the change in the topological relations that hold between the two regions at the beginning and end of the interval, and use them to categorize motion by constructing a set of classes that it may belong to.

The topology at t and $t + \Delta$ is described by the RCC8 relations that hold between x and y at t and $t + \Delta$ respectively. Because RCC8 has 8 relations, all adhering to a continuous neighborhood structure, there exists a total of $|RCC8|^2 = 64$ possible forms of motion. In this section, we construct the set MC, which is an exhaustive set of motion classes, determined by the 64 possibilities.

Hence, our calculus consists of a topological spatial base (RCC8) and a set of motion classes derived from it (MC). Before we introduce MC, we reformulate the properties of the RCC8 set to make it compatible with our calculus.

3.1 The Spatial Base

Consists of the set $RCC8 = \{DC, EC, PO, EQ, TPP, TPPI, NTPP, NTPPI\}$. In this paper, we adopt the notation $rcc_{[t, t + \Delta]}(x, y)$ to denote the following:

1. RCC8 relation rcc holds between two regions x and y during the entire duration of the interval $[t, t + \Delta]$.

2. It is not prohibited for rcc to hold between x and y before $[t, t + \Delta]$ or to persist after it. The knowledge of rcc before or after $[t, t + \Delta]$, however, is not readily available.

Elements of the RCC8 set possess the following properties:

1. They are Jointly Exhaustive: Any topological relation that may hold between two regions of space x and y during the interval $[t, t + \Delta]$ must belong to RCC8.

$$\forall rcc, rcc_{[t, t + \Delta]}(x, y) \rightarrow rcc \in \text{RCC8}$$

2. They are Pairwise Disjoint: No two RCC8 relations can hold between two spatial regions x and y at the same time.

$$\forall rcc_1, rcc_2 \in \text{RCC8} : (rcc_{1[t, t + \Delta]}(x, y) \wedge rcc_{2[t, t + \Delta]}(x, y)) \rightarrow rcc_1 = rcc_2$$

3. They Adhere to a Conceptual Neighborhood structure:

$$\forall rcc_1 \in \text{RCC8}, \exists rcc_2 \in \text{RCC8} : rcc_1 \neq rcc_2 \wedge \mu(rcc_1, rcc_2) = 1$$

Where $\mu(rcc_1, rcc_2)$ is the number of arcs separating rcc_1 and rcc_2 in RCC8's conceptual neighborhood graph.

3.2 The Set MC of Motion Classes

As explained above, the primitives of our calculus, the elements of $\text{MC} = \{mc_1, \dots, mc_n\}$ are characterized by the RCC8 relations that hold at the beginning and end of the interval $[t, t + \Delta]$ during which they hold.

For this purpose, we define the predicates **starts**($mc_i, [t, t + \Delta], x, y$) and **ends**($mc_i, [t, t + \Delta], x, y$), where **starts**($mc_i, [t, t + \Delta], x, y$) returns a subset of RCC8 corresponding to the relations that can hold between spatio-temporal objects x and y at the beginning of the interval $[t, t + \Delta]$ during which the class mc_i correctly describes the change taking place (i.e. at time t), while **ends**($mc_i, [t, t + \Delta], x, y$) gives the set of RCC8 relations that hold between spatio-temporal objects x and y at the end of the interval $[t, t + \Delta]$ during which the class mc_i holds (i.e. at $t + \Delta$). They are both given in definitions 1 and 2 below:

Definition 1. **starts**($mc_i, [t, t + \Delta], x, y$) = $\{rcc \mid rcc \in \text{RCC8} \wedge rcc_t(x, y)\}$.

Definition 2. **ends**($mc_i, [t, t + \Delta], x, y$) = $\{rcc \mid rcc \in \text{RCC8} \wedge rcc_{(t + \Delta)}(x, y)\}$.

It is important to note before continuing, that the interval $[t, t + \Delta]$ is defined as the minimal interval required to inflict spatial change. In other words, for some motion class mc , if **starts**($mc, [t, t + \Delta], x, y$) = {DC} and **ends**($mc, [t, t + \Delta], x, y$) = {PO}, then the only relations allowed during the interval are those relations that lie between DC and PO in RCC8's conceptual neighborhood graph, which are the relations necessary for the change to occur. Also, these relations should only hold in the order that allows for mc to hold during the interval.

3.3 Constructing the Set MC

The construction of MC is initiated by the fact that the elements of the set $RCC8^2$ do not uniquely describe $|RCC8|^2=64$ classes of motion. For example, any motion that starts with the regions being in any configuration except disconnected and ends with the regions being disconnected represents that one region left the other. Hence, there exists a one-to-many mapping between MC and $RCC8^2$. This mapping is defined below.

We define the set $j = \{j_1, ..., j_n\}$, where for each motion class $mc_i \in MC$, there exists $j_i = \{(rcc_1, rcc_2), ... (rcc_r, rcc_s)\} \in j$ representing the subset of $RCC8^2$ that describes all the possible pairs of spatial relations that hold at the beginning and end of the interval during which the class mc_i holds (e.g. $rcc_1 \in \mathbf{starts}(mc_i, [t, t + \Delta], x, y)$ and $rcc_2 \in \mathbf{ends}(mc_i, [t, t + \Delta], x, y)$ for two regions x and y moving during interval $[t, t + \Delta]$). Unlike the case with $RCC8$, there exists a one-to-one correspondence between j and MC. Property 3.3.1 formalizes the above concept.

Property 3.3.1. $\forall mc_i \in MC, \exists j_i = \{(rcc_1, rcc_2), ..., (rcc_r, rcc_s)\} \in j: \forall (rcc_l, rcc_m) \in j_i, rcc_l \in \mathbf{starts}(mc_i) \wedge rcc_m \in \mathbf{ends}(mc_i)$.

As a result, nine motion classes have been identified and are given in table 1. The motion classes are LEAVE, REACH, HIT, SPLIT, PERIPHERAL, EXPAND, SHRINK, INTERNAL and EXTERNAL. In the table, the rows correspond to the $RCC8$ relation which belongs to the set $\mathbf{starts}(mc_i, [t, t + \Delta], x, y)$ while the column corresponds to the $RCC8$ relation which belongs to the set $\mathbf{ends}(mc_i, [t, t + \Delta], x, y)$. Each intersection of a row and a column presents the resulting motion class when the corresponding $RCC8$ relations at the beginning and end of the interval hold.

Table 1. The Set MC of Motion Classes

		ENDS												
		DC	EC	PO	TPP	NTPP	EQ	TPPi	NTPPi					
STARTS	DC	EXTERNAL	HIT	REACH										
	EC	SPLIT	PERIPHERAL											
	PO	LEAVE OR REACH			EXPAND									
	TPP	LEAVE												
	NTPP													
	EQ													
	TPPi													
	NTPPi	SHRINK		INTERNAL										
				INTERNAL										

3.4 Properties of MC

In order for the elements of the set MC to be useful in being primitives in a calculus of motion classes, they must, along with the spatial base $RCC8$, exhaustively describe all possible forms of motion between two regions x and y during interval $[t, t + \Delta]$, and possess continuity properties by having a conceptual neighborhood structure. Below we outline these properties and show that elements of MC indeed possess them.

Completeness

Claim 3.4.1 *MC exhausts all forms of spatio-temporal change during a given interval. In other words, MC covers RCC8².*

Justification:

By construction, the set $\mathcal{J}$ covers RCC8². As a result

$$\bigcup_{j_i \in \mathcal{J}} j_i = \text{RCC8}^2$$

Since there exists a one-to-one correspondence between $\mathcal{J}$ and MC, MC in turn covers RCC8².

Continuity

Claim 3.4.2 *The set of motion classes MC adheres to a conceptual neighborhood structure. In other words, for every class $mc_i \in \text{MC}$, there must exist another class $mc_j \in \text{MC}$ which represents the linguistic continuation of mc_i .*

Justification:

1. Existence:

Dictated by definitions 1 and 2, for all motion classes mc_i and $mc_j \in \text{MC}$, $\text{starts}(mc_i)$, $\text{ends}(mc_i)$, $\text{starts}(mc_j)$, $\text{ends}(mc_j) \in \text{RCC8}$.

However, because the RCC8 adheres to a conceptual neighborhood structure, for every RCC8 relation rcc_i , there exists another RCC8 relation rcc_j such that rcc_i and rcc_j are conceptual neighbors in RCC8's conceptual neighborhood graph.

As a result, mc_i and mc_j are conceptual neighbors if **ends**(mc_i) and **starts**(mc_j) are conceptual neighbors in RCC8's conceptual neighborhood graph. Now, since members of MC cover RCC8², the pair (**ends**(mc_i), **starts**(mc_j)) must belong to RCC8². Hence, the motion class mc_j that represents the continuous extension of mc_i must exist.

2. Necessity:

Assume that for three motion classes mc_i , mc_j and $mc_k \in \text{MC}$, mc_i and mc_j are conceptual neighbors, mc_j and mc_k are conceptual neighbors, while mc_i and mc_k are not conceptual neighbors. Also, assume that mc_i has evolved to mc_k without going through the motion class mc_j .

This means that **ends**(mc_i) and **starts**(mc_k) must be conceptual neighbors in the RCC8's conceptual neighborhood graph. However, this forms a contradiction, because if this is true, then mc_i and mc_k would have been conceptual neighbors.

Therefore, members of MC adhere to constraints specified by their conceptual neighborhood but forming only continuous transitions.

Having justified the existence of MC's conceptual neighborhood graph, we now give a formal definition of two motion classes being conceptual neighbors. The definition is given in 3 and is read as follows: motion classes mc_i holding during

interval $[t, t + \Delta]$ and mc_j holding during the interval which immediately follows $[t, t + \Delta]$ (i.e. interval $[t + \Delta, t + \Delta + \delta]$ in the definition) are conceptual neighbors if one of the RCC8 relations that may hold at the end of mc_i is a conceptual neighbor of one of the RCC8 relations that may hold at the beginning of mc_j in RCC8's conceptual neighborhood graph.

Definition 3. *conc_neigh* ($mc_i [t, t + \Delta] x, y$), ($mc_j [t + \Delta, t + \Delta + \delta] x, y$) if $\exists s_1 = \mathbf{starts}(mc_j [t + \Delta, t + \Delta + \delta] x, y)$, $s_2 = \mathbf{ends}(mc_i [t, t + \Delta] x, y) \wedge \exists r_1 \in s_1 \wedge r_2 \in s_2: 0 \leq \mu(r_1, r_2) \leq 1$.

We show the conceptual neighborhoods of the set MC in table 2 below. We resort to the table notation instead of the well-known conceptual neighborhood graph for visibility purposes. The table should be read as follows. The rows represent

Table 2. MC's Conceptual Neighbors

Motion Class	Conceptual Neighbors
LEAVE	All
REACH	All but Hit and External
HIT	Leave, Reach, Split, Peripheral, External
SPLIT	Reach, Hit, Peripheral, External
PERIPHERAL	Leave, Reach, Hit, Split, External
EXPAND	Leave, Reach, Shrink, Internal
SHRINK	Leave, Reach, Expand, Internal
INTERNAL	Leave, Reach, Expand, Shrink
EXTERNAL	Reach, Hit, Split, Peripheral

4 Compound Motion Classes

As the name implies, a compound motion consists of two primitive motions. However, to be able to compose two primitive motions, the following conditions should be satisfied:

1. Motion continuity dictates that the spatial relation that holds at the end of the first motion also holds for the start of the second motion. Formally, to compose mc_i and mc_j , there should exist $rcc_i \in \text{RCC8}$ such that $rcc_i \in \mathbf{ends}(mc_i)$ and $rcc_i \in \mathbf{starts}(mc_j)$. For example, it is possible to form a compound motion from HIT and SPLIT but it is not possible to combine HIT and INTERNAL.
2. To eliminate redundancy, the two motions forming the compound motion should not be subsumed by a primitive motion. Formally, this condition implies that for the sequence of RCC8 relationships visited by the compound motion composed of mc_i and mc_j does not correspond to the sequence generated from any single motion mc_k . For example, REACH subsumes the composition of HIT and REACH.

Fig. 5. Compound Motion Classes

Figure 5 illustrates five compound motion classes satisfying the two above conditions. These compound motions include REACH-LEAVE (or CROSS), PERIPHERAL -REACH, LEAVE-REACH, PERIPHERAL-LEAVE, and HIT- SPLIT (or TOUCH). In the figure, the dotted lines represent the trajectory of the moving region with respect to another region. The start and end of each constituent motion are marked with dark bullets.

5 Queries About Motion Classes

Now that we have established the set of motion classes, and we have shown that they are exhaustive to the set $RCC8^2$, and that they adhere to a conceptual neighborhood structure, we can now use composition tables to reason about the set MC^2 to obtain spatio-temporal information regarding three objects in motion as illustrated in section 5.1 below.

Table 3. Composition Table Representing Spatio-temporal Queries

Motion	LEAVE	REACH	HIT	SPLIT	PERIPHERAL
LEAVE	Ext. Leave Int.	Ext.	Ext.	Ext. Hit Reach	Ext. Split
REACH	Ext.	Reach Int.	Hit Periph. Ext.	Ext.	Hit Ext.
HIT	Ext.	Int. Periph. Hit	Int.	Ext. Leave	Ext. Reach
SPLIT	Leave Int.	Ext.	Ext.	Int. Ext.	Leave Ext.
PERIPHERAL	Ext. Leave	Hit Reach	Ext. Reach	Ext. Leave	Ext. Int.
EXPAND	Ext. Leave Exp.	Reach	Reach	Hit Reach Ext.	Reach
SHRINK	Leave	Leave, Reach Ext.	Ext.	Leave Ext.	Leave
INTERNAL	Leave Ext. Split	Reach Ext. Hit	Ext.	Ext.	Ext.
EXTERNAL	All but Shrink	All but Expand	All but Expand	All but Shrink	All but Exp. & Shrink

Table 4. Composition Table Representing Spatio-temporal Queries

Motion	INTERNAL	EXTERNAL	EXPAND	SHRINK
LEAVE	Int. Leave	Reach Ext. Hit	Leave Int.	Leave Peri. Split Int. Ext.
REACH	Reach Int.	Leave Split Ext.	Reach	Reach
HIT	Reach Int.	Leave Ext. Split	External	Reach
SPLIT	Int. Leave	Reach Hit Ext.	Leave	Ext.
PERIPHERAL	Int.	Leave Reach Hit Split Ext.	Leave	Reach
EXPAND	Int. Leave	Ext. Reach Hit	Expand	Int. Reach Expand
SHRINK	Shrink Int.	Ext. Leave Split	Int. Leave	Shrink
INTERNAL	Int.	Ext.	Leave Int. Expand	Reach Int. Shrink
EXTERNAL	Ext. Int. Leave Reach Hit Split Shrink	All	Ext. Leave Split	Ext. Reach Hit

5.1 Queries About Spatio-temporal Knowledge: Composition Tables

During interval $[t, t + \Delta]$, given three objects x , y and z undergoing motion, where $mc_i(x, y)$ holds and $mc_j(y, z)$ holds, what are the possibilities for the motion class between x and z ?

Tables 3 and 4 answer this question for all members of MC. In the table, the rows represent the possibilities for the motion class $mc_i(x, y)$ while the columns represent $mc_j(y, z)$. The intersection of a row and a column represent the possible motion classes $mc_k(x, z)$ during the interval $[t, t + \Delta]$.

6 Conclusions and Future Research

Continuity of spatio-temporal evolutions as encoded in the conceptual neighborhood graph is an important property that can be exploited to represent and reason about motion. This paper has proposed a framework for exploiting continuity in the regions connection calculus RCC8 to define an abstract framework that represents motion based on classes. The framework distinguishes between a set of simple or primitive motion classes that can be combined to form compound motion. In addition, the paper shows how to use the set of motion class to answer queries in the spatio-temporal domain based on composition tables. Extending the proposed framework to handle probabilistic and uncertain spatio-temporal evolutions; and exploring applications for the proposed framework are two directions of future investigation.

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