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Surprise-Based Qualitative Probabilistic Networks

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Abstract. This paper discusses a modification of the kappa measure of surprise and uses it to build semi-qualitative probabilistic networks. The new measure is designed to enable the definition of partial-order relations on its conditional values and is hence used to define qualitative influences over the edges of the network, similarly to Qualitative Probabilistic Networks. The resulting networks combine the advantages of kappa calculus of robustness and ease of assessment and the efficiency of Qualitative Probabilistic Networks. The measure also enables a built-in tradeoff resolution mechanism for the proposed network.

1 Introduction

Qualitative probabilistic networks (QPNs) [6,13] abstract Bayesian Networks (BNs) by replacing the numerical relations defined on the arcs of a BN (i.e. the conditional probability tables) by relations that describe how evidence given for one node influences other nodes in the network [13]. These influences are qualitative in nature, in the sense that the only information they capture is the direction of the influence (i.e. whether the evidence makes a node more or less likely) and is hence represented by its sign, being positive, negative, zero (constant) or unknown.

Despite the efficiency in reasoning with QPNs [3] (propagating influences along the arcs is efficient compared to the NP-hard reasoning in BNs), QPNs may suffer from over-abstraction because its reasoning mechanism is only concerned with finding the effect of new evidence on each node in terms of the sign of the change in belief (increase or decrease) [3] which may lead to problems when a node receives two influences of conflicting signs and a tradeoff must be resolved in order to continue reasoning. This issue has been addressed by several and solutions have been found using various tradeoff resolution strategies [9,10,14].

Apart from QPNs, there exist other formalisms for qualitatively reasoning about uncertain beliefs. One such formalism is the κ calculus, which uses the order of magnitude of probability to establish a measure, called the κ measure, where integers capture the relative degree of surprise associated with the event occurring.

The κ calculus has been used to create what is known as κ networks [4] to perform reasoning similarly to BNs [2]. κ networks have the advantage of being more robust and easier create than regular BNs as κ values are less easily affected by change and are easier to estimate compared to numerical probabilities [2]. However, κ networks still suffer from the inefficiency resulting from the polynomial size of the conditional tables associated with the nodes of the network, and as a result remain NP-hard in terms of reasoning.

To combine the efficiency of QPNs and the robustness and ease of estimation of the κ calculus, [9] proposes the use of κ values as measures of strength of the influences of a QPN. The approach retains the efficiency of arc-based reasoning of QPNs while reducing the unwanted coarseness in the representation by using κ values as measures of strength of QPN influences and resorting to them for tradeoff resolution. Despite the advantages of this work, it is only capable of capturing situations where it is possible to categorize the influences on the edges (i.e. as being positive, negative or constant). When it is not possible to do so (i.e. the influence is unknown), one must resort to quantitative probabilities to complete the reasonings task, which is contrary to the advantages of using QPNs in the first place.

Inspired by the ideas presented in [9] and the problem of unknown influences, we aim at creating a qualitative network in which the nodes represent an order of magnitude abstraction of probabilities and (i.e. as in κ networks) for which reasoning is possible on the arc-level (via qualitative influences) when the types of influences are known and in which it is possible to resort to node-based reasoning on a qualitative level when it is not possible to decide the type of the influences, only losing out on efficiency and retaining the robustness of qualitative calculi.

For this purpose, we define a new abstraction of probability, namely κ^{++} , which is based on the same concepts of κ but has additional semantics that enable its use to establish qualitative influences. For example, in the κ calculus, the rules governing the relation between $\kappa(g)$ and $\kappa(\neg g)$ are not rich enough to numerically deal with compliments in a manner that enables the propagation of their values through conditioning [5], in contrast to κ^{++} as the paper will show.

This paper is structured as follows. After providing preliminary concepts relating to qualitative probabilistic networks in section 2 and the κ calculus in section 3, we introduce the new qualitative measure, κ^{++} , in section 4. We present the details of constructing κ^{++} -based networks and using them to perform influence-based reasoning in section 5 and illustrate how reasoning can be performed if there is no information available about qualitative influences in section 6. We conclude in section 7 and outline some of our future research.

2 Qualitative Probabilistic Networks

Qualitative Probabilistic Networks (QPNs) are directed acyclic graphs that represent a qualitative abstraction of Bayesian Networks. The structure of a QPN captures the conditional independence among the variables the network represents and the arcs encode the relationships that exist between the nodes. Unlike

in BNs however, the arcs of a QPN capture qualitative relations instead of conditional probabilities [6,13].

Formally, a QPN is given by a pair $G = (V, Q)$, where V is the set of variables it represents and Q is the set of qualitative relations among the variables. There are two types of qualitative relations in Q , qualitative influences and synergies.

Influences describe how the change of the value of one variable effects that of another. There are essentially four types of influences, positive, negative, constant and unknown.

A positive influence exists between two variable X and Y (X is said to positively influence Y) if observing higher values for X makes higher values of Y more probable regardless of the value of any other variable which may directly influence Y . The inequality given below describes the notion of a positive influence more formally. The inequality assumes that the variables X and Y are binary and places a partial order on their values such that for a variable X with two values x and $\neg x$, $x > \neg x$.

$$I^+(X, Y) \text{ iff } Pr(y|x, W) \geq Pr(y|\neg x, W)$$

$I^+(X, Y)$ reads: X positively influences Y , and W denotes all the other variables other than X that may directly influence Y . Negative, constant and unknown influences are defined analogously.

An example of a QPN is given in figure 1. In the figure, $V = \{A, B, C, D\}$ and $Q = \{(B, C), (A, C), (C, D)\}$. The only information encoded in the arcs are the signs of the influences from one node to another. For instance, the figure shows that node A positively influences node C, while B on the other hand has a negative influence on C.

Fig. 1. A Qualitative Probabilistic Network

Observed evidence is propagated through the network via qualitative operators that combine influences and produce their net effect. Essentially, there are two such operators, each is used for a specific topology of arcs. When evaluating the net effect of two influences in a chain (for example, in order to obtain the effect of A on D, we have to examine a chain of two influences, that of A on C and of C on D), the sign multiplication operator, given in the left portion of table 1, is used. On the other hand, parallel connections (for example, two influences in parallel are required to establish the net effect of nodes A and B

on node C, that of B on C and of A on C) are evaluated using the sign addition operator given in the right portion of the table. The signs propagate through the network until the net effect of the evidence is observed on the required node or all the nodes are known to be visited twice by the sign-propagation algorithm given in [3].

Table 1. Sign multiplication ($\otimes$) and sign addition ($\oplus$) Operators [13]

$\otimes$	+	-	0	?	$\oplus$	+	-	0	?
+	+	-	0	?	+	+	?	+	?
-	-	+	0	?	-	?	-	-	?
0	0	0	0	0	0	+	-	0	?
?	?	?	0	?	?	?	?	?	?

3 Overview of the Kappa Calculus

The kappa calculus [4,11] is a system that abstracts probability theory by using order of magnitude of probabilities as an approximation of probability values. It does so by capturing the degree of disbelief in a proposition g , or the degree of incremental surprise or abnormality associated with finding g to be true [4], labeled $\kappa(g)$. The value of $\kappa(g)$ is assigned so that probabilities having the same order of magnitude belong to the same κ class, and that $\kappa(g)$ grows inversely to the order of magnitude of the probability value $p(g)$.

The abstraction is achieved via a procedure which begins by representing the probability of a proposition g , $p(g)$, by a polynomial function of one unknown, ϵ , an infinitesimally small positive number ($0 < \epsilon < 1$). The rank κ of a proposition g is represented by the power of the most significant ϵ -term in the polynomial representing $p(g)$ (the lowest power of ϵ in the polynomial). Accordingly, the relation between probability and κ values is that $p(g)$ is of the same order as ϵ^k , where $k = \kappa(g)$ [11], that is:

$$\epsilon < \frac{p(g)}{\epsilon^k} \leq 1 \text{ or equivalently: } \epsilon^{k+1} < p(g) \leq \epsilon^k$$

Where ϵ^k is the most significant ϵ -term of the polynomial representing $p(g)$.

The κ -calculus is useful because it provides an abstraction that only requires specifying the κ values of propositions, which is an easier task than specifying the exact probabilities associated with the specific value of the proposition. The κ values are in turn representative of the interval in which the probability falls [2].

$$\kappa(g) = \begin{cases} \min\{k \text{ such that } \lim_{\epsilon \rightarrow \infty} \frac{p(g)}{\epsilon^k} \neq 0\} & \text{iff } p(g) > 0 \\ \infty & \text{iff } p(g) = 0 \end{cases}$$

The above defines $\kappa(g)$ as the power of the most significant term (the term with the least power of ϵ , since $\epsilon < 1$) in the polynomial representing $p(g)$. In other words, $\kappa(g) = k$ if $p(g)$ has the same order of magnitude as ϵ^k [4].

A direct consequence of how $\kappa(g)$ is obtained is that since the most significant term is that with the smallest k it corresponds to the inverse of the likelihood of g , and is therefore representative of the degree of surprise associated with believing g . This can be seen in a more intuitive manner in the table below (obtained from [4]), which shows an example of how kappas can be mapped to linguistic quantifiers of beliefs.

$p(g) = \epsilon^0$	g and $\neg g$ are possible	$\kappa(g) = 0$
$p(g) = \epsilon^1$	$\neg g$ is believed	$\kappa(g) = 1$
$p(g) = \epsilon^2$	$\neg g$ is strongly believed	$\kappa(g) = 2$

The above abstraction yields an integer-based calculus which enables combining κ 's via rules that are derived from those of probability theory by replacing multiplication by addition and addition by minimum [11]. The resulting properties are given below, along with their probability-theory equivalents.

$$\begin{aligned}
 \kappa(g_1) &= \min_{\omega \models g_1} & p(g_1) &= \sum_{\omega \models g_1} p(\omega) \\
 \kappa(g_1) \vee \kappa(\neg g_1) &= 0 & p(g_1) + p(\neg g_1) &= 1 \\
 \kappa(g_2|g_1) &= \kappa(g_2 \wedge g_1) - \kappa(g_1) & p(g_2|g_1) &= p(g_2 \wedge g_1)/p(g_1)
 \end{aligned}$$

4 A New Ranking Function, $\kappa^{++}(\cdot)$

We propose a ranking function that is based on taking the order of magnitude of the surprise associated with observing a certain event. The function is based on the surprise measure proposed by Weaver [12] which calculates the surprise associated with an event G having specific value g_r (where G has a total of I possible values) by dividing the expected value of probability of G by the probability $p(G = g_r)$ as given by equation 1 below.

$$\mathcal{W}(g_r) = \frac{\sum_{i=1}^I p(g_i)^2}{p(g_r)} \quad (1)$$

Given a distribution ζ , the abstraction we form here is similar to that of the $\kappa(\cdot)$ function in that it depends on the idea of writing the probability of every event $g \in \zeta$ as a polynomial χ of infinitesimally small numbers, ϵ (for example, $p(g) = 1 - c_1\epsilon^2 + c_2\epsilon^4$) and obtaining the most significant term whose power is representative of the order of magnitude of $p(g)$. Moreover, since we are only interested in the most significant term of the polynomial, we adapt the notation χ_g^n to denote the polynomial representing $p(g)$, where n is the lowest exponent of ϵ in the polynomial.

Given the above, a ranking function is formulated to reflect the order of magnitude to which $\mathcal{W}(g_r)$, the surprise associated with the event g_r , belongs. Hence, we redefine the κ -measure [4] by associating it with the surprise measure of an event instead of the probability. The result is a qualitative measure of the surprise associated with an event g_r . The new measure, we call it κ^{++} to distinguish it from the original κ measure of [4], is derived below.

Let $\chi_{g_r}^n$ be the polynomial representing $p(g_r)$, and for every other value g_i of g , let $\chi_{g_i}^{\beta_i}$ denote the polynomial corresponding to $p(g_i)$, with β_i representing the minimum power of ϵ in the polynomial. According to equation 1, the surprise associated with $p(g_r)$, namely $\mathcal{W}(g_r)$, is:

$$\mathcal{W}(g_r) = \frac{\sum_{i=1}^I p(g_i)^2}{p(g_r)} = \frac{\sum_{i=1}^I (\chi_{g_i}^{\beta_i})^2}{\chi_{g_r}^n}$$

Where I , as given previously, is the number of possible values of G . Since all the polynomials χ are to the base ϵ , it is possible to add the terms that have equal exponents. This makes the summation:

$$\frac{\sum_{i=1}^I (\chi_{g_i}^{\beta_i})^2}{\chi_{g_r}^n} = \frac{\alpha_1 \epsilon^{2\beta_1} + \dots + \alpha_I \epsilon^{2\beta_I} + \alpha_{I+1} \epsilon^{2\alpha_1} + \dots + \alpha_l \epsilon^{2\phi_k}}{\chi_{g_r}^n}$$

$\forall \beta_i, 1 \leq i \leq I$, $\alpha_i \epsilon^{2\beta_i}$ is a term whose power is a candidate to be the minimum power of the polynomial representing $\sum_{i=1}^I (\chi_{g_i}^{\beta_i})^2$ as each $2\beta_i$ is the minimum power of $(\chi_{g_i}^{\beta_i})^2$. The α terms in the equation above are non-minimum terms and therefore, their number ($k = l - (I + 1)$) and values are irrelevant for our purpose.

Let m be such term, i.e. $m = \beta_i$ is the minimum of the minimum powers of the polynomials $\chi_{g_i}^{\beta_i}$. The surprise measure $\mathcal{W}(g_r)$ can now be represented only in terms of polynomials as:

$$\mathcal{W}(g_r) = \frac{\chi_{g_i}^{2m}}{\chi_{g_r}^n} \quad (2)$$

Having represented $\mathcal{W}(g_r)$ as a fraction of two polynomials, we now construct an abstraction of $\mathcal{W}(g_r)$ which maybe regarded as the order of magnitude class to which $\mathcal{W}(g_r)$ belongs. The integer value resulting from the abstraction is denoted by $\kappa^{++}(g_r)$ and can be used to rank the belief g_r . $\kappa^{++}(g)$ is given below.

Definition 1. For an event g_r whose probability is given by the polynomial $\chi_{g_r}^n$, the qualitative degree of surprise associated with g_r , namely $\kappa^{++}(g_r)$ is the power of the most significant ϵ -term in the polynomial representing the numerical surprise associated with g_r , $\mathcal{W}(g_r)$. In other words:

$$\kappa(g) = \begin{cases} \min\{k \text{ such that } \lim_{\epsilon \rightarrow \infty} \frac{\mathcal{W}(g)}{\epsilon^k} \neq 0\} & \text{iff } \mathcal{W}(g) > 0 \\ \infty & \text{iff } \mathcal{W}(g) = 0. \end{cases}$$

According to the above, $\kappa^{++}(g_r) = \log \mathcal{W}(g_r) = 2m - n$, where m is the minimum of all minimum powers in the polynomials $p(g_i)$, $1 \leq i \leq I$, and n is the minimum power in $p(g_r)$.

4.1 Semantics of κ^{++}

The $\kappa^{++}(\cdot)$ function can now be understood as a function which ranks events according to the surprise associated with finding that the event has occurred. $\kappa^{++}(g_r) = 2m - n$ returns a signed integer whose value and sign are representative of the degree of surprise. Accordingly, the larger the value of $\kappa^{++}(g_r)$, the greater the difference between its constituent quantities ($2m$ and n), and as a result the more surprising the event in question, g_r , is. Therefore, the signed integer produced by $\kappa^{++}(\cdot)$ carries the semantics defined by three possible classes for its value.

Positive: $(\kappa^{++}(g_r) = 2m - n) > 0$ implies that the event g_r is a lot less likely than the other events g_i ($1 \leq i \leq I$) of the distribution, i.e. $2m > n$. Hence, the occurrence of g_r indicates a surprise. Moreover, the larger the value of $\kappa^{++}(g_r)$ (the greater the difference is between $2m$ and n), the more surprising the event g_r is.

Zero: $\kappa^{++}(g_r) = 0$ represents the normal world where both g_r and $\neg g_r$ are likely to occur as the order of magnitude of the probability of the variable g_r is comparable to that of the distribution, i.e. $2m = n$.

Negative: $\kappa^{++}(g_r) < 0$ refers to the case in which having the event g_r to be false is surprising as g_r becomes more likely than unlikely compared to other events in the distribution (because $n > 2m$), which implies that $\neg g_r$ is unlikely and its $\kappa^{++}(\cdot)$ should indicate a surprise. In this case, the smaller the value of $\kappa^{++}(\cdot)$, the more surprising $\neg g_r$ is.

Because $\kappa^{++}(\cdot)$ is obtained through order-of-magnitude abstraction, its rules can be derived from those of probability theory by replacing multiplication by addition and addition by minimum, and can be summarized below.

1. $\kappa^{++}(G_1|G_2) = \kappa^{++}(G_1 \wedge G_2) - (G_2)$
2. $\kappa^{++}(G_1 \wedge G_2) = \kappa^{++}(G_1) + \kappa^{++}(G_2)$
Given that G_1 and G_2 are two independent variables.
3. $\kappa^{++}(G_1) + \kappa^{++}(G_2) = 0$
4. $\kappa^{++}(G_1 \vee G_2) = \min(\kappa^{++}(G_1), \kappa^{++}(G_2))$

It is worth noting that the κ^{++} semantics introduced earlier permit the creation of a correspondence between κ^{++} and linguistic quantifiers such as the one given below.

g is strongly believed	$\kappa^{++}(g) = -2$
g is believed	$\kappa^{++}(g) = -1$
g and $\neg g$ are possible	$\kappa^{++}(g) = 0$
$\neg g$ is believed	$\kappa^{++}(g) = 1$
$\neg g$ is strongly believed	$\kappa^{++}(g) = 2$

5 Efficient Reasoning in κ^{++} Networks

Similarly to the κ calculus, κ^{++} can be used to abstract probabilistic networks to construct a semi-qualitative equivalent whose nodes have κ^{++} s as values instead of probabilities. Despite the added robustness of the resulting network, inference will remain NP-hard, i.e. of the same complexity as its quantitative equivalent [2].

Alternatively, we use κ^{++} to perform Qualitative Probabilistic Network-like inference by utilizing the sign and magnitude of κ^{++} to define partial order relations governing the conditional κ^{++} values of pairs of nodes in the κ^{++} network. In other words, we define notions of influences [13] on the arcs of the network by, when possible, ordering the conditional probabilities of the nodes connecting the arc.

The influences defined using κ^{++} values will not only be identified by their signs, which designates the type of influence, but also by a signed integer that can be used to evaluate their relative strength and to propagate them across the network. Tradeoff resolution comes natural in this case because conflicting signs can be resolved by assessing the magnitude of the influences in conflict. The result is a κ^{++} network capturing the semantics of conditional independence that can be used to propagate beliefs qualitatively and has a built-in conflict-resolution mechanism. In what follows, we define the notion of κ^{++} -based influences.

5.1 κ^{++} -Based Influences

We define four types of influences analogous to those defined over QPNs, positive, negative, zero and unknown. In this section, we restrict our discussion to the first three types of influences and delay the discussion of unknown influences to section 6.

Positive Influences: A binary variable X is said to positively influence another binary variable Y if the degree of conditional surprise associated with Y being true given X is observed, $\kappa^{++}(y|x)$, is lower than that of Y being true given that X is not observed $\kappa^{++}(y|\neg x)$ regardless of the value of any other variable which may directly influence Y . Definition 2 formally states this notion.

Definition 2. $I_{\kappa^{++}}^+(X, Y)$ iff $\kappa^{++}(y|\neg x, W) - \kappa^{++}(y|x, W) > 0$.

W represents any other variable other than X that directly influences Y , which maybe written as $\pi(Y) \setminus X$ (where there is more than one such variable, W is thought of as the conjunction of the possible values of such variables [7]). We denote the influence by a subscript κ^{++} to enforce the idea that they are defined over κ^{++} values and not probability values as in QPNs, and will follow the nomenclature for negative, zero and unknown influences.

It is important to see that the semantics of κ^{++} guarantee that the constraint given by the definition holds, which is what we show in proposition 1.

Proposition 1. *For two binary variables X and Y :*

$$\kappa^{++}(y|x, W) < \kappa^{++}(y|\neg x, W) \rightarrow \kappa^{++}(y|\neg x, W) - \kappa^{++}(y|x, W) \in \mathbb{Z}^+.$$

Proof

There are essentially two cases that result from the inequality $\kappa^{++}(y|x, W) < \kappa^{++}(y|\neg x, W)$:

- Case 1: $\kappa^{++}(y|x, W) \in \mathbb{Z}^-$ and $\kappa^{++}(y|\neg x, W) \in \mathbb{Z}^+$
In this case, the fact that $\kappa^{++}(y|\neg x, W) - \kappa^{++}(y|x, W) \in \mathbb{Z}^+$ is intuitive.
- Case 2: Both $\kappa^{++}(y|x, W)$ and $\kappa^{++}(y|\neg x, W) \in \mathbb{Z}^+$
In this case, the semantics of κ^{++} enforces that for $\kappa^{++}(y|x, W)$ to be less surprising than $\kappa^{++}(y|\neg x, W)$, it must possess a higher magnitude, which will guarantee the result.

Negative Influences. Similarly to positive influences, a binary variable X negatively influences another binary variable Y if the degree of conditional surprise associated with Y being true given X is observed, $\kappa^{++}(y|x)$, is higher than that of Y being true given that X is not observed $\kappa^{++}(y|\neg x)$ regardless of the value of any other variable which may directly influence Y as given in definition 3 below.

Definition 3. $I_{\kappa^{++}(X,Y)}^-$ iff $\kappa^{++}(y|\neg x, W) - \kappa^{++}(y|x, W) < 0$.

Zero Influences are defined in the same manner and is given in definition 4.

Definition 4. $I_{\kappa^{++}(X,Y)}^0$ iff $\kappa^{++}(y|x, W) - \kappa^{++}(y|\neg x, W) = 0$.

Although the influences given in this work are defined over binary variables, the definitions can be naturally extended to multi-valued variables as we have adopted the order of $x > \neg x$ to denote that a true value has a higher value than a false one.

5.2 Influence Propagation

To combine influences, we redefine the $\oplus$ and $\otimes$ operators to accommodate the sign and magnitude properties of the κ^{++} -based influences.

Chained Influences. As done in [3,9,13], we propagate influences along chains using the order of magnitude multiplication operator. Since our influences include sign and magnitude components, these components are handled separately to obtain the net effect on the variables.

The sign portion of the influence is dealt with using sign multiplication as in [13] while the magnitude portion is handled in accordance with the rules of order of magnitude multiplication by adding the corresponding values (since the magnitude represent the difference between two κ^{++} values, which are in essence order of magnitude abstractions of the numerical surprise associated with the variable). The result is presented in the table below.

$\otimes$	$+ve$	$-ve$	0	?
$+u$	$+(u+v)$	$-(u+v)$	0	?
$-u$	$-(u+v)$	$+(u+v)$	0	?
0	0	0	0	0
?	?	?	?	?

Parallel Influences. For influences in parallel chains, the net effect is decided by that of the strongest influence incident on the node. Accordingly, the effect is achieved via the $\oplus$ operator, presented in the table given below.

$\oplus$	$+ve$	$-ve$	0	$?$	
$+u$	$+\min\{u, v\}$	$a)$	$+u$	$?$	$a) = +u, \text{ if } u < v$
$-u$	$b)$	$-\min\{u, v\}$	$-u$	$?$	$= -v, \text{ otherwise}$
0	$+v$	$-ve$	0	$?$	$b) = -u, \text{ if } u < v$
$?$	$?$	$?$	$?$	$?$	$= +ve, \text{ otherwise}$

Combining influences in chains and in parallel can be illustrated via an example such as the one given in the network of figure 2. In the figure, when nodes A and C are received as evidence, the discovery of the influences in the network propagates as follows. The net influence of node A on node E through B is given by $-1 \otimes +2 = -3$ because this influence consists of two influences in a chain whose effect is obtained via the $\otimes$ operator. Similarly, Node D receives evidence from both A and C with the net influence being evaluated as $+4 \oplus -5 = +4$ because node D has two arcs incident on it, which implies that the net effect on D is obtained through the discovery of the combined influences in parallel, which is achieved through the $\oplus$ operator. Similarly, the net influence of A and C on E through D is given by $+4 \otimes +5 = +9$. Finally, node E receives as a net influence $-3 \oplus +9 = -3$. As a result, the net influence of observing A and C on E is a negative one.

Fig. 2. Reasoning with a κ^{++} -based Qualitative Probabilistic Network

6 The Case of Unknown Influences

Because influences only exist when one is able to establish a partial order on the conditional κ^{++} of two variables [7], it is a weak concept that may not be defined when such order does not exist. In this case, it is imperative to resolve to methods at a finer level of resolution. In our approach, since our network are based on κ^{++} values, it is not necessary to go to probabilities and is sufficient to go back to node-based inference on the κ^{++} -level. Although this reduces the efficiency of the inference, it is a necessary last resort when orders are not definable. Moreover, the network retains its qualitative nature as we are still dealing with κ^{++} 's, which are easier to assess than numerical probabilities.

7 Conclusions and Future Work

We presented κ^{++} , a qualitative measure that uses sign and magnitude to designate the degree of surprise associated with an event and used it to construct a qualitative system with two levels of resolution. The resulting system enables the definition of qualitative influences and when not possible, can be used to reason on the node level. The system has its built-in conflict resolution mechanism and is as efficient as previous QPN systems when used with known influences. When the influences are unknown, the system we presented has the advantage of providing the option of reasoning (although on the node, and not arc level) qualitatively without having to resort back to numerical probabilities. Our current work involves an empirical analysis of the κ^{++} calculus and its possible application to study qualitative patterns in gene regulatory networks [8].

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