Towards Safer Obstacle Avoidance for Continuum-Style Manipulator in Dynamic Environments*

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Abstract—The flexibility and dexterity of continuum manipulators in comparison with rigid-link counterparts have become main features behind their recent popularity. Despite of that, the problem of navigation and motion planning for continuum manipulators turns out to be demanding tasks due to the complexity of their flexible structure modelling which in turns complicates the pose estimation. In this paper, we present a real-time obstacle avoidance algorithm for tendon-driven continuum-style manipulator in dynamic environments. The algorithm is equipped with a non-linear observer based on an Extended Kalman Filter to estimate the pose of every point along the manipulator’s body. The overall algorithm works well for a model of a single-segment continuum manipulator in a real-time simulation environment with moving obstacles in the workspace of manipulators.

I. INTRODUCTION

During the last decade, continuum manipulators, mostly inspired by biological properties of snake [1] or octopus [2], have become an emerging technology in robotics. Diverse continuum manipulators design and structures have been explored recently [3], [4]. Researches on modeling and control have also been reported and summarized in [5]. The flexibility and dexterity of such manipulators in comparison with rigid-link counterparts become main features behind their recent popularity. Their inherent bending ability makes them more suitable to be applied in cluttered environments such as surgery, or other medical applications [6].

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Fig. 1. A tendon-driven single-segment arm used as a model in this paper. To measure the tip’s pose and the obstacle’s pose, the NDI Aurora tracker can be adopted.

Despite of all this favorable features, the problem of modelling and pose estimation of continuum manipulators turns out to be daunting tasks due to their highly flexible structure. These will lead to a challenging problem of obstacle avoidance in which the whole body of manipulators needs to be kept safe from colliding with the surrounding environment, especially when the environment of the manipulators changes over time in an unpredictable way. Therefore, to have a real-time pose estimation strategy, combined with reactive obstacle avoidance, is crucial to achieve safe collision-free motion in dynamic environments.

Researches on continuum manipulators pose estimation have been reported in recent years. Most of the works have been designed for snake-like robots [7], [8], [9]. Others used pose estimation for steerable catheter [10] and tendon-driven manipulators [11], [12]. Among other estimation techniques, the Kalman-Filter [7], [9] or a variance [13] have been predominantly employed. From the sensor side, various type of measurement strategy have been reported, such as inertial sensor [8] and visual sensing approach [13], [10], [12]. While the inertial sensor suffers from an accumulated integration error, visual sensing approach can be unreliable for dynamically changing environment. Other emerging approach in this field is the application of electro-magnetic based sensor to track the position and orientation of a point on the robot body [7], [9]. Yet, the current research lacks a further utilization of the pose estimation approach to improve safety of continuum manipulators from collision.

For unpredictable dynamically changing environment, a real-time reactive obstacle avoidance is needed. Several
obstacle avoidance approaches have been implemented in the field of continuum robot. A real-time adaptive motion planning approach was used for octopus-inspired continuum manipulator with moving obstacles [14]. However, the manipulator is assumed to move in planar case only while, at the same time, the algorithm has yet to produce solution in actuator space, limiting its application. An alternative work used an inverse-Jacobian-based planning to yield solutions in actuator space [15]. Yet, the avoid-obstacle approach is designed for a specific workspace inside a tubular environment. A well-known potential field method was successfully implemented for a steerable needle in soft tissue environment [16]. Although having a well-documented local minima problem, the reactive property of this approach makes it perfectly suited for dynamic environments.

In this paper, we propose a real-time obstacle avoidance for tendon-driven continuum manipulator in dynamic environments. The obstacle avoidance algorithm is derived from classical potential field and modified to be used in the kinematic model of continuum manipulators as shown in Figure 1 [17]. To enhance the safety of the whole body of manipulators, the algorithm is equipped with a non-linear observer based on an Extended Kalman Filter to estimate the pose of every point along the manipulator’s body based on the model and measurement data from an electro-magnetic-based position sensor embedded at the tip. To our knowledge, this is the first attempt which further utilizes the pose of every point along the manipulator’s body based on classical potential field and modified to be used in the field of continuum robot. A real-time adaptive motion planning approach was used for octopus-inspired continuum manipulator with moving obstacles [14].

II. MODELLING OF CONTINUUM MANIPULATOR

A. Kinematic Model

Throughout this paper, we assume that the segment of the manipulator behaves like the arc of a circle with a constant radius of curvature. For manipulator whose task space is in $\mathbb{R}^3$, the manipulator’s segment can be specified by three configuration space variables $\mathbf{k} = [\kappa \phi s]^T$ where $\kappa$, $\phi$, and $s$ denote the curvature, rotational deflection angle, and arc length of a segment respectively. The homogeneous transformation matrix describing the pose of the tip with respect to the base, $\mathbf{T}(\mathbf{k}) \in \mathbb{SE}(3)$, is a function of the component of $\mathbf{k}$ as reported in [5].

Further mapping is needed to formulate the relationship between the previous configuration space variables $\mathbf{k}$ and the actuator space variables $\mathbf{q}$. This mapping depends on the manipulator’s design and actuating mechanism. For a tendon-driven continuum manipulator, like the one presented in [17], a segment consists of three tendons whose length can be modified through the DC Motor system. Hence, the actuator space variables is specified by the length of each tendon and can be expressed as $\mathbf{q} = [l_1 l_2 l_3]^T$ where $l_i$ represents the length of tendon-$i$. The relation between the configuration space variables $\mathbf{k}$ and the actuator space variables $\mathbf{q}$ depends on the distance between the centre of the cross-section surface to each tendon $d$ as presented in [5]. $\xi$ is defined as a scalar specifying a unique point located along the backbone of the manipulators. $\xi = 0$ stands for the base position and $\xi = 1$ stands for the tip of manipulator. Thus, any value of $\xi \in [0,1]$ stands for a point locating between the base and the tip along the arc of the segment.

Hence, the complete forward kinematic relation can be expressed in the following form

$$\mathbf{T}(\mathbf{q}, \xi) = \begin{bmatrix} \mathbf{R}(\mathbf{q}, \xi) & \mathbf{p}(\mathbf{q}, \xi) \\ \mathbf{0}_{1 \times 3} & 1 \end{bmatrix} \quad (1)$$

where $\mathbf{R}(\mathbf{q}, \xi) \in \mathbb{SO}(3)$ denotes the rotation matrix and $\mathbf{p}(\mathbf{q}, \xi) \in \mathbb{R}^3$ denotes the position vector of the point along the body of manipulator. A Jacobian of a point in the backbone of manipulator, $\mathbf{J}(\mathbf{q}, \xi) = \frac{\partial \mathbf{p}(\mathbf{q}, \xi)}{\partial \mathbf{q}} \in \mathbb{R}^{3 \times 3}$, relates the velocity vector in the actuator space $\dot{\mathbf{q}}$ to the task space $\dot{\mathbf{p}}$ as follows

$$\dot{\mathbf{p}} = \mathbf{J}(\mathbf{q}, \xi) \dot{\mathbf{q}}. \quad (2)$$

B. State-Space Representation

The state space representation is used to describe the kinematic model of the continuum manipulator. For the kinematic model in Section II-A, the tendon’s length $\mathbf{q} \in \mathbb{R}^3$ is used as a state variable $x$. The actuator that we use in our manipulator is DC motor; each is connected to a tendon which will governs the pose of manipulator. So, our input signal $\mathbf{u}$ in this case is given by the DC motor’s rotational speed which is identical to the tendon length’s rate of change $\mathbf{q} \in \mathbb{R}^3$.

$$\mathbf{x} = \mathbf{q} = [l_1 l_2 l_3]^T, \quad (3)$$

$$\mathbf{u} = \dot{\mathbf{q}} = [l_1 l_2 l_3]^T. \quad (4)$$

For the time sampling of $\Delta t$, the state equation in discrete form can be written as

$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) = \mathbf{x}_k + \Delta \mathbf{u}_k. \quad (5)$$

In this paper, we assume that a 3-DOF electro-magnetic-based tracker is embedded in the tip of manipulator. This sensor will give a position of the tip with respect to the base. The measurement rate is assumed to be identical to the sampling frequency $\frac{1}{\Delta t}$ such that new data is always available at every iteration. Therefore, the output variable $\mathbf{y}$ is denoted by the tip’s position with respect to the base, $\mathbf{p}$.

$$\mathbf{y} = g(\mathbf{x}_k) = \mathbf{p}(\mathbf{q}, \xi) = 1, \quad (6)$$

Thus, the output equation is simply a forward kinematics relation.

III. THE PROPOSED OBSTACLE AVOIDANCE ALGORITHM

A. Observability Analysis of a Linearized Kinematic Model

In this section, we present an observability analysis on a linearized version of the non-linear kinematic model presented in previous section. A linearization is used to check the local observability of a non-linear model. This process is achieved by computing the following matrix $\mathbf{A} = \frac{\partial f(\mathbf{x}_k, \mathbf{u}_k)}{\partial \mathbf{x}_k}$, $\mathbf{B} = \frac{\partial f(\mathbf{x}_k, \mathbf{u}_k)}{\partial \mathbf{u}_k}$, and $\mathbf{C} = \frac{\partial g(\mathbf{x}_k)}{\partial \mathbf{x}_k}$ from the non-linear state equation in (5)-(6).
To check the observability, we only need matrix $A_k$, which turns out to be an identity matrix, and $C_k$, which turns out to be a Jacobian of the tip. Given by

$$A_k = \frac{\partial f(\mathbf{x}_k, \mathbf{u}_k)}{\partial \mathbf{x}_k} = I \in \mathbb{R}^{3 \times 3},$$

$$C_k = \frac{\partial g(\mathbf{x}_k)}{\partial \mathbf{x}_k} = \frac{\partial \mathbf{p}(\mathbf{q}_k, \xi) = 1}{\partial \mathbf{q}_k} = J(\mathbf{q}_k, \xi = 1).$$

The Jacobian matrix $J(\mathbf{q}, \xi)$ is derived as follows

$$J(\mathbf{q}, \xi) = \frac{\partial \mathbf{p}(\mathbf{k}, \xi)}{\partial \mathbf{k}} \frac{\partial \mathbf{k}}{\partial \mathbf{q}} = J_k \mathbf{q}.$$

$$J_k = \begin{bmatrix}
\cos \phi (\cos \kappa \sin \kappa + \cos \kappa - 1) & -\sin \phi (1 - \cos \kappa) \\
\sin \phi (\cos \kappa \sin \kappa + \cos \kappa - 1) & \cos \phi \sin \kappa
\end{bmatrix},$$

$$J_q = \begin{bmatrix}
\frac{3}{2} \frac{d\phi}{d\theta} \frac{l_2}{l_1} & \frac{3}{2} \frac{d\phi}{d\theta} \frac{l_3}{l_1} & -3 \frac{d\phi}{d\theta} \frac{l_1}{l_1} \frac{l_2}{l_1} + \frac{l_3}{l_1}
\end{bmatrix},$$

where $l_1 = l_1 + l_2 + l_3, l_2 = \sqrt{l_1^2 + l_2^2 + l_3^2 - l_1 l_2 - l_1 l_3 - l_2 l_3}$.

An observability matrix $O = [C \quad CA \quad \ldots \quad CA^{n-1}]^T$ can be used to check whether the system with $n$ number of state is observable. For our linearized system, the observability matrix $O \in \mathbb{R}^{9 \times 3}$ is expressed as

$$O = [J(\mathbf{q}_0, \xi = 1) \quad J(\mathbf{q}_1, \xi = 1) \quad J(\mathbf{q}_2, \xi = 1)]^T.$$

The condition for observability is rank($O$) = $n = 3$. From the above equation, the following proposition is concluded.

**Proposition 1:** The linearized system of the non-linear model described in (5)-(6) is observable at all state except the singular state.

**Proof:** From the form of the matrix $O$, we can easily conclude that its rank is equal to the rank of the Jacobian matrix itself, i.e. rank($O$) = rank($J(\mathbf{q}, \xi)$). We can see from Equation (10)-(11) that each Jacobian component (and equally their products) will always have a full rank aside from the singular configuration where $l_1 = l_2 = l_3$ and $\kappa = 0$ or a zero-bending configuration. Hence, if we assume that the manipulator is never at the singular configuration during the whole movement, both $J_k$ and $J_q$ will always have a full rank of $n = 3$, and so does matrix $O$. So, the linearized system is concluded to be observable at every state except the singular configuration.

**B. Pose Estimation**

Knowing that the linearized system is observable, the well-known Extended Kalman Filter (EKF) is designed to estimate the state value, i.e. the tendon’s length. The EKF formulation is given by

$$\dot{\hat{\mathbf{x}}}_{k+1|k} = f(\hat{\mathbf{x}}_k, \mathbf{u}_k),$$

$$\mathbf{P}_{k+1|k} = \mathbf{A}_k \mathbf{P}_{k|k} \mathbf{A}_k^T + \mathbf{Q}_k,$$

$$\mathbf{K}_k = \mathbf{P}_{k+1|k} \mathbf{C}_k^T (\mathbf{C}_k \mathbf{P}_{k+1|k} \mathbf{C}_k^T + \mathbf{R}_k)^{-1},$$

$$\hat{\mathbf{x}}_{k+1|k+1} = \hat{\mathbf{x}}_{k+1|k} + \mathbf{K}_k (\mathbf{y}_k - g(\hat{\mathbf{x}}_{k+1|k})), $$

$$\mathbf{P}_{k+1|k+1} = (I - \mathbf{K}_k \mathbf{C}_k) \mathbf{P}_{k+1|k}.$$

**C. Modified Potential Field**

The obstacle avoidance algorithm employed in this paper is a modification of reactive potential field method [18]. The manipulator’s workspace will be filled with a potential field $F$, derived from a negative gradient of potential function $U$. This will attract the tip towards a defined target position while at the same time avoid the manipulator’s body from collisions. A modification is made such that the standard potential field method can be applied to a kinematic model of a continuum manipulator by using the field $F$ as a manipulator’s task-space velocity, $\mathbf{p}$.

$$\dot{\mathbf{p}} = -\nabla U(\mathbf{p}).$$

The potential function in the task space $U(\mathbf{p})$ used in this paper is identical to the one utilized in [18]. Hence, the corresponding attractive velocity in task space is given by

$$\dot{\mathbf{p}} = -c(\mathbf{p} - \mathbf{p}_d),$$

where $\mathbf{p}_d$ and $c$ stand for a desired position and a positive constant gain respectively. The corresponding repulsive velocity by an obstacle $\mathbf{p}$ can be calculated as follows

$$\dot{\mathbf{p}} = \begin{cases} \eta (\frac{1}{\rho} - \frac{1}{\rho_0}) \frac{1}{\rho^2} \frac{\partial \rho}{\partial \mathbf{p}} & \text{if } \rho < \rho_0 \\ 0 & \text{if } \rho \geq \rho_0 \end{cases},$$

where $\rho = \sqrt{\mathbf{p} - \mathbf{p}_0}$ represents the closest distance from an obstacle to the manipulator’s body. $\eta$ is positive constant, and $\rho_0$ denotes the limit distance of the
In the obstacle avoidance part, a pose estimator is used to estimate the pose of the tip and the PSPs along the chosen path. The potential influence. Figure 2 shows an illustration of the potential function in planar case.

To make the whole body of the manipulator safe from any collision, a number of points, called point subjected to potential (PSP), is picked from the body of the manipulators. The closest PSP to the neighboring environment will be potential function in planar case.

The pose of the tip (\(p(q, \xi = 1)\)) as well as the PSPs (\(p(q, \xi \in [0, 1])\)) are all estimated by the EKF at every iteration as expressed by

\[
\hat{p}_k(\xi) = p(\hat{x}_{kk}, \xi).
\]  

Lastly, the task-space velocity applied in the tip and the chosen PSP are mapped to the corresponding actuator space velocity \(\dot{q}\) via an inverse Jacobian relation given in (2) and fed to the kinematic model as an input signal \(u\). The resulting velocity is given by a linear combination of attractive and repulsive velocity in actuator space as follows

\[
u_k = \dot{q} = J_e^{-1} \dot{p}_p + J_o^{-1} \dot{p}_\theta.
\]

TABLE I

<table>
<thead>
<tr>
<th>Param</th>
<th>Value</th>
<th>Param</th>
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</thead>
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<td>(\sigma)</td>
<td>0.0134 m</td>
<td>(c)</td>
<td>2</td>
</tr>
<tr>
<td>(P_{OD})</td>
<td>(I_{3\times3} m^2)</td>
<td>(\Delta)</td>
<td>2.5 (\times 10^{-2}) s</td>
</tr>
<tr>
<td>(Q)</td>
<td>(10^{-10} I_{3\times3} m^2)</td>
<td>(\eta)</td>
<td>2 (\times 10^{-4})</td>
</tr>
<tr>
<td>(R)</td>
<td>(2.5 \times 10^{-5} I_{3\times3} m^2)</td>
<td>(\rho_0)</td>
<td>0.065 m</td>
</tr>
</tbody>
</table>

In the first simulation, the input signal is chosen to be zero (\(u = 0 \in \mathbb{R}^3\)). Starting from a random initial state value, the EKF receives the measurement data of the tip pose \(y_k\) and starts to estimate the state value as shown in Figure 4a. Using this estimate of tendon length, by employing the forward kinematics, the tip pose is estimated.

The comparison between the true pose with added Gaussian noise (which acts as a measurement data) and the estimated tip pose is shown in Figure 4b. In the second simulation, a circular path is given as a reference for the manipulator’s tip. Using an inverse Jacobian in (2), the varying input signal \(u_k\) is derived. The true state and tip pose measurement data will then vary over time. The results, as depicted in Figure 5a and 5b, show that the EKF can cope with dynamically-changing input signal and tip pose’s measurement.

In the obstacle avoidance part, a pose estimator is used to estimate the pose of the tip and the PSPs along the body of the manipulator to avoid obstacles and reach a desired target. The obstacle is drawn as a black sphere while the tip’s target, assumed to be fixed, is drawn as a red dot. In the first simulation, the obstacle moves at a height close to the tip’s position as depicted in Figure 6. The obstacle avoidance works well, but the main contribution of the whole algorithm can be seen when the obstacle moves at a lower height, such as close to the manipulator’s centre of mass as depicted in Figure 7. This height is chosen to check whether the proposed algorithm is capable of steering not only the tip but also the manipulator’s body in a way that it is safe.

![Diagram](image-url)
from collisions. Figure 7 shows how the obstacle avoidance stage exploits the pose estimation result to make the body of manipulator safer from colliding with moving obstacle.

B. Discussion

The simulation results show that the combined pose estimator and obstacle avoidance algorithm works well to keep a single segment continuum manipulator safe from collision in a real-time scenario with dynamic obstacle. One important assumption that we make is that the obstacle is still in a considerably large distance from the body of manipulator during the initial stage of estimation. This assumption is important because the pose estimate at that time, used in the motion planner stage, still does not match the real pose value due to the rise time needed by the EKF. Otherwise, the planner might produce a repulsive potential field which does not correspond to a real avoid-the-obstacle behavior. This can even disturb the performance of the pose estimator.

The results also illustrate that the proposed method can be implemented in a real manipulator. In that case, an electromagnetic tracker such as a 6-DOF NDI Aurora Tracker can be used to measure the tip’s pose, as shown in Figure 1. Looking at these promising results, the proposed strategy can also be extended easily for a multi-segment continuum manipulator. In this case, we might need more than one electromagnetic tracker to ensure the observability of the entire robot body.

Despite its fast execution time for real-time applications, the potential field itself is only a local reactive planner which does not guarantee completeness in complex environments. Running a global planner approach which produces an initial trajectory with global property on top of the reactive obstacle avoidance can also be explored in the future - this may, though, be too slow to provide suitable paths sufficiently quickly, especially in highly-dynamic environments.

V. CONCLUSIONS AND FUTURE WORKS

In this paper, a real-time pose estimator and obstacle avoidance for a tendon-driven continuum-style manipulator moving in dynamic environments is proposed. To our knowledge, this is the first attempt which combines together pose estimation and obstacle avoidance to improve the safety of a tendon-driven continuum manipulator, minimizing the risk of collisions between the manipulator body and obstacles in the robot workspace. The proposed algorithm is shown to perform in a real-time simulation.

This algorithm can be extended and applied to real manipulators, even in the case of multi-segment continuum arms. In the future, a more advanced dynamic model of the continuum manipulator can also be explored. A combination with a global motion planner approach when operating in more complex environments can also be designed.

REFERENCES

Fig. 6. The movement of the single segments continuum manipulator with a static target position (small red dot) when obstacle (black sphere) moves close to the tip of the manipulator. The order of movement is as follows: upper left picture, upper right picture, lower left picture, and finally lower right picture.


