A no-scale inflationary model to fit them all

This content has been downloaded from IOPscience. Please scroll down to see the full text.

JCAP08(2014)044
(http://iopscience.iop.org/1475-7516/2014/08/044)

View the table of contents for this issue, or go to the journal homepage for more

Download details:

IP Address: 137.73.15.135
This content was downloaded on 20/02/2017 at 16:53

Please note that terms and conditions apply.

You may also be interested in:

Two-field analysis of no-scale supergravity inflation
John Ellis, Marcos A.G. García, Dimitri V. Nanopoulos et al.

Resurrecting quadratic inflation in no-scale supergravity in light of BICEP2
John Ellis, Marcos A.G. García, Dimitri V. Nanopoulos et al.

Starobinsky-like inflation and neutrino masses in a no-scale SO(10) model
John Ellis, Marcos A.G. García, Natsumi Nagata et al.

Calculations of inflaton decays and reheating: with applications to no-scale inflation models
John Ellis, Marcos A.G. García, Dimitri V. Nanopoulos et al.

Moduli inflation in five-dimensional supergravity models
Hiroyuki Abe and Hajime Otsuka

Inflating with large effective fields
C.P. Burgess, M. Cicoli, F. Quevedo et al.

No-scale ripple inflation revisited
Tianjun Li, Zhijin Li and Dimitri V. Nanopoulos

Planck-suppressed operators
Valentin Assassi, Daniel Baumann, Daniel Green et al.

Hybrid inflation in the complex plane
W. Buchmüller, V. Domcke, K. Kamada et al.
A no-scale inflationary model to fit them all

John Ellis\textsuperscript{a,b}, Marcos A.G. García\textsuperscript{c}, Dimitri V. Nanopoulos\textsuperscript{d,e,f} and Keith A. Olive\textsuperscript{c}

\textsuperscript{a}Theoretical Particle Physics and Cosmology Group, Department of Physics, King's College London, WC2R 2LS London, U.K.
\textsuperscript{b}Theory Division, CERN, CH-1211 Geneva 23, Switzerland
\textsuperscript{c}William I. Fine Theoretical Physics Institute, School of Physics and Astronomy, University of Minnesota, 116 Church Street SE, Minneapolis, MN 55455, U.S.A.
\textsuperscript{d}George P. and Cynthia W. Mitchell Institute for Fundamental Physics and Astronomy, Texas A&M University, College Station, 77843 Texas, U.S.A.
\textsuperscript{e}Astroparticle Physics Group, Houston Advanced Research Center (HARC), Mitchell Campus, Woodlands, 77381 Texas, U.S.A.
\textsuperscript{f}Division of Natural Sciences, Academy of Athens, 28 Panepistimiou Avenue, 10679 Athens, Greece

E-mail: john.ellis@cern.ch, garciagarcia@physics.umn.edu, dimitri@physics.tamu.edu, olive@physics.umn.edu

Received May 23, 2014
Accepted June 30, 2014
Published August 19, 2014

Abstract. The magnitude of B-mode polarization in the cosmic microwave background as measured by BICEP2 favours models of chaotic inflation with a quadratic $m^2 \phi^2/2$ potential, whereas data from the Planck satellite favour a small value of the tensor-to-scalar perturbation ratio $r$ that is highly consistent with the Starobinsky $R + \dot{R}^2$ model. Reality may lie somewhere between these two scenarios. In this paper we propose a minimal two-field no-scale supergravity model that interpolates between quadratic and Starobinsky-like inflation as limiting cases, while retaining the successful prediction $n_s \simeq 0.96$.

Keywords: inflation, supersymmetry and cosmology, gravitational waves and CMBR polarization

ArXiv ePrint: 1405.0271
1 Introduction

The theory of inflationary cosmology has received important boosts from the first release of data from the Planck satellite [1] — which confirmed the infrared tilt of the scalar perturbation spectrum $n_s$ [2, 3] expected in slow-roll models of inflation [4–7] — and the discovery by BICEP2 of B-mode polarization fluctuations [8] — which may be interpreted as primordial tensor perturbations with a large ratio $r$ relative to the scalar perturbations, as would be generated in models with a large energy density during inflation. On the other hand, whereas Planck and BICEP2 agree with earlier WMAP data that $n_s \simeq 0.96$, there is tension between the BICEP2 measurement $r = 0.16^{+0.06}_{-0.05}$ (after dust subtraction) and the Planck upper limit on $r$ obtained indirectly from the temperature fluctuations at low multipoles [9]. Theoretically, this tension could be reduced by postulating large running of the scalar spectral index, but this would require a large deviation from the hitherto successful slow-roll inflationary paradigm. Experimentally, it is known that the temperature perturbations in the Planck data [1] lie somewhat below the standard inflationary predictions at low multipoles, and there is a hint of a hemispherical asymmetry, so perhaps the inflationary paradigm does indeed require some modification [10, 11]. On the other hand, verification of the dust model used by BICEP2 is desirable, and other possible sources of foreground contamination such as magnetized dust associated with radio loops need to be quantified [12]. The good news is that the experimental situation should be clarified soon, with additional data from Planck, the Keck Array and other B-mode experiments.

In the mean time, inflationary theorists are having a field day exploring models capable of accommodating the BICEP2 and/or Planck data. Taken at face value, the BICEP2 data are highly consistent with the simplest possible chaotic $m^2\phi^2/2$ potential [13, 14] that predicts $r \simeq 0.16$, whereas the Planck data tend to favour the Starobinsky $R + R^2$ model [15–17] that predicts $r \simeq 0.003$. It seems very likely that experimental measurements of $r$ may settle down somewhere between these limiting cases, so it is interesting to identify models that interpolate between them, while retaining the successful prediction $n_s \simeq 0.96$. Desirable features of such models would include characteristic predictions for other inflationary observables and making connections with particle physics.

With the latter points in mind, we consider the natural framework for formulating models of inflation to be supersymmetry [18–26], specifically local supersymmetry, i.e., supergravity [27, 28]. Moreover, in order to avoid holes in the effective potential with depths
that are $\mathcal{O}(1)$ in Planck units and to address the $\eta$ problem [29, 30], we favour no-scale supergravity [31–33], which has the theoretical advantage that it emerges naturally as the low-energy effective field theory derived from compactified string theory [34]. In the past we [35–40] and others [41–51] have shown how no-scale supergravity with a Wess-Zumino or other superpotential [52] leads naturally to Starobinsky-like inflation, and more recently we [40] and others [53–64] have given examples how quadratic inflation may be embedded in no-scale supergravity.

In this paper we introduce a minimal two-field no-scale supergravity model with a Kähler potential motivated by orbifold compactifications of string theory [65, 66]. The initial condition for the inflaton field has a free parameter that which can be regarded as an angle in the two-field space. Varying this angle, we can interpolate between the quadratic and Starobinsky-like limits: $0.16 \gtrsim r \gtrsim 0.003$,\(^1\) while $n_s \simeq 0.96$ for most values of the interpolating parameter. We follow numerically the evolution of the inflaton in this space, including the possibility of initial values of the inflaton fields that are larger than the minimal values need to obtain sufficient e-folds. The key model predictions are insensitive to the mechanism of supersymmetry breaking, as long as it occurs at some scale much less than the inflaton mass. We illustrate this via a Polonyi example of supersymmetry breaking, and comment on the connection to particle phenomenology in this model.

The structure of this paper is as follows. In section 2 we specify our model, and in section 3 we provide numerical analyses of inflationary scenarios with different initial conditions for the inflaton field, discussing its predictions for $n_s$ and $r$. Then, in section 4 we discuss briefly supersymmetry breaking, and in section 5 we draw some conclusions.

## 2 Specification of the model

The original, minimal no-scale supergravity model has a Kähler potential of the form [31, 32]

$$K = -3 \ln(T + \bar{T}) + \ldots, \quad (2.1)$$

where the dots represent a possible superpotential, terms involving additional matter fields, etc. Subsequent to its discovery, it was shown that no-scale supergravity emerges naturally as the low-energy effective field theory in generic string compactifications [34]. In general, these contain the complex moduli $T_i : i = 1, 2, 3$, and (2.1) becomes

$$K = -\sum_{i=1}^{3} \ln(T_i + \bar{T}_i) + \ldots. \quad (2.2)$$

In the specific case of orbifold compactifications of string, matter fields $\phi$ have non-zero modular weights and appear in a Kähler potential term of the form [65]

$$\Delta K = \frac{|\phi|^2}{\prod_{i=1}^{3}(T_i + \bar{T}_i)}. \quad (2.3)$$

For simplicity, we consider here the case where the ratios of the three orbifold moduli are fixed at a high scale by some unspecified mechanism so that, neglecting irrelevant constants,

\(^1\)One might have thought that [49] would provide a suitable framework for achieving this. However, the Starobinsky-like limit is lost when the Kähler potential is stabilized as discussed in [40, 50, 51].
the Kähler potential may be written in the form [66]:

\[ K = -3 \ln(T + \bar{T}) + \frac{|\phi|^2}{(T + \bar{T})^3}. \] (2.4)

Thus we assume that the field \( \phi \) has modular weight 3, and stress that the special properties of the model we describe below depend on this choice of modular weight. More general constructions of Minkowski and de Sitter vacua were discussed in a similar context in [67], which focused on models where the modular weight of \( \phi \) is 0.

The specification of our no-scale supergravity model of inflation is completed by specifying the superpotential:

\[ W = \sqrt{\frac{3}{4}} \frac{m}{a} \phi(T - a), \] (2.5)

and we identify \( T \) as the chiral (two-component) inflaton superfield. Although the superpotential (2.5) is the same as in the model [52] derived in a realization of \( R + R^2 \) gravity in a SU(2,1)/SU(2) \( \times \) U(1) no-scale model, the Kähler potential (2.4) manifests only the minimal SU(1,1)/U(1) no-scale structure. The scalar potential is derived from the Kähler potential (2.4) and the superpotential (2.5) through

\[ V = e^K (K^{ij} D_i W \bar{D}_j \bar{W} - 3|W|^2), \] (2.6)

where \( D_i W \equiv \partial_i W + K_{ij} W \). We will work in Planck units \( M_P^2 = 8\pi G_N = 1 \). The motion of the scalar field \( \phi \) is constrained by the exponential factor \( e^K \):

\[ V \propto e^{\frac{|\phi|^2}{(T+\bar{T})^3}} \approx e^{(2a)^{-3}|\phi|^2}. \] (2.7)

For \( a \leq 1 \), it can be assumed (and we have verified) that \( \phi \) is rapidly driven to zero at the onset of inflation, and we assume that \( a = 1/2 \) so that \( \phi \) has a canonical kinetic term. For vanishing \( \phi \), the scalar potential takes the simple form

\[ V = \frac{3m^2}{4a^2} |T - a|^2 \] (2.8)

which can be shown to be equivalent to the Starobinsky model along the real direction of the canonical field associated with \( T \) [36, 41, 52]. For other choices of the modular weight \( w \) of \( \phi \), the potential (at \( \phi = 0 \)) is

\[ V = \frac{3m^2}{4a^2} |T - a|^2(T + T^*)^{(w-3)}. \] (2.9)

It is convenient to decompose \( T \) into its real and imaginary parts defined by \( \rho \) and \( \sigma \), respectively, where \( \rho \) is canonical and \( \sigma \) is canonical at the minimum when \( \rho = 0 \):

\[ T = a \left( e^{-\sqrt{\frac{2}{3}} \rho} + i \sqrt{\frac{2}{3}} \sigma \right) \] (2.10)

The scalar component of \( T \) minimizes the potential when \( T = a \), and the resulting Lagrangian is given by

\[ L = \frac{1}{2} \partial_{\mu} \rho \partial^{\mu} \rho + \frac{1}{2} e^{\sqrt{\frac{2}{3}} \rho} \partial_{\mu} \sigma \partial^{\mu} \sigma - \frac{3}{2(5-w) a (3-w)} m^2 e^{\sqrt{\frac{2}{3}} (3-w) \rho} \left( 1 - e^{-\sqrt{\frac{2}{3}} \rho} \right)^2 \]

\[- \frac{1}{2(4-w) a (3-w)} m^2 e^{\sqrt{\frac{2}{3}} (3-w) \rho} \sigma^2. \] (2.11)
In the particular case $w = 3$ this reduces to
\begin{equation}
\mathcal{L} = \frac{1}{2} \partial_\mu \rho \partial^\mu \rho + \frac{1}{2} e^{2\sqrt{\frac{3}{2}} \rho} \partial_\mu \sigma \partial^\mu \sigma - \frac{3}{4} m^2 \left(1 - e^{-\sqrt{\frac{3}{2}} \rho}\right)^2 - \frac{1}{2} m^2 \sigma^2, 
\end{equation}
which is the starting-point for our analysis.

Although this is similar to the Lagrangian for the no-scale model of \cite{49, 52}, it differs in an important way. In that SU(2,1)/SU(2) × U(1) model, the mass term for $\sigma$ contained a coupling to $\rho$ of the form $e^{2\sqrt{\frac{2}{3}} \rho}$. In (2.12), the real and imaginary parts of $T$ are decoupled in the potential and only mix through their kinetic terms (we return later to the effect of this mixing).

The minimum of the effective potential (2.12) in the $(\rho, \sigma)$ plane is located at
\begin{equation}
\rho_0 = \sigma_0 = 0. 
\end{equation}
When $\rho$ is at the minimum, the effective Lagrangian for $\sigma$ is
\begin{equation}
\mathcal{L} = \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m^2 \sigma^2, 
\end{equation}
and we recover the minimal quadratic inflationary model. Conversely, when $\sigma$ is at the minimum, the effective Lagrangian for $\rho$ is
\begin{equation}
\mathcal{L} = \frac{1}{2} \partial_\mu \rho \partial^\mu \rho - \frac{3}{4} m^2 \left(1 - e^{-\sqrt{\frac{3}{2}} \rho}\right)^2, 
\end{equation}
which is of the same form as the Starobinsky model \cite{15}.

Various slices through the potential (2.12) for the canonical choice $a = 1/2$ are shown in figure 1. In the upper left panel we see the characteristic Starobinsky form in the $\rho$ direction (2.15) and the simple quadratic form in the $\sigma$ direction (2.14). In the upper right panel we see that both the real and imaginary parts of $\phi$ are indeed stabilized at zero, the value that was assumed in the upper left panel. The lower left panel shows that the effective potential is well-behaved in the $(\sigma, \text{Re } \phi)$ plane for $\rho = \text{Im } \phi = 0$, and the lower right panel makes the same point for the $(\rho, \text{Re } \phi)$ plane for $\sigma = \text{Im } \phi = 0$. In both cases, the effective potential is identical when the rôles of $\text{Re } \phi$ and $\text{Im } \phi$ are reversed. The Starobinsky form of potential is visible again in the lower right panel of figure 1.

The model described by (2.4) and (2.5) has two dynamical fields, and a correct discussion of their behaviour during inflation requires a more sophisticated analysis than single-field models of inflation \cite{68–72}. We leave such a discussion for future work \cite{73}. Instead, here we modify the Kähler potential (2.4) so as to reduce it to a family of nearly single-field models characterized by an angle $\theta$ in the $(\text{Re } T, \text{Im } T)$ plane defined in figure 2. This is accomplished by introducing a $\theta$-dependent stabilization term of the same general form as introduced in \cite{74}:
\begin{equation}
K = -3 \log \left(T + T^* - c \left(\cos \theta (T + T^*) - 1 - i \sin \theta (T - T^*)\right)^4\right) + \frac{|\phi|^2}{(T + T^*)^3},
\end{equation}
It is clear that, for a large enough coefficient $c$ of the quartic stabilization term, the inflaton trajectory is confined to a narrow valley in field space, much like a bobsleigh confined inside a narrow track.
3 Numerical analysis of the model

The classical motion of the inflaton field for the model (2.12), (2.16), can be numerically calculated solving the equations

\[ H^2 = \frac{1}{3} \left[ K_{ab} \dot{\Psi}^a \dot{\Psi}^b + V(\Psi) \right] = (\dot{N})^2, \]

\[ \ddot{\Psi}^a + 3H \dot{\Psi}^a + \Gamma^a_{bc} \dot{\Psi}^b \dot{\Psi}^c + K_{ab} \frac{\partial V}{\partial \bar{\Psi}^b} = 0, \]

where \( \Psi \equiv (T, \phi) \), \( K_{ab} \) is the Kähler metric, \( \Gamma^a_{bc} \equiv K^{ad} \partial_b K_{cd} \), and \( N \) is the number of e-foldings. The figures that follow show the resulting evolution for the \( T \) and \( \phi \) fields for four different choices of initial conditions.

In order to calculate the values of the scalar tilt \( n_s \) and the tensor-to-scalar ratio \( r \) one cannot use the usual single field formulae, since isocurvature perturbations are generally present when more than one scalar field evolves during inflation. In order to simplify the analysis, we will calculate \( n_s \) and \( r \) assuming that \( \phi \) starts at zero or is driven very quickly to the origin using the techniques in [68–71]. The scalar tilt and the tensor to scalar ratio are then calculated from their definitions

\[ n_s = 1 + \frac{d \log \mathcal{P}_T}{d \log k}, \quad r = \frac{\mathcal{P}_T}{\mathcal{P}_R}. \]
where $P_R$ is the power spectrum of the adiabatic perturbations. The tensor perturbations have the same form as in the single field case, $P_T = \frac{2}{3} H^2 |k=aH|$, since at linear order the scalar field perturbations decouple from vector and tensor perturbations \[75, 76\]. For the scale $k$ we choose a perturbation that leaves the horizon at the start of the last 50 or 60 e-foldings of inflation, assuming that its value corresponds to the Minkowski-like vacuum 10 e-foldings before the scale leaves the Hubble radius.

Figures 3 and 4 display the numerical solution for $\theta = 0$ and $c = 1000,^2$ with the initial conditions $\rho_0 = 0$, $\sigma_0 = 5$ and $\phi_0 = 0$, corresponding to the case of quadratic inflation. The top panel of figure 3 shows the evolution of $\sigma$, which is the inflaton field in this case. The second panel of figure 3 displays the evolution of $\rho$. We note a perturbation of $\rho$ that is due to its coupling with $\sigma$ through the kinetic term (see (2.12)). However, the value of $\rho$ remains small and does not affect substantially the inflationary dynamics of the inflaton field other than allowing for more e-folds of inflation at smaller values of $\sigma$ (which is not canonically normalized when $\rho \neq 0$). The third panel shows that $\text{Re } \phi$ remains zero (and the same is true for $\text{Im } \phi$). Finally, the bottom panel of figure 3 displays the growth of the number of e-folds for this choice of boundary conditions. We find the following values of $n_s$ and $r$ for this case:

\begin{align}
N &= 50: \quad (n_s, r) = (0.951, 0.088), \\
N &= 60: \quad (n_s, r) = (0.959, 0.074). \quad (3.4)
\end{align}

As already commented, small values of $\rho$ are generated during the evolution of the inflaton field, and figure 4 displays the joint evolution of $\rho$ and $\sigma$, where we see a ‘circling the drain’ phenomenon towards the end of inflation. This has the effect of reducing the value of $r$ by a factor of 2.6, without having a significant effect on $n_s$. A smaller reduction factor, and hence a larger value of $r \sim 0.16$, is possible with a different stabilization term in the Kähler potential \[73\].

---

\[2\] As discussed later, we have checked that these and subsequent results are insensitive to the value of $c$. 

---
Figure 3. Numerical solution for the choice $\theta = 0$, $c = 1000$, with initial conditions $\rho_0 = 0$, $\sigma_0 = 5$ and $\phi_0 = 0$: field evolution and e-folds.

Figure 5 displays the numerical solution for $\theta = 0$ and $c = 1000$, with the initial conditions $\rho_0 = 0$, $\sigma_0 = 5$ and $\phi_0 = 0.4 + 0.4i$, corresponding to a modification of the previous case of quadratic inflation, allowing for non-trivial evolution of $\phi$. However, we see in the top panels of figure 5 that both $\text{Re} \phi$ and $\text{Im} \phi$ evolve rapidly to zero, as expected, and $\rho$ behaves similarly to the first case above (third panel). Correspondingly, the behaviour of
the inflaton $\sigma$ is also similar (fourth panel), and the number of e-folds grows in a similar way as in the first case. We find that

\begin{align}
N = 50 : \quad & (n_s, r) = (0.943, 0.077), \\
N = 60 : \quad & (n_s, r) = (0.945, 0.058),
\end{align}

results that are very similar to the first case.

As a third example, displayed in figure 6, we show the evolution of the fields and the number of e-folds for Starobinsky-like initial conditions: $\rho_0 = 6$, $\sigma_0 = 0$, $\phi_0 = 0$, with $\theta = \pi/2$ and $c = 1000$. The top panel of figure 6 shows that, as expected, the inflaton field ($\rho$ in this case) moves slowly initially, but then accelerates rapidly towards zero as it rolls down the steepening potential, before exhibiting damped oscillations. The second, third and fourth panels of figure 6 show that, also as expected, the other fields $\sigma$, $\text{Re}\phi$ and $\text{Im}\phi$ all remain fixed at their minima, and the bottom panel displays the growth in the number of e-folds. We find for this set of initial conditions that

\begin{align}
N = 50 : \quad & (n_s, r) = (0.960, 0.004), \\
N = 60 : \quad & (n_s, r) = (0.967, 0.003).
\end{align}

Finally, figure 7 displays the results of including a small non-zero value of $\phi$ in the initial conditions: $\rho_0 = 6$, $\sigma_0 = 0$, $\phi_0 = 0.001 + 0.001i$, for $\theta = \pi/2$ and $c = 1000$. We choose $|\phi_0| \ll 1$ for $\rho > 1$ because, as seen in the lower right panel of figure 1, the effective potential rises very steeply as a function of $\phi$ when $\rho$ is large. We see in the top two panels that $\rho$ and $\sigma$ evolve almost identically as before, whereas the third and fourth panels show that $\text{Re}\phi$ and $\text{Im}\phi$ exhibit small oscillations as inflation comes to an end. However, this has negligible effect of the growth in the number of e-folds, as seen in the bottom panel of figure 7. We
Figure 5. Numerical solution for $\theta = 0$, $c = 1000$, with initial conditions $\rho_0 = 0$, $\sigma_0 = 5$ and $\phi_0 = 0.4 + 0.4i$.

find for this case that

\[
N = 50 : \quad (n_s, r) = (0.961, 0.004),
\]

\[
N = 60 : \quad (n_s, r) = (0.968, 0.003),
\]

results that are very similar to the previous pure Starobinsky case.
It is clear from the above results that the scalar tilt and the tensor-to-scalar ratio depend on the initial condition for the complex inflaton field $T$. In order to quantify this dependence more generally, we consider general initial conditions in the $(\rho, \sigma)$ plane parametrized by the angle $\theta$, as shown in figure 8, restricting our attention to the case $\phi_0 = 0$. For definiteness
Figure 7. Numerical solution for $\theta = \pi/2$, $c = 1000$, with initial conditions $\rho_0 = 6$, $\sigma_0 = 0$ and $\phi_0 = 0.001 + 0.001i$.

we have considered initial conditions on the curve in the $(\rho, \sigma)$ plane that leads to $N + 10$ e-foldings of inflation, for $N = 50, 60$. The resulting $\theta$ dependences of the inflationary observables $n_s$ and $r$ are displayed in figures 9 and 10. We see in the upper panel of figure 9 that $n_s$ is almost independent of $\theta$, and always within the 68% CL range favoured by WMAP [2, 3],
Figure 8. Parameterization of initial conditions in the $(\rho, \sigma)$ plane for $\phi_0 = 0$ and $c = 200$.

Figure 9. Upper panel: the scalar tilt as function of $\theta$. Lower panel: the tensor-to-scalar ratio as function of $\theta$.

Planck [1] and BICEP2 [8], except for a region centered around $\theta \sim 0.25$. This can be tracked to a sharp enhancement in the curvature power spectrum around these values of $\theta$. In the lower panel of figure 9 we notice that, as expected, $r$ decreases monotonically from the large BICEP2-friendly values $r \gtrsim 0.08$ at $\theta = 0$ to the much smaller Planck-friendly values at $\theta = \pi/2$. We note that the results are symmetric under reflection in the $\rho$ axis: $\theta \rightarrow \pi - \theta$. 
and that initial conditions $0 > \theta > -\pi/2$ (and their reflections) would give larger values of $r$ than quadratic inflation or simply not inflate due to the exponential nature of the potential when $\rho < 0$. We complete this discussion of the $\theta$ dependence of $r$ and $\theta$ by displaying in figure 10 the parametric curve $(n_s(\theta), r(\theta))$.

Up to now, we have fixed the value of $c \gg 1$. Figure 11 shows the dependence of $(n_s, r)$ on the constant $c$ for $\rho_0 = 0$. The initial condition $\sigma_0$ is fixed by the requirement of a total of $N + 10$ e-foldings; it is dependent on the value of $c$. As one can see, for small $c$, $n_s$ deviates significantly from the range favoured by WMAP, Planck and BICEP2, while for $c \gtrsim 10$, it falls within acceptable values. The tensor-to-scalar ratio $r$ rises monotonically and eventually plateaus at the values (3.4) when $c \gtrsim 100$.

So far, we have considered initial conditions for the fields with vanishing time derivatives — a ‘standing start’ — with larger field values than the minimum needed to obtain sufficient e-folds, as reflected in the numbers of e-folds $N = \mathcal{O}(100)$ in figures 2, 4, 5 and 6. In such models, the number of e-folds sufficient to generate the observable universe effectively follow a ‘rolling start’, and the inflationary observables $r$ and $n_s$ may take different values, in general. We display in figure 12 the dependences of the scalar tilt (upper panel) and the tensor-to-scalar ratio (lower panel) on the initial value of $\sigma$.\footnote{The dependence on the initial value of $\rho$ is equally insignificant.} Despite the coupling between the real and imaginary parts of the complex field $T$ which perturbs the field $\rho$, even if it is initially stationary at its minimum $\rho = 0$, there is very little dependence of $r$, on the initial value of $\sigma$, as seen in figure 12. The scalar tilt is also independent of $\sigma$.

4 Supersymmetry breaking

In the above analysis we have neglected the possible effects of supersymmetry breaking, which one would expect, in general, to have little importance for $m_{3/2} \ll m \sim 2 \times 10^{13}$ GeV. There
Figure 11. Upper panel: the scalar tilt as function of the stabilization parameter $c$. Lower panel: the tensor-to-scalar ratio as function of $c$. The initial conditions correspond to $\rho_0 = 0$ and $N_{\text{tot}} = N + 10$, for $N = 50, 60$.

Figure 12. Upper panel: the scalar tilt $r$ as a function of the initial value of $\sigma$. Lower panel: the tensor-to-scalar ratio $n_s$ as a function of the initial value of $\sigma$. Here $\rho_0 = 0$ and $c = 200$.

is certainly plenty of room between this upper limit and the lower limits imposed by Big Bang nucleosynthesis and the absence of supersymmetric particles so far at the LHC.

In principle, one could imagine that adding a constant term, $W_0$, to the superpotential (2.5) would suffice to induce supersymmetry breaking as in [37]. However, doing so shifts
the minimum from $\phi = 0, T = 1/2$, very slightly to a supersymmetry preserving AdS vacuum state [61, 77, 78] as in KKLT [79] and KL [80–82] models. Thus, some form of ‘uplifting’ is necessary and the restoration of a Minkowski (or slightly dS) vacuum with supersymmetry breaking is possible with simple examples of F-term uplifting [67, 83–87].

As a toy example, we consider an unstabilized Polonyi modulus [88] as the source of supersymmetry breaking. Thus, we add to our previous Kähler potential (2.4) the following terms

$$\Delta K = |Z|^2 + \frac{|Y|^2}{(T + \bar{T})^n},$$

(4.1)

where $Z$ is the Polonyi field and the $\{Y\}$ are generic matter fields with unspecified modular weights. We also add to our superpotential (2.5) the terms

$$\Delta W = \mu(Z + \nu) + W(Y).$$

(4.2)

The scalar potential is minimized along the imaginary directions of $T, \phi$ and $Z$ for

$$\text{Im} \phi = \text{Im} Z = \sigma = 0.$$  (4.3)

Along the real directions, the minimization must be performed numerically in general, since it depends on the value of $\mu$. However, for $\mu \ll m$, the conditions

$$\partial_{\text{Re} \phi} V = \partial_{\text{Re} Z} V = \partial_{\rho} V = V = 0$$

(4.4)

for the minimum yield, to second order in $\mu/m$, in Planck units,

$$\text{Re} \phi \simeq \sqrt{3} \frac{\mu}{m}, \quad \rho \simeq \sqrt{6}(1 - \sqrt{3}) \left( \frac{\mu}{m} \right)^2, \quad \text{Re} Z \simeq -1 + \sqrt{3}, \quad \nu \simeq 2 - \sqrt{3}. \quad (4.5)$$

We explicitly see that the shifts in $\text{Re} \phi$ and $\rho$ induced by the parameter $\mu$ are small when $\mu \ll m$. Finally, we recall that since

$$D_Z W \simeq \sqrt{3} \mu,$$

(4.6)

supersymmetry is broken with

$$m_{3/2} \simeq \mu,$$

(4.7)

and the induced masses, trilinear and bilinear terms for the matter fields $\{Y\}$ correspond to (after a constant rescaling of the superpotential $W \rightarrow e^{\sqrt{3}/2}W$)

$$m_0 \simeq m_{3/2}, \quad A_0 \simeq (3 - \sqrt{3}) m_{3/2}, \quad B_0 \simeq (2 - \sqrt{3}) m_{3/2},$$

(4.8)

as in models of minimal supergravity [89]. The dependence of these parameters on the modular weight $n$ appears at higher order in $\mu/m$.

In order to avoid the well-known problems associated with the minimal Polonyi model [90–94], we can extend this analysis by considering stabilization [78, 87, 95–101] of the Polonyi field via the Kähler potential

$$\Delta K = |Z|^2 - \frac{|Z|^4}{\Lambda^2} + \frac{|Y|^2}{(T + \bar{T})^n},$$

(4.9)

with the same superpotential (4.2). In this case, the scalar potential is also minimized along the imaginary directions of $T, \phi$ and $Z$ with $\text{Im} \phi = \text{Im} Z = \sigma = 0$. 

- 15 -
Once again, along the real directions, the minimization must be performed numerically since it is dependent on the values of both $\mu$ and $\Lambda$. For $\Lambda \ll 1$ and $\mu \ll m$, where $m$ is the inflaton mass, the conditions (4.4) can now be solved approximately to give

\[
\text{Re } \phi \simeq \frac{\mu}{m}, \quad \rho \simeq -2\sqrt{\frac{2}{3}} \left( \frac{\mu}{m} \right)^2, \quad \text{Re } Z \simeq \frac{\Lambda^2}{\sqrt{12}}, \quad \nu \simeq \frac{1}{\sqrt{3}} \quad (4.10)
\]

with higher order terms at most $O(\frac{\mu}{m} \Lambda^2)$. Since

\[
D_Z W \simeq \mu, \quad (4.11)
\]

supersymmetry is broken, with

\[
m_{3/2} \simeq \frac{\mu}{\sqrt{3}}, \quad (4.12)
\]

and the induced masses, bilinear and trilinear terms for the matter fields $\{Y\}$ are

\[
m_0 \simeq m_{3/2}, \quad A_0 \simeq 0, \quad B_0 \simeq -m_{3/2}, \quad (4.13)
\]

as in minimal supergravity models with vanishing $A$ terms or models of pure gravity mediation [102, 103]. In this case the dependence on the modular weight $n$ appears at $O(\frac{\mu}{m} \Lambda^2)$.

5 Summary and conclusions

We have proposed in this paper a simple two-field no-scale supergravity model of inflation whose predictions for the scalar-to-tensor perturbation ratio $r$ interpolate between limits that are BICEP2-friendly and Planck-friendly, Starobinsky-like: $0.09 \lesssim r \lesssim 0.003$. As we have shown, this model also yields $n_s \sim 0.96$ in most of field space, as indicated by WMAP, Planck and BICEP2 data. Our model is based on the form of effective low-energy field theory derived from orbifold compactifications of string theory, and can accommodate a Polonyi mechanism for supersymmetry breaking that is suitable for particle phenomenology.

We await with interest confirmation of the B-mode polarization measurement made by BICEP2, and verification that is mainly of primordial origin. In the mean time, we note that over a region of its parameter space our no-scale model yields values of $r$, which may be compatible at the 68% CL with each of the WMAP, Planck and BICEP2 measurements. In that sense, our model may indeed ‘fit them all’.

Acknowledgments

We thank Krzysztof Turczyński for emphasizing the importance of two-field effects in the absence of a stabilizing term in the Kähler potential. The work of J.E. was supported in part by the London Centre for Terauniverse Studies (LCTS), using funding from the European Research Council via the Advanced Investigator Grant 267352 and from the U.K. STFC via the research grant ST/J002798/1. J.E. also thanks Hong-Jian He for his kind hospitality at the Center for High Energy Physics, Tsinghua University, and S.-H. Henry Tye for his kind hospitality at the Institute for Advanced Study, Hong Kong University of Science and Technology, while this work was being carried out. The work of D.V.N. was supported in part by the DOE grant DE-FG03-95-ER-40917 and would like to thank A.P. for inspiration. The work of M.A.G.G. and K.A.O. was supported in part by DOE grant DE-FG02-94-ER-40823 at the University of Minnesota.
References


– 17 –


