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A Novel Stabilization Condition for T-S Polynomial Fuzzy System with Time-delay: A Sum-of-Squares Approach

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Abstract—A novel stabilization problem for T-S polynomial fuzzy system with time-delay is investigated in this paper. Firstly, a polynomial fuzzy controller for T-S polynomial fuzzy system with time-delay is proposed. In addition, based on polynomial Lyapunov–Krasovskii function and the developed polynomial slack variable matrices, a novel stabilization condition for T-S polynomial fuzzy system with time-delay is presented in terms of sum-of-square (SOS) form. Lastly, nonlinear system with time-delay and a well-known T-S fuzzy system with time-delay are illustrated to demonstrate the feasibility and effectiveness of the proposed results.

I. INTRODUCTION

During the past few decades, Takagi-Sugeno (T-S) fuzzy-model-based control method has aroused immense attention in the study and application of engineering and physical problems. The main reason is that T-S fuzzy-model-based control method provides a systematic design procedure for nonlinear systems. By examination of the modeling problem, T-S fuzzy model can approximate any smooth nonlinear function to any degree of accuracy in any convex compact region [1], [2]. For controller design, the fuzzy controller can be designed through the parallel distributed compensation (PDC) scheme [1], [3] to stabilize the T-S fuzzy system. The output of the overall fuzzy controller is a fuzzy blending of each individual linear controller. Furthermore, linear matrix inequality (LMI) technique has been adopted to address the stability and stabilization problems of T-S fuzzy systems. Therefore, T-S fuzzy-model-based control method has been widely applied to investigate complex nonlinear systems [4]–[6].

Despite the significant successes of the T-S fuzzy-model-based control scheme, there still exist some spaces to further improve some existed results. Therefore, T-S fuzzy model has been extended to T-S polynomial fuzzy model presented in recent years. Each T-S fuzzy model can be expressed as a polynomial model to represent the nonlinear model. Different from the existing LMI approach, sum-of-squares (SOS) technique is adopted to explore the stability and stabilization problem of T-S polynomial fuzzy system. The SOS-based stability/stabilization conditions were thus derived in many studies [7], [8] to ensure the stability of T-S fuzzy polynomial system and facilitate the controller synthesis. In addition,

some SOS-based relaxed stability/stabilization conditions are proposed in [9]–[11].

On another research field, it is known that time-delay phenomenon is frequently encountered in many dynamical systems. Recently, in view of possible time-delays in practical systems, analysis and control of fuzzy time-delay systems in T-S fuzzy model have been also addressed in literature [12]–[14]. Therefore, the stability problem for system with time-delay has received much attention and various analysis methods have thus been proposed. For example, in [15], an augmented Lyapunov-Krasovskii function with a triple integral and some augmented vectors was employed to explore the stability problem of T-S fuzzy systems with time-delay. [16] developed a novel augmented Lyapunov function and a delay-dependent stability condition is proposed. In addition, to further reduce the conservativeness of the results, the various constructions of parameter-dependent Lyapunov-Krasovskii functional methods are developed in many studies [15], [17]–[19]. Nevertheless, few studies explore the stability/stabilization problem for T-S polynomial fuzzy system with time-delay. Therefore, based on SOS technique, the stabilization condition for T-S polynomial fuzzy system with time-delay is investigated in this paper.

Motivated by above discussion, this study explores the stabilization problem of T-S polynomial fuzzy time-delay systems by SOS technique. The main contributions of this paper includes: i) a polynomial fuzzy controller for T-S fuzzy system with time-delay is proposed; and ii) a stabilization condition for T-S polynomial fuzzy time-delay systems is proposed in SOS form. The rest of this paper is organized as follows. In Section II, a polynomial fuzzy controller for T-S polynomial fuzzy system with time-delay is proposed. In Section III, based on some slack variable matrices and variable transformations, the delay-dependent stabilization condition for T-S polynomial fuzzy system with time-delay is proposed. In Section IV, a well-known numerical example and a nonlinear model are given to demonstrate that the proposed stabilization condition provides longer allowable delay times than existing ones. Lastly, conclusions are given in Section V.

Notations : $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ represent the set of real n -vector, $m \times n$ matrices. The notation $P > 0 (\geq 0)$ means that matrix P is positive (semi) definite. I_n denotes an identity matrix with dimension n , and $0_{m \times n}$ denotes an $m \times n$ dimension

zero matrix. The symbol $\star$ indicates transposed elements in SOS, which can be obtained via transpose operation, denoted by T . $Diag$ denotes a block diagonal matrix.

II. PRELIMINARIES

Consider the following i^{th} rule of the T-S polynomial fuzzy model with time-delay.

Rule i : **IF** $s_1(t)$ is $M_{i1}(t)$ and $\dots$ and $s_g(t)$ is $M_{ig}(t)$
THEN : $\dot{x}(t) = A_i(x(t))x(t) + A_{di}(x(t))x(t-d(t)) + B_i(x(t))u(t)$ (1)

where $s_1(t), s_2(t), \dots, s_g(t)$ are premise variables, $M_{ij}(t)$ is fuzzy set, $i, j = 1, 2, \dots, r$, where r is the number of fuzzy rule. $A_i(x(t)) \in \mathbb{R}^{n \times n}$, $A_{di}(x(t)) \in \mathbb{R}^{n \times n}$ and $B_i(x(t)) \in \mathbb{R}^{n \times 1}$ are polynomial matrices in $x(t)$. $x(t) = \phi(t)$, $t \in [-d_M, 0]$, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t)$ is the control input, $\phi(t)$ is the given initial value, and d_M is the upper bound of delay time. $d(t)$ is the time-varying delay in the state and satisfies that

$$0 \leq d(t) \leq d_M, \quad 0 \leq \dot{d}(t) \leq d_D. \quad (2)$$

By using a center average defuzzifier, product inference, and a singleton fuzzifier, the overall T-S polynomial fuzzy system with time-delay can be expressed as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r \mu_i(s(t)) (A_i(x(t))x(t) + A_{di}(x(t))x(t-d(t)) \\ &\quad + B_i(x(t))u(t)) / \sum_{i=1}^r \mu_i(s(t)) \\ &= \sum_{i=1}^r h_i(s(t)) (A_i(x(t))x(t) + A_{di}(x(t))x(t-d(t)) \\ &\quad + B_i(x(t))u(t)) \end{aligned} \quad (3)$$

where, $\mu_i(s(t)) = \prod_{j=1}^p M_{ij}(s(t))$, $h_i = \mu_i(s(t)) / \sum_{i=1}^r \mu_i(s(t))$. Two basic properties of $\mu_i(s(t))$ are $\mu_i(s(t)) \geq 0$ and $\sum_{i=1}^r \mu_i(s(t)) > 0$. It is clear that $h_i(s(t)) \geq 0$, and $\sum_{i=1}^r h_i(s(t)) = 1$.

A. Polynomial Fuzzy Controller Design

Based on PDC scheme, the polynomial fuzzy controller for the polynomial fuzzy system with time-delay (1) is designed as follow.

Controller Rule i :

IF $s_1(t)$ is $M_{i1}(t)$ and $\dots$ and $s_g(t)$ is $M_{ig}(t)$
THEN $u(t) = K_i(x)x(t)$ (4)

Substituting (4), the closed-loop polynomial fuzzy system (3) can be represented as:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^r h_i(s(t)) \sum_{j=1}^r h_j(s(t)) ((A_i(x) + B_i(x)K_j(x))x(t) \\ &\quad + A_{di}(x)x(t-d(t))). \end{aligned} \quad (5)$$

In next section, the stabilization problem for (3) will be discussed.

III. MAIN RESULTS

In this section, the stabilization condition for T-S polynomial fuzzy system with time-delay will be explored. The main result is proposed in the following theorem.

Theorem 1: For T-S polynomial fuzzy time-delay system (3), there exists a polynomial fuzzy controller (4) such that closed-loop T-S polynomial fuzzy time-delay system (5) is asymptotically stable if there exists matrices P , symmetric polynomial matrices $\bar{E}(x)$, $\bar{W}(x)$, $\bar{Z}(x) \in \mathbb{R}^{n \times n}$, polynomial matrix $\bar{S}(x) \in \mathbb{R}^{n \times n}$, scalar $\bar{\sigma} > 0$, and $0 \leq d(t) \leq d_M$, $0 \leq \dot{d}(t) \leq d_D$ satisfying the following SOS conditions, where $i, j = 1, \dots, r$, $i < j \leq r$ are nonnegative polynomials for all x .

$$\begin{aligned} \bar{v}_1^T (P - \bar{e}_1(x)I) \bar{v}_1 & \text{ is SOS} \\ \bar{v}_2^T (\bar{E}(x) - \bar{e}_2(x)I) \bar{v}_2 & \text{ is SOS} \\ \bar{v}_3^T (\bar{W}(x) - \bar{e}_3(x)I) \bar{v}_3 & \text{ is SOS} \\ \bar{v}_4^T (\bar{Z}(x) - \bar{e}_4(x)I) \bar{v}_4 & \text{ is SOS} \\ \bar{v}_5^T (\bar{W}(x) - \bar{Z}(x) - \bar{e}_5(x)I) \bar{v}_5 & \text{ is SOS} \\ -\bar{v}_6^T (\bar{\Lambda}_{ii}(x) + \bar{e}_{6ii}(x)I) \bar{v}_6 & \text{ are SOS} \\ -\bar{v}_6^T (\bar{\Lambda}_{ij}(x) + \bar{\Lambda}_{ji}(x) + \bar{e}_{6ij}(x)I) \bar{v}_6 & \text{ are SOS} \end{aligned}$$

where

$$\bar{\Lambda}_{ij}(x) = \begin{bmatrix} \bar{\Lambda}_{ij}(1,1) & \bar{\Lambda}_{ij}(1,2) & \bar{\Lambda}_{ij}(1,3) & 0_{N \times N} \\ \star & \bar{\Lambda}_{ij}(2,2) & \bar{\Lambda}_{ij}(2,3) & \bar{\Lambda}_{ij}(2,4) \\ \star & \star & \bar{\Lambda}_{ij}(3,3) & 0_{N \times N} \\ \star & \star & \star & \bar{\Lambda}_{ij}(4,4) \end{bmatrix} \leq 0$$

$$\bar{\Lambda}_{ij}(1,1) = (A_i(x)P + \star) + (B_i(x)\bar{K}_j(x) + \star) + \bar{E}(x)$$

$$\bar{\Lambda}_{ij}(1,2) = A_{di}(x)P + \bar{S}^T(x)$$

$$\bar{\Lambda}_{ij}(1,3) = \sigma P A_i^T(x) + \sigma \bar{K}_j(x) B_i^T(x)$$

$$\bar{\Lambda}_{ij}(2,2) = -(1-d_D)\bar{E}(x) - (\bar{S}(x) + \star)$$

$$\bar{\Lambda}_{ij}(2,3) = \sigma P A_{di}^T(x), \quad \bar{\Upsilon}_{3ii}(2,4) = \bar{S}(x)$$

$$\bar{\Lambda}_{ij}(3,3) = d_M \bar{W}(x) - 2\sigma P$$

$$\bar{\Lambda}_{ij}(4,4) = -d_M^{-1} \bar{Z}$$

$\bar{v}_i \in \mathbb{R}^n$, $i = 1, \dots, 6$ denote the vectors that are independent of $x(t)$ and $x(t-d(t))$.

Proof: Choose the following Lyapunov-Krasovskii functional candidate

$$V(x) = V_1(x) + V_2(x) + V_3(x) \quad (6)$$

where

$$V_1(x) = x^T(t)P^{-1}x(t)$$

$$V_2(x) = \int_{t-d(t)}^t x^T(\alpha) E x^T(\alpha) d\alpha$$

$$V_3(x) = \int_0^{-d_M} \int_{t+v}^t \dot{x}^T(\alpha) W \dot{x}(\alpha) d\alpha dv.$$

E and W are positive symmetric matrices.

By Newton-Leibniz formula, the time derivative of (6) becomes:

$$\dot{V}(x) = \dot{V}_1(x) + \dot{V}_2(x) + \dot{V}_3(x) \quad (7)$$

where

$$\begin{aligned}
\dot{V}_1(x) &= 2\dot{x}^T(t)P^{-1}x(t) \\
&= 2\sum_{i=1}^r h_i(s(t))\sum_{j=1}^r h_j(s(t))((A_i(x) \\
&\quad + B_i(x)K_j(x))x(t) + A_{di}(x)x(t-d(t)))^T P^{-1}x(t) \\
&= \sum_{i=1}^r h_i(s(t))\sum_{j=1}^r h_j(s(t))(\hat{x}^T(t)((A_i^T(x)P^{-1} + \star) \\
&\quad + (K_j^T(x)B_i^T(x)P^{-1} + \star))\hat{x}(t) \\
&\quad + (\hat{x}^T(t-d(t))(A_{di}^T(x)P^{-1})x(t) + \star)) \quad (8)
\end{aligned}$$

$$\begin{aligned}
\dot{V}_2(x) &= x^T(t)Ex(t) \\
&\quad - (1-d(t))x^T(t-d(t))Ex(t-d(t)) \quad (9)
\end{aligned}$$

$$\begin{aligned}
\dot{V}_3(x) &= d_M \dot{x}^T(t)W\dot{x}(t) \\
&\quad - \int_{t-d_M}^t \dot{x}^T(x(\alpha))W\dot{x}(x(\alpha))d\alpha. \quad (10)
\end{aligned}$$

From (9) and $0 \leq d(t) \leq d_D$, we have

$$\begin{aligned}
\dot{V}_2(x) &\leq x^T(t)Ex(t) \\
&\quad - (1-d_D)x^T(t-d(t))Ex(t-d(t)). \quad (11)
\end{aligned}$$

Similarly, from (10) and $0 \leq d(t) \leq d_M$, yield

$$\begin{aligned}
\dot{V}_3(x) &\leq d_M \dot{x}^T(t)W\dot{x}(t) \\
&\quad - \int_{t-d(t)}^t \dot{x}^T(x(\alpha))W\dot{x}(x(\alpha))d\alpha. \quad (12)
\end{aligned}$$

Furthermore, the following matrix equations are held

$$\begin{aligned}
\Pi_5 &= 2x^T(t-d(t))S(x)(x(t) - x(t-d(t))) \\
&\quad - \int_{t-d(t)}^t \dot{x}(\alpha)d\alpha = 0 \quad (13)
\end{aligned}$$

$$\begin{aligned}
\Pi_6 &= d_M x^T(t-d(t))S(x)W^{-1}S^T(x)x(t-d(t)) \\
&\quad - \int_{t-d_M}^t x^T(t-d(t))S(x)W^{-1}S^T(x) \\
&\quad \times x(t-d(t))d\alpha = 0 \quad (14)
\end{aligned}$$

$$\begin{aligned}
\Pi_7 &= \sum_{i=1}^r h_i(s(t))\sum_{j=1}^r h_j(s(t))2(\dot{x}^T(t)\sigma P^{-1}((A_i(x) \\
&\quad + B_i(x)K_j(x))x(t) \\
&\quad + A_{di}(x)x(t-d(t)) - \dot{x}(t))) = 0. \quad (15)
\end{aligned}$$

According to (14) and $0 \leq d(t) \leq d_M$, we can obtain

$$\begin{aligned}
\Pi_5 &\leq d_M x^T(t-d(t))S(x)W^{-1}S^T(x)x(t-d(t)) \\
&\quad - \int_{t-d(t)}^t x^T(t-d(t))S(x)W^{-1}S^T(x) \\
&\quad \times x(t-d(t))d\alpha = 0 \quad (16)
\end{aligned}$$

Concluding by above results with $Z < W$, we can get

$$\begin{aligned}
\dot{V}(x) &\leq \sum_{i=1}^r h_i(s(t))\sum_{j=1}^r h_j(s(t))(\Upsilon_{ij}(x) \\
&\quad + d_M x^T(t-d(t))S(x)W^{-1}S(x)x(t-d(t)) \\
&\quad - \int_{t-d(t)}^t ((\dot{x}^T(t)W + x^T(t-d(t))S(x))W^{-1} \\
&\quad \times (W\dot{x}(t) + S^T(x)x(t-d(t))))ds) \\
&\leq \sum_{i=1}^r h_i(s(t))\sum_{j=1}^r h_j(s(t))(\Upsilon_{1ij}(x) \\
&\quad + d_M x^T(t-d(t))S(x)Z^{-1}S^T(x)x(t-d(t))) \\
&= \sum_{i=1}^r h_i^2(s(t))(\Upsilon_{1ii}(x) + d_M x^T(t-d(t))S(x)Z^{-1} \\
&\quad \times S^T(x)x(t-d(t))) + \sum_{i < j}^r h_i(s(t))h_j(s(t))(\Upsilon_{1ij}(x) \\
&\quad + \Upsilon_{1ji}(x) + d_M x^T(t-d(t))S(x)Z^{-1} \\
&\quad \times S^T(x)x(t-d(t))) \quad (17)
\end{aligned}$$

where

$$\begin{aligned}
\Upsilon_{1ij}(x) &= d_M \dot{x}^T(t)W\dot{x}(t) + x^T(t)Ex(t) - (1-d_D) \\
&\quad \times x^T(t-d(t))Ex(t-d(t)) + x^T(t)(P^{-1}(A_i(x) \\
&\quad + B_i(x)K_j(x)) + \star)x(t) + (x^T(t)P^{-1}A_{di}(x) \\
&\quad \times x(t-d(t)) + \star) + 2x^T(t-d(t))S(x)(x(t) \\
&\quad - x(t-d(t))) + 2\sigma\dot{x}^T(t)P^{-1}(A_i(x)x(t) \\
&\quad + A_{di}(x)x(t-d(t)) - \dot{x}(t)) \\
&= \eta^T(t)\Upsilon_{2ij}(x)\eta(t)
\end{aligned}$$

$$\begin{aligned}
\Upsilon_{2ij}(x) &= \begin{bmatrix} \Upsilon_{2ij}(1,1) & \Upsilon_{2ij}(1,2) & \Upsilon_{2ij}(1,3) \\ \star & \Upsilon_{2ij}(2,2) & \Upsilon_{2ij}(2,3) \\ \star & \star & \Upsilon_{2ij}(3,3) \end{bmatrix} \\
\Upsilon_{2ij}(1,1) &= (P^{-1}A_i(x) + \star) + (P^{-1}B_i(x)K_j(x) + \star) + E \\
\Upsilon_{2ij}(1,2) &= (P^{-1}A_{di}(x) + S^T(x)) \\
\Upsilon_{2ij}(1,3) &= \sigma A_i^T(x)P^{-1} + \sigma K_j^T(x)B_i(x)P^{-1} \\
\Upsilon_{2ij}(2,2) &= -(1-\sigma_D)E - (S(x) + \star) \\
\Upsilon_{2ij}(2,3) &= \sigma A_{di}^T(x)P^{-1} \\
\Upsilon_{2ij}(3,3) &= d_M W - 2\sigma P^{-1} \\
\eta^T(t) &= [x^T(t) \quad x^T(t-d(t)) \quad \dot{x}^T(t)]^T
\end{aligned}$$

In order for $\dot{V}(x) \leq 0$, the following conditions should be satisfied

$$\begin{aligned}
&\eta^T(t)\Upsilon_{2ii}(x)\eta(t) + d_M x^T(t-d(t))S(x)Z^{-1} \\
&\quad \times S^T(x)x(t-d(t)) \leq 0 \quad (18)
\end{aligned}$$

$$\begin{aligned}
&\eta^T(t)(\Upsilon_{2ij}(x) + \Upsilon_{2ji}(x))\eta(t) + d_M x^T(t-d(t))S(x) \\
&\quad \times Z^{-1}S^T(x)x(t-d(t)) \leq 0. \quad (19)
\end{aligned}$$

Firstly, let us consider (18). Applying Schur complement

formula to (18), we can obtain

$$\Upsilon_{3ii}(x) = \begin{bmatrix} \Upsilon_{2ii}(1,1) & \Upsilon_{2ii}(1,2) & \Upsilon_{2ii}(1,3) & 0_{N \times N} \\ \star & \Upsilon_{2ii}(2,2) & \Upsilon_{2ii}(2,3) & S(x) \\ \star & \star & \Upsilon_{2ii}(3,3) & 0_{N \times N} \\ \star & \star & \star & -d_M^{-1}Z \end{bmatrix} \leq 0. \quad (20)$$

Pre- and post-multiplying both sides of (20) with $\text{Diag}[P \ P \ P \ P]$ and its transpose, and defining $PE(x)P = \bar{E}(x)$, $PWP = \bar{W}(x)$, $PZP = \bar{Z}$, $PS(x)P = \bar{S}(x)$ and $K_j(x)P = \bar{K}_j(x)$, yield

$$\bar{\Lambda}_{ii}(x) = \begin{bmatrix} \bar{\Lambda}_{ii}(1,1) & \bar{\Lambda}_{ii}(1,2) & \bar{\Lambda}_{ii}(1,3) & 0_{N \times N} \\ \star & \bar{\Lambda}_{ii}(2,2) & \bar{\Lambda}_{ii}(2,3) & \bar{\Lambda}_{ii}(2,4) \\ \star & \star & \bar{\Lambda}_{ii}(3,3) & 0_{N \times N} \\ \star & \star & \star & \bar{\Lambda}_{ii}(4,4) \end{bmatrix} \leq 0$$

where

$$\begin{aligned} \bar{\Lambda}_{ii}(1,1) &= (A_i(x)P + \star) + (B_i(x)\bar{K}_i(x) + \star) + \bar{E}(x) \\ \bar{\Lambda}_{ii}(1,2) &= A_{di}(x)P + \bar{S}^T(x) \\ \bar{\Lambda}_{ii}(1,3) &= \sigma P A_i^T(x) + \sigma \bar{K}_i(x) B_i^T(x) \\ \bar{\Lambda}_{ii}(2,2) &= -(1 - d_D)\bar{E}(x) - (\bar{S}(x) + \star) \\ \bar{\Lambda}_{ii}(2,3) &= \sigma P A_{di}^T(x), \quad \bar{\Upsilon}_{3ii}(2,4) = \bar{S}(x) \\ \bar{\Lambda}_{ii}(3,3) &= d_M \bar{W}(x) - 2\sigma P \\ \bar{\Lambda}_{ii}(4,4) &= -d_M^{-1}\bar{Z} \end{aligned}$$

Through a similar procedure, we can obtain $\bar{\Lambda}_{ij}(x) + \bar{\Lambda}_{ji}(x) \leq 0$ from (19). Therefore, if SOS conditions in Theorem 1 are satisfied, then $\dot{V}(x) \leq 0$. In addition, it is easy to see that if SOS conditions in Theorem 1 are satisfied for $x \neq 0$ then $\bar{\Lambda}_{ii}(x) < 0$ and $\bar{\Lambda}_{ij}(x) + \bar{\Lambda}_{ji}(x) < 0$. Thus, $\dot{V}(x) < 0$ for all $x \neq 0$. Hence, the closed-loop polynomial fuzzy time-delay system (5) is asymptotically stable. Moreover, if the SOS conditions are satisfied for $x \neq 0$, then $\dot{V}(x) < 0$. This completes the proof. ■

IV. SIMULATION

In this section, a numerical example and a nonlinear model are provided to illustrate the effectiveness of the previously developed methods.

Example 1:

Consider the following T-S fuzzy time-delay system given in [18], [20]–[26].

Rule 1 : IF $x_1(t)$ is $M_1(x(t))$

THEN $\dot{x}(t) = A_1x(t) + A_{d1}x(t - d(t)) + B_1u(t)$

Rule 2 : IF $x_1(t)$ is $M_2(x(t))$

THEN $\dot{x}(t) = A_2x(t) + A_{d2}x(t - d(t)) + B_2u(t)$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \end{aligned}$$

TABLE I. COMPARISONS AMONG VARIOUS METHODS

Method	Maximum allowed d_2
Theorem 2 of [20]	0.152
Theorem 1 of [21]	0.230
Corollary 2 of [22]	0.257
Theorem 1 of [22]	0.266
Theorem 1 of [23]	0.491
Theorem 3 of [18]	0.842
Theorem 1 of [18]	0.900
Theorem 2 of [24]	1.050
Corollary 2 of [25]	1.095
Theorem 1 of [26]	1.380
Theorem 2 of [27]	1.432
Theorem 2 of [28]	1.482
Theorem 1 of this paper	1.5003

The membership functions are defined as: $M_1(x(t)) = 1/(1 + \exp(-2x_1(t)))$, $M_2(x(t)) = 1 - M_1(x(t))$. By Theorem 1 with $\sigma = 0.1$, we can obtain the maximum allowable delay time 1.5003 sec. with P and controllers $\bar{K}_1$ and $\bar{K}_2$ where $K_1 = \bar{K}_1 P^{-1}$ and $K_2 = \bar{K}_2 P^{-1}$.

Table I shows a comparison of the maximal allowable delay times obtained in ten studies. It can be seen from Table I that the proposed stabilization conditions can provide the largest allowable delay time. The state responses for T-S fuzzy time-delay system with $x(0) = [2 \ 1]^T$ is shown in Fig. 1. Simulation results show that the trajectories of fuzzy time-delay system converge to the equilibrium state after some transient times.

Example 2:

Consider the following nonlinear system.

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) + x_1^2(t) + x_1^2(t)x_2(t) + x_2(t) + x_1(t)u(t) \\ \dot{x}_2(t) &= -\sin x_1(t) - x_2(t) + u(t) \end{aligned}$$

To illustrate the proposed results, we assume that the system $x_2(t)$ is perturbed by time-delay and the delayed model is given as

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) + x_1^2(t) + ax_1^2(t)x_2(t) + ax_2(t) + x_1(t)u(t) \\ &\quad + (1 - a)x_1^2(t)x_2(t - d(t)) + (1 - a)x_2(t - d(t)) \\ \dot{x}_2(t) &= -\sin x_1(t) - ax_2(t) - (1 - a)x_2(t - d(t)) + u(t) \end{aligned} \quad (21)$$

The constant a is the retarded coefficient, which satisfies the conditions $a \in [0, 1]$. The limits 1 and 0 correspond to no delay term and to a complete delay term, respectively. In this example, we assume $a = 0.7$. By the fuzzy modeling method in [7], (21) can be expressed as the following T-S polynomial fuzzy model with time-delay.

Rule 1 : IF $x_1(t)$ is 0

THEN $\dot{x}(t) = A_1(x)x(t) + A_{d1}(x)x(t - d(t)) + B_1(x)u(t)$

Rule 2 : IF $x_1(t)$ is $-\pi$ or π

THEN $\dot{x}(t) = A_2(x)x(t) + A_{d2}(x)x(t - d(t)) + B_2(x)u(t)$

Fig. 1. State response for T-S fuzzy system with time-delay

Fig. 2. State responses of Example 2.

where

$$A_1(x) = \begin{bmatrix} -1 + x_1(t) & ax_1^2(t) + a \\ -1 & -a \end{bmatrix}, \quad B_1(x) = \begin{bmatrix} x_1(t) \\ 1 \end{bmatrix},$$

$$A_2(x) = \begin{bmatrix} -1 + x_1(t) & ax_1^2(t) + a \\ 0.2172 & -a \end{bmatrix}, \quad B_2(x) = \begin{bmatrix} x_1(t) \\ 1 \end{bmatrix},$$

$$A_{d1}(x) = A_{d2}(x) = \begin{bmatrix} 0 & (1-a)x_1^2 + (1-a) \\ 0 & -(1-a) \end{bmatrix}.$$

The delay time $d(t) = 1 + \sin(t)$. The membership functions are defined as: $M_1(x(t)) = (\sin(x_1(t)) + 0.2172x_1(t))/(1.2172x_1(t))$, $M_2(x(t)) = 1 - M_1(x(t))$. From $d(t)$, it can be seen that $d_M = 2$ and $d_D = 1$. By utilizing the MATLAB SOS toolbox to solve the convex optimization problem in Theorem 1 with $\sigma = 1$, the polynomial fuzzy controller are obtained as follows

$$P = \begin{bmatrix} 1.797 \times 10^{-8} & 8.645 \times 10^{-10} \\ 8.645 \times 10^{-10} & 2.912 \times 10^{-8} \end{bmatrix}$$

$$K_1(x) = [-0.8049x_1 \quad -0.5800x_1]$$

$$K_2(x) = [-1.4259x_1 \quad -0.5641x_1].$$

Fig. 2 shows the state responses of (21) under initial values $[2 \ 1]^T$, and it can be seen that the trajectories of the trajectories converge to equilibrium state after few seconds.

By utilizing the polynomial fuzzy controller, the state responses for the closed-loop nonlinear system (21) with initial values $[2 \ 1]^T$, $[1 \ 2]^T$, $[2 \ -1]^T$, $[1 \ -2]^T$, $[-2 \ -1]^T$, $[-1 \ -2]^T$, $[-1 \ 2]^T$ and $[-2 \ 1]^T$ are shown in Fig. 3.

V. CONCLUSION

In this paper, a stabilization problem for T-S polynomial fuzzy system with time-delay is investigated. By utilizing PDC concept, a polynomial fuzzy controller for T-S polynomial fuzzy system with time-delay is proposed. In addition, based on some slack variable matrices, a delay-dependent stabilization condition for T-S polynomial fuzzy system with time-delay is proposed. Furthermore, the results can be formulated in terms of SOS forms. A nonlinear system with time-delay and a well-known numerical example are given to illustrate the effectiveness of the proposed methods.

Fig. 3. Behaviors in $x_1 - x_2$ plane (with feedback).

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