Abstract—A novel stabilization problem for T-S polynomial fuzzy system with time-delay is investigated in this paper. Firstly, a polynomial fuzzy controller for T-S polynomial fuzzy system with time-delay is proposed. In addition, based on polynomial Lyapunov–Krasovskii function and the developed polynomial slack variable matrices, a novel stabilization condition for T-S polynomial fuzzy system with time-delay is presented in terms of sum-of-squares (SOS) form. Lastly, nonlinear system with time-delay and a well-known T-S fuzzy system with time-delay are illustrated to demonstrate the feasibility and effectiveness of the proposed results.

I. INTRODUCTION

During the past few decades, Takagi-Sugeno (T-S) fuzzy-model-based control method has aroused immense attention in the study and application of engineering and physical problems. The main reason is that T-S fuzzy-model-based control method provides a systematic design procedure for nonlinear systems. By examination of the modeling problem, T-S fuzzy model can approximate any smooth nonlinear function to any degree of accuracy in any convex compact region [1], [2]. For controller design, the fuzzy controller can be designed through the parallel distributed compensation (PDC) scheme [1], [3] to stabilize the T-S fuzzy system. The output of the overall fuzzy controller is a fuzzy blending of each individual linear controller. Furthermore, linear matrix inequality (LMI) technique has been adopted to address the stability and stabilization problems of T-S fuzzy systems. Therefore, T-S fuzzy-model-based control method has been widely applied to investigate complex nonlinear systems [4]–[6].

Despite the significant successes of the T-S fuzzy-model-based control scheme, there still exist some spaces to further improve some existed results. Therefore, T-S fuzzy model has been extended to T-S polynomial fuzzy model presented in recent years. Each T-S fuzzy model can be expressed as a polynomial model to represent the nonlinear model. Different from the existing LMI approach, sum-of-squares (SOS) technique is adopted to explore the stability and stabilization problem of T-S polynomial fuzzy system. The SOS-based stability/stabilization conditions were thus derived in many studies [7], [8] to ensure the stability of T-S fuzzy polynomial system and facilitate the controller synthesis. In addition, some SOS-based relaxed stability/stabilization conditions are proposed in [9]–[11].

On another research field, it is known that time-delay phenomenon is frequently encountered in many dynamical systems. Recently, in view of possible time-delays in practical systems, analysis and control of fuzzy time-delay systems in T-S fuzzy model have been also addressed in literature [12]–[14]. Therefore, the stability problem for system with time-delay has received much attention and various analysis methods have thus been proposed. For example, in [15], an augmented Lyapunov-Krasovskii function with a triple integral and some augmented vectors was employed to explore the stability problem of T-S fuzzy systems with time-delay. [16] developed a novel augmented Lyapunov function and a delay-dependent stability condition is proposed. In addition, to further reduce the conservativeness of the results, the various constructions of parameter-dependent Lyapunov-Krasovskii functional methods are proposed in many studies [15], [17]–[19]. Nevertheless, few studies explore the stability/stabilization problem for T-S polynomial fuzzy system with time-delay. Therefore, based on SOS technique, the stabilization condition for T-S polynomial fuzzy system with time-delay is investigated in this paper.

Motivated by above discussion, this study explores the stabilization problem of T-S polynomial fuzzy time-delay systems by SOS technique. The main contributions of this paper includes: i) a polynomial fuzzy controller for T-S fuzzy system with time-delay is proposed; and ii) a stabilization condition for T-S polynomial fuzzy time-delay systems is proposed in SOS form. The rest of this paper is organized as follows. In Section II, a polynomial fuzzy controller for T-S polynomial fuzzy system with time-delay is proposed. In Section III, based on some slack variable matrices and variable transformations, the delay-dependent stabilization condition for T-S polynomial fuzzy system with time-delay is proposed. In Section IV, a well-known numerical example and a nonlinear model are given to demonstrate that the proposed stabilization condition provides longer allowable delay times than existing ones. Lastly, conclusions are given in Section V.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ represent the set of real $n$-vector, $m \times n$ matrices. The notation $P > 0$ ($\geq 0$) means that matrix $P$ is positive (semi) definite. $I_n$ denotes an identity matrix with dimension $n$, and $0_{m \times n}$ denotes an $m \times n$ dimension
zero matrix. The symbol $\star$ indicates transposed elements in SOS, which can be obtained via transpose operation, denoted by $^T$. $\text{Diag}$ denotes a block diagonal matrix.

II. Preliminaries

Consider the following $i^{th}$ rule of the T-S polynomial fuzzy model with time-delay.

**Rule $i$:** IF $s_1(t)$ is $M_{i1}(t)$ and \cdots and $s_g(t)$ is $M_{ig}(t)$ THEN $\dot{x}(t) = A_i(x(t))x(t) + A_{di}(x(t))x(t - d(t)) + B_i(x(t))u(t)$

(1)

where $s_1(t)$, $s_2(t)$, \cdots, $s_g(t)$ are premise variables, $M_{ij}(t)$ is fuzzy set, $i, j = 1, 2, \cdots, r$, where $r$ is the number of fuzzy rule. $A_i(x(t)) \in \mathbb{R}^{n \times n}$, $A_{di}(x(t)) \in \mathbb{R}^{n \times n}$ and $B_i(x(t)) \in \mathbb{R}^{n \times 1}$ are polynomial matrices in $x(t)$. $x(t) = \phi(t), t \in [-d_M, 0]$, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t)$ is the control input, $\phi(t)$ is the given initial value, and $d_M$ is the upper bound of delay time. $d(t)$ is the time-varying delay in the state and satisfies that $0 \leq d(t) \leq d_M$, $0 \leq d(t) \leq d_D$.

(2)

By using a center average defuzzifier, product inference, and a singleton fuzzifier, the overall T-S polynomial fuzzy system with time-delay can be expressed as

$$\dot{x}(t) = \sum_{i=1}^r \mu_i(s(t))(A_i(x(t))x(t) + A_{di}(x(t))x(t - d(t)) + B_i(x(t))u(t))/\sum_{i=1}^r \mu_i(s(t))$$

$$= \sum_{i=1}^r h_i(s(t))(A_i(x(t))x(t) + A_{di}(x(t))x(t - d(t)) + B_i(x(t))u(t))$$

(3)

where, $\mu_i(s(t)) = \prod_{j=1}^r M_{ij}(s(t))$, $h_i = \mu_i(s(t))/\sum_{i=1}^r \mu_i(s(t))$. Two basic properties of $\mu_i(s(t))$ are $\mu_i(s(t)) \geq 0$ and $\sum_{i=1}^r \mu_i(s(t)) = 1$. It is clear that $h_i(s(t)) \geq 0$, and $\sum_{i=1}^r h_i(s(t)) = 1$.

A. Polynomial Fuzzy Controller Design

Based on PDC scheme, the polynomial fuzzy controller for the polynomial fuzzy system with time-delay (1) is designed as follow.

**Controller Rule $i$:** IF $s_1(t)$ is $M_{i1}(t)$ and \cdots and $s_g(t)$ is $M_{ig}(t)$ THEN $u(t) = K_i(x(t))x(t)$

(4)

Substituting (4), the closed-loop polynomial fuzzy system (3) can be represented as:

$$\dot{x}(t) = \sum_{i=1}^r h_i(s(t)) \sum_{i=j}^r h_j(s(t))((A_i(x) + B_i(x)K_j(x))x(t) + A_{di}(x)x(t - d(t))).$$

(5)

In next section, the stabilization problem for (3) will be discussed.

III. Main Results

In this section, the stabilization condition for T-S polynomial fuzzy system with time-delay will be explored. The main result is proposed in the following theorem.

**Theorem 1:** For T-S polynomial fuzzy time-delay system (3), there exists a polynomial fuzzy controller (4) such that closed-loop T-S polynomial fuzzy time-delay system (5) is asymptotically stable if there exists matrices $P$, symmetric polynomial matrices $E(x), W(x), Z(x) \in \mathbb{R}^{n \times n}$, polynomial matrix $S(x) \in \mathbb{R}^{n \times n}$, scalar $\sigma > 0$, and $0 \leq d(t) \leq d_M$, $0 \leq d(t) \leq d_D$ satisfying the following SOS conditions, where $i, j = 1, \cdots, r$, $i < j \leq r$ are nonnegative polynomials for all $x$.

$$\bar{v}_1^T(P - \bar{e}_1(x)I)\bar{v}_1$$

is SOS

$$\bar{v}_2^T(E(x) - \bar{e}_2(x)I)\bar{v}_2$$

is SOS

$$\bar{v}_3^T(W(x) - \bar{e}_3(x)I)\bar{v}_3$$

is SOS

$$\bar{v}_4^T(Z(x) - \bar{e}_4(x)I)\bar{v}_4$$

is SOS

$$\bar{v}_5^T(W(x) - Z(x) - \bar{e}_5(x)I)\bar{v}_5$$

is SOS

$$-\bar{v}_6^T(\bar{A}_{ij}(x) + \bar{e}_{6ij}(x)I)\bar{v}_6$$

are SOS

$$-\bar{v}_6^T(\bar{A}_{ij}(x) + \bar{A}_{ji}(x) + \bar{e}_{6ij}(x)I)\bar{v}_6$$

are SOS

where

$$\bar{A}_{ij}(1, 1) \hspace{1cm} \bar{A}_{ij}(1, 2) \hspace{1cm} \bar{A}_{ij}(1, 3) \hspace{1cm} 0_{N \times N}$$

$$\begin{bmatrix}
\bar{A}_{ij}(2, 1) & \bar{A}_{ij}(2, 2) & \bar{A}_{ij}(2, 3) & 0_{N \times N} \\
\star & \bar{A}_{ij}(3, 2) & \bar{A}_{ij}(3, 3) & 0_{N \times N} \\
\star & \star & \bar{A}_{ij}(4, 4) & 0_{N \times N} \\
\star & \star & \star & \bar{A}_{ij}(4, 4)
\end{bmatrix} \leq 0$$

$$\bar{A}_{ij}(1, 1) = (A_i(x)P + \star) + (B_i(x)\bar{K}_j(x) + \star) + \bar{E}(x)$$

$$\bar{A}_{ij}(1, 2) = A_{di}(x)P + S^T(x)$$

$$\bar{A}_{ij}(1, 3) = (\sigma P A_i^T(x) + \sigma \bar{K}_j(x)B_i^T(x)$$

$$\bar{A}_{ij}(2, 2) = -(1 - d_M)\bar{E}(x) - (\bar{S}(x) + \star)$$

$$\bar{A}_{ij}(2, 3) = \sigma P A_i^T(x), \bar{Y}_{3ii}(2, 4) = \bar{S}(x)$$

$$\bar{A}_{ij}(3, 3) = d_M W(x) - 2\sigma P$$

$$\bar{A}_{ij}(4, 4) = -d_M^1 Z$$

$$\bar{v}_i \in \mathbb{R}^n, i = 1, \ldots, 6$$

denote the vectors that are independent of $x(t)$ and $x(t - d(t))$.

**Proof:** Choose the following Lyapunov-Krasovskii functional candidate

$$V(x) = V_1(x) + V_2(x) + V_3(x)$$

(6)

where

$$V_1(x) = x^T(t)P^{-1}x(t)$$

$$V_2(x) = \int_{t-d(t)}^t x^T(\alpha)Exx^T(\alpha)d\alpha$$

$$V_3(x) = \int_0^{d_M} \int_{t+d}^{t+d} x^T(\alpha)Wxx^T(\alpha)d\alpha$$

$E$ and $W$ are positive symmetric matrices.

By Newton-Leibniz formula, the time derivative of (6) becomes:

$$\dot{V}(x) = \dot{V}_1(x) + \dot{V}_2(x) + \dot{V}_3(x)$$

(7)
where

\[ \dot{V}_1(x(t)) = 2\dot{x}^T(t)P_1(t)x(t) \]
\[ = 2\sum_{i=1}^r h_i(s(t))\sum_{j=1}^r h_j(s(t))(A_i(x) + B_i(x)K_j(x)x(t) + A_{di}(x)x(t - d(t)))^T P_1(t)x(t) \]
\[ = \sum_{i=1}^r h_i(s(t))\sum_{j=1}^r h_j(s(t))(\dot{A}_i^T(x)P_1(t) + *) \]
\[ + (K_i^T(x)B_i^T(x)P_1(t) + *)\dot{x}(t) \]
\[ + (\dot{A}_i^T(t - d(t))(A_{di}^T(x)P_1(t) + *)) \]
\[ \dot{V}_2(x) = x^T(t)E x(t) \]
\[ = -(1 - d(t))x^T(t - d(t))E x(t - d(t)) \]
\[ \dot{V}_3(x) = d_M \dot{x}^T(t)W \dot{x}(t) \]
\[ = \int_{t-d(t)}^t \dot{x}^T(x(\alpha))W \dot{x}(x(\alpha))d\alpha. \]

From (9) and 0 \leq d(t) \leq d_D, we have

\[ \dot{V}_2(x) \leq x^T(t)E x(t) \]
\[ = -(1 - d(t))x^T(t - d(t))E x(t - d(t)). \]

Similarly, from (10) and 0 \leq d(t) \leq d_M, yield

\[ \dot{V}_3(x) \leq d_M \dot{x}^T(t)W \dot{x}(t) \]
\[ = \int_{t-d(t)}^t \dot{x}^T(x(\alpha))W \dot{x}(x(\alpha))d\alpha. \]

Furthermore, the following matrix equations are held

\[ \Pi_5 = 2x^T(t - d(t))S(x)x(t - x(t - d(t))) \]
\[ - \int_{t-d(t)}^t \dot{x}(\alpha)d\alpha = 0 \]
\[ \Pi_6 = d_M x^T(t - d(t))S(x)W^{-1}S^T(x)x(t - d(t)) \]
\[ - \int_{t-d(t)}^t x^T(t - d(t))S(x)W^{-1}S^T(x)x(t - d(t))d\alpha = 0 \]
\[ \Pi_7 = \sum_{i=1}^r h_i(s(t))\sum_{j=1}^r h_j(s(t))2(\dot{x}^T(t)\sigma P^{-1}(A_i(x) + B_i(x)K_j(x)x(t) + A_{di}(x)x(t - d(t))) - \dot{x}(t)) = 0. \]

According to (14) and 0 \leq d(t) \leq d_M, we can obtain

\[ \Pi_5 \leq d_M x^T(t - d(t))S(x)W^{-1}S^T(x)x(t - d(t)) \]
\[ - \int_{t-d(t)}^t x^T(t - d(t))S(x)W^{-1}S^T(x)x(t - d(t))d\alpha = 0 \]

Concluding by above results with Z < W, we can get

\[ \dot{V}(x) \leq \sum_{i=1}^r h_i(s(t))\sum_{j=1}^r h_j(s(t))(\Pi_{ij}(x) \]
\[ + d_M x^T(t - d(t))S(x)W^{-1}S(x)x(t - d(t)) \]
\[ - \int_{t-d(t)}^t (\dot{x}^T(t)W + x^T(t - d(t))S(x))W^{-1} \]
\[ \times (W \dot{x}(t) + S^T(x)x(t - d(t)))d\alpha \]
\[ \leq \sum_{i=1}^r h_i(s(t))\sum_{j=1}^r h_j(s(t))(\Pi_{ij}(x) \]
\[ + d_M x^T(t - d(t))S(x)Z^{-1}S^T(x)x(t - d(t)) \]
\[ = \sum_{i=1}^r h_i^2(s(t))((\Pi_{ij}(x) + d_M x^T(t - d(t))S(x)Z^{-1} \]
\[ \times S^T(x)x(t - d(t))) + \sum_{i<j}^r h_i(s(t))h_j(s(t))(\Pi_{ij}(x) \]
\[ + \Pi_{ji}(x) + d_M x^T(t - d(t))S(x)Z^{-1} \]
\[ \times S^T(x)x(t - d(t))) \]

where

\[ \Pi_{ij}(x) = \text{d}_M \dot{x}^T(t)W \dot{x}(t) + x^T(t)E x(t) - (1 - d_D) \]
\[ \times x^T(t - d(t))E x(t - d(t)) + x^T(t)(P^{-1}(A_i(x) + B_i(x)K_j(x))x(t) + \sigma A_{di}^T(x)x(t) + \sigma A_{di}^T(x)x(t - d(t))) \]
\[ + A_{di}(x)x(t - d(t)) - \dot{x}(t) \]
\[ = \eta^T(t)\eta(t) \]

According to (14) and 0 \leq d(t) \leq d_M, we can obtain

\[ \Pi_5 \leq \text{d}_M x^T(t - d(t))S(x)W^{-1}S^T(x)x(t - d(t)) \]
\[ - \int_{t-d(t)}^t x^T(t - d(t))S(x)W^{-1}S^T(x)x(t - d(t))d\alpha = 0 \]

In order for \dot{V}(x) \leq 0, the following conditions should be satisfied

\[ \eta^T(t)\Pi_{ii}(x)\eta(t) + \text{d}_M x^T(t - d(t))S(x)Z^{-1} \]
\[ \times S^T(x)x(t - d(t)) \leq 0 \]

Firstly, let us consider (18). Applying Schur complement
formula to (18), we can obtain

$$
\begin{align*}
\mathbf{Y}_{3ii}(x) &= \begin{bmatrix}
\mathbf{Y}_{2ii}(1,1) & \mathbf{Y}_{2ii}(1,2) & \mathbf{Y}_{2ii}(1,3) & 0_{N \times N} \\
* & \mathbf{Y}_{2ii}(2,2) & \mathbf{Y}_{2ii}(2,3) & \mathbf{S}(x) \\
* & * & \mathbf{Y}_{2ii}(3,3) & 0_{N \times N} \\
* & * & * & -d_M^{-1}\mathbf{Z}
\end{bmatrix} \\
& \leq 0.
\end{align*}$$

\quad \text{(20)}

Pre- and post-multiplying both sides of (20) with \( \text{Diag}[P P P P] \) and its transpose, and defining \( PE(x)P = \overline{E}(x), PWP = W(x), PZP = \overline{Z}, PS(x)P = \overline{S}(x) \) and \( K_j(x)P = \overline{K}_j(x) \), yield

$$
\mathbf{A}_{ii}(x) = \begin{bmatrix}
\overline{A}_{ii}(1,1) & \overline{A}_{ii}(1,2) & \overline{A}_{ii}(1,3) & 0_{N \times N} \\
* & \overline{A}_{ii}(2,2) & \overline{A}_{ii}(2,3) & \overline{A}_{ii}(2,4) \\
* & * & \overline{A}_{ii}(3,3) & 0_{N \times N} \\
* & * & * & \overline{A}_{ii}(4,4)
\end{bmatrix} \leq 0
$$

where

\begin{align*}
\overline{A}_{ii}(1,1) &= (A_i(x)P + \star) + (B_i(x)\overline{K}_i(x) + \star) + \overline{E}(x) \\
\overline{A}_{ii}(1,2) &= A_{di}(x)P + S\overline{E}(x) \\
\overline{A}_{ii}(1,3) &= \sigma \mathbf{P}^T_i(x) + \sigma \mathbf{K}_i(x)B_i^T(x) \\
\overline{A}_{ii}(2,2) &= -((1-d_D)\overline{E}(x) - (\overline{S}(x) + \star) \\
\overline{A}_{ii}(2,3) &= \sigma \mathbf{P}^T_i(x), \quad \overline{Y}_{3ii}(2,4) = \overline{S}(x) \\
\overline{A}_{ii}(3,3) &= d_M \overline{W}(x) - 2\sigma \mathbf{P} \\
\overline{A}_{ii}(4,4) &= -d_M^{-1}\mathbf{Z}
\end{align*}

Through a similar procedure, we can obtain \( \overline{A}_{ij}(x) + \overline{A}_{ji}(x) \leq 0 \) from (19). Therefore, if SOS conditions in Theorem 1 are satisfied, then \( \overline{V}(x) \leq 0 \). In addition, it is easy to see that if SOS conditions in Theorem 1 are satisfied for \( x \neq 0 \) then \( \overline{A}_{ii}(x) < 0 \) and \( \overline{A}_{ij}(x) + \overline{A}_{ji}(x) < 0 \). Thus, \( \overline{V}(x) < 0 \) for all \( x \neq 0 \). Hence, the closed-loop polynomial fuzzy timedelay system (5) is asymptotically stable. Moreover, if the SOS conditions are satisfied for \( x \neq 0 \), then \( \overline{V}(x) < 0 \). This completes the proof.

IV. SIMULATION

In this section, a numerical example and a nonlinear model are provided to illustrate the effectiveness of the previously developed methods.

Example 1:

Consider the following T-S fuzzy time-delay system given in [18], [20]–[26].

**Rule 1:** IF \( x_1(t) \) is \( M_1(x(t)) \) THEN \( \dot{x}(t) = A_1 x(t) + A_{d1} x(t - d(t)) + B_1 u(t) \)

**Rule 2:** IF \( x_1(t) \) is \( M_2(x(t)) \) THEN \( \dot{x}(t) = A_2 x(t) + A_{d2} x(t - d(t)) + B_2 u(t) \)

where

\[
A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \end{bmatrix}, \\
A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.
\]

Table I. Comparisons among various methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Maximum allowed ( d_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Theorem 2 of [20]</td>
<td>0.152</td>
</tr>
<tr>
<td>Theorem 1 of [21]</td>
<td>0.230</td>
</tr>
<tr>
<td>Corollary 2 of [22]</td>
<td>0.257</td>
</tr>
<tr>
<td>Theorem 1 of [22]</td>
<td>0.266</td>
</tr>
<tr>
<td>Theorem 1 of [23]</td>
<td>0.491</td>
</tr>
<tr>
<td>Theorem 3 of [18]</td>
<td>0.842</td>
</tr>
<tr>
<td>Theorem 1 of [18]</td>
<td>0.900</td>
</tr>
<tr>
<td>Theorem 2 of [24]</td>
<td>1.050</td>
</tr>
<tr>
<td>Corollary 2 of [25]</td>
<td>1.095</td>
</tr>
<tr>
<td>Theorem 1 of [26]</td>
<td>1.380</td>
</tr>
<tr>
<td>Theorem 2 of [27]</td>
<td>1.432</td>
</tr>
<tr>
<td>Theorem 2 of [28]</td>
<td>1.482</td>
</tr>
<tr>
<td>Theorem 1 of this paper</td>
<td>1.5003</td>
</tr>
</tbody>
</table>

The membership functions are defined as: \( M_1(x(t)) = 1/(1 + \exp(-2x_1(t))) \), \( M_2(x(t)) = 1 - M_1(x(t)) \). By Theorem 1 with \( \sigma = 0.1 \), we can obtain the maximum allowable delay time 1.5003 sec. with \( P \) and controllers \( K_1 \) and \( K_2 \) where \( K_1 = K_1 P^{-1} \) and \( K_2 = K_2 P^{-1} \).

Table I shows a comparison of the maximal allowable delay times obtained in ten studies. It can be seen from Table I that the proposed stabilization conditions can provide the largest allowable delay time. The state responses for T-S fuzzy time-delay system with \( x(0) = [2 \ 1]^T \) is shown in Fig. 1. Simulation results show that the trajectories of fuzzy time-delay system converge to the equilibrium state after some transient times.

**Example 2:**

Consider the following nonlinear system.

\[
\begin{align*}
\dot{x}_1(t) &= -x_1(t) + x_1^2(t) + x_2^2(t) x_2(t) + x_2(t) + x_1(t) u(t) \\
\dot{x}_2(t) &= -\sin x_1(t) - x_2(t) + u(t)
\end{align*}
\]

To illustrate the proposed results, we assume that the system \( x_2(t) \) is perturbed by time-delay and the delayed model is given as

\[
\begin{align*}
\dot{x}_1(t) &= -x_1(t) + x_1^2(t) + ax_1^2(t) x_2(t) + ax_2(t) + x_1(t) u(t) \\
&+ (1-a)x_1^2(t) x_2(t - d(t)) + (1-a) x_2(t - d(t)) \\
\dot{x}_2(t) &= -\sin x_1(t) - ax_2(t) - (1-a) x_2(t - d(t)) + u(t)
\end{align*}
\]

(21)

The constant \( a \) is the retarded coefficient, which satisfies the conditions \( a \in [0, 1] \). The limits 1 and 0 correspond to no delay term and to a complete delay term, respectively. In this example, we assume \( a = 0.7 \). By the fuzzy modeling method in [7], (21) can be expressed as the following T-S polynomial fuzzy model with time-delay.

**Rule 1:** IF \( x_1(t) = 0 \) THEN \( \dot{x}(t) = A_1 x(t) + A_{d1} x(t - d(t)) + B_1 u(t) \)

**Rule 2:** IF \( x_1(t) = -\pi \) or \( \pi \) THEN \( \dot{x}(t) = A_2 x(t) + A_{d2} x(t - d(t)) + B_2 u(t) \)
where
\[
A_1(x) = \begin{bmatrix}
-1 + x_1(t) & ax_1^2(t) + a \\
-1 & -a
\end{bmatrix},
B_1(x) = \begin{bmatrix}
x_1(t) \\
1
\end{bmatrix},
A_2(x) = \begin{bmatrix}
-1 + x_1(t) & ax_1^2(t) + a \\
0.2172 & -a
\end{bmatrix},
B_2(x) = \begin{bmatrix}
x_1(t) \\
1
\end{bmatrix},
A_d1(x) = A_d2(x) = \begin{bmatrix}
0 & (1 - a)x_1^2 + (1 - a) \\
0 & -(1 - a)
\end{bmatrix}.
\]

The delay time \(d(t) = 1 + \sin(t)\). The membership functions are defined as: \(M_1(x(t)) = (\sin(x_1(t)) + 0.2172x_1(t))/(1.2172x_1(t))\), \(M_2(x(t)) = 1 - M_1(x(t))\). From \(d(t)\), it can be seen that \(d_M = 2\) and \(d_D = 1\). By utilizing the MATLAB SOS toolbox to solve the convex optimization problem in Theorem 1 with \(\sigma = 1\), the polynomial fuzzy controller are obtained as follows
\[
P = \begin{bmatrix}
1.797 \times 10^{-8} & 8.645 \times 10^{-10} \\
8.645 \times 10^{-10} & 2.912 \times 10^{-8}
\end{bmatrix},
K_1(x) = \begin{bmatrix}
-0.8049x_1 \\
-0.5800x_1
\end{bmatrix},
K_2(x) = \begin{bmatrix}
-1.4259x_1 \\
-0.5641x_1
\end{bmatrix}.
\]

Fig. 2 shows the state responses of (21) under initial values [2 1]^T, and it can be seen that the trajectories of the trajectories converge to equilibrium state after few seconds.


V. CONCLUSION

In this paper, a stabilization problem for T-S polynomial fuzzy system with time-delay is investigated. By utilizing PDC concept, a polynomial fuzzy controller for T-S polynomial fuzzy system with time-delay is proposed. In addition, based on some slack variable matrices, a delay-dependent stabilization condition for T-S polynomial fuzzy system with time-delay is proposed. Furthermore, the results can be formulated in terms of SOS forms. A nonlinear system with time-delay and a well-known numerical example are given to illustrate the effectiveness of the proposed methods.

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