Performance Enhancement of Cognitive Radio Networks via Multi-Power Level Transmission

Khomejani, Shabnam

Awarding institution: King's College London

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Performance Enhancement of Cognitive Radio Networks via Multi-Power Level Transmission

by

Shabnam Khomejani

This dissertation is submitted for the degree of

Doctor of Philosophy

at

King’s Collage London

May 2017
I would like to dedicate this thesis to my loving parents
Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements.

by

Shabnam Khomejani

May 2017
Acknowledgements

I would like to express my special gratitude and appreciation to my supervisor Professor Arumugan Nallanathan who supported me during my Ph.D. His advice and encouragements motivated me through.

I would also like to specially thank my secondary supervisor Professor Hamid Aghvami for his continued support, patience and valuable insights. I have learned a lot from him during the past years.

I also acknowledge Dr. Huan X. Nguyen for the fruitful and excellent meetings and comments. I am extremely grateful for his kind help throughout my research years.

I also like to use this opportunity to express my appreciation for my friends, colleagues and the academic staff at the Department of Informatics, formerly known as the Centre for Telecommunication Research (CTR). Their motivational assistance during my Ph.D. years contributed greatly to the fulfilment of my research.

My special appreciation and love go to my parents who cared for me the most in the past years. A simple ‘thank you’ is far too little in return for their exceptional and continued support and understanding which helped me in striving towards this academic goal of mine.
Abstract

The ubiquitous trend to the “next generation communication” is a symbol of the communication arena’s need for independence, efficiency and flexibility. Cognitive radio was introduced in the late 1990s as a concept to improve efficiency of spectrum use. Cognitive users would sense spectral holes and exploit unused spectrum. The original strategy suggested that spectrum being employed by licensed cognitive users should be strictly avoided in order to ensure no interference to the licensed users. However, this strict “white space” approach is also inherently spectrally inefficient.

This Ph.D. thesis focuses its contributions to researching into technologies and solutions intended for cognitive radio networks that may lead to improvements in the coming wireless communication generations. Due to different technical challenges for spectrum sensing, power allocation and security in the physical layer, the contribution of this thesis is the proposition of a realistic scenario for cognitive radio systems. In this thesis, we suggest several strategies that offer limited interference to primary users while significantly improving the throughput of cognitive users. A novel cognitive radio scheme is proposed which exhibits improved achievable throughput levels and spectrum sensing capabilities compared to the conventional opportunistic spectrum access cognitive radio networks studied so far. The proposed cognitive radio strategy can overcome the sensing throughput trade-off problem in the opportunistic spectrum access cognitive radio systems. In addition, it also provide its cognitive users with increased levels of average achievable throughput.
Furthermore, in most of the studies on power allocation, schemes for the secondary users have been proposed that limit such users’ average achievable rates. Therefore, to address this problem and also to maximize the achievable system throughput, we propose an algorithm for multi-power allocation. The scheme proves to be faster than the conventional algorithms and it increases the average achievable throughput. Finally, there are several challenges for cryptographic approaches of several layers’ in the protocol stack. These include the private key management complexity and the key transmission security issues. As a result, the issue of physical layer security, as an alternative for cryptographic approaches has attracted researchers and scientists. The security parameters in the proposed cognitive radio scheme are therefore analysed.
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<tr>
<td>ITU</td>
<td>International Telecommunication Union</td>
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<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
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<td>QoS</td>
<td>Quality of Service</td>
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<td>CR</td>
<td>Cognitive Radio</td>
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<tr>
<td>OSA</td>
<td>Opportunistic Spectrum Access</td>
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<tr>
<td>HetNets</td>
<td>Heterogeneous Networks</td>
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<tr>
<td>MAC</td>
<td>Media Access Control</td>
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<tr>
<td>PHY</td>
<td>Physical Layer</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>ASR</td>
<td>Achievable Secrecy Rate</td>
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<td>CDF</td>
<td>Cumulative Distribution Function</td>
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<td>ETSI</td>
<td>European Telecommunications Standards Institute</td>
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<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
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<tr>
<td>SU</td>
<td>Secondary User</td>
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<td>PU</td>
<td>Primary User</td>
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<td>Abbreviation</td>
<td>Description</td>
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<tr>
<td>SNR</td>
<td>Signal-to-Noise-Ratio</td>
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<tr>
<td>LO</td>
<td>Local Oscillator</td>
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<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
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<tr>
<td>SS</td>
<td>Spectrum Sharing</td>
</tr>
<tr>
<td>SSS</td>
<td>Sensing-based Spectrum Sharing</td>
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<tr>
<td>OLA</td>
<td>Overlay Spectrum Access</td>
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<tr>
<td>CSI</td>
<td>Channel State Information</td>
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<tr>
<td>P2P</td>
<td>Point to Point</td>
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<tr>
<td>SIMO</td>
<td>Single-Input Multiple-Output</td>
</tr>
<tr>
<td>LTE-A</td>
<td>LTE-Advanced</td>
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<tr>
<td>DoD</td>
<td>Department of Defence</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>LTR</td>
<td>Likelihood Radio Test</td>
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<tr>
<td>CSCG</td>
<td>Circularly Symmetric Complex Gaussian</td>
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<td>PT</td>
<td>Primary Transmitter</td>
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<tr>
<td>PR</td>
<td>Primary Receiver</td>
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<tr>
<td>ST</td>
<td>Secondary Transmitter</td>
</tr>
<tr>
<td>SR</td>
<td>Secondary Receiver</td>
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<tr>
<td>PKI</td>
<td>Primary Key Infrastructure</td>
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<td>PER</td>
<td>Primary Exclusive Region</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<td>OFDM</td>
<td>Orthogonal Frequency-Division Multiplexing</td>
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<td>ECC</td>
<td>Electronic Communications Committee</td>
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<td>Ofcom</td>
<td>Office of Communications</td>
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Chapter 1

Introduction

1.1 Background and motivation

The quick proliferation of wireless devices and applications of all types in the communication industry in the recent past has been explosive. This includes the sheer number of legacy and new users, wide and local area networks, commercial and government users and communication and sensing applications. According the International Telecommunication Union (ITU), the actual mobile traffic in 2010 is more than five times greater than an official forecast made by this organization in 2005 [1]. Moreover, Cisco forecasts that the global mobile data traffic will grow nearly eightfold between the years 2015 and 2020 [2]. As the demand for spectrum continues to increase, it will become increasingly difficult to meet cognitive user Quality of Service (QoS) demands through conventional spectrum policies. Such existing schemes operate based on the assignment of solid and exclusive use of spectrum bands to particular applications. On the other hand, the communication market conditions aggravate the spectrum scarcity associated with old fashioned conventional regulatory frameworks. This is while the legacy management regime is inflexible making it increasingly difficult to transition spectrum resources to new users and technologies. Therefore, spectrum regulators are resorting to wide infrastructural and fundamental changes to boost their services. To
1.1 Background and motivation

meet this objective, a paradigm is needed which shares the spectrum more intensively and
dynamically among all users. This includes incumbent users that have access rights to
spectrum and the other users that seek access to additional spectrum.

The cognitive radio (CR), as one of the improvements in designing next-generation
wireless communications systems in the recent past is becoming increasingly popular. The
main objectives of the cognitive radio systems are:

- Improving the utilization of the frequency spectrum;

- Achieving high reliability and efficiency in wireless communications.

Dynamic spectrum access technique, as shown in Fig.1.1, is employed to obtain the best
available spectrum through its cognitive capability. In the last decade, different aspects of
cognitive radio networks are being implemented in wireless systems. Meanwhile, noticeable
study efforts have been made to address its challenges. Dynamic spectrum access based
cognitive radio networks pose many considerable challenges in the design, analysis, optimi-
mization, development and deployment that involve both regulatory and technology [3–5].

The main technical challenges include spectrum sensing, multiple access, resource allo-
cation and transmission power control techniques for cognitive radio systems. Additionally,
measurements and statistical modelling of radio spectrum usage, self-configuration, dis-
tributed learning, adaptation, coexistence and cooperation techniques as well as development
of network control and management protocols are amongst such challenges. Further to the
above, routing in multi-hop cognitive radio networks, cross-layer optimization, modelling
the emergent systems’ behaviour, security and robustness in cognitive radio systems are
also identified areas needing thorough research and refinement [6–10]. Moreover, from a
spectrum regulatory perspective, there are fundamental research issues demanding attention.
Such areas are: developments in dynamic spectrum access policies that lead to efficient
spectrum use, maintaining the quality of service and development of certification of license holders in cognitive radio networks [11]. In addition to a huge number of research papers that have appeared in the literature, a number of unique issues on cognitive radio have been organized and addressed in both IEEE and non-IEEE journals.

Traditionally, different design criteria of dynamic spectrum access schemes have been designed based on two major views: horizontal and vertical spectrum access. In the horizontal spectrum access case, all users in the cognitive radio network have equal regulatory statuses. This is not the case in the vertical case. In fact, in vertical spectrum access, the unlicensed users opportunistically access the spectrum without affecting the licensed users’ performances. Dynamic spectrum access in vertical spectrum access is referred to as opportunistic spectrum access (OSA) which is a method for the unlicensed user to operate within a frequency band that is designated to the licensed user.
1.2 Scope of the work

Generally, in cognitive radio systems, through the spectrum sensing phase, the cognitive (unlicensed) user obtains the information of the target radio spectrum such as the current activity of the licensed user. Then, through the spectrum management function phase, the spectrum sensing information is extracted to analyse the spectrum opportunities and decisions are made on spectrum access. If changes are detected in the status of the target spectrum, the operating frequency bands for the unlicensed user can be switched by the spectrum mobility function. In conventional spectrum sensing schemes, to determine the activity of licensed users and the spectrum, the target frequency band is sensed periodically. In fact, a transceiver detects an unused spectrum and the related information. This information can include the band, location and the method of accessing the spectrum such as access duration and transmission power.

1.2 Scope of the work

Cognitive radio provides a novel method to address the spectrum scarcity problem and spectrum inefficiency issues in wireless networks. The recent improvements in regulatory domains and the developments in spectrum policy have allowed the cognitive radio technology to support a variety of applications [12] such as broadband cellular, Ad-hoc networks, smart grids [13] and heterogeneous networks (HetNets). Although the cognitive radio technology exploits alternative spectrum opportunities through dynamic spectrum access capabilities, it causes various challenges for existing communication systems. This is due to high fluctuations in the available spectrum, diverse QoS requirements and overall system throughput [14]. Moreover, some of the main factors of cognitive radio networks such as the time and location varying spectrum availability and the dynamic network topology make it incompatible with the existing wireless protocols or current platforms. Contrary to this challenging background, a large volume of research has been performed in the cognitive radio area in the last decade [15]. However, the most recent research works emphasise only
on the upper layers of the protocol stack and their related issues. For example, research on
the upper layer design such as the Network and the Media Access Control (MAC) layers
is well mature in terms of energy efficiency, reliability and scalability. On the other hand,
emerging applications of cognitive radio technology within various types of networks are
growing at a great rate [16], and create new research areas for future wireless communication
networks.

Therefore, novel solutions related to the lower layers are still needed to meet key re-
quirements of the cognitive radio networks. Spectrum sensing and power allocation as the
front line functionalities of the physical layer (PHY) are amongst such requirements. Proper,
efficient and accurate treatments are needed for the above to facilitate enhancements on the
opportunistic cognitive radio network architecture.

1.3 Contributions and organization of the thesis

In this dissertation, we argue that to the specific mindset we have when we consider the
research and propose our scheme for cognitive radio networks. Reviewing the literature
suggests that the majority of the research in the cognitive radio arena is theoretical and make
unrealistic assumptions that may not hold in practice. Although these works are arguably
important and can provide scientific intuitions, they are not particularly effective in driving
practical implementation, commercial adaptation and success. As it is elaborated in the
following sections, important assumptions that are prevalent in the literature can have two
implications. They can either expose the licensed receiver to harmful interference or limit
the throughput of cognitive users and the overall performance of a cognitive radio network or
limit licensed user’s access do the paid spectrum.

Most of the existing works focus on the detection of the signal from a licensed transmitter
that transmits with two levels of power using threshold schemes. However, effort has been
made to consider realistic scenario assumptions and settings in proposing the guidelines and
algorithms throughout the research phases. Major considerations in achieving maximum
system performance as well as to adhere to realistic scenario through three technical chapters
(3, 4 and 5) were pursued by:

- Protecting licensed users from harmful interference.

- Providing maximum QoS for the licensed users.

- Assuming multiple levels of power for both licensed and cognitive users.

The main contribution of this Ph.D. dissertation includes providing methods and algo-
rithms that can improve the achievable throughput. In addition, studies of a number of
candidate throughput improvement schemes have been conducted. Further, methods to
enhance spectrum sensing capabilities in cognitive radio systems have also been investigated.
Moreover, certain significant physical layer security parameters, as one of the largest and most
rapidly growing application areas for cognitive radio communications, are subject to analysis
for our proposed cognitive radio system. Both contributions made and research gaps identi-
ﬁed have been derived on some of the most key pre-cognitive radio throughput-increasing
technologies such as spectrum sensing and power allocation. This Ph.D. thesis comprises of
six chapters. The ﬁrst main chapter is dedicated to providing a technical background into
key relevance and the motivation behind the research undertaken in this work. The main
challenges for improving the achievable throughput with each of the technologies studied
in this chapter are evaluated. Additionally, schemes are established for potential candidate
solutions for obtaining the same objective of system throughput enhancement. The main
technical chapters 3 and 4 are focused at providing strategies and algorithms for achievable
throughput improvement whilst chapter 5 deals with analysing the security parameters of the
proposed cognitive radio scheme.

Chapter 3 begins with addressing the different ways in which cognitive users can transmit
without causing interfering to licensed users. Drawing upon the insufficiencies of some
of the most relevant state-of-the-art technology available in the literature, it proposes a novel and comprehensive combined spectrum sensing method to further improve spectrum opportunities. Centralized spectrum sensing with a sensing controller (e.g. a base station) senses the target frequency band and shares the information obtained from sensing with other nodes. However, such centralized strategy suffers from location diversity (for instance, the sensing controller may not be able to detect licensed users at the cell-edge). In this thesis, we assume distributed spectrum sharing in which unlicensed users perform spectrum sensing independently and the spectrum sensing results are used by users individually. In this case, the proposed cognitive spectrum sensing mechanism does not incur communication and processing overhead. Since each cognitive user is responsible for its own decision, the proposed cognitive spectrum strategy has minimal communication requirements and less overhead subsequently. However the spectrum utilization may be low. Its focus however, remains on obtaining sufficient and practical results in the utilisation of the scarce time and frequency resources as well as reflecting on the joint benefits of the two methods of sensing. Since the propagation of radio signals is affected by a plenty of variables such as geometry and nature of the environment, the environmental conditions (e.g temperature) as well as presence of obstacles responsible for shadowing or multipath fading. Thus different transmit powers should be used for different transmission environments, primary user power is changeable and varies based on environment’s situation. To this end, unlike the conventional binary hypothesis model (two-level power model), the licensed user is assumed to transmit with multiple levels of powers representing a more realistic network scenario. Therefore, cognitive users are required to recognise the licensed user’s actual power and designing better cognitive user-side transmission strategies would be achievable. This leads to the need to address throughput maximising whilst minimising the cognitive user outage probability in conjunction with providing the licensed users with interference protection. Hence, through the assumed cognitive radio scheme, implementation of a power
allocation strategy is facilitated. Cognitive users’ power limitations and licensed/cognitive users’ average transmit power are also constraints in this work for improved secondary user achievable rate. This objective is pursued ensuring guarantees on licensed user quality of service through protecting it against harmful interferences. In addition, using a single-slot frame structure for spectrum sensing is identified as a tool to avoid the sensing throughput trade-off problem. The conventional schemes using frame structures with separated slots generally face complications rising from this problem. A detailed algorithm structure is given following a mathematical representation of the objective it follows. Key performance indicators for the algorithm are benchmarked against other cognitive radio schemes and results are analysed.

Multi-level power allocation for the cognitive users in cognitive radio systems is the research impetus in chapter 4. Transmissions are scheduled using multiple levels of power for both cognitive and primary users. The work addresses the joint problem of multi-level power allocation and maximizing overall throughput for cognitive users in the proposed cognitive radio scheme. Having conducted a critical review of the related work, the direction for research concentration is determined by the gap in the work existent in the literature. A paramount component to the selection of power levels by cognitive users is the maximisation of the achievable rate. Power constraints such as the average transmit power at the cognitive users and the average interference power to the licensed user are also strong considerations. Additionally, improvements in cognitive users’ throughput should be guaranteed. This is while minimizing harmful interference to the licensed user and its quality of service is achieved at reduced complexity of computation. The interweave, the underlay and the sensing based spectrum sharing are identified as the three principle strategies for spectrum access. Drawbacks are discussed for each method and a common cause is derived for such disadvantages. The inabilities of the schemes to perform are therefore determined on the
cognitive side of the network and considerations are put in place to tackle them in this research process.

Allowing for a multi-level power allocation factor at the cognitive user allows cognitive users to vary their power level based on their receiving energy from the licensed user. This in turn results in better performance on the cognitive side. Additionally, adjusting the transmit power from the cognitive user side does not impose considerable interference upon the licensed user side in a cognitive radio network. Moreover, to address the sensing throughput trade-off problem and increasing the cognitive radio system throughput simultaneously, a single-slot frame structure is considered in the cognitive side of the proposed scheme. The average achievable rate has also been studied in this chapter. The problem is formulated under the constraints of the average transmit power at the cognitive user and the average interference power at the licensed user. Further, energy threshold and the power levels are optimized to maximize the average achievable rate at the cognitive user. Analytical expressions are derived for the optimal power policy through which, the cognitive side will be able to instantaneously guarantee the throughput balance and the licensed user QoS. Finally, to provide an optimized solution to the problem, a k-means method’s algorithm is applied due to its speed superiority to conventional algorithms. This is because a classification technique is needed to classify the transmit power from licensed user in cognitive side. Finally, the performance of the proposed algorithm is evaluated and analysed against various spectrum access methods. Such comparisons are drawn under different receiving energies and are benchmarked with conventional methods.

Chapter 5 shifts the focus of the previous chapters towards the topic of physical layer security for the proposed cognitive radio system. Although the physical layer security in cognitive radio networks has been considered in a number of studies, there is still no research with multi-level power transmission considerations for the cognitive radio networks in the literature. To develop a secure multi-power level strategy for cognitive radio systems, a
number of assumptions have been put into place. Firstly, as with previous chapters, a multi-level transmission for the licensed user is identified as a suitable scheme for the system deployment explained. This calls for a realistic strategy that is aware of such limitation. It is also used as a potential countermeasure for the purpose of security in our proposed cognitive radio system. The advantage is that malicious users will find it difficult to recognize the exact licensed user’s power level to detect information. Moreover, recognition time and power budget of the malicious users are two significant factors in the malicious users’ recognition process. Therefore, the process has to be completed during a limited time so as not to miss the transmitted power from the licensed user. In addition, in accordance to the literature review in chapter 5, the assumption of both cognitive radio and malicious users having Poisson process distributions does not appear anywhere else in previous research. Additionally, the existing literature’s scope is limited to the licensed user’s transmissions to be based on a single-level transmit power as opposed to our proposed multi-power level scheme. Both of these factors are integral in achieving a more realistic scheme and enhanced validity of the proposed strategy. To this end, the stochastic interference from the cognitive users to both licensed and malicious users is analysed. On the other hand, the probability density function (pdf) of the interference powers on the licensed and cognitive users is studied. Additionally, the achievable secrecy rate (ASR) and the outage probability of secrecy capacity of the primary user are investigated for our proposed cognitive radio system. The former is known as one of the most important physical layer security parameters and the latter is done from a secure communication graph point of view. In this way, the expression for the licensed users’ ASR in the presence of cognitive users is derived. Also, factors like the influence of the stochastic distribution of licensed, cognitive and malicious users on the ASR is also investigated. Additionally, the cumulative distribution function (CDF) of ASR and outage probability of the licensed user’s secrecy capacity are analysed. Then, the existence and outage probability of the secrecy capacity in the presence of a Possion field
of the licensed, cognitive and malicious users are derived. Finally, chapter 6 provides the concluding remarks on the entire research conducted and highlights the future improvements identified for individual phases.
1.4 List of publications

\((C_1)\) S. Khomejani, A. Nallanathan, H. Nguyen and A. H. Aghvami, “Enhancing throughput of cognitive radio system with multiple power levels for primary users” in proceedings of the 22nd IEEE International Conference on Telecommunications (ICT), Sydney, Australia, April 2015, pp. 1-5.


Chapter 2

Background and Preliminaries

2.1 Motivation

Spectrum is considered to be the most valuable commodity and resource in wireless communications due to its scarcity. In order to provide essential services and systems without imposing harmful interferences, international restrictions have been introduced on spectrum utilization. Traditionally, in large geometric areas or based on the employed radio technologies, exclusive spectrum allocation is generally implemented statically. In particular, spectrum regulatory bodies such as the European Telecommunications Standards Institute (ETSI) and the Federal Communications Commission (FCC) have allocated spectrum blocks for specific usages and also assigned licenses for these blocks to particular companies or groups. Therefore, it may seem that emerging services do not enjoy equal spectrum availability as existing ones. On the other hand, several careful reports and studies have revealed that spectrum is vastly underutilized under the fixed spectrum allocation policy. For example, a statistical report regarding spectrum utilization showed that during a high demand period of a political convention in the USA, only about 13% of spectrum opportunities were utilized [17], [18]. In addition, accommodating a fast growing amount of data traffic with a restricted amount of
spectrum and supporting an increasing number of devices and users are daunting challenges for modern wireless communications.

These facts suggest that devices using advanced radio technologies should be able to exploit underutilized spectrum. The early motivation for the cognitive radio technology was to accomplish opportunistic spectrum usage and alleviate spectrum scarcity. Cognitive radio offers a promising technology that aims to relieve the spectrum scarcity problem in wireless communication networks. This is achieved by allowing unlicensed users to access frequency bands that are allocated to licensed users. This allocation protects and ensures the QoS of the licensed network users [4], [19]. Inefficient resource utilisation realised under the traditional fixed spectrum allocation policy lead to the FCC revisiting the scheme. Endeavours to welcome improvements were attempted by allowing unlicensed user access to the broadcast television spectrum (a.k.a White Space spectrum). To keep interference levels within acceptable levels, a strict condition for this access is therefore the vacancy of the frequency resources on such bands. In other words, unlicensed users will only be served if resource blocks have not previously been assigned to licensed users [20]. For example, to develop the air interference for opportunistic secondary access to TV bands, the IEEE 802.22 working group [21] was formed. Among other advantages, it would provide added bandwidth to support the demand for higher data rate and quality wireless services well into the future.

2.2 Cognitive radio

As a generic term, cognitive radio is used to describe an intelligent radio that is aware of its surrounding environment and can adapt to it. This adaptation is done by making corresponding changes in certain operating parameters (e.g. transmit power, carrier frequency and modulation strategy), in order to achieve highly reliable communications and efficiently use the available radio spectrum [3]. In the simplest instances, cognitive radio can recognize the available systems around them and adjust their protocols and frequencies to access those
systems efficiently [22]. Cognitive radio is equipped with the agility that differentiates it from normal radios. This agility is exhibited on multiple levels:

- Spectrum agility which refers to the acquisition strategies for available spectrum and also opportunistic transmission in the recognized spectrum. Well designed algorithms and protocols are required to achieve such operation for suitable selection of transmission frequencies, cooperation and coordination. Also, advanced sensing capabilities are addressed by the concept of agility.

- Technology agility which refers to a single radio device’s operation among various access technologies. This seamless interoperability can be performed by multi-platform radios.

- Protocol agility which refers to setting up a reconfigurable protocol stack on radio devices so they can adapt their protocols according to the devices they interact with.

2.3 Cognitive radio networks’ user hierarchy

Due to opportunistic use of spectrum and the users’ associated rights to transmit over the spectrum, users can be classified according to multiple hierarchies. I) Classification based upon spectrum license. In this scenario, licensed users are those users that occupy the spectrum and unlicensed (cognitive) users communicate either with the underlying infrastructure or other unlicensed users without interfering with the active licensed users. An example of this hierarchy relates to cognitive users’ operation in TV bands. In this paradigm, the licensed user is the television receiver. Since the spectrum may be used depending on the broadcast schedule of the television channel, a cognitive user can sense the spectrum and use the band if the spectrum is not pre-allocated. In cognitive radio systems, the cognitive and the licensed users are often termed "secondary users" (SUs) and "primary users" (PUs), respectively. II) Classification based upon user technological capability. Capable user
have access to the information concerning the transmission of the non-capable users [23]. Therefore, the capable users are able to use of the side information to avoid interfering with the less capable users.

2.4 Functional blocks of cognitive radio networks

There are four main functional blocks for cognitive radio networks. The blocks consist of spectrum sensing, spectrum management, spectrum sharing and spectrum mobility. Spectrum sensing is to recognize spectrum availability and the presence of the licensed users. Spectrum management aims to estimate how long the white spaces are remained for use by cognitive users. Spectrum sharing also enables distributing the white space frequency resources fairly among the unlicensed users. On the other hand, spectrum mobility is to keep the same permanent communication requirements during the transmission to better spectrum [24].

2.5 Spectrum sensing

To avoid collision with the licensed user and improving spectrum utilization efficiency, spectrum sensing has a noticeable role in cognitive radio networks [25], [26] and [27]. Spectrum sensing can be described as taking measurements on a part of the radio spectrum and making a decision about the spectrum usage based on data measured earlier [28]. Thus, Spectrum sensing is to detect the presence (or absence) of a licensed user to give the unlicensed users permission to access the channel. In fact, the unlicensed users need to continuously monitor the status of the licensed users and find the spectrum bands that can be used by the cognitive users without interfering with the licensed users. These spectrum bands are known as spectrum holes and are divided to two types: temporal and spatial spectrum holes [25]. A temporal spectrum hole happens when licensed user choose not to transmit during a certain time period during which cognitive users can access the spectrum...
Figure 2.1 Spectrum sensing.

and use it for transmission. A temporal spectrum hole is presented when the licensed user transmits within a region and the unlicensed user can use the spectrum outside that region. The principle of spectrum sensing is displayed in Fig. 2.1. As can be seen, data is sent from the primary transmitter to the primary receiver in a licensed spectrum band while the cognitive users aim to access the spectrum. In order to protect the primary transmitter, the secondary transmitter is required to detect whether there are any primary receivers within the coverage area of the secondary transmitter. To detect the presence or absence of the primary receiver, secondary transmitter only needs to detect the presence of the primary signals. Since the situation is not always ideal, for example the primary power level is below the noise power. By properly thresholding the auto-correction function of the received signal, the presence of licensed users can be detected. However, the radius of primary transmitter and receiver detections are different and this imposes certain complications and challenges. When the primary receiver is outside the primary transmitter detection radius, the spectrum holes may be missed. Instead of detecting a primary receiver, most works focus on primary
transmitter detection [5] and [9]. There are many signal detection techniques that are used in spectrum sensing to enhance the detection probability. These techniques are classified as:

2.5.1 Non cooperative detection (Primary transmitter detection)

This technique is used to determine a signal from a licensed transmitter to determine whether it is active or not. In fact, it uses the detection of the signal from a licensed transmitter. Since it is usually assumed that there is no real-time interaction between cognitive users and licensed transmitters/receivers, cognitive users cannot have accurate information on current transmissions within the licensed networks. Therefore, to distinguish between used and unused spectrum bands, cognitive users need to detect the primary transmitter’s signals. For transmitter detection, based on a basic hypothesis model, the received signal at the SU can be expressed as follows [29]:

\[
\int_l = \begin{cases} 
hr_l + n_l & H_1 \quad l = 1, \ldots, N \\
n_l & H_0
\end{cases}
\] (2.1)

where \( \int_l \) denotes received signal by cognitive users, \( n_l \) is the additive white Gaussian noise (AWGN), \( r_l \) is the primary users’s transmitted signal and \( h \) is the channel’s amplitude gain. \( H_0 \) is a null hypothesis which means the absence of licensed user and \( H_1 \) is an alternative hypothesis that shows the licensed user is present. The objective for spectrum sensing is to decide between \( H_0 \) and \( H_1 \) based on the observation \( \int_l \). In addition, according to hypothesis model, three schemes are utilized for transmitter detection [30].

- Matched filter detection [31]: when cognitive users have the licensed user signal information, the matched filter is an optimal detector for maximizing the received signal-to-noise ratio (SNR) in the presence of additive noise [24]. Similar to other methods, it has some advantages and disadvantages. Less detection time is required by this method for higher processing gains. A disadvantage to this technique however
is that prior knowledge of licensed (primary) user signals are needed as well as synchronisation between the cognitive users and the licensed transmitters.

- **Energy detection [32]:** It is an optimal scheme for detecting signals if the receiver cannot gather sufficient information about the primary user signal. In this approach, cognitive users sense the presence of the licensed users based on the energy of the received primary signal. To measure the energy, the received signal is squared and integrated. Then, to make a decision about the presence of the licensed user, the output of the integrator is compared to a pre-defined threshold. This approach has some disadvantages in that it is not able to recognise the difference between the interference from other cognitive users sharing the same channel and that of the licensed users. In addition, under low SNR conditions, it shows a poor performance because the noise power is not certain and can take any value (in the range of "X" dB). That is called as SNR wall [33].

- **Cyclostationary feature detection [28]:** Due to statistical properties of a cyclostationary process that vary periodically over time, this method deals with the cyclostationary features of the signal. These features have periodic statistics that cannot be found in any interference signal or noise. To identify the presence of licensed users, it extracts this periodicity in the received primary signal. Because of its noise rejection capability, it performs better than other detection schemes in low SNR regions [28]. At the same time, this scheme has some disadvantages too. It needs complex computations and longer observation time and also higher costs.

In addition, other spectrum sensing scenarios such as the waveform-based sensing [34], [35] and the wavelet-based detection [25], [36] and some proposed spectrum sensing technique use the statistic covariance of the received signal [37], [38]. Among spectrum sensing techniques, energy detection [39], [40] requires the least prior knowledge of licensed user and is simple to implement and as a consequence, it has attracted intensive attention due to
2.5 Spectrum sensing

its performance. In [41], a sensing technology was designed where the secondary user is equipped with multiple antennas and to recognise the status of licensed users, the eigenvalues of the received covariance matrix are used. Admittedly, when wireless terminals size impose limitations on the employment of multiple antennas, the cooperative spectrum sensing is preferred over the single-SU to enhance the sensing performance [42], [43]. Moreover, there are a number of studies discussing different aspects of cognitive networks over the past decade. For instance, cognitive radio system design with imperfect sensing is investigated in [44], parameter uncertainty based spectrum sensing is presented in [45] and spectrum sharing in [46]. Liang and et al. in [47] proposed a sensing throughput trade-off scheme. On the other hand, there are many hybrid scenarios between cognitive radio and other technologies such as sensing in relay networks [48], [49] and games among cognitive users [50]. Furthermore, sensing in OFDM systems is proposed in [51] and authors in [52] present a sensing technique with a MAC layer protocol.

Figure 2.2 Concept of cooperative Spectrum sensing.
2.5 Spectrum sensing

2.5.2 Cooperative transmitter detection

Cooperative detection was proposed due to the lack of interaction between the cognitive user and the licensed user that causes transmitter detection schemes relay on the weak signals from the primary transmitters. Therefore because of this shortage, transmitter detection schemes cannot avoid causing interference to primary receivers. In addition, a cognitive transmitter may be able to have a good line of sight to a cognitive receiver, but may not be able to detect the licensed transmitter because of shadowing. Hence, for more careful primary transmitter detection, sensing information from the other user can be useful. In fact, cooperative detection refers to spectrum sensing methods where information from multiple cognitive users is incorporated for licensed user detection. Using this scheme, the uncertainty of a user’s detection can be minimized. Also multipath and shadowing effects can be relived so that in a shadowed environment, the probability of detection is increased [33]. As can be seen in Fig. 2.2, in the case for secondary user 2, primary signal is blocked by obstacles and the received signal at the secondary user is too weak to be detected. When the primary receiver is within the cognitive users’ transmission range, the transmission of the primary user will be interfered if the cognitive transmitter cannot detect the presence of the primary transmitter. To address the problem of decreased spectrum sensing performance due to noise levels, multipath fading and shadowing, cooperative spectrum sensing [53] has been proposed that allows cognitive users to collaborate and make decisions. With the collaboration of several cognitive users for sensing and by exploiting the independent fading channels and multi-user diversity, the detection performance will be improved.

2.5.3 Primary receiver detection

The best way to detect the white spaces is to detect the licensed users that are receiving data in the communication range of a cognitive user [33]. When a primary receiver receives signals from the corresponding transmitter, local oscillator (LO) leakage power is usually
emitted from the primary receiver’s RF front-end. This LO leakage power is used instead of the received signal from the primary transmitter to detect the presence of primary receivers directly.

2.5.4 **Interference temperature management**

Interference takes place at the receiver users and can be controlled at the secondary user transmitter. FCC introduced interference temperature as a model for measuring interference. In this model, an interference temperature limit is suggested which is the amount of the interference that primary receivers can tolerate. Cognitive users can use the spectrum band since they do not exceed the limit [33]. Although this method seems like an ideal model for spectrum sensing, estimating the interference temperature limit based on the users’ location is hard. Also, there is not a specific way to estimate the interference temperature thus cognitive users have difficulties in distinguishing between noise and the licensed users’ signals.

Based on the results from spectrum sensing, the cognitive users have knowledge about the channels that they can access. Since the channel status and the behaviour of the licensed users are not constant and may change rapidly, to use the spectrum bands effectively, spectrum sharing and allocation techniques are important [26], [54]. When cognitive users co-exist with the primary users, to manage the interference generated by the cognitive users’ transmissions, the power control schemes should be carefully designed. To achieve interference-free co-exiting transmissions, some advanced technologies such as multiple-input multiple-output (MIMO) and beamforming with smart antennas are proposed and utilized [55]. In multi-hop cognitive radio networks, relays that generate spatial spectrum holes can achieve more communication opportunities and assist cognitive users’ transmissions. Additionally, the resource competition among unlicensed users as one of the important issues that have been addressed in some studies [56], [57].
2.6 Spectrum access

Based on the results of the sensing phase, the cognitive users have knowledge about the channels that they are able to access. Since the conditions of the channels and the status of the primary users might change, to achieve better performance, cognitive users need to decide which channels can be used for transmission. In addition, should also decide when and how to access channels. In some literature, depending on spectrum bands that the cognitive users use, the spectrum sharing schemes can be divided into two types: open and licensed spectrum sharing [5], [9]. In the licensed spectrum sharing which is also known as the hierarchical spectrum access model, licensed users have higher priorities to access the channels than the cognitive users. Here, since the licensed users all have their own licensed bands, the cognitive users require to adjust their parameters such as their transmit power and transmission strategy to avoid harmful interferences to the licensed users. In the other hand, all users in the open spectrum sharing scenario have an equal chance to access the channels and the spectrum sharing among cognitive users for the unlicensed bands. This is a typical example for this model.

In a cognitive radio network that accesses spectrum allocated to licensed services, some different approaches have been proposed as methods through which cognitive users gain access to. These cognitive radio, spectrum access methods are listed as follows:

I Spectrum sharing (SS)

II Sensing-based spectrum sharing (SSS)

III Overlay spectrum access (OLA)

IV Opportunistic spectrum access
2.7 Spectrum underlay sharing

The spectrum underlay [9] technique permits the cognitive users to co-exist with the licensed users in the same frequency bands as long as secondary users consider the impact of their transmissions on the reception quality of the primary licensee [58]. In this scheme, a threshold is assumed as an interference power constraint to keep the harmful incurred interference at the primary receivers below the threshold. In the literature, this threshold is called the interference temperature limit [3]. Figure 2.3 illustrates an overview of the spectrum sharing.

Figure 2.3 Spectrum underlay sharing.

where S-T denotes the secondary transmitter, S-R is the secondary receiver and P-R is the primary receiver. As can be seen, the secondary users (S-R and S-T) coexist with the primary users (P-R) by keeping the harmful interference at the primary receiver (P-R) below a certain level. To achieve this, cognitive users require to know the channel state information (CSI)
of the channel $X_1$ between the S-T and P-R. Since dynamic resource allocation (bandwidth, transmit power, etc.) has a key role in cognitive systems, spectrum sharing has been an area of intensive research over the last decade. In [58] they studied the outage capacity of a single antenna point to point (P2P) system under average and peak interference power constraints. Kang and et al. investigated the outage capacity under joint transmit and interference power constraints and the research in [59] was under joint peak and average interference power constant. Moreover, to prevent the interference to the licensed users, spectrum sharing cognitive radio systems with multiple antennas was proposed. In [60] the throughput of a MIMO and single-input multiple-output (SIMO) link was proposed under joint transmit and interference power constraints. Due to avoiding the trade-off between throughput maximization and interference, the multiple antennas at the secondary transmitter were used. In addition, the impacts of employing multiple antenna terminals on the network performance under several interference constraints of licensed networks were studied [22], [61]. In [22], maximum rate of SIMO has been investigated further. Fundamental capacity constraints of a MIMO network regarding the capacity of spectrum underlay cognitive radio system have been analysed in [62].

![Figure 2.4 Opportunistic spectrum access.](image-url)
2.8  Opportunistic spectrum access

In opportunistic spectrum access or otherwise known as the interweave method, cognitive users are given access to licensed frequency bands only when the bands are not being used by primary users [63], [64]. In order to avoid causing irreparable interference to the licensed users, secondary users require to sense the channel carefully to detect the status of the primary users. Secondary users use the available unused spectrum when the primary users are detected idle. Therefore, spectrum sensing proves to be a crucial process in the OSA cognitive radio systems. Figure 2.4 shows a very simple example of an OSA scheme in which a secondary user senses the white spaces and transmits over the detected spaces opportunistically. It can be seen that the secondary user can access the licensed spectrum through spectrum holes without impacting the primary users. Thus, the opportunistic spectrum access employs the available limited spectrum in a more effective way. Due to the importance of the OSA scheme, spectrum sensing has been an attractive research area over the past few years [65], [66].

2.9  Sensing based spectrum sharing

Sensing-based spectrum sharing scheme was proposed as a combined method of the spectrum sharing and opportunistic spectrum access scenarios. In this scheme, the cognitive users sense the spectrum and adapt their transmit power based on the spectrum sensing decision. When the primary users are detected to be absent, the secondary users transmit with higher transmit power levels. Conversely, they use a lower transmit power below a certain threshold if the primary users are detected to be present to protect the primary users from any potential interference. In [67], the throughput of a P2P secondary link was discussed under joint average transmit and interference power constraints for the cases of perfect and imperfect spectrum sensing. In [67], the sensing-based spectrum sharing method was benchmarked
2.10 Overlay spectrum access

In the OLA scheme, side information such as channel gain, primary users’ codebooks and messages are available for the secondary users. This a priori knowledge of the primary users will allow for a type of asymmetric cooperation between the primary and secondary transmitters [68]. The asymmetric form of transmitter cooperation that has been a motivation in cognitive radio research, was first introduced in [69]. For instance, if $S \rightarrow T$ is geographically close to $P \rightarrow R$, the radio channel ($P \rightarrow T \rightarrow S \rightarrow T$) could have a higher capacity than the channel ($P \rightarrow T \rightarrow P \rightarrow R$). Therefore $S \rightarrow T$ could listen to the channel in a fraction of the transmission time and get the message transmitted by $P \rightarrow T$. Figure 2.5 shows an overlay

Figure 2.5 Overlay spectrum access

against the opportunistic spectrum access method and was shown to achieve higher system throughput.

2.10 Overlay spectrum access

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access scheme where the big arrow indicates that the secondary transmitter has knowledge of the primary transmitter’s message. In this scenario, a part of secondary user’s transmit power is allocated for its transmissions and the remaining part is used to aid the communication of the primary user. Most of the studies in the overlay spectrum access include complex coding schemes such as the cooperative method and are based on the information theory [69], [70]. There are several attempts in literature to enhance the achievable throughput of overlay cognitive radio networks which have obtained impressive results [62], [69] and [71].

Among those spectrum access methods, the sensing-based spectrum sharing is able to achieve the highest rate for the cognitive users [67], [72]. In [67], kang and et al. designed a sensing algorithm to find the optimal sensing time and transmit power in order to maximize the ergodic throughput of the secondary link under two constraints: the average transmit and interference power levels. In [72], stotas and et al. extended this idea [67] to a multi-band scheme and proposed an algorithm that achieves an optimal sensing time and power allocation that can be applied even when not all frequency bands are used. Additionally, in [73] an optimal power allocation scheme is developed to maximize the energy efficiency of a secondary user transmitter under instantaneous transmit power and interference power constraints. In [67], [73], sensing-based spectrum sharing is studied for networks where secondary users have only one antenna. This work has been extended to a multi-antenna scheme to optimize power allocation in [74].

Recently, in [75] a hybrid scheme has been proposed to enhance the secondary user transmission opportunity allowing it to transmit over any time slot. In this work, the studies in [47], [67] and [76] are extended and apply a mixture of methods and allow transmission in two phases. A time slot is split in two parts by the secondary user. In phase I, the secondary user transmits through the underlay scheme and it senses the status of the licensed user. In phase II, the secondary transmits depending on the result from sensing. The user transmits under the underlay method in the case of a licensed user being detected or it will transmit
using the interweave technique otherwise. Motivated by the results in [75] and [77], ridouani and et al. continued with a scheme called the "always transmit" under the QoS management for cognitive and primary system over a Nakagami-m fading channel [78]. Although, this scheme is proposed to guarantee the QoS at the primary side, it does not offer any secondary user side QoS guarantees.

In some literature (e.g. [5]), to manage spectrum allocation and access procedure, a central node is assumed. This central node is in control of the whole procedure. This method is called the distributed spectrum sharing whereby cognitive users make their own decisions based on the observations of the local spectrum dynamics. In this case, if several cognitive users cooperate with one another, it is called the cooperative spectrum sharing [5]. These centralized techniques may be not practical in some systems due to the cost of information feedback and the existence of a central node.

2.11 Spectrum allocation and sharing

In this section, some important techniques on spectrum allocation and sharing will be discussed.

2.11.1 Resource allocation and power control

To protect the primary users from the cognitive users' interference, as one of the main aims of the cognitive radio networks, various resource allocation and power control techniques have been proposed. Although these techniques cannot totally avoid the impacts of interference, they focus on limiting the interference to the primary user created by the cognitive users in several ways such as: single-carrier and single-antenna [79], multi-carrier and multi-channel [80], multi-antenna [22], [81] and [82] and the multi-hop systems [57]. Figure 2.6 illustrates the spectrum sharing model in a single-carrier and single-antenna system also known as
a P2P system with a single antenna where secondary transmitters can transmit as long as their interference imposed on to the primary receivers are below a pre-defined threshold [83]. This fact can be shown as a formula: $E[P(C_1, C_2)] \leq P_{av}, \forall(C_1, C_2)$ where $C_1$ and $C_2$ denote the channel gain from the secondary transmitter to the primary receiver and the secondary receiver, respectively and $P_{av}$ is the average transmit power limit. In [79], [84], the power control for systems with a single primary and secondary user pair have been studied. In [79], different kinds of capacities such as the outage and ergodic for secondary user networks are determined in a cognitive radio system under interference powers constraints. In [85], the performance of a primary user network has been investigated under average and peak interference powers constraints. According to this work, the average interference power constraints show performance superiorities over the peak interference power constraints. Since the secondary user systems exhibit better performance under the average interference power constraints, it should be preferred in practice to protect primary and secondary user networks.

In addition, to protect the primary transmission, outage probability constraint can be used [86] in that the outage probability of the primary transmissions should not be below a pre-defined threshold. Similar to the case of the P2P systems with single antenna, the peak and average interference power constraints have been considered in the multi-carrier and multi-channel systems. Moreover, In these systems, interference generated by the cognitive users to the licensed users can be considered across all channels or each sub-channel separately [83]. Capacity maximization for the cognitive user system under the peak and average interference power constraints is also investigated in [80]. The existing results demonstrated that the average interference power constraints resulted in better performance for the cognitive user networks than the peak interference power constraints.
2.12 Applications of cognitive radio

For the unlicensed use of TV white spaces, the main regulatory agencies such as: the FCC in the United States, the Electronic Communications Committee (ECC) of the conference of the European Post and Telecommunications in Europe and the Office of Communications (Ofcom) in the United Kingdom have been conducting extensive studies in the field of cognitive radio. After a decade of investigation and research in September 2010, the FCC released the rules for using the TV white space resources based on the idea of sensing and obtaining spectrum holes within TV bands to utilize for secondary users communication [87]. Moreover, the applications of cognitive radio systems in cellular networks have been developed in recent years. For example, small cells have been proposed in 3GPP LTE-Advanced (LTE-A) [88] and IEEE 802.16m [89] to overcome the indoor coverage
problem. In this way, PicoChip as one of the globally well-known companies have introduced femtocell which makes the task of managing interference in a centralized manner somewhat impractical due to the self-deployment property of such nodes. In femtocell, cognitive radio system is used to estimate the spatio-temporal characteristic of spectrum usage traffic and opportunistically use the available spectrum during times of high data demands. In military, cognitive radio systems have been used by soldiers to recognize enemies’ communications whilst protecting their own as well as search for more transmission opportunities. For instance, the SPEAKeasy radio system has been established by the US department of defence (DoD) to take advantage of the cognitive radio technology.

Moreover, emergency networks as one of the cognitive radio applications have been used under severe conditions, e.g., accidents and natural disasters. In these extreme situations, responders require to detect and locate survivors as well as maintain reliable communications between responders and public safety agencies [83]. To realize reliable emergency network transmission, the opportunistic spectrum usage techniques could be used. For emergency use, a 700 MHz (698-806 MHz) frequency band has been designed by the FCC [90] and [91]. Still, there remain some possible applications for the cognitive radio such as improving reliability and efficiency of communications. In emergency situations, energy efficiency should be considered because battery life limits potentially successful operations [83].

2.13 Security in cognitive radio networks

The concepts of security and privacy in wireless networks have taken on an important role as these networks continue to grow all over the world. In addition, next multi-tier heterogeneous networks have become more and more weak from the security perspective and more vulnerable to serious security threats and attacks of eavesdropping [92]. Since cognitive radio networks are wireless in nature, they are susceptible to all common security threats found in the traditional wireless communication networks. Moreover, due to the cognitive
radio networks’ open nature, these networks face several attacks and hazards targeting the physical or MAC layers [93]. We use the term open nature to describe cognitive radio that is aware of the environment around itself and is able to adapt its transmission according to the interferences. The most common security objectives for wireless communication systems are [94]: confidentiality, availability, integrity and access control. Except for the named threats inherited from their wireless nature, there are some security challenges that have risen because of these systems’ unique cognitive characteristics. Moreover, as wireless devices become pervasive and personalized, maintaining secrecy of information is a serious issue for cognitive radio networks. In particular, security of cognitive radio systems is exposed to the external hazards [95], [96]. Broadly, the purpose of secure communication is to guarantee that the legitimate receivers can gain the accurate message whereas the users cannot.

Traditionally, security is mentioned based on cryptographic approaches which can be classified into public and private key protocols [97]. These approaches have been established at the upper layers and independent of the physical medium [98]. In these scenarios, a protocol is designed which is computationally intensive for the eavesdroppers to decode the message. In fact, these approaches use the limited computational power at the eavesdroppers. However, due to the broadcast characteristic of the wireless channels, key exchange algorithms can be vulnerable to eavesdropping attacks [99] and [100]. To guarantee that sent messages cannot be decoded by malicious nodes, the information theoretic security as the strictest notion of security calls for the combination of channel coding methods with cryptographic techniques that exploit the randomness of the communication channels [93]. As shown in figure 2.7, node \( N_2 \) senses the transmission of node \( N_1 \) and may obtain confidential information. Node \( N_1 \) can guarantee the confidentiality of its transmitted data by exploiting the channel’s physical properties to secure data by coding against node \( N_2 \). This information theoretic approach builds on Shannon’s notion of perfect secrecy [101]. Moreover, information-theoretic security
as a physical-layer technique that provides secure wireless communication has been widely studied in [102] and [103].

Generally, for cognitive radio networks as a kind of wireless networks, physical layer is not as robust as that in wired communications. The physical layer in cognitive radio networks is more complex than its counterpart in other network models. On the other hand, physical layer security can be a solution to support existing cryptographic protocols [104]. The main idea of communication security in cognitive radio systems is to strengthen the main channel (between legitimate transmitters and receivers) relative to the eavesdroppers’ channel for obtaining perfect secrecy [92].

There have been some influential motivations in studying the security issues in the physical layer. For example, in underlay cognitive spectrum sharing networks, as a complex environment in which the primary and secondary users are allowed to transmit concurrently in the same spectrum [105], [106], protecting the broadcast channel against eavesdropping
is a big challenge. On the one hand, spectrum sharing can be beneficial for the licensed users in the sense that primary users can achieve some motivations such as: relaying licensed users’ transmission by cognitive users or monetary rewards (e.g. the Property-rights model or spectrum leasing) [107]. On the other hand, spectrum sharing may result in some security problems such as eavesdropping. For example, cognitive users may try to access and decode licensed users’ messages from the sensing results [108]. Generally, information-theoretic secrecy is considered to prevent eavesdropping impacts in the sense that information-theoretic secrecy allows the legitimate users to protect their confidential messages from eavesdropping by using wiretap channel coding [109], [110]. The notion of information-theoretic secrecy was first addressed by Shannon [101] and later continued by Wyner [103], Csiszar and Korner [109], Leung and Hellman [110] and others [111]. There are many approaches and schemes to develop the notion of information-theoretic secrecy in [112], [113]. The secrecy capacity is one of the major concepts which is mentioned in these studies and defined as the largest achievable rate from the legitimate transmitter to the receiver assuming the condition that the malicious user achieves no information from a confidential message sent to a receiver. A novel work has shown that perfect secrecy can be achieved when an eavesdropper channel is a degraded version of the main source to destination channel [114]. Moreover, a higher secrecy rate can be achieved as the quality of the channel between the legitimate transmitter and receiver is getting better than the channel between legitimate transmitter and the malicious user [108]. In some works [115], [116], this concept has been applied and cooperation methods between the licensed and reliable secondary users were proposed for the case that licensed users try to send a confidential message in the presence of eavesdroppers. In these works, the secrecy rate of the licensed users is improved by sharing primary spectrum band with the cognitive users. In this case, the cognitive users’ signal operates as interference to the eavesdropper providing a higher secrecy rate for the primary users. One scenario has investigated the use of cooperative relays in the physical layer security [117], [118], [119].
In [120], [121], a beamforming technique combined with the injection of artificial noise has been used that declines the use of cooperative relays in the physical layer security. In [122], Yu and et. al. have introduced the security issues on the physical layer in a cognitive radio network in detail. Due to rapid advances in multi-antenna techniques, security in wiretap channels has attracted widespread attention where legitimate users and/or malicious users are equipped with multiple antennas [123], [124]. In [123], a single input multi output wiretap channel is considered and the secrecy outage probability is calculated. In [125], wiretap channel is defined as: “a setting where one aims to provide information theoretic privacy of communicated data based solely on the assumption that the channel from transmitter to eavesdropper is noisier than the channel from transmitter to receiver”. [126], as an extension of [123], is a practical scenario of multiple eavesdropper. In [126], to enhance the security, authors introduced transmit antenna selection. In [127], to secure cognitive radio network with a single eavesdropper, a relay selection strategy was proposed. This strategy assumes a trusted relay and maximizes the achievable secrecy rate subject to interference power constraints at the licensed user under available channel information. In [124], Huang and et. al. evaluated the secrecy outage probability in the presence of an untrusted relay. In [128], [129] and [130], to protect the transmit signal from the eavesdropper in cognitive radio networks, multiple antennas have been exploited and the secrecy capacity of the secondary system was investigated. In [131] to protect the cognitive transmission and enhance its security against eavesdropping attacks, a multi-user scheduling scheme was presented.

The concept of secrecy capacity has been defined as a maximum achievable rate at which information can be transmitted secretly from the source to its destination [98] and [132]. In the calculation of secrecy capacity, it is very important to establish whether the CSI of the malicious user is available at the source node or not. In the case that the malicious user is active, its CSI may be known at the source [132]. In [133], Simon and et. al. considered the assumption of perfect CSI for the eavesdroppers and analysed the secrecy capacity of
their proposed cognitive radio network. Since the malicious user is generally passive and its location is unknown to the transmitter, obtaining the CSI of the malicious user in a realistic environment is very difficult [98]. Due to this fact, only practical knowledge on the eavesdropper channel which is available at the transmitter has been assumed in some conventional studies [133]. In [133], interference between active primary and cognitive users was not considered.

In [98], the authors proposed a combined scheme that used beamforming technique and artificial noise at the primary transmitter and have analysed the ergodic secrecy capacity of the primary side.
Chapter 3

Multi-Power Level Strategy for Primary Users in Cognitive Radio Networks

3.1 Abstract

In this chapter, we propose a more practical scheme that exhibits a throughput improvement in addition to enhancing spectrum sensing capabilities. Such improvements are in comparison to the conventional OSA cognitive radio systems with multiple primary transmit power levels studied so far. In [134], a new frame structure is introduced to improve throughput and spectrum sensing capabilities compared to conventional systems that use frame structure with periodic sensing. However, employing only a dual-level transmission power for primary users can be theoretically considered but in practice, it may be necessary for primary users to transmit with multiple levels of power. In this chapter, we propose a cognitive radio system with multiple primary transmit powers and a new frame structure which provides sensing and transmission at the same time in a single time slot. Then, the optimal spectrum sensing algorithm at the cognitive user which be able to discriminate the licensed users’ transmitting power is proposed and its performance is analysed. Moreover, a cognitive user transmission strategy is proposed where one transmit power is founded for each estimation of
the licensed user’s power. The way that this is achieved is described in more detail in Section 3.4 and our proposed strategy exhibits improved achievable throughput and spectrum sensing capabilities compared to the conventional opportunistic spectrum access cognitive radio systems. The proposed algorithm provides optimal power allocation for secondary users to protect the primary user from the harmful interferences. Finally, at the end of this chapter, the proposed system is validated by computer simulations that illustrate the enhancement of overall throughput of the cognitive radio networks.

3.2 Introduction

Various methods and schemes have been developed in cognitive radio systems that provide different ways in which secondary users can transmit without causing interfering to licensed users. Among them, there are two main methods: (i) Spectrum Sharing [37], [135] in which the licensed and unlicensed users coexist with appropriate interference protection mechanisms in place and (ii) Opportunistic Spectrum Access [136], [137] whereby the secondary users sense the medium for a certain time and will only transmit if they find the transmission channel free. In this chapter, we propose a scheme that combines the two above-mentioned methods to further [138] improve spectrum management. With efficient spectrum sensing, more spectrum opportunities could be discovered. In this way, spectrum sensing is an undeniably promising technology of cognitive radio [47].

There are two performance probabilities which are of interest in spectrum sensing: probability of false alarm $P_{fa}$ that declares falsely the presence of a primary signal under hypothesis $H_0$ and the probability of detection $P_d$ which denotes the correct detection of primary users (under hypothesis $H_1$) [47]. In fact, the detection performance is characterized by $P_{fa}$ and $P_d$. $P_d$ is the probability that the decision is $H_1$ while $H_0$ is true and $P_d$ denotes the probability that the decision is $H_1$ while $H_1$ is true. From the licensed users’ perspective, the higher the $P_d$, the better protection it can receive. From the cognitive users’ perspective,
the lower the $P_{fa}$, higher the chances of cognitive users to access the not-in-use frequency bands. Evidently, for a good detection scheme, the $P_d$ should be as high as possible while the $P_{fa}$ should be kept to a minimum [47].

Therefore, in order to improve spectrum utilization and solve the spectrum scarcity problem, the cognitive radio networks implement a scheme in which the secondary users are allowed to use the spectrum which is licensed to the primary users when unused. As a result, there have been many attempts in designing efficient spectrum sensing and power allocation schemes in cognitive radio networks.

For power allocation, there are some factors that need to be considered when designing a suitable policy. These include secondary user power limitations and secondary/primary user peak and average transmit power constraints. These constraints are incorporated and considered in designing algorithms that aim at maximising the secondary user achievable rate whilst ensuring primary users the quality of service through protecting them against harmful interferences. The works in [139] and [140] assume that optimal power allocations were used to maximize the secondary achievable rate for different combinations of the power sensing limitations.

In most of the existing works in the literature, the conventional binary hypothesis model “0” or “1” is assumed as the underlying sensing model (two-level power model). According to the assumption, the constant transmit power of a primary user is either present or not. However, this “0-1” model does not obtain sufficient and practical results in the utilisation of the scarce time and frequency resources [136], [141]. Therefore, if secondary users could recognise the primary users’ actual power (which could be of multiple levels), designing better secondary user-side transmission strategies would be achievable. Consequently, the above has been the main motivation for us to consider the case whereby the primary users transmit with multiple levels of power.
For spectrum sensing, various conventional methods consider the frame structure in Fig. 3.1.a where spectrum sensing is carried out at the beginning of each frame (during $\tau$ units of time). The second part of the frame is then used for data transmission (in $T-\tau$ units of time).

On the one hand, the increase in sensing time $\tau$ results in better detection of primary users’ status and increase in the probability of detection but it leads to a decrease in the data transmission time. On the other hand, the increase in the transmission time results in an increase of false alarm probability and better protection of primary users from harmful interferences. Thus, based on this conventional structure, there are sensing-transmission trade-off problems [63]. As a result, the joint problem of maximising throughput [47] and minimising the cognitive user outage probability [142] has been investigated and researched whilst providing licensed users with interference protection. Moreover, the impact of various trade-offs in sensing-transmission are very critical specially at the MAC layer where such
sensing events are scheduled (i.e. when and how to sense the channel). In literature, some works do address this aspect of spectrum sensing in depth [143]. For example, in [143], a set of integrated MAC and physical layer spectrum sensing techniques are proposed that guarantee the detection of the licensed users while meeting the secondary user QoS. However, the main focus of this thesis lies on the enhancement of the sensing throughput considering physical layer parameters. To avoid the sensing throughput trade-off problem and to increase cognitive radio users’ throughput, we employ a frame structure as in [134] called that Frame Structure with a Single Slot (SST) as shown in Fig. 3.1.b.

With multiple levels of power for licensed users and a SST structure, we propose a combined scheme that shows improvements in throughput and avoids the sensing trade-off at the same time. This combined scheme is referred to as the multi-power level for primary users with Simultaneous Sensing and Transmission Slot (MSST) in this chapter. We will then benchmark our proposed algorithm’s performance to the conventional Two-level PU power with Simultaneous Sensing and Transmission slot (TSST) and other existing frame structure models.

The rest of this chapter is organized as follows: In Section 3.3, the MSST scheme is presented as the system model. In Section 3.4, we study the dynamics of transmission rate and allocation power. We then present the throughput of the proposed scheme in conjunction with the energy detection scheme. Throughput of our proposed model as a key performance indicator is then benchmarked against that of the respective conventional cognitive radio systems. Finally, simulation results are presented and analysed in detail in Section 3.5, followed by the concluding remarks in Section 3.6.
3.3 The system model

In this model we have a primary and a secondary unit with each unit being equipped with a receiver and a transmitter. The primary unit consists of primary transmitter P-T and a primary receiver P-R while the secondary unit has two elements: a secondary transmitter S-T and a secondary receiver S-R, as shown in Fig. 3.2.

The secondary user is allowed to use the channel provided the interference level it poses onto the primary user does not surpass a predefined level. In this model, the channel gain between the four elements of the network are $X_1, X_2, \alpha$ and $\beta$ where $X_1$ shows the channel gain from P-T to S-T and $X_2$ denotes the channel gain from P-T to S-R while $\alpha$ and $\beta$ denote the power gains of channels between S-T and P-R, and between S-T and S-R, respectively.

We consider the case that the channel gains are assumed to be constant and known at the secondary side. We assume the probability that the PU is idle is $P(H_0)$ and the probability that PU is transmitting is $P(H_1) = 1 - P(H_0)$. 
In conventional OSA schemes with time frames of length \( T \), periodical spectrum sensing is performed in which secondary users sense the transmission channels for a fixed period \( \tau \) (duration of sensing slot). If a primary user is found to be idle, transmission would occur in the remaining time till the end of the time slot \( T-\tau \), that is also known as the transmission slot. If the secondary user senses a busy channel, it will not transmit and it will only proceed to transmission if the channel is sensed to be idle. The problem is therefore to determine whether the channel is busy using sensing samples.

As a result, there is a trade-off between the sensing time and the transmission time, hence the throughput of the system [140]. According to the classical detection theory [144] and [139], an increase in the sensing time results in higher probability of detection and lower probability of false alarm [139]. This trade-off is known as the sensing throughput trade-off. In [47], Liang and et. al. tried to find the optimal sensing time that maximizes the average achievable throughput of an OSA for the protection of the QoS of the primary users.

In this chapter, a frame structure which is shown in 3.1.b is used for the proposed cognitive system [139]. The mentioned frame structure consists of a single slot during which both spectrum sensing and data transmission are performed at the same time. Under this system both the spectrum sensing and data transmission are simultaneously maximized. Both of these functions will last exactly an amount of time equal to the duration of a frame \( T \).

The assumed frame structure introduces some advantages that enable us to achieve the goal. The significance of the results is twofold.

First, the increased sensing time enables the detection of weak signals from licensed users, this detection performed under the frame structure depicted in Fig. 3.1.a would reduce the data transmission time and hence, the throughput of the cognitive radio system. Additionally, the increased sensing time leads to an enhanced detection probability. The sensing time covers the whole duration of the frame that leads to a higher detection probability and a better protection of primary user QoS as well as a lower probability of false alarm. All of
these factors will consequently make spectrum utilisation more efficient that help achieve higher throughput levels. The increased sensing time simplifies the use of complex spectrum sensing techniques that perform more sophisticated sensing capabilities but require higher sensing time (e.g. cyclostationary [145]) [37]. When some of the signal’s features are known, then one may use this knowledge to form a cyclostationary test closely matched to the signal. In fact, cyclostationary detection consists of analysing the cyclic auto-correlation function of the received signal. This scheme works well in very low SNR regions and where energy detection might fail due to uncertainty in the value of noise power [141]. In fact, the application of such techniques with periodical spectrum sensing will be somewhat limited under the conventional frame structure. Thereby, the calculation of the optimal sensing time will not be a problem when sensing time is maximized and equal to the frame duration $T$.

Moreover, this frame enables us to employ continuous spectrum sensing that leads to higher QoS levels for licensed users.

The second significant aspect is related to the increased data transmission time that results in an increased throughput of the cognitive radio network [146]. This is due to the simplified continuity in data transmission as a result of the sensing time slot $\tau$, of the frame structure illustrated in Fig. 3.1.a.

### 3.3.1 Energy detection

From the concept of local spectrum sensing, we know that the secondary transmitter requires to perform spectrum sensing to detect whether there is a licensed receiver in the coverage of the cognitive transmitter that aims to protect the primary transmission. A schematic of the conventional receiver structure in [139] is shown in Fig. 3.3. Now we briefly touch upon the action of the cognitive radio system in the next frame. When the secondary receiver receives information from the secondary transmitter, it decodes the signal and it strips it away from the received signal. The remaining signal is then used for spectrum sensing. At the end
of the frame, if the primary user is present, to protect the licensed user from interference, data transmission will be ceased. Otherwise, if the primary user is detected to be idle, the secondary user will access the frequency band again in the next frame. Then, the process is repeated.

![Figure 3.3 Receiver structure of the opportunistic spectrum access](image)

In this scenario, the received signal at the secondary receiver is given by:

\[ S'_{k} = \phi \sqrt{P_{p,i}} \sqrt{X_{1} r_{k}} + \beta S_{su} + n_{k}, \quad \forall i = 1, \ldots, N \]  

(3.1)

where \( \phi \) represents the actual status of the frequency band (\( \phi = 1 \) when it is active and \( \phi = 0 \) when it is idle) and \( S_{su} \) denotes the signal from the secondary transmitter. As shown in this figure, the received signal is passed through the decoder where the signal from \( S - T \) is obtained. Then, the signal from \( S - T \) is cancelled out from the received signal \( S''_{k} \). Therefore, the remaining signal used for spectrum sensing is given by:

\[ S''_{k} = \phi \sqrt{P_{p,i}} \sqrt{X_{1} r_{k}} + n_{k}, \quad \forall i = 1, \ldots, N \]  

(3.2)
This is the signal S-R would receive if the S-T kept quite, which is the common scenario in conventional spectrum sensing strategies [47]. However, instead of sensing for a limited duration $\tau$, we use the whole frame duration for sensing. Accordingly, it is possible to perform sensing and transmission at the same time in a cognitive radio system using the whole duration of the frame structure $T$.

Generally, it is easier for the secondary transmitter to detect the presence of the primary signals than detecting the primary receiver directly.

Moreover, it is difficult for the cognitive users to differentiate primary signals from other existing secondary transmitter signals. Thus, we consider them all as one received signal at the cognitive user $S_k$. As mentioned before in the chapter, we assume a multi-level transmission power for licensed users. Therefore, the received signal at the $k^{th}$ sample at the secondary user can be represented by:

$$S_k = \begin{cases} \sqrt{P_{p,i}} \sqrt{X_1} r_k + n_k & H_1 \quad i = 1, \ldots, N \\ n_k & H_0, \end{cases}$$

(3.3)

where $r_k$ is the $j^{th}$ symbol transmitted from the primary transmitter which is assumed to follow, Gaussian distribution with zero mean and unit variance, i.e., $r_k \sim \mathcal{N}(0, 1)$, $N$ is the number of power levels. $P_{p,i}$ denotes the primary discrete transmit power where $0 < P_{p,i} < P_{p,i+1}$, $H_0$ and $H_1$ represent the hypothesis that P-T is absent (idle) and present (active), respectively and $n_k \sim \mathcal{N}(0, N_0)$ is the Additive white Gaussian Noise (AWGN). In fact, the secondary user must decide whether or not the primary user is transmitting whilst the objective for spectrum sensing is to detect between $H_0$ and $H_1$ based on the observation $S_k$.

In the hypothesis testing criteria in spectrum sensing, the optimal detector that uses the Likelihood Ratio Test (LRT) can be written as:
\(O(S_1, \ldots, S_M) = \frac{L(H_1|S_1, \ldots, S_M)}{L(H_0|S_1, \ldots, S_M)} = \sum_{i=1}^{N} \left( \frac{N_0}{N_0 + X_1P_{p,i}} \right)^{(\frac{M}{2})} \times \exp \left( \frac{X_1P_{p,i} \sum_{k=1}^{M} |S_k|^2}{2N_0(N_0 + X_1P_{p,i})} \right) \times \frac{P(P_{p,i})}{P(H_0)}, \tag{3.4} \)

in which we assume that the primary users can transmit at \(N\) power levels \(P_{p,1}, P_{p,2}, \ldots, P_{p,N}\), the probability for each of these levels is denoted by \(P(P_{p,i})\) satisfying \(\sum_{i=1}^{N} P(P_{p,i}) = P(H_1)\) and \(M\) represents the total number of samples at the secondary transmitter in one frame.

In this chapter, the energy detection \([136]\) is chosen as a local spectrum sensing technique that identifies spectrum holes, protects primary transmissions and provides low complexity and robustness against dispersed channels and fading. Energy detection is easy to implement and does not need any prior knowledge about the primary signal. Moreover, since from (3.4) we know that, \(O(S_1, \ldots, S_M)\) is increasing over \(E = \frac{1}{N} \sum_{k=1}^{M} |S_k|^2\), the hard decision is \(E \geq H_0 \delta\) where \(\delta\) is a pre-defined threshold. In fact, in the energy detector the samples are squared and their sum is compared against \(\delta\) (detection threshold). If \(E\) (which is the energy of the received signal) is greater than the sensing threshold, the null hypothesis is rejected and SU will not transmit \([139]\).

From \([144]\), the pdf of \(E \) conditioned on \(P_{p,i}, H_0\) and \(H_1\) can be written as:

\[
f(E|P_{p,i}) = \frac{E^{(\frac{M}{2}-1)}e^{-\frac{E}{2N_0 + X_1P_{p,i}}}}{\Gamma(\frac{M}{2})(2N_0 + 2X_1P_{p,i})^{(\frac{M}{2})}}, \tag{3.5} \]

\[
f(E|H_0) = \frac{E^{(\frac{M}{2}-1)}e^{-\frac{E}{2N_0}}}{\Gamma(\frac{M}{2})(2N_0)^{(\frac{M}{2})}}, \tag{3.6} \]

\[
f(E|H_1) = \sum_{i=1}^{N} f(E|P_{p,i}) \times \frac{P(P_{p,i})}{P(H_1)}, \tag{3.7} \]
where $\Gamma(.)$ shows the gamma function.

Therefore, based on the used energy detection scheme [136] as a spectrum sensing technique, the probability of detection is calculated as [138]:

$$P_d(\delta) = \sum_{i=1}^{N} \int_{\delta}^{+\infty} f(E|P_{p,i})dE \times \frac{P(P_{p,i})}{P(H_1)} = \sum_{i=1}^{N} \frac{\gamma(M/2, \frac{\delta}{2N_0+2P_{p,i}X_1}) P(P_{p,i})}{\Gamma(M/2)} P(H_1),$$

(3.8)

where $\gamma$ denotes the lower incomplete Gamma function, $\delta$ symbolises the decision threshold of the energy detector as already defined. Moreover, the probability of false alarm can be written as [138]:

$$P_{fa}(\delta) = \int_{\delta}^{+\infty} f(E|H_0)dE = \frac{\gamma(M/2, \frac{\delta}{2N_0})}{\Gamma(M/2)}.$$

(3.9)

We know from [134], [47] that the number of samples should be less than or equal to the product of sensing time and the sampling frequency $M \leq T_{fs}$, where $f_s$ represents the sampling frequency. Now, based on the pdf of the test statistic, the probability of detection can be formulated as:

$$P_d(\delta) = Q\left(\left(\frac{\delta}{N_0^2} - \omega - 1\right)\sqrt{\frac{T_{fs}}{2\omega + 1}}\right),$$

(3.10)

where $\omega$ denotes the received SNR from the primary user at the secondary detector. We can determine the detection threshold $\delta$ for a target probability of detection as follows:

$$\left(\frac{\delta}{N_0^2} - \omega - 1\right)\sqrt{\frac{T_{fs}}{2\omega + 1}} = Q^{-1}(P_d),$$

(3.11)

where $\bar{P}_d$ is the target probability of detection. In addition, the threshold is related to the probability of false alarm can be written as [3]:

...
3.3 The system model

\[ Q^{-1}(P_{fa}(\delta)) = (\frac{\delta}{N_0^2} - 1)\sqrt{T f_s}. \]  

(3.12)

Therefore, the probability of false alarm is related to the target probability of detection and can be approximated by:

\[ P_{fa}(\delta) = Q\left(\left(\frac{\delta}{N_0^2} - 1\right)\sqrt{\tau f_s}\right) = Q\left(\sqrt{2\omega + 1}Q^{-1}(P_d) + \sqrt{T f_s}\omega\right). \]  

(3.13)

Similar to the work in [47], we consider a high probability constraint due to the protection of the licensed users from harmful interference. Since the QoS protection of the primary side is a priority in cognitive radio systems, a high target detection probability is required. That can ensure that no detrimental interference is caused to the primary users by the secondary side. For example, in the IEEE 802.22 WARAN standard [135], the target probability of detection is chosen to be 90% for a SNR as low as \(-20\) \(dB\) for the licensed users’ signal at the secondary detector.

Moreover, the detection threshold \(\delta\) for a given target detection probability \(P_d = \bar{P}_d\) and a given sensing time (T) is formulated as [47]:

\[ \delta = N_0^2\left(\sqrt{\frac{2\omega + 1}{T f_s}}Q^{-1}(\bar{P}_d) + \omega + 1\right) \]  

(3.14)

Therefore, the required number of samples can be determined from (3.8) and (3.13):

\[ M \leq \frac{1}{\omega^2} \left[Q^{-1}(P_{fa}(\delta)) - Q^{-1}(P_d(\delta))\sqrt{2\omega + 1}\right]^2. \]  

(3.15)

Furthermore, in order to show the superiority of the proposed system (MSST), in the following remark, we address the probability of false alarm. Specifically, we discuss that under the energy detection scheme, this probability in conventional systems (TSST) is an increasing and concave function of the \(P_d\).
3.4 Primary Transmission with Multi-Power Level Estimation

As mentioned earlier, in the OSA setting, the secondary transmitter senses the channel for a certain length of time at the beginning of a time slot and sends the data if it does not detect any signal from the primary users. In [146], authors showed that there is no need for the cognitive users to be allocated a time period to sense and a separate period to transmit. They claim that both tasks can be carried out simultaneously. In this method, the secondary unit senses the channel at the beginning of the first time frame and transmits if the channel is free. But while transmitting, the secondary unit continues to sense the channel, then removes its own signal from the received signal and infers the transmitting status of the primary unit based on the remaining signal. If at any time the cognitive user detects that the licensed user is using the channel, it will stop transmitting in order not to interfere with the licensed user. Using this method, the cognitive radio can achieve better sensing accuracy and a higher transmission rate because both sensing and transmission times are increased. This scheme however, was proposed to overcome the trade-off between sensing accuracy and throughput.

3.4.1 Power allocation and transmission rate

In the two level transmission power system, the power allocation scheme allocates transmit power levels for the presence and absence of the primary users respectively.

It should be noticed that in many cases, the primary user transmits in more than one power level when it is active and the secondary user can take advantage of this and adjust its transmit power accordingly to achieve a higher throughput. When sensing, the secondary user not only determines whether the primary user is active or not, but also it attempts to detect the power level at which the primary user is transmitting. In fact, in the case of multiple primary transmit power levels, we use the multi-level power allocation. In multi-level power
allocation, a specified transmit power will be assigned due to one estimation of primary transmit power.

Table 3.1 Power allocation for the multiple primary transmit power

<table>
<thead>
<tr>
<th>Status of PU</th>
<th>Possibility</th>
<th>S-T’s Transmission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idle ((P_{p,0}))</td>
<td>(H_0)</td>
<td>((P_{s,0}))</td>
</tr>
<tr>
<td>Active ((P_{p,i}))</td>
<td>(H_1)</td>
<td>((P_{s,j}))</td>
</tr>
</tbody>
</table>

Table 3.1 illustrates the multi-level power allocation and the cognitive user status in the cases of the licensed user being active or idle. False alarms or a miss-detections of primary user could occur due to the constraints of the spectrum sensing. Therefore, \(P_{p,0}\) means the licensed user transmits with power zero and \(P_{p,i}\) means the licensed user is present with transmission power \(P_{p,i}\).

In each time slot, the primary transmitter’s power gain is \(X_2\) and \(\beta\) denotes the secondary transmitter’s power gain. Thus we calculate the secondary user’s instantaneous transmission rate using the formula below \([137]\):

\[
Tr_{i,j} = \log_2 \left(1 + \frac{\beta P_{s,j}}{N_0 + X_2P_{p,i}}\right),
\]

where \(i = 1, \ldots, N\) (\(N\) is the number of power levels) and \(j = 1, \ldots, N’\) (\(N’\) is the number of corresponding power levels). The first index means the actual status and the second index represents the decision result. \(P_{s,j}\) denotes the transmit power of S-T. Thus the index “0” indicates the status that the licensed user is idle i.e. \(P_{p,0} = 0\) and \(P(P_{p,0}) = P(H_0)\). In the following sections of this chapter, we use the instantaneous transmission rate in calculating the average achievable rate and the overall achievable throughput of our proposed cognitive radio system.

Therefore, it is important to estimate the primary transmit power that can be employed to decide the cognitive user’s power resulting in the licensed users’ transmissions being protected. Since, it is a hypothesis testing problem, the decision rule can be written as:
3.4 Primary Transmission with Multi-Power Level Estimation

\[ f(P_{p,i}|E) > f(P_{p,j}|E) \quad \forall j \neq i, \quad (3.17) \]

where \( P_{p,i} \) is the decision. If we substitute \( f(P_{p,i}|E) = \frac{f(E|P_{p,i})P(P_{p,i})}{f(E)} \) and (3.5) into (3.17), the decision space for the \( P_{p,i} \) can be written as:

\[ D_S(P_{p,i}) = \left\{ E | \max_{j \in [1,i]} d(j,i) < E < \min_{j \in [i,N]} d(j,i) \right\}, \quad (3.18) \]

where

\[ d(j,i) = \frac{2(N_0 + P_{p,j}X_1)(N_0 + P_{p,i}X_1)}{P_{p,j}X_1 - P_{p,i}X_1} \times \ln \left( \frac{P(P_{p,j})}{P(P_{p,i})} \frac{(N_0 + P_{p,j}X_1)^M}{(N_0 + P_{p,i}X_1)^M} \right). \quad (3.19) \]

Additionally, \( P(P_{p,i}) \) in (3.18) should not be very small because in this case, \( \max_{j \in [1,i]} d(j,i) \) can be greater than \( \min_{j \in [i,N]} d(j,i) \) which means the decision space for \( P_{p,i} \) is empty. In the following, we use this fact to initialize \( k \). For this, we assume an example where \( i \in \{3,4,5,6\} \), calculate \( f(P_{p,i}|E) \) and plot \( f(P_{p,i}|E) \) versus \( E \). As the result, for \( f(P_{p,i}|E) \) at any \( E \), it is not the greatest value. Therefore, its decision space is empty. We define \( \lambda_i, i = 1, \ldots, j + 1 \) as the break point and \( \alpha_j, i = 1, \ldots, j \) as the corresponding power estimation that is in the \( [\lambda_i, \lambda_{i+1}) \) interval and \( j \leq N \). Letting, \( \lambda_1 = 0 \) and \( \lambda_{j+1} = +\infty \), we can write the estimation of the primary user’s transmission power as follows:

\[ \hat{P}_{p,i} = \alpha_i, \quad \text{if} \quad E \in [\lambda_i, \lambda_{i+1}), \quad i = 1, \ldots, j. \quad (3.20) \]

Now, We require to decide on optimal values for \( \lambda_i \) and \( \alpha_i \).

**Lemma 3.4.1.** If \( E_1 < E_2 \) and \( f(P_{p,i}|E_1) < f(P_{p,j}|E_1), i < j \) then \( f(P_{p,i}|E_2) < f(P_{p,j}|E_2) \).

**Proof.** See Appendix A.1
Now, from (3.19), we know that $d(j,i)$ represents point $E$ for which $f(P_{pj}|E) < f(P_{pj}|E')$. Besides 3.4.1, optimal solution of $\lambda_i$ and $\alpha_i$ can be obtained as follows:

Step 1: at point $E = 0$, calculate $f(P_{pj}|0, k \in [1,N])$, also $i = \arg\max_{1 \leq j \leq N} f(P_{pj}|0)$ and set $\alpha_1 = P_{pj}$.

Step 2: in each iteration, calculate $\lambda_k = \min_{i < j \leq N} d(j,i)$, choose the minimum one and set $\alpha_k = P_{pj}$. The iteration continues till $i = N$. Algorithm 4.2 details the algorithm.

**Algorithm 3.1 Power estimation and optimal threshold**

1: Calculate $\lambda_1 = 0$, $i = \arg\max_{1 \leq j \leq N} f(P_{pj}|0)$, 
2: $\alpha_1 = P_{pj}$, $k = 4$. 
3: For $i = 1$ to $N$ 
4: $\lambda_k = \min_{i < j \leq N} d(j,i)$, $i = \arg\min_{i < j \leq N} d(j,i)$, 
5: $\alpha_k = P_{pj}$, $k = k + 1$. 
6: EndFor 
7: $\lambda_k = +\infty$, $j = k - 1$. 
8: Stop

The total detection probability is formulated as [146]:

$$P(T_d) = \sum_{i=0}^{N} P(\hat{P}_{pj}|P_{pj})P(P_{pj}),$$ (3.21)

where $\hat{P}$ denotes the optimal estimated primary transmit power and the $P(\hat{P}_{pj}|P_{pj})$ is the performance of the primary transmit power estimation.

### 3.4.2 Throughput of the Proposed Cognitive Radio System

In this section, we focus on the average achievable throughput of our scheme (MSST) and compare it with the conventional OSA method that operates based on two levels of power depicted in Fig. 3.3. Since a cognitive user is allowed to access a frequency band belonging to a primary user, spectrum sensing is the most significant task of a cognitive system that aims for OSA. When the frequency band is correctly detected idle ($H_0$), the instantaneous transmission rate of the cognitive radio network is given by:
3.4 Primary Transmission with Multi-Power Level Estimation

\[ c_0 = \log_2 \left(1 + \frac{\beta P_s}{N_0^2}\right). \]  \hspace{1cm} (3.22)

However, in practice, due to the nature of wireless communications that includes phenomena such as shadowing and fading, perfect spectrum sensing may not be achievable and a missed detection may occur. It may happen when the frequency band is incorrectly detected to be idle (i.e. the actual status of the primary users might be incorrectly detected). Therefore, we also consider the more realistic scenario of imperfect spectrum sensing. This is when the frequency band is incorrectly detected to be idle when in fact it is active \((H_1)\). In this case, following the proposed methods in [139], [147], the instantaneous transmission rate is given by:

\[ c_1 = \log_2 \left(1 + \frac{\beta P_s}{N_0^2 + P_p^2}\right). \]  \hspace{1cm} (3.23)

The average achievable throughput of a cognitive radio system that uses the conventional frame structure of Fig. 3.1.a is given by:

\[ \tilde{C}(\tau) = \tilde{C}_0(\tau) + \tilde{C}_1(\tau), \]  \hspace{1cm} (3.24)

where \( \tilde{C}_0(\tau) \) and \( \tilde{C}_1(\tau) \), the average throughput when the frequency band is idle and busy but detected to be idle, respectively are given by:

\[ \tilde{C}_0 = \frac{T - \tau}{T} P(H_0)(1 - P_{fa}(\tau)) c_0, \]  \hspace{1cm} (3.25)

\[ \tilde{C}_1 = \frac{T - \tau}{T} P(H_1)(1 - P_d(\tau)) c_1. \]  \hspace{1cm} (3.26)

Under the proposed cognitive radio system scheme (MSST) that assumes multi-level of power for primary users, spectrum sensing is performed simultaneously with data transmis-
sion in the frame duration $T$. Additionally, this scheme operates based on the frame structure with a simultaneous sensing and transmission slot as exhibited in Fig. 3.1.b. Therefore, when the frequency band is correctly detected idle ($H_0$), the instantaneous transmission rate of the proposed cognitive radio network is given by:

$$r_0 = \log_2(1 + \frac{\beta P_s}{N_0}).$$ \hspace{1cm} (3.27)

However, in the case that the frequency band is incorrectly detected to be idle when in fact it is active ($H_1$) whilst the primary user transmits with multiple levels of power, the instantaneous transmission rate is given by:

$$r_1 = \sum_{i=1}^{N} \sum_{j=1}^{N} \log_2\left(1 + \frac{\beta P_s}{N_0 + X_2 P_{p,i}}\right),$$ \hspace{1cm} (3.28)

where the first index means the actual status and the second index represents the decision result while $P_{s,j}$ denotes the transmit power of $S-T$. Thus the index '0' refers to the status that the licensed user is idle, i.e. $P_{p,0} = 0$ and $P(P_{p,0}) = P(H_0)$.

Therefore, the average achievable throughput of the proposed cognitive radio system (MSST) is given by:

$$\bar{Tr} = \bar{Tr}_0 + \bar{Tr}_1,$$ \hspace{1cm} (3.29)

where $\bar{Tr}_0$ and $\bar{Tr}_1$ denote the average achievable throughput when the frequency band is actually idle and busy (but incorrectly detected to be idle), respectively, and are given by:

$$\bar{Tr}_0 = P(H_0)(1 - P_fa(T))r_0$$ \hspace{1cm} (3.30)

$$\bar{Tr}_1 = P(H_1)(1 - P_d(T))r_1$$ \hspace{1cm} (3.31)
Now, to show the preference of the proposed cognitive radio system (MSST), we can show that for a target probability of detection $\bar{P}_d$, the proposed scheme exhibits higher average achievable throughputs compared to the conventional TSST scheme. According to the classical detection theory [148], [149], it is known that for a target probability of detection, higher sensing times lead to lower probabilities of false alarm. Therefore, for a given target probability of detection $P_d = \bar{P}_d$, $0 < \tau \leq T$ and also from the equation (3.13), it can be deduced that:

$$Q\left(\sqrt{2\omega + 1}Q^{-1}(\bar{P}_d) + \sqrt{f_s}\omega\right) \geq Q\left(\sqrt{2\omega + 1}Q^{-1}((\bar{P}_d) + \sqrt{Tf_s}\omega\right) \iff P_f(\tau) \geq P_f(T).$$

(3.32)

We know that the complementary cumulative distribution function of the standard Gaussian $Q(a)$ is a decreasing function of $a$. Considering this fact, for a sensing time $\tau$ where $0 < \tau \leq T$ and from the equations (3.24)-(3.32), it follows that:

$$\tilde{C}(\tau) = \tilde{C}_0(\tau) + \tilde{C}_1(\tau)
= \frac{T - \tau}{T} P(H_0)(1 - P_f(\tau))c_0 + \frac{T - \tau}{T} P(H_1)(1 - \bar{P}_d(\tau))c_1
< P(H_0)(1 - P_f(\tau))c_0 + P(H_1)(1 - \bar{P}_d(\tau))c_1
\leq P(H_0)(1 - P_f(T))c_0 + P(H_1)(1 - \bar{P}_d(T))c_1
= \tilde{T}r_0 + \tilde{T}r_1 = \tilde{T}r. \quad (3.33)$$

Obviously, for a sensing time, $0 < \tau \leq T$, it results in:

$$\tilde{T}r > \tilde{C}(\tau) \quad (3.34)$$
As a result, for a target detection probability \( P_d = \tilde{P}_d \), the average achievable throughput of the proposed cognitive radio system MSST is higher compared to the respective cognitive radio system TSST.

Therefore, considering earlier discussions and formulae, the average achievable throughput of the MSST scheme can be obtained as follows:

\[
\tilde{Tr}_0(\delta) = (1 - \frac{\gamma(M/2, \frac{\delta}{2N_0})}{\Gamma(M/2)}) \times \log_2 \left( 1 + \frac{\beta P_s}{N_0} \right) P(H_0), \quad (3.35)
\]

\[
\tilde{Tr}_1 = \sum_{i=1}^{N} \sum_{j=1}^{N} \log_2 \left( 1 + \frac{\beta P_{s,j}}{N_0 + X_2 P_{p,i}} \right) \times \left( 1 - \sum_{i=1}^{N} \frac{\gamma(M/2, \frac{\delta}{2N_0+2P_{p,i}X_1})}{\Gamma(M/2)} \right) P(P_{p,i}), \quad (3.36)
\]

where \( P(P_{p,i}) \) as mentioned before, denotes the probability of the primary user transmitting with power \( P_{p,i} \) which \( \sum_{i=1}^{N} P_{p,i} = P(H_1) \). As the result of this, we can obtain:

\[
\tilde{Tr} = \sum_{i=1}^{N} \sum_{j=1}^{N} \log_2 \left( 1 + \frac{\beta P_{s,j}}{N_0 + X_2 P_{p,i}} \right) \times \left( 1 - \sum_{i=1}^{N} \frac{\gamma(M/2, \frac{\delta}{2N_0+2P_{p,i}X_1})}{\Gamma(M/2)} \right) P(P_{p,i}) \]

\[
+ (1 - \frac{\gamma(M/2, \frac{\delta}{2N_0})}{\Gamma(M/2)}) \times \log_2 \left( 1 + \frac{\beta P_s}{N_0} \right) P(H_0). \quad (3.37)
\]

In addition, from (3.21) and (3.29), the average achievable rate at the secondary user can be modelled as:

\[
Tr_{av} = \sum_{i=0}^{N} \sum_{j=0}^{N} P(P_{p,j}) \tilde{Tr} P(\hat{P}_{p,i} | P_{p,j}). \quad (3.38)
\]

Due to the interference from the secondary user onto the primary user when \( P(H_1) \), the average interference power constraint under the pre-defined maximum interference \( I_{Max} \) can be written as:

\[
\sum_{i=0}^{N} \sum_{j=0}^{N} \alpha P(P_{p,j}) P_{s,j} P(\hat{P}_{p,i} | P_{p,j}) \leq I_{Max}. \quad (3.39)
\]
In our proposed scheme, the SU detects the power levels that PU uses and adjusts its transmit power while using the proposed strategy to sense the channel and transmit at the same time.

### 3.5 Simulation Results

![Graph showing Probability of detection versus the number of samples](image)

**Figure 3.4 Probability of detection versus the number of samples**

In this section, computer simulation results are presented to evaluate the performance of our proposed method MSST using the energy detection scheme as a spectrum sensing technique. In this way, we compare the MSST with the cognitive radio system with two levels of power that uses the frame structure presented in Fig. 3.1.a. We assume five levels of power ($N = 5$) for the primary transmitter with random probabilities. In MSST, the secondary tries to detect the primary’s power levels and adjust it’s transmission power. Whereas in TSST, the secondary user transmits just in the two statuses ON and OFF. The frame duration is set to $T = 100\, ms$, the probability that the frequency band is idle is considered to be $P(H_0) = 0.5$ whilst the sampling frequency $f_s$ is assumed to be 6 MHz. Additionally the
average power at the transmitter is set to 10 dB. The channels $\alpha$ and $\beta$ are assumed to follow the Rayleigh fading model and they are the squared norms of the independent circularly symmetric complex Gaussian (CSCG), with mean zero and variance $\delta_n^2$; namely $CN(0, \delta_n^2)$, random variables that are distributed as $CN(0, 1)$ and $CN(0, 10)$, respectively. The average interference power at the primary receiver is considered to be $P_s = 3 dB$ and the received SNR from the licensed user is set to $\omega = -20 dB$. The received SNR from the secondary transmitter is set to be $SNR_{su} = 20 dB$ whereas the ratio of $\tau$ is considered to be $\frac{\tau}{T} = 0.3$ in the simulations.

In Fig. 3.4, the probability of detection versus the number of samples is presented for our proposed scheme MSST (solid line) and the cognitive radio with two levels of power TSST (dashed line). One can clearly see that in both schemes, the probability of detection increases dramatically as the number of samples increase. As a result, the increase in total probabilities of detection of our scheme MSST (solid line) is significantly higher compared to the probability of detection of the two-level power system TSST (dashed line). This is because the number of samples used for detection are multiplied by $\frac{\tau}{T}$ as in [139]. The proposed scheme MSST defines the $\frac{\tau}{T}$ fraction. Therefore, in this case the number of samples are multiplied by $\frac{1}{\tau}$. More number of samples for detection means better detection of the primary user and lower probabilities of harmful interference. In addition, the average achievable rate is increased which can be explained by the fact that the whole duration of the frame $T$ is used for spectrum sensing in the proposed system MSST. This is contrary to the dual-level transmission power scheme TSST under the conventional frame structure of Fig. 3.1.a in which only a part of the frame is utilised for transmission.

In Fig. 3.5, the average achievable rate versus the target detection SNR from the primary user is presented for all four cases, which include five-level and two-level OSAs that employ the conventional frame structure in Fig. 3.1.a. The figure also depicts the proposed cognitive radio system and the conventional two-level power scheme for the primary users as well as
Figure 3.5 The average achievable rate (bits/sec/Hz) versus SNR from the primary user simultaneous sensing and transmission slot, MSST and TSST, respectively over a range of $X_1$. Further, the probability of target detection is equal to $\bar{P}_d = 0.9999$. In fact, we assume the following four states: dual and multiple levels of power under the frame structure as in Fig. 3.1.a as well as dual and multiple levels of power under the frame structure in Fig. 3.1.b. We will also compare the behaviours of such methods against each other. It can be clearly seen that the achievable throughput of both schemes that employ the frame structure with a single slot SST is significantly higher compared to the respective achievable throughput of the cognitive radio systems that employ the conventional frame structure of Fig. 3.1.a. This improvement in throughput can be explained by the argument that $T$ as the the whole duration of the frame is used for data transmission. This is in contrast to the conventional frame structure with two separated slots where only a part of the frame (i.e. $T - \tau$) is used for data transmission. Clearly, improvements in sensing and efficiency in the usage of the unused spectrum will result in the throughput enhancements in the system. Additionally, it
can be extracted from the equation (3.13) where $\tau < T$ for the same $\bar{P}_d$. That the $P_f$ under the conventional frame structure is higher compared to the respective $P_f$ of the proposed system MSST. Moreover, as can be seen, when the receiving power gain from the primary user increases, the achievable rate first raises and then stays constant. The latter remark can be explained and justified as follows: when the mean value of the primary transmission power increases, the detection performance is enhanced. This is also because the difference in the values of the pdf between $H_0$ and $H_1$ (from formulae (3.6) and (3.7)) becomes larger. This therefore has a positive effect on the detection of the primary transmission power in $H_1$.

![Figure 3.6 The average achievable rate (bits/sec/Hz) versus the probability of detection](image)

In Fig. 3.6, the average achievable throughput is illustrated versus the target probability of detection $P_d$ for all mentioned cases namely five-level and two-level OSAs that employ the conventional frame structure of Fig. 3.1.a as well as the proposed cognitive radio system and the conventional two-level power scheme for the primary users and simultaneous sensing and transmission slot, MSST and TSST. From Fig. 3.6, it can be seen that the average
3.6 Conclusions

achievable throughput under the proposed cognitive radio scheme MSST is significantly higher compared to the respective achievable throughput of the systems that employ the conventional frame structure in Fig. 3.1.a and the two-level scheme under the single slot frame structure TSST. This is whilst the reduction in the achievable throughput is small as the target probability of detection $P_d$ reaches higher values. This is especially prominent when compared to the respective cognitive users that use the conventional frame structure with two separated slots as in Fig. 3.1.a. As a result, MSST achieves an increased throughput at very high detection probabilities and can even detect weak signals transmitted by primary users with a better recognition of primary users’ presence. On the other hand, better protection for the licensed users is also provided.

3.6 Conclusions

In this chapter, we proposed a combined scheme for cognitive radio networks that significantly improves the achievable throughput of cognitive radio systems. In addition, we used the transmission method with multiple levels of power for primary users and a frame structure that provides secondary-user spectrum sensing and data transmission simultaneously. We studied the average achievable throughput of the proposed cognitive radio system under various target detection probabilities and illustrated that it can achieve remarkably improved throughput levels compared to the respective conventional cognitive radio systems. More specifically, a multilevel transmit power technique for the secondary users was proposed which resulted in maximizing the secondary users’ achievable rate. In this case, secondary users can adjust their transmission rates in accordance to their detection of primary users’ transmission power. That also was shown to provide better protection for the primary users from harmful interferences. Moreover, due to using the frame structure with one slot, the proposed multi-power level scheme can overcome the sensing throughput trade-off problem.
Furthermore, simulation results were provided to prove significant improvements in the overall throughput of the proposed cognitive radio system.
Chapter 4

Maximizing Achievable Rate of Cognitive Radio Networks for Both Primary and Secondary Users Through Efficient Spectrum Sensing and Multi-Level Power Allocation

4.1 Abstract

In this chapter, a multi-level power allocation scheme for the secondary users in cognitive radio systems is proposed that use a new frame structure to improve overall throughput of the system. The proposed cognitive radio scheme is different from the conventional strategies. The difference is in that the primary users can now transmit with multiple levels of power rather than a constant level of power. This is a more realistic scenario on existing practical systems. Moreover, different from previous chapter, our proposed strategy also allows the
secondary users to choose different power levels according to their receiving energy from the licensed user. In this chapter, we investigate the prospects of finding a solution to the problem of secondary user power level selection. Optimising such a functionality will lead to the maximisation of the achievable rate under the constraints of average transmit power at cognitive users. This will in turn keep average interference power to the primary users within bounds. This allows the throughput of secondary users to be improved while minimizing harmful interference to the primary user and maintaining the quality of service in the primary user side at the reduced complexity of computation. Finally, simulation results demonstrate that the proposed scheme can enhance the overall throughput of the system compared to conventional schemes.

4.2 Introduction

Power allocation as a function of cognitive radio networks has an important role in the usage of primary band under interference constraints. Spectrum access can be classified by different approaches: I) the interweave (the opportunistic spectrum access) [150]; II) the underlay [151]; III) the sensing-based spectrum sharing [152], [153] and IV) the multi-level power allocation [154]. In the opportunistic spectrum access, cognitive users can only access the licensed band when the primary band is detected to be idle [150]. When primary user QoS does not need to be protected in a cognitive radio system, the cognitive users can use the underlay scheme to transmit together with the licensed user’s signal at a lower level of power [151]. In addition, since the aim of spectrum sensing in sensing-based spectrum sharing is to determine the status of licensed users to access the PU band, if the primary user is recognised absent, the SU occupies the primary band with a high level power of transmission, otherwise with a low power [153].

In the three mentioned approaches, power allocation at the cognitive user is considered either constant or binary which causes low performance in the secondary side of the cognitive
radio network. To avoid degradation in performance of secondary users, Chen and et al. proposed a multi-level power allocation for secondary users in [154] that allows secondary users to adjust their power level based on their receiving energy from the primary user during the sensing period. In the multi-level power allocation strategy, from a secondary user’s perspective, the cognitive user should adjust its transmit power properly in order to select a transmission rate without causing too much interference to the licensed user. Therefore, the multi-level power allocation is designed in two stages:

- **Sensing stage**: the receiving energy from the primary user is gathered and the secondary users’ transmit power is decided.
- **Transmission stage**: with the corresponding power level, the cognitive user transmits and sends its data.

<table>
<thead>
<tr>
<th>Frame $\beta$</th>
<th>Frame $\beta+1$</th>
</tr>
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<tbody>
<tr>
<td>Sensing/Transmission</td>
<td>Sensing/Transmission</td>
</tr>
<tr>
<td>$T$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

Figure 4.1 Frame structure with a single slot

It can be easily understood that the binary power allocations are especial cases of this multi-level power allocation method.

In the first three sensing models, the transmit power of the primary user is constant and the sensing decision is one of the two values “0” or “1”. In [154], Chen and et al. consider the case in which the licensed user transmits with constant power however, cognitive users adopt a multi-power level. In practice, the binary modelling does not achieve a sufficient scenario in the spectrum scarcity situation [136], [141]. In this regard, we study sensing and power allocation for the scenario whereby the primary user transmits with multiple levels of
power. In this case, when the secondary users recognize the actual power of the primary user then a better design of transmission strategy can be applied at the secondary user side. In this chapter, we propose a multiple-level the power allocation strategy for cognitive users where power allocation strategy depends on the receiving energy from primary users on secondary side. Moreover, to address the problem of sensing throughput trade-off and increase the cognitive system throughput, we use a frame structure as in [134] which is shown in Fig. 4.1. Due to the use of the mentioned frame structure, our proposed scheme shows throughput improvements on the secondary side whilst simultaneously avoiding the sensing trade-off problem. Later in this chapter, we compare a multi-power level cognitive radio scheme that employs the single slot frame structure with the case that assumes constant power for the primary user transmission.

4.2.1 Advantages of the Proposed Scheme

From the secondary users’ perspective the object of achieving an acceptable high transmission rate is to be considered. In addition to this, a prospective scheme should aim at minimising interference to the primary users whilst adhering to the transmission power limitations. Such limitations are those of cognitive users and the average transmit power at the secondary ones. Moreover, under the average transmit power at the secondary user and the average interference power at the primary user constraints, energy threshold and power levels are optimised to maximise average achievable cognitive user rates. In addition, We compare our proposed cognitive radio strategy to the opportunistic spectrum access, the underlay and sensing-based spectrum sharing all of which adopt frame structures with a single slot. In this chapter, to maximise the average achievable rate, the Elkan algorithm [155] is employed as a K-means method. The scheme is much faster than the conventional algorithms such as the Lloyd’s algorithm used in [154].
As shown in Fig. 4.1, the frame structure used in our proposed cognitive radio scheme contains a single slot. Both the data transmission and the spectrum sensing tasks are carried out simultaneously during the mentioned slot. In this case, the advantage is the maximization of spectrum sensing and data transmission at the same time.

The maximization has two advantages: 1) the increased sensing time; 2) increase in the throughput of the cognitive radio network due to the sensing time slot $\tau$ of the frame structure of Fig. 4.2 that is now used for data transmission.

Using this frame structure, the system is enabled to detect very weak signals from the primary user, the detection of which under the frame structure of Fig. 4.1 would increase the transmission time and the throughput of the proposed cognitive radio system. The increased sensing time leads to better protection of the primary user from harmful interference and a decrease in false alarm probability results in a better use of the available unused spectrum.

Moreover, the calculation of the optimal sensing time is not an issue since the sensing time is maximized and equal to the frame duration $T$. Additionally, continuous spectrum sensing leads to better protection of licensed user quality of service.

4.2.2 Motivation

In our proposed multi-level power allocation strategy for the secondary users, the power level used at the secondary user varies based on its receiving energy during the sensing period. For our proposed cognitive radio scheme, we only assume a single phase due to the
use of the frame structure with one slot (Fig. 4.1). As already mentioned, this is different from the conventional multiple level power schemes. During this sensing/transmission phase, the receiving energy is accumulated and the transmit power of the secondary user is decided. On the consequent transmission stage, the secondary user sends its own data with the corresponding power level. Moreover, we maximize the average achievable rate at the secondary user under the secondary user average transmit power and the primary user average interference power level constraints. In this way, we optimize the power allocation ($P_{s,i}$) and the optimal space division ($D_i$) and unlike the conventional scheme, in our proposed strategy, optimizing the sensing period $\tau$ is not an issue. Afterwards, we compare our proposed cognitive radio system scheme with some of the conventional strategies. These strategies consist of the sensing-based spectrum sharing, opportunistic spectrum access, the underlay approach as well as the multiple level power allocation strategy. The multi-power level scheme applies the conventional frame structure with two separated slots as in Fig. 4.2. Results show improvements in throughput of the secondary users. However, in most of the conventional multi-level power allocation schemes [154], similar to the conventional opportunistic spectrum access schemes, frame structures with two separate slots are employed as shown in Fig. 4.2. In the mentioned frame structure, at the beginning of each frame (during $\tau$ units of time), spectrum sensing is carried on and the rest of the frame is used for transmission data (in $T-\tau$ units of time). On one hand, the increase in the sensing time, $\tau$, results in better detection of the primary user’s status and an increase in the probability of detection. However, a decrease in data transmission time will be a consequence. On the other hand, the increase in the transmission time results in an increase in false alarm probability and better protection of the primary user from harmful interferences. Therefore, there has been work on sensing trade-off [63] in the use of this conventional frame structure [150] and to address this research gap, we use the frame structure in Fig.4.1.
The rest of this chapter is organised as follows: In Section 4.3, the system model of the proposed scheme is presented. In Section 4.4, the proposed multi-level power allocation is described. In Section 4.5, the optimization problem, the applied solution and the proposed algorithm are studied. In Section 4.6, simulation results are provided. Finally, Section 4.7 concludes the chapter.

4.3 System Model

![Frame structure with a single slot](image)

In our proposed cognitive radio system model as shown in Fig. 4.3, two separate units are considered. The first is a primary unit that contains the primary transmitter (PT) and the primary receiver (PR). The second unit is one with the secondary transmitter (ST) and the secondary receiver (SR). The channel power gains between the elements of the network are $X_1$, $X_2$, $\alpha$ and $\beta$ where $X_1$ shows the channel gain from PT to ST and $X_2$ denotes the
channel gain from PT to SR. Additionally, $\alpha$ and $\beta$ denote the instantaneous channel power gains between ST and PR, and between ST and SR, respectively. To avoid the complexity of the channels’ issues, the channel gains are considered to be constant and known at the secondary side. We therefore mainly focus on multi-level power allocation method. Moreover, we concentrate on the multi-level power allocation strategy. Unlike various conventional schemes, we use the frame structure as in Fig. 4.1 with a single slot. In conventional cognitive radio systems, a frame structure with separated slots (i.e Fig. 4.2) is used with duration $\tau$ for the sensing slot and transmission slot with duration $T - \tau$. Due to periodical spectrum sensing in the conventional frame structure that have time frames of length, $T$, the secondary users would sense the channel for a fixed period $\tau$ (duration of sensing slot). Decisions will be then made on the status of the primary user. If the primary user is idle, it would transmit till the end of the time slot $T - \tau$.

It can be suggested that the secondary user listens to the primary user’s channel and collects the primary user’s accumulated energy during the sensing slot. Using the frame structure with a single slot, the system would be able to sense the channel obtaining the accumulated energy from the primary user and transmit at the same time. That leads to overcoming the sensing trade-off [153] and therefore increase the throughput of the cognitive radio system. According to the classical detection theory [144], an increase in the sensing time results in higher probability of detection and a lower probability of false alarm [139]. This trade-off is known as the sensing throughput trade-off. Additionally, due to the use of a single slot frame structure, our proposed cognitive radio scheme represents some advantages. These advantages include increasing detection probability that results in a better protection of primary user and lower probabilities of false alarm leading to a better use of the unused spectrum. The effects will be an increase in system throughput. In this way, the complexity of the proposed scheme is decreased. Since in our proposed cognitive radio system, the
primary user transmits with different levels of power, the received signal at the $l^{th}$ sample at secondary user can be given by:

\[
S_l = \begin{cases} 
\sqrt{P_{p,i}} \sqrt{X_l} e^{i\phi} r_l + n_l & H_1 \quad i = 1, \ldots, N \\
 n_l & H_0, 
\end{cases}
\]  \hspace{1cm} (4.1)

where $H_0$ and $H_1$ represent the hypothesis that the primary transmitter is idle and active, respectively. $N$ is the number of power levels. $\phi$ is the instant channel phase; $r_l \sim \mathcal{N}(0, \mathcal{P}_{p,i})$ denotes the $l^{th}$ transmitted symbol from primary transmitter that is used in the computation of the achievable channel rate. $\mathcal{P}_{p,i}$ shows the primary discrete transmit power for the $i^{th}$ power level; $n_l \sim \mathcal{N}(0, N_0)$ is the additive noise which follows a circularly symmetric complex Gaussian distribution with zero mean and variance $N_0$. Without loss of generality, $r_l$ and $n_l$ are assumed to be independent of each other.

During the slot period $(T)$, the sum of the accumulated received samples energy results in the detection statistic $\mathcal{W}$ which can be given by:

\[
\mathcal{W} = \sum_{l=0}^{T f_s} S_l^2. 
\]  \hspace{1cm} (4.2)

This way, the secondary user can decide whether or not the primary user is active. The formula in 4.2 shows the energy of the received signal and $T f_s$ is the number of samples at the secondary transmitter in one frame. We consider, similar to the works in [47] and [134], the number of samples at the secondary transmitter denoted by $M$. This should be equal or less that the product of the sensing time and the sampling frequency, i.e. $M \leq T f_s$. In this chapter, we assume the equality case where $M = T f_s$.

Moreover, the pdf of the received signal conditioned on $H_0$ and $H_1$ are given by [146]:

\[
f(W | H_0) = \frac{W^{M-1} e^{-\frac{W}{2N_0}}}{\Gamma\left(\frac{M}{2}\right)(2N_0^2)^{M/2}}, \hspace{1cm} (4.3)
\]
4.3 System Model

\[
f(W|H_1) = \sum_{i=1}^{N} \frac{W^{\frac{M}{2}-1} e^{-\frac{W}{2} (2N_0 + 2X_1 P_{p,i})}}{\Gamma\left(\frac{M}{2}\right) (2N_0 + 2X_1 P_{p,i})^{\frac{M}{2}}} P(P_{p,i}) P(H_1) \tag{4.4}
\]

From Eq. 4.4, the pdf of \(W\) conditioned on \(P_{p,j}\) can be given by:

\[
f(W|P_{p,j}) = \frac{W^{\frac{M}{2}-1} e^{-\frac{W}{2} (2N_0 + 2X_1 P_{p,j})}}{\Gamma\left(\frac{M}{2}\right) (2N_0 + 2X_1 P_{p,j})^{\frac{M}{2}}} \tag{4.5}
\]

where \(\Gamma(\cdot)\) denotes the Gamma function and is defined as \(\Gamma(a) = \int_0^{+\infty} t^{a-1} e^{-t} dt\). This way, \(f(W|H_0)\) and \(f(W|H_1)\) are variants of the Gamma distribution. Due to the assumption of that multi-power levels for the primary user, the primary user can transmit at \(N\) power levels, \(P_{p,1}, P_{p,2}, \ldots, P_{p,N}\) and the probability for each of these levels is denoted by \(P(P_{p,i})\). However, \(P(P_{p,i})\) is the probability that the primary user transmits with power \(P_{p,i}\) that satisfies \(\sum_{i=1}^{N} P(P_{p,i}) = P(H_1)\). In the conventional cognitive radio systems, the secondary user compares \(W\) with a threshold \(\delta\) and makes decisions according to \(W \gtrless H_1 \delta\). The following describes how different cognitive radio systems make decisions:

- In an OSA scheme: secondary transmitters can only access the primary band when the primary user is idle, which means \(W < \delta\).

- In a sensing based spectrum sharing scheme: secondary transmitter transmits with a higher power when \(W < \delta\) and with a lower power when \(W > \delta\).

- In an underlay scheme: secondary transmitters transmit with a constant power for all \(W\) according to the interference constraint at the primary transmitter.
4.4 Proposed Multi-Level Power Allocation for Secondary Users

In this chapter, we describe our proposed multi-level power allocation scheme where more than one level of power is available and assumed for the primary users. The scheme ensures that the multi-level transmission power for the primary users satisfies practical standards such as the GSM, LTE, LTE-A, etc. According to the current standards such as GSM [156], LTE, LTE-A [157] and the future standards, primary users are required to be equipped with different transmit power levels. Other reasons demanding such capability are environmental factors which place both practical and theoretical demands on primary users. As can be derived from much of the literature studying the power allocation problem [158], [159], different transmit power levels of the primary users is a natural functionality and should be considered [160]. In this scenario, when the licensed user operates under various transmit levels of power, the secondary user tries to recognise the primary user’s transmit power level. In addition to this, the presence of the primary user is also detected. Identifying the power level of the primary user achieves more cognition results in different interference levels to protect different powered primary users. In the conventional two-level power system, the power allocation scheme allocates two transmit power levels for the presence and absence of the primary users, respectively. In the case of multiple levels of primary transmit power, we use the multi-level power allocation when the PU transmits with different powers. In this chapter specifically, we concentrate on obtaining a reliable scheme for allocating power in the secondary users’ side to improve the average achievable rate. Therefore, we define a set of $N$ disjoint spaces of receiving energy $\mathcal{W}$ like $\{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_N\}$ and a set of corresponding allocated power levels of cognitive users $\{\mathcal{P}_{s,1}, \mathcal{P}_{s,2}, \ldots, \mathcal{P}_{s,N}\}$. The power allocation can be given by:
4.4 Proposed Multi-Level Power Allocation for Secondary Users

\[ P(\mathcal{W}) = \sum_{i=0}^{N} P_{s,i} I_{\mathcal{W} \in \mathcal{D}_i}, \]  

(4.6)

where \( I_f \) is an indicating function that if \( f \) is true then \( I_f = 1 \) and if \( f \) is not true, \( I_f = 0 \). Obviously, especial cases arise when \( N = 1 \) or \( 2 \) which indicates the conventional binary power allocation rules.

Using 4.6, the secondary users’s instantaneous transmission rate [138] with receiving \( \mathcal{W} \) can be obtained through below formulae in the presence and the absence of the primary user.

The instantaneous rate of cognitive user when the primary user is recognized idle is:

\[ R_{\text{inst}}(\mathcal{W})|_{H_0} = \sum_{i=0}^{N} \log_2 \left( 1 + \beta \frac{P_{s,i}}{N_0} \right) I_{\mathcal{W} \in \mathcal{D}_i}, \]  

(4.7)

where \( \beta \) denotes the secondary transmitter’s power gain.

The instantaneous rate of the cognitive user when the primary user is active can be given by:

\[ R_{\text{inst}}(\mathcal{W})|_{H_1} = \sum_{i=0}^{N} \log_2 \left( 1 + \beta \frac{P_{s,i}}{N_0 + X_2 P_{p,i}} \right) I_{\mathcal{W} \in \mathcal{D}_i}. \]  

(4.8)

Therefore, from (4.7) and (4.8), using the frame structure with a single slot as shown in 4.1 and the total probability formula, the average achievable rate of the proposed scheme can be calculated as follows:

\[ R_{\text{av}} = \sum_{i=1}^{N} \left\{ P_{i,0} \log_2 \left( 1 + \frac{P_{s,i} \beta}{N_0} \right) P(H_0) + P_{i,1} \log_2 \left( 1 + \frac{P_{s,i} \beta}{N_0 + X_2 P_{p,i}} \right) P(H_1) \right\}, \]  

(4.9)

where \( P(H_0) \) and \( P(H_1) \) define the idle and busy probabilities of the primary user, respectively and \( P(H_0) + P(H_1) = 1 \). \( P_{i,0} \) and \( P_{i,1} \) are functions of the pdf which are defined in the next section.
4.5 Optimization Problem

As mentioned earlier, in the schemes based on conventional OSA, the secondary transmitter senses the channel for a certain time (sensing time = $\tau$) at the beginning of a time slot and sends the data if it does not detect any signals from the licensed users. In our proposed cognitive radio system, we use the fact [161] that periodical sensing is not necessary and the cognitive user does not require to allocate a period to sense and a separate period to transmit; it can do both tasks at the same time. Through this strategy, at the beginning of the first time frame, the cognitive user senses the channel and transmits if the channel is free. The cognitive user will stop transmitting in order not to interfere with the primary user if at any time it detects that the primary user is using the channel. A higher transmission rate and a better sensing accuracy are the main results achieved due to increasing both sensing time and transmission time. Using our proposed strategy for the cognitive radio scheme results in overcoming the trade-off between sensing accuracy and throughput. Moreover, in this chapter, we assume that the licensed users transmit in more than one power level and the cognitive user can adjust its transmit power accordingly as well as transmit in multiple power levels to achieve higher throughputs.

It should be noticed that in the design of cognitive radio schemes, there are some restrictions that should be considered such as:

- Cognitive users’ power budget is limited.

- Protection needs to be provided to licensed users’ quality of service.

To keep the long-term power budget of the secondary users, the sum of powers allocated to the cognitive users should be equal or lower than the average transmit power. This limitation can be represented as:

$$\sum_{i=1}^{N} \left\{ P_{s,i}P(H_0)P_{i,0} + P_{s,i}P(H_1)P_{i,1} \right\} \leq P_{av}, \quad (4.10)$$
where $\mathcal{P}_{s,i}$ denotes the corresponding allocated power of the cognitive user; $\mathcal{P}_{i,0}$ means secondary transmitter chooses a power from $\{\mathcal{P}_{s,1}, \mathcal{P}_{s,2}, \ldots, \mathcal{P}_{s,N}\}$ when the primary user is idle and $\mathcal{P}_{i,1}$ has the same meaning when the primary user is active. Therefore, $\mathcal{P}_{i,0}$ and $\mathcal{P}_{i,1}$ are functions of the pdf and can be given by:

$$\mathcal{P}_{i,j} = P(W \in \mathcal{D}_i | H_j) = \int_0^{\infty} \mathcal{I}_{W \in \mathcal{D}_i} f(W | H_j) dx, \quad j = 0, 1.$$ (4.11)

In addition, since protecting primary users’ quality of service is an obligatory requirement, an assumption for the interference constraint should also be in place. According to (4.6), obviously the interference is affected only when the PU is active ($H_1$). Therefore, the average interference power can be given by:

$$\sum_{i=1}^{N} \alpha \mathcal{P}_{s,i} P(H_1) \mathcal{P}_{i,1} \leq I_{\text{max}}$$ (4.12)

where $I_{\text{max}}$ denotes the maximum average allowed interference at the primary user. Moreover, in the optimization problem, we are interested to find: i) the space division of the PU’s powers $\{\mathcal{P}_i\}$; ii) the power allocation $\{\mathcal{P}_{s,i}\}$ in order to maximize the secondary users’ average achievable rate under the power constraints.

Therefore, the optimization problem for the power allocation can be given by:

$$\max_{\mathcal{P}_{s,i}} \quad R_{av} = \sum_{i=1}^{N} \log_2 \left( 1 + \frac{\mathcal{P}_{s,i} h}{N_0} P(H_0) \mathcal{P}_{i,0} + \log_2 \left( 1 + \frac{\mathcal{P}_{s,i} h}{N_0 + X_2 \mathcal{P}_{i,1}} P(H_1) \mathcal{P}_{i,1} \right) \right)$$

s. t. $\sum_{i=1}^{N} \{ \mathcal{P}_{s,i} P(H_0) \mathcal{P}_{i,0} + \mathcal{P}_{s,i} P(H_1) \mathcal{P}_{i,1} \} \leq \mathcal{P}_{av},$ (4.13)

$$\sum_{i=1}^{N} \alpha \mathcal{P}_{s,i} P(H_1) \mathcal{P}_{i,1} \leq I_{\text{max}}$$

$$\mathcal{P}_{s,i} \geq 0 \quad \forall i, \quad T > 0.$$
4.5.1 The Applied Solution

Due to the problem’s characteristic, we need to decide the allocated power in secondary users according to the receiving powers from the primary user. However, as shown in Fig. 4.4, two stages are proposed to solve the problem:

i) Stage 1: assuming some optimal domains (ranges) for the primary user’s powers followed by determining the power allocations ($P_{s,i}$); ii) Stage 2: assuming a completed power allocation phase followed by determining the optimal domains ($D_i$). Since we have random data samples, a K-means method is deployed to solve this problem in our proposed cognitive radio system. In our case due to receiving power from PU randomly, the simplest initialization method is to sample $N$ levels of power at random from the receiving power and use them as initial values for the cluster centre to make domains.
4.5 Optimization Problem

4.5.2 The Algorithm

One of the common K-means methods is the Elkan’s algorithm [155]. While jointly optimizing clusters and assignment is difficult, optimizing one given the other is easy. In fact, the Elkan’s algorithm is a variation of the Lloyd’s algorithm [162] alternate optimization algorithm that uses the triangular inequality to avoid many distance calculations when assigning powers to domains. Although it is much faster than Lloyd’s, using a storage proportional to the number of domains by data points makes it impractical for a huge number of domains. However, in our proposed cognitive radio scheme, the Elkan’s algorithm is proposed to be applied. This is because recognizing primary users’ power level in a short period of time is very important and also the number of primary user power levels is limited.

The algorithm alternates the following stages until convergence: First, space divisions of the primary users’ powers \( \{ \mathcal{D}_i \} \) are obtained when the power allocations \( \{ \mathcal{P}_{s_i} \} \) are given. Secondly, power allocations \( \{ \mathcal{P}_{s_i} \} \) are determined when space divisions \( \{ \mathcal{D}_i \} \) are given. To solve this problem, we start from a feasible solution as the initial value. Here, space divisions \( \{ \mathcal{D}_i \} \), are obtained that satisfy \( P_{i,0} = \frac{1}{N} \) which means the possibilities of choosing between the space divisions are equal. We show that both the design of the space divisions, \( \{ \mathcal{D}_i \} \) and the transmission of the power allocations \( \{ \mathcal{P}_{s_i} \} \), are equivalent to the design of a modified distortion measure [163]. Details of the proposed algorithm is shown in Algorithm 4.2.
Algorithm 4.2 The proposed algorithm

1: **Procedure** Elk \((\mathcal{W}, P)\)
2: **Initialization**
3: \(P \leftarrow [P_1, \ldots, P_k] \), \(i \leftarrow \text{aremax} R(0, P_j)\), \(\phi = \phi / i, L(\mathcal{W}, P) \leftarrow 0\) \(\triangleright\) Set the lower band
4: Each time \(d(\mathcal{W}, P)\) is computed, set \(L(\mathcal{W}, P) \leftarrow d(\mathcal{W}, P), U(\mathcal{W}) \leftarrow \min_P d(\mathcal{W}, P)\)
5: **Repeat** until convergence:
6: **For** all \(P_i\) and \(P_k\) do
7: Compute \(d(P_i, P_k)\) \(\triangleright d : \text{distance}\)
8: **For** all \(P\) do
9: Compute \(\mathcal{L}_{i,k}(P) \leftarrow \frac{1}{2} \min_{P_i \neq P_k} d(P_i, P_k)\)
10: **End For**
11: **End For**
12: **Identity** all points \(\mathcal{W}\) such that \(\eta_l \leq \mathcal{L}_{i,k}(P(\mathcal{W}))\)
13: **For** all remaining points \(\mathcal{W}\) and \(P\) such that
14: A) \(P \neq P(W)\) and B) \(U(W) > L(W, P)\) and C) \(U(W) > \frac{1}{2} d(P(W), P)\) do
15: **If** \(r(\mathcal{W})\) then
16: Compute \(d(\mathcal{W}, P(\mathcal{W})))\) \& \& \(r(\mathcal{W}) \leftarrow false\)
17: **Else** \(d(\mathcal{W}, P(\mathcal{W})) \leftarrow U(\mathcal{W})\)
18: **End If**
19: **If** \(d(\mathcal{W}, P(\mathcal{W})) > L(\mathcal{W}, P)\) or \(d(\mathcal{W}, P(\mathcal{W})) > \frac{1}{2} d(P(\mathcal{W}), P)\) then
20: **End If**
21: **End If**
22: **End For**
23: **For** each \(P\), let \(m(P)\) be the mean of points \(P\)
24: \(P_i \leftarrow m(P_i)\)
25: **End For**
26: **For** each point \(W\) and \(P\) do
27: \(L(\mathcal{W}, P) \leftarrow \text{Max}[L(\mathcal{W}, P_i) - d(P, m(P)), 0]\)
28: **End For**
29: **For** each point \(W\) do
30: \(U(\mathcal{W}) \leftarrow U(\mathcal{W}) + d(m(P(\mathcal{W})), P(\mathcal{W}))\), \(r(\mathcal{W}) \leftarrow True\)
31: **End For**
32: Replace each \(P\) by \(m(P)\)
33: **End Procedure**
We introduce $\lambda$ and $\mu$ as the Lagrange multipliers for the average transmit power constraint and the average interference power. The distortion measure for optimising secondary users' throughput (rate) is given by:

$$R_{av}(W, P_{s,i}) = \log_2 \left( 1 + \frac{P_s \beta}{N_0} \right) P(H_0) f(W | \mathcal{H}_0) - \mu \alpha P_s, \mathcal{H}_1 f(W | \mathcal{H}_1) + P(H_1) \times \log_2 \left( 1 + \frac{P_s \beta}{N_0 + X_2 P_{p,i}} \right) f(W | \mathcal{H}_1) - \lambda P_s, \mathcal{H}_1 f(W | \mathcal{H}_1) + P(H_1) f(W | \mathcal{H}_1).$$

(4.14)

Stage 1 and 2 are repeatedly applied until convergence. According to [162], Lloyd’s algorithm does not guarantee that it converges to the global optimal. Here, the local convergence has been proved for some cases in one dimensional space. In simulations, they are repeated through 10 iterations ($N$ times, number of disjoin space divisions), every single time starting with a set of initial partition centroids ($\{1, \ldots, N\}$). At the end of Algorithm 4.2, the one achieving the greatest rate is chosen. The optimal power allocation strategy is given by the generalized partition centroid. During each iteration of the proposed algorithm, the lower bands $L(W_i, P_i)$ are updated for all receiving energy $W_i$ and power levels $P_i$. These updates take $O(nk)$ time, where $n$ is the number of data points (energies), $k$ is the number of domains to be found and $e$ is the number of iterations required. Experimentally, $e$ grows with $n$ and $k$. So the number of distance computations is $nke$ and the time complexity of the algorithm remains at $O(nke)$ despite the number of distance calculations being $O(n)$ [155]. The optimization problem in (4.13) is equivalent to choosing $\{D_i\}$ and $\{P_{s,i}\}$ to maximize the average distortion as:

$$R_{av} = \sum_{i=1}^{N} \int_{W \in D_i} R_{av}(W, P_{s,i}) dW.$$

(4.15)
4.5 Optimization Problem

Since we want to assume a general case as \( N \) spaces, to obtain the optimal space divisions \( \mathcal{W} \in \mathcal{D}_i \), the farthest neighbour rule is used as below [163]:

\[
\mathcal{D}_i(\mathcal{W}) = \{ \mathcal{W} : R_{av}(\mathcal{W}, \mathcal{P}_{s,i}) \geq R_{av}(\mathcal{W}, \mathcal{P}_{s,j}); \forall i, j \in \mathbb{R}, i \neq j \}, \quad (4.16)
\]

where \( R_{av}(\mathcal{W}, \mathcal{P}_{s,i}) \) is the rate of signal \( \mathcal{W} \) with power \( \mathcal{P}_{s,i} \) and \( R_{av}(\mathcal{W}, \mathcal{P}_{s,j}) \) is the rate of the same signal with a different power \( \mathcal{P}_{s,j} \). \( \mathcal{D}_i \) denotes \( i^{th} \) division domain. To obtain the optimal division domains \( \{ \mathcal{D}_i \} \) can be possible under some conditions such as:

**Lemma 4.5.1.** If \( \mathcal{W}_1 < \mathcal{W}_2 < \mathcal{W}_3 \) where \( \mathcal{W}_1 \in \mathcal{D}_i, \mathcal{W}_2 \in \mathcal{D}_j \) and \( i \neq j \), then we should have \( \mathcal{W}_3 \notin \mathcal{D}_i \).

**Proof:** See B.1.

In addition, by choosing \( (\mathcal{W}_i) \), we can find \( \mathcal{L}_{i,j}(\mathcal{W}_i) = 0 \). In this way, it can be proven that \( \mathcal{D}_i, i = 1, \ldots, N \) are continuous intervals and satisfy \( \cup_{i=1,\ldots,N} \mathcal{D}_i = [0, +\infty] \).

Here, the contradiction method can be used to obtain the proof. First, assume that \( \mathcal{D}_i \) can have more than two non-continuous intervals which contradicts Lemma 4.5.1. Second, define \( N + 1 \) thresholds \( \psi_0, \psi_1, \ldots, \psi_N \) with \( \psi_0 = 0 \) and \( \psi_N = +\infty \). Therefore, \( \mathcal{D}_i \) corresponds to one of \([\psi_{j-1}, \psi_j], j = [1, \ldots, N]\). Based on Lemma 4.5.1, the answer of \( \mathcal{W}_k \) that satisfies \( \mathcal{L}_{i,j}(\mathcal{W}_k) = 0 \) is given by:

\[
\mathcal{W}_k = \frac{2N_0(N_0 + X_2 \mathcal{P}_{p,i})}{X_2 \mathcal{P}_{p,i}} \times \ln \left[ \left( -Pr(H_0) \left( \log_2 \left( 1 + \frac{\beta \mathcal{P}_{p,i}}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) - \lambda (\mathcal{P}_{s,i} - \mathcal{P}_{s,k}) \right) \right) \right] - Pr(H_0) \log_2 \left( 1 + \frac{\beta \mathcal{P}_{p,k}}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) - \lambda (\mathcal{P}_{s,i} - \mathcal{P}_{s,k}) \right) \right] \times (2N_0 + X_2 \mathcal{P}_{p,i})^{\frac{M}{2}}, \quad (4.17)
\]

where
4.5 Optimization Problem

\[ D_k = P(H_1) \left[ \log_2 \left( 1 + \frac{\beta \mathcal{P}_{s,i}}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) - \log_2 \left( 1 + \frac{\beta \mathcal{P}_{s,k}}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) \right] - \]

\[ P(H_1) \lambda (\mathcal{P}_{s,i} - \mathcal{P}_{s,k}) - \mu P(H_1) \alpha (\mathcal{P}_{s,i} - \mathcal{P}_{s,k}) = \]

\[ P(H_1) \left[ \log_2 \left( 1 + \frac{\beta \mathcal{P}_{s,i}}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) - \log_2 \left( 1 + \frac{\beta \mathcal{P}_{s,k}}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) - (\mathcal{P}_{s,i} - \mathcal{P}_{s,k})(\lambda - \mu \alpha) \right]. \]

To obtain the endpoints of the subdomain’s interval, we have: \( \psi_l = \min_{\forall k \in \theta} \mathcal{W}_k. \) Assign \( D_i = [\psi_{l-1}, \psi_l) \forall l = [1, ..., M - 1] \) and \( i = \arg \min_{\forall k \in \theta} \mathcal{W}_k. \)

4.5.3 Power Allocation

After obtaining the optimal sub domain \( D_i = [\psi_{l-1}, \psi_l) \) and the endpoint of the interval as the threshold \( \psi_l, \) the equation of probabilities (4.11) can be re-written as:

\[ \mathcal{P}_{i,j} = \Pr(\mathcal{W} \in D_i | H_j) = \int_0^\infty I_{\mathcal{W} \in D_i} f(\mathcal{W} | H_j) d\mathcal{W}, \quad j = 0, 1. \]  (4.18)

According to the optimization problem (4.13) and under it’s constraints (4.10) and (4.12), the Lagrangian for the problem can be written as:

\[ L(\mathcal{P}_{s,i}, \lambda, \mu) = \sum_{i=1}^{N} \log_2 \left( 1 + \frac{\mathcal{P}_{s,i} h}{N_0} \right) P(H_0) P_{i,0} + \log_2 \left( 1 + \frac{\mathcal{P}_{s,i} h}{N_0 + X_2 \mathcal{P}_{p,i}} \right) P(H_1) P_{i,1} + \]

\[ \lambda \left[ \mathcal{P}_{av} - \sum_{i=1}^{N} \mathcal{P}_{s,i} P(H_0) P_{i,0} + \mathcal{P}_{s,i} P(H_1) P_{i,1} \right] + \mu \left[ I_{av} - \sum_{i=1}^{N} \alpha \mathcal{P}_{s,i} P(H_1) P_{i,1} \right], \]  (4.19)

where \( \lambda \) and \( \mu \) are dual variables. To determine the dual variables and solve the Lagrangian, the Lagrangian dual optimization can be formulated as:

\[ \min_{\lambda, \mu \geq 0} \mathcal{H}(\lambda, \mu) \triangleq \sup_{\mathcal{P}_{s,i} \geq 0} L(\mathcal{P}_{s,i}, \lambda, \mu). \]  (4.20)
From (4.13), we know:

\[
\frac{\partial^2 R_{av}}{\partial^2 P_{s,i}} = -\frac{P(H_1)P_{i,1}^2 \log_2 \left( \frac{P_{i,1} \beta}{N_0 + X_2 P_{Pj}} + 1 \right) (N_0 + X_2 P_{Pj})^2}{\log_2 \left( \frac{P_{i,1} \beta}{N_0} + 1 \right) N_0^2} - \frac{P(H_0)P_{i,0}^2 \log_2 \left( \frac{P_{i,0} \beta}{N_0 + X_2 P_{Pj}} + 1 \right) (N_0 + X_2 P_{Pj})^2}{\log_2 \left( \frac{P_{i,0} \beta}{N_0} + 1 \right) N_0^2}.
\]

So, \( \frac{\partial^2 R_{av}}{\partial P_{s,i}} < 0 \) and \( \frac{\partial^2 R_{av}}{\partial P_{s,i} \partial P_{s,j}} = 0 \forall i \neq j \). Considering that constraints \((4.10)\) and \((4.12)\) are linear functions, problem (4.13) is concave over \( P_{s,i} \). Therefore, to obtain the optimal value \( P_{s,i} \) of the Lagrangian dual optimization (4.20) that is equal to that of the main optimization problem (4.13), we can solve (4.20) instead of (4.13). To obtain the supremum of (4.20) \( L(P_{s,i}, \lambda, \mu) \), we have to take the derivative of \( L(P_{s,i}, \lambda, \mu) \) with respect to \( P_{s,i} \) as follows:

\[
\frac{\partial L(P_{s,i}, \lambda, \mu)}{\partial P_{s,i}} = \frac{P(H_1)P_{i,1} \beta}{\log_2 \left( \frac{P_{i,1} \beta}{N_0 + X_2 P_{Pj}} + 1 \right) (N_0 + X_2 P_{Pj})} - \lambda \left( P(H_0)P_{i,0} + P(H_1)P_{i,1} \right) - \mu P(H_1)P_{i,1} \alpha + \frac{P(H_0)P_{i,0} \beta}{\log_2 \left( \frac{P_{i,0} \beta}{N_0} + 1 \right) N_0}. \tag{4.21}
\]

Since the constraint \( P_{s,i} \geq 0 \) and by setting the equation (4.21) to “0”, the optimal power allocation \( P_{s,i} \) for the mentioned Lagrange multipliers \( \mu \) and \( \lambda \) can be calculated as:

\[
P_{s,i} = \left[ \frac{2P(H_0)P_{i,0} \beta + P(H_1)P_{i,1} \beta}{\mathcal{A}_1} - \frac{2(\mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4 + \mathcal{A}_5)}{\mathcal{A}_1} \right]^+ \tag{4.22}
\]
where \([x]^+\) denotes \(\max(0,x)\), and

\[
\mathcal{A}_1 = 4P(H_0)P_{i,0}\beta\lambda \log_2(e) + 4P(H_1)P_{i,1}\beta\lambda \log_2(e), \\
\mathcal{A}_2 = 4P(H_1)P_{i,1}\lambda X_2 \mathcal{P}_{P_i}\log_2(e), \\
\mathcal{A}_3 = 2P(H_0)P_{i,0}\lambda X_2 \mathcal{P}_{P_i}\log_2(e), \\
\mathcal{A}_4 = 4N_0P(H_1)P_{i,1}\lambda \log_2(e), \\
\mathcal{A}_5 = 4N_0P(H_0)P_{i,0}\lambda \log_2(e).
\]

To solve the above quadratic equation (4.22), we have:

\[
\mathcal{P}_{s,i} = \left[ \frac{\mathcal{A}_i + \sqrt{\Delta_i}}{2} \right]^+, i = \{1, 2, 3, 4, 5\}, \tag{4.23}
\]

where:

\[
\mathcal{A}_i = \frac{\log_2(e)\left(P(H_0)P_{i,0} + P(H_1)P_{i,1}\right)}{\lambda \left(P(H_0)P_{i,0} + P(H_1)P_{i,1}\right) + \mu \lambda P(H_1)P_{i,1}} - \frac{2N_0 + X_2 \mathcal{P}_{P_i}}{\beta} \tag{4.24}
\]

\[
\Delta_i = \mathcal{A}_i^2 + \frac{4}{\beta} \left\{ \frac{\log_2(e)[P(H_0)P_{i,0}(2N_0 + X_2 \mathcal{P}_{P_i}) + P(H_1)P_{i,1}N_0]}{\lambda [P(H_0)P_{i,0} + P(H_1)P_{i,1}] + \mu \alpha P(H_1)P_{i,1}} - \frac{N_0(N_0 + X_2 \mathcal{P}_{P_i})}{\beta} \right\} \tag{4.25}
\]

We have the following observation:

**Remark 1.** Power levels \(\mathcal{P}_{s,i}\) are non-increasing over \(i\).

**Proof:**
From (4.3) and (4.4), it can be written:

$$
\frac{f(W|H_1)}{f(W|H_0)} = \frac{\frac{e^{-\frac{W}{2}N_0 + X_1 \mathcal{P}_{P_i}}}{\Gamma\left(\frac{M}{2}\right) (2N_0 + X_1 \mathcal{P}_{P_i})}}{\frac{e^{-\frac{W}{2}N_0}}{\Gamma\left(\frac{M}{2}\right) (2N_0^2)}} = \frac{2N_0^2 e^{-\frac{W}{2}N_0 - X_1 \mathcal{P}_{P_i}}}{2(N_0 + X_1 \mathcal{P}_{P_i})} = \frac{e^{\frac{X_1 \mathcal{P}_{P_i}}{N_0 + X_1 \mathcal{P}_{P_i}}}}{N_0 + X_1 \mathcal{P}_{P_i}}.
$$

(4.26)

From the above equation, it can be concluded that (4.26) is an increasing function over $W$. To determine the monotonicity of $A_i$ easily, a term $Z_i$ can be defined to show that it’s monotonicity is equivalent to the monotonicity of $A_i$ as follows:

$$
Z_i = 1 + \frac{P[H_1]}{P[H_0]} \frac{P_{i+1}}{P_{i}} \left( 1 + \frac{\mu \alpha}{\lambda} \right) \frac{P_{i+1}}{P_{i}}.
$$

(4.27)

From (4.26), we have:

$$
P_{i+1} > P_{i+1} \frac{P_{i+1}}{P_{i+1}}, \quad \forall i
$$

(4.28)

In (4.28) and using d’Alembert’s formula [146], it can be found that the equation (4.28) is a decreasing function over $i$. In addition, by increasing the counter $i$, the fraction $\frac{P_{i+1}}{P_{i}}$ decreases. It can be deduced that fraction $\frac{P_{i+1}}{P_{i}}$ is a decreasing function. The claim can clearly be justified in equation (4.26) that fraction $\frac{P_{i+1}}{P_{i}}$ is an exponential function and with increasing $i$, $W$ decreases.

From (4.27), (4.28) and the above discussion, $A_i$ is a decreasing function over $i$. Now, to determine the monotocity of $A_i$, we define the following term to show that it’s monotonicity is equivalent to the $A_i$’s:
\[ Y_i = \frac{1 + \left( \frac{N_0}{N_0 + X_2 P_{p,i}} \right) \left( \frac{P_{H_1}}{P_{H_0}} \right) \left( \frac{P_{p,i}}{P_{p,0}} \right)}{1 + \left( \frac{P_{H_1}}{P_{H_0}} \right) \left( 1 + \frac{4 \alpha}{\lambda} \right) \left( \frac{P_{p,i}}{P_{p,0}} \right)}. \]  

(4.29)

In a similar way to \( \mathcal{A}_i \), it can be found that \( \Delta_i \) is a decreasing function over \( i \). Therefore, from (4.23), it can be concluded that \( \mathcal{P}_{s,i} \) is a non-increasing function with respect to \( i \).

### 4.6 Simulation results

In this section, simulation results are displayed to evaluate the performance of the proposed power allocation method with multi-level power allocation that employs a single slot frame structure. In this multi-level power allocation strategy, the system parameters are chosen as follows: \( T = 100ms \) (the frame duration) and the target detection probability is \( P_{d} = 0.9 \) in the opportunistic spectrum access method. We take \( X_1 = X_2 = N_0 = 1dB \), \( \alpha = \beta = 1dB \), \( I_{max} = \mathcal{P}_{p,i} = 0.5 \). The sampling frequency is assumed to be \( f_s = 1MHz \) and \( P_{av} = 10dB \).

Figure 4.5 illustrates a comparison in the power allocation policies under the proposed multi-level scheme and the conventional strategies. The mentioned comparison is carried out under the same assumption where the single slot frame structure is used for all schemes. It can be seen that in Fig. 4.5, the proposed multi-level power allocation strategy is compared with three other allocation schemes: namely, the opportunistic spectrum access [164], [165], the underlay scheme [154] and the sensing-based spectrum sensing [139]. In all mentioned schemes, a constant power for the different receiving energies is allocated. Fig. 4.5 exhibits that the proposed scheme allocates various power levels corresponding to the receiving energy from the primary user that reduces the probability of interference to the licensed user. Due to the above-mentioned adjustment when the receiving energy \( (W) \) is small, the proposed scheme allocates more power than the conventional strategies. However, when the receiving energy is large, the algorithm allocates less power. In addition, comparisons are provided on the proposed multi-level power allocation for different levels of energy such as \( N = 4 \) and...
Figure 4.5 Power allocation versus receiving energy under the conventional strategies and different levels ($N = 4, 8$) in proposed method

$N = 8$. Observations show that increasing the number of levels leads to larger allocations in small amounts of received energy. This behaviour is quite predictable. The opportunistic spectrum access has the lowest average achievable rate among all the schemes. This is because in this scheme, the secondary transmitter compares the accumulated received energy ($W$) with a threshold $\delta$ and makes decisions according to $W \lesssim H_0 \delta$. In the opportunistic spectrum access strategy, ST can only access the primary channel when $W < \delta$ which corresponds to the event $H_0$ (the primary user is idle).

Figure 4.6 illustrates the average achievable rate of the secondary user versus the average transmit power. The proposed multi-power level allocation, underlay method and sensing based spectrum sharing have all the same values of rate in the low average transmit power region. Nevertheless, in higher average transmit powers, the proposed multi-level power allocation reaches higher throughput margins. When the average transmit power increases,
4.6 Simulation results

Figure 4.6 The average achievable rate (bits/sec/Hz) versus average transmit power

since the rate (throughput) is decided by the average interference power constraint, the average achievable rates of all schemes remain constant. When the number of levels rise, the throughput of the proposed multi-level power allocation grows. One of the most impressive advantages of our proposed strategy appears when the number of levels increases. When this happens, the throughput views an upper limit and does not experiment trade-off complexity and performance. Therefore, choosing an optimal number of power levels is not necessary.

Figure 4.7 displays the comparison in average achievable throughput between our proposed multi-level power allocation system with multi-level receiving power capabilities equipped with the single-slot frame structure and the conventional power allocation that using the classic frame. The comparison is done under the same condition assuming $N = 4$, four levels of energy for both systems. As can be seen, in the low average transmit range, our proposed system and the conventional one have the same rates. However, when the average transmit is high, our proposed system achieves a higher average achievable rate.
Figure 4.7 The comparison in the achievable rate (bits/Hz/sec) between the conventional multi-level and our proposed multi-level power allocation which uses the single slot frame structure.

Figure 4.8 shows the relation between optimal space divisions and power allocation in different levels of receiving energy. As can be seen, an increase in optimal space division results in a decrease in optimal power allocation ($D_4 > D_8 \rightarrow P_4 > P_8$). This conclusion can be drawn by considering the sampling intervals. When this interval decreases, the pdf reduces. This means that when we have more (e.g. $N = 8$) levels of energy, we have more optimal space division. Therefore, the probability of existing $W$ (receiving energy from the primary user) is located in one of those regions (pdf) is less than when we have fewer regions (e.g. $N = 4$), ($pdf_{N=4} > pdf_{N=8}$). The above-mention issue forces secondary transmitters to have smaller allocations than when the number of levels is larger (e.g. $N = 8$).

Figure 4.9 presents the average achievable rate versus the permitted average interference power for both, the conventional allocation system and our proposed multi-level power allocation strategy. As can be seen, for low interference powers there is not a significant
4.7 Conclusion

In this chapter, we propose a combined scheme for cognitive radio networks. We employ a multi-level power transmission method for the primary user, where receiving signals from

difference between the two schemes. On the other hand, when the interference power reaches a threshold (maximum value), the throughput tends towards a constant value. However, the maximum value of the permitted average interference power in our proposed strategy is greater than the maximum permitted average interference power in the conventional system. This shows a better tolerance and demonstrated that our proposed system can provide better protection of QoS for the primary users.

Figure 4.8 The relation between optimal space divisions and power allocation in different levels of receiving energy
Figure 4.9 The average achievable throughput (bits/sec/Hz) versus the allowable average interference power

the primary users are grouped into different categories of power ranges. By maximizing the average achievable rate under two major constraints, namely the cognitive user average transmit power and the primary side average interference power, optimal power levels at the secondary user are determined. In this way, we can reduce the computational complexity and provide benchmarks against all power allocation strategies.
Chapter 5

Achievable Secrecy Rate of Multi-Power Level Cognitive Radio Networks

5.1 Abstract

In this chapter, we investigate the ASR as one of the most important PHY security parameters. The study is carried out for a multi-power level cognitive radio network based on stochastic geometry distributions. We consider multi-power level transmission for the primary user for the two following reasons. Firstly, it is more realistic to transmit with more than one power level and secondly, this can be used as a potential countermeasure for the purpose of security. In this case, eavesdroppers will find it difficult to detect the exact primary users’ power level which will consequently make extracting information very difficult. In particular, we consider the Poisson process of both the secondary and malicious users. We then analyse the effect of cognitive users’ interference on the achievable secrecy rate of the primary user. Derivations of closed-form achievable secrecy rate in an additive white Gaussian noise channel will follow. Furthermore, the outage probability of secrecy capacity of the primary user from a secure communication graph point of view is investigated. The cumulative distribution function of the achievable secrecy rate between the primary transmitter and the primary receiver
is studied. This helps achieve intuitive findings on the PHY security under a multi-power level scheme within the cognitive radio networks. All users are assumed to have Poisson distributions.

## 5.2 Introduction

Cognitive radio technology has attempted to minimise the adverse effects of the spectrum storage problem. This is due to advancements in wireless communication. Dynamically use of free spectrum in wireless channels allows secondary user to access the licensed bands opportunistically. This can be done without causing interference to the communications of the primary user. This is considered a scientific advantage of cognitive radio networks [166]. Cognitive radio networks can change their transmission or reception parameters based on interaction with environments in which they operate. This results in an efficient communication by avoiding interference with licensed or unlicensed users. There are two main characteristics of cognitive radio [167]: cognitive capability and reconfigurability. Cognitive capability refers to the ability of the radio system to sense information from its environment. Through this capability, the cognitive radio system can detect the spectrum resources that are not used by the primary user. Hence, the secondary user can select best spectrum allocation schemes and transmission parameters. Reconfigurability allows a cognitive user to change the transmitting channel adaptability according to the radio network environment. The spectrum sharing scheme of cognitive radio enables cognitive users to operate as long as they do not harm the licensed users’ transmission. In the spectrum sharing scheme, the secondary users’ transmission power is controlled optimally so no extra interference power can be imposed onto the primary user. Cognitive radio networks are more flexible in terms of wireless networks compared to other traditional radio networks. While cognitive radio is known as an efficient method to relieve the pressure of spectrum scarcity, there are at the same time, many security threats to cognitive radio networks over and above
other traditional radio environments. Special characteristics of cognitive radio networks have introduced entirely new types of security threats and challenges in networks. Since then, cognitive radio networks have made security more challenging and several major issues have not yet been investigated in the field of security for cognitive radio networks.

Due to the vulnerable nature of cognitive radio networks, providing strong security can be the most difficult aspect of making the cognitive radio a long-term commercially-viable concept [168]. Secure routing and other proposals in typical ad-hoc networks are achieved by a typical public key infrastructure. However, because of limited communication and computation resources in cognitive radio environments, a public key infrastructure (PKI) scheme cannot guarantee the security of cognitive radio systems [166]. Conventional security systems are based on cryptographic schemes that generally ignore: 1) the spatial configuration of the transceiver nodes; 2) the physical properties of channels [169]. In [167], a secrecy capacity study for a cognitive radio network model is presented. Primary users are assumed to have two-level transmit power capabilities. When the primary user is not active, its transmission power is assumed to be “0” and when it is active, it transmits with a constant power. In this chapter, we analyse the achievable secrecy rate of a cognitive radio network by considering multiple levels of power for the primary user. Due to the nature of cognitive radio systems, a primary user usually transmits with more than one power level, thus the assumption of multi-power level is more realistic. Moreover, in security issues which assume multi-power level for the primary user, this can be used as a countermeasure. Generally, eavesdroppers need to receive the primary user’s transmission power to allocate a corresponding power maliciously. However, when the primary user transmits with more than one level of power, it becomes more difficult for eavesdroppers to recognise the exact power level of the primary user. In addition, in the recognition process, time has a central role and the process has to be completed during a limited time so as not to miss the transmitted power from the primary user. In secrecy topics, Wyner proposed that the theoretical information
of secrecy capacity in [103] indicates how perfect secrecy can be obtained when a receiver uses a better channel than the eavesdropper. The secrecy capacity is one of the most major parameters of physical security in cognitive radio networks [170]. The term is defined as the difference between the Shannon capacity of the main channel (between the source and destination) and the Shannon capacity of the eavesdropper channel (between the source and the eavesdropper) [93]. In [171], Gopala et al. discussed the secrecy capacity of fading channels and studied the secure transmission of information over an ergodic fading channel in the presence of an eavesdropper. The outage secrecy capacity in additive white Gaussian noise channels was also investigated. In [172], the secrecy capacity in wireless networks was studied and it was found that the assumed network was a random extended network. According to Poisson Point Processes (PPP), the legitimate nodes (the primary and secondary users) and malicious nodes (eavesdroppers) are located in a square region. In [173], Negi et al. used MIMO communications and presented the achievable secret communication rate. In [112] and [174], the ergodic secrecy capacity of fading channels is studied. In [175], the secrecy capacity of cognitive radio networks was studied. In addition, primary user nodes are assumed to be static and a Primary Exclusive Region (PER) is considered. Further in their considerations, secondary nodes are not allowed inside the mentioned region and only a single eavesdropper is modelled. In [176], Devroye and et al. analysed the secrecy capacity and obtained expressions for the PER of a primary transmitter in a cognitive radio network without fading. In most works in the existing literature on PHY security, configurations are addressed with a limited number of nodes. Since the spatial location of the nodes can be modelled stochastically or deterministically, our proposed model can be extended to large-scale wireless networks. Deterministic models can be used in networks in which node locations are precisely known or nodes with regular structures. In the two-dimensional plane, deterministic models such as: square, hexagonal and triangular lattices [177] and [76], a stochastic spatial model is more suitable when just a statistical description of the location of
the nodes is known. To study network characteristics and to analyse interference dynamics of wireless networks [178], [179] and [180], the Poisson spatial model is general employed. This model also allows for thorough examination of connectivity and coverage [76], PHY security [181] and sensor cooperation [182] in such networks. Based on stochastic geometry and the theory of random geometric graphs, different methods such as the point process theory have shown results on fundamental limits of wireless networks. Such limits are generally associated with connectivity [76], the capacity and the outage probability of such networks [183]. In addition, other methods namely the percolation theory and probabilistic combinatorics have been employed to extract similar results on the above bounds. In all of the mentioned references, both the secondary users and the eavesdroppers have been assumed to only have Poisson distributions. In [184], Shu et al. considered the impact of the combination of the Poisson process of both the secondary users and the eavesdroppers.

In this chapter, we analyse the stochastic interference from the cognitive users to both the primary and malicious nodes (eavesdroppers). We also combine the Poisson processes of both the cognitive users and the eavesdroppers when the primary user transmits with multiple levels of power. Moreover, we describe the spatial distribution of network nodes and characterise the spatial location of both the cognitive users and eavesdroppers as Poisson Point Processes. The main contributions of this chapter can be listed as follows:

I . *Impact of interference from the secondary users:* the interference impact of the secondary users in a cognitive radio network model is investigated. In particular, the total interference of the secondary users on the primary users and eavesdroppers is obtained. Moreover, the PDF of the interference powers on the primary user and the secondary users is also analysed.

II . *Achievable rate of the primary users:* the expression of primary users’ achievable secrecy rate in the presence of secondary users is derived. The influence of the
stochastic distribution of primary users, secondary users and eavesdroppers on the achievable secrecy rate is investigated.

III. Outage probability of secrecy rate: the cumulative distribution function of achievable secrecy rate and outage probability of the primary users’ secrecy capacity are analysed. Then, expressions are derived for the probabilities of existence and outage probability of the secrecy capacity. This is carried out for a Poisson field of primary, secondary and malicious users in the network.

The rest of this chapter is organised as follows: In Section 5.3, the system model is presented. In Section 5.4, the information-theoretic description of the proposed scheme is studied. In Section 5.5, the ASR is discussed and then, the distribution of secrecy rate is derived. Finally, simulation results are presented and analysed in detail in Section 5.6, followed by the concluding remarks in Section 5.7.

5.3 The System Model

Figure 5.1 The system model
PT: Primary transmitter, PR: Primary receiver,
ST: Secondary transmitter, SR: Secondary receiver
Eve: Eavesdropper
In our proposed model, we consider a cognitive radio network where legitimate users and malicious users are randomly scattered in space as illustrated in Fig. 5.1. In some previous works including [181], a random geometric graph is applied to the study of the physical layer security that takes a geometrical point of view. However, the model does not explore the fundamental concepts of information theoretic security. On networks with randomly scattering nodes, S-graph (a.k.a Poisson secure communication graph) can be established as an appropriate geometrical representation of the information of secure links theoretically. Based on the notion of secrecy capacity, the S-graph can be defined as follows. Let \( \Pi_{PU} = \{ pu_i \} \subset \mathbb{R}^d \) denote the set of primary nodes, \( \Pi_{SU} = \{ su_i \} \subset \mathbb{R}^d \) gives the set of secondary nodes and \( \Pi_{EVE} = \{ eve_i \} \subset \mathbb{R}^d \) shows the set of eyedropper nodes where nodes are inside a region \( \mathbb{R} \) and \( d \) shows the region’s dimension. The S-graph is a directed graph \( G = (\Pi, E) \) with vertex set \( \Pi_{PU} \) and edge set:

\[
E = \{ pu_i, pu_j : \mathcal{C}_s(pu_i, pu_j) > \delta \},
\]

where \( \mathcal{C}_s(pu_i, pu_j) \) is the achievable secrecy rate of the link between the transmitter \( pu_i \) while receiver \( pu_j \) and \( \delta \) is a threshold that denotes the predefined infimum achievable secrecy rate for each link. There are two kinds of models to represent the spatial location of the nodes: statistical and deterministic model. In ad-hoc networks, only a statistical description of the node location is available and therefore a stochastic spatial model should be used. In such scenarios when there is no a priori information about the positions of the node, they can be treated as randomly distributed according to a homogeneous Poisson point access process [185]. The spatial Poisson process is a natural choice in such a situation because, given that a node is inside a region \( \mathbb{R} \), the pdf of its position is uniform over \( \mathbb{R} \) [169]. Among all homogeneous processes, the Poisson process has maximum entropy and corresponds to a simple and useful model for the location of nodes in a cognitive radio network [185]. In our proposed model, legitimate and malicious nodes coexist in a cognitive radio network.
5.3 The System Model

where $\Pi_{PU}$, $\Pi_{SU}$ and $\Pi_{EVE}$ follow a mutually independent homogeneous Poisson Point Processes [185] with densities $\lambda_{PU}$, $\lambda_{SU}$ and $\lambda_{EVE}$, respectively. The density indicates the average number of points (nodes) per unit area.

As illustrated in Fig. 5.1, primary receiver 1 ($PR_1$) is the closest neighbour (node) of primary transmitter 1 ($PT_1$) due to its distance to $PT_1$. $PR_2$ is the second closest neighbour of $PT_1$ as its distance to $PT_1$ is the second shortest. The proposed network is assumed to be a circular region of radius $R$. The features of the mentioned network are analysed and our results are extended to large networks with $R \to \infty$. The received powers at the primary users and eavesdroppers can be formulated by the propagation laws of wireless transmission[167].

In a quasi-static wireless environment, the received powers associated with the link $\overrightarrow{a_ia_j}$ can be written as:

$$P_{ra}(a_i, a_j) = \left( \sum_{k=0}^{N} \mathcal{P}_k I_{f_k} \right) \frac{|h(a_i, a_j)|^2}{L_{ij}},$$  \hspace{1cm} (5.2)

where $\mathcal{P}_k$ is the $k^{th}$ received power of the PU, $k$ is the index of power levels, $N$ is number of power levels and $I_{f_k}$ is an indicating function. If $f_k$ is true (i.e. primary user with transmission power level $k$ is active), then $I_{f_k} = 1$ and if however $f_k$ is not true (i.e. primary user is idle), then $I_{f_k} = 0$. Additionally, $h(a_i, a_j)$ is the complex fading coefficient of the primary link $\overrightarrow{a_ia_j}$ which is considered constant during the communication interval. In a quasi-static wireless environment, received power at the primary receiver increases with a rise in the transmission power of the primary transmitter. Following this, an increase in the amplitude of the complex fading coefficient of the primary link between the primary transmitter and primary receiver will also be witnessed.

In our case, due to assuming the multi levels of power for the primary user, the received power (i.e. $P_{ra}(a_i, a_j)$) at the primary receiver (i.e. $a_j$) in our proposed scenario (multi-power level) is potentially high. In addition, the value of the amplitude of the complex fading coefficient (i.e. $h(a_i, a_j)$) does not pose a problem. To calculate interference, we assume a
special case in $\mathbb{R}^2$ considering that the wireless environment only introduces path loss, i.e. $h(a_i, a_j) = 1$ for all $i \neq j$. In addition, the noise powers of the legitimate and the malicious users are equal. In such scenario, as described in [186], a transmitter $a_i$ is connected to a receiver $a_j$ if and only if $a_j$ is closer to $a_i$ than any other eavesdropper. $L_{ij} = ||a_i - a_j||$ is the distance between node $a_i$ and $a_j$ and $\alpha$ is the loss exponent of medium which varies $0.8 < \alpha < 4$ due to different communication environments. In our proposed system model, it is assumed that $\alpha > 2$.

5.4 Information-Theoretic Description of the Proposed System

5.4.1 The Shannon Capacity

In information theory, the maximum amount of information that can be transmitted through a channel is called Shannon information transmission capacity or generally known as Shannon capacity. The Shannon capacity’s equation is $C = B \log_2 \left( \frac{S}{N} + 1 \right)$ where $B$ is the channel bandwidth, $S$ is the signal energy and $N$ is the noise power. In general, in wireless scenarios, the Shannon capacity is written as $C = \log_2 (1 + \text{SNIR})$; $\text{SNIR} = \frac{P}{W + I}$, where $W$ and $I$ are the noise and interference powers, respectively.

In our proposed cognitive radio system, we deploy Poisson S-graph in $\mathbb{R}^2$, thus the Shannon capacity (i.e. $C_{\mathcal{P}}$) between primary transmitter and primary receiver with interference can be formulated as:

$$C_{\mathcal{P}} = \log_2 \left( 1 + \frac{P_{ra}(a_i, a_j)}{N_p + I_p} \right),$$

where $I_p$ denotes the interference powers of the primary receiver from the secondary users, $N_p$ is the noise powers introduced by the primary receivers and $C_{\mathcal{P}}$ is measured in bps/Hz.
Correspondingly, the Shannon capacity between the primary transmitter and the eavesdropper (i.e. eve) with interference can be given by:

$$C_{Eve} = \log_2 \left(1 + \frac{P_{ra}(a_i, eve)}{N_E + I_E}\right),$$

(5.4)

where $I_E$ is the interference power of the eavesdroppers from the secondary users and $N_E$ is the noise power introduced by the eavesdropper receiver.

### 5.4.2 The Achievable Secrecy Rate

Secrecy capacity is defined as the difference in the Shannon capacity of the channel between the source and the destination (a.k.a main channel) [93]. Therefore, we have $C_S = \max\{C_P - C_{Eve}, 0\}$. In this equation zero, (i.e.”0”) means $C_P - C_{Eve} > 0$ otherwise ($C_P - C_{Eve} < 0$) denotes there is no connection. Further, the noise affecting network nodes is Gaussian and independent. Equations (5.3) and (5.4) will lead to the following formulation of the achievable secrecy rate of the main link (i.e. $\vec{a}_i\vec{a}_j$):

$$C_S(a_i, a_j) = \max \left\{ \log_2 \left(1 + \frac{P_{ra}(a_i, a_j)}{\mathcal{N}_P + \mathcal{I}_P}\right) - \log_2 \left(1 + \frac{P_{ra}(a_i, eve)}{N_P + I_E}\right), 0 \right\},$$

(5.5)

where eve is the nearest eavesdropper with the strongest received signal from the primary transmitter $a_i$, $eve = \arg\min_{eve} P_{ra}(a_i, eve)$. As previously stated, a special case is studied in which path loss (i.e. $h(a_i, a_j)$) is the only type of fading assumed in our wireless environment.

Thus $h(a_i, a_j) = 1$ for all “i”s which are not equal to j (i.e. $i \neq j$). In addition, the thermal noise powers at the primary user and the eavesdropper can be assumed to be independent from the location of secondary users. This means noise powers are maintained equal even if secondary user receivers change position. So, Such noise levels can be assumed to be the equal, $\mathcal{N}_P = \mathcal{N}_E = \mathcal{N}$. In this case, the received powers of the primary users and the eavesdroppers become:
5.5 Achievable Secrecy Rate: Analysis and Discussions

The achievable secrecy rate's formula (5.5) can be simplified as:

\[
P_{ra}(a_i, a_j) = \frac{\sum_{k=0}^{N} P_k I_{f_k}}{|a_i - a_j|^{\alpha}},
\]

(5.6)

\[
P_{ra}(a_i, eve) = \frac{\sum_{k=0}^{N} P_k I_{f_k}}{|a_i - eve|^{\alpha}}.
\]

(5.7)

Therefore, the achievable secrecy rate’s formula (5.5) can be simplified as:

\[
C_s(a_i, a_j) = \max \left\{ \log_2 \left( 1 + \frac{\sum_{k=0}^{N} P_k I_{f_k}}{|a_i - a_j|^{\alpha (N + \mathcal{P})}} \right) - \log_2 \left( 1 + \frac{\sum_{k=0}^{N} P_k I_{f_k}}{|a_i - eve|^{\alpha (N + \mathcal{E})}} \right), 0 \right\},
\]

(5.8)

where \( \{d_i\}_{i=0}^{\infty} \) and \( \{g_i\}_{i=0}^{\infty} \) denote the random distances to the origin of the nodes in \( \Pi_{PU} \) and \( \Pi_{EVE} \), respectively. In general, the received power \( P_{ra}(a_i, a_j) \) decreases when the distance between nodes \( |a_i - a_j| \) increases. However, in our supposed scheme, due to assuming the multi levels of power for primary users and using average of received powers, the distance between nodes does not introduce a major challenge.

5.5 Achievable Secrecy Rate: Analysis and Discussions

In this section, we analyse the secrecy capacity by considering the main link (between the primary transmitter and its \( i^{th} \) closest neighbour that is a primary receiver node) and the eavesdropper. Since \( C_s(a_i, a_j) \) is a random variable, we need to derive the pdf of the secrecy capacity as an important property of the random variable \( C_s(a_i, a_j) \).
5.5 Achievable Secrecy Rate: Analysis and Discussions

5.5.1 Interference from SUs

To evaluate the pdf of the interference from all of the secondary users to the primary receiver, the pdf of the interference from the secondary users to the primary receiver can be given by:

\[ I_{pu} = \sum_{i=1}^{m} P_{su,i} L_{\alpha,pr,i} \]  

(5.9)

where \( m \) shows the number of secondary users in the system model, \( P_{su,i} \) is the transmitting power of the \( i^{th} \) secondary users (is assumed that \( P_{su,i} = 1 \) Watt in our proposed cognitive radio system). Further, \( L_{pr,i} \) indicates the distance between the \( i^{th} \) secondary user and the primary receiver. Moreover, we can formulate the pdf of interference from the secondary and primary users to the eavesdropper as follows:

\[ I_{eve} = \sum_{i=1}^{m} P_{su,i} L_{\alpha,eve,i} + \sum_{k=0}^{N} P_{pu,k} L_{\alpha,eve,pu} \]  

(5.10)

where \( P_{pu,k} \) is the transmitting power of the \( k^{th} \) received power of primary user, \( L_{eve,i} \) and \( L_{eve,pu} \) indicate the distance between the \( i^{th} \) secondary user and the eavesdropper and the distance between the primary user and the eavesdropper, respectively. As previously demonstrated, SUs are assumed to satisfy a Poisson process in the \( \mathbb{R}^2 \) (the two dimensional plane) with the densities equal to \( \lambda_{SU} \). Based on the Poisson process’s properties, the distributions of the locations \( L_{pr,i} \) of secondary users independent and the secondary nodes are distributed points with uniform distributions [187]. In addition, the characteristic function of the sum of a number of independent random variables is the product of the individual characteristic functions. Therefore, the following results can be obtained:

**Lemma 5.5.1.** The pdf of interference is found as:

...
5.5 Achievable Secrecy Rate: Analysis and Discussions

\[ f_X(z; \alpha) = \frac{1}{\pi \sqrt{z}} \sum_{k=1}^{\infty} \frac{\Gamma\left(\frac{1}{2}(k+1)\right)}{k!} \left(\frac{q}{z}\right)^k \sin\left(\frac{k\pi}{2}\right) = \frac{\pi}{2} \lambda_{SU} z^{-3} e^{-\frac{\pi^2 z^3}{2}}. \tag{5.11} \]

**Proof:** See Appendix C.1.

**Lemma 5.5.2.** The pdf of the ASR \( C_{\mathcal{PU},i} \), between PU and its \( i \)th closest neighbour is obtained as

\[
    f_{C_{\mathcal{PU},i}}(b) = (F_{C_{\mathcal{PU},i}}(b))' = - \left( F_{\mathcal{W}} \left( \frac{\sum_{k=0}^{N} \mathcal{P}_k I_k}{2^b - 1} \right) \right) \times \frac{\sum_{k=0}^{N} \mathcal{P}_k I_k \times 2^b \ln(2)}{(2^b - 1)^2} \times \frac{\sum_{k=0}^{N} \mathcal{P}_k I_k \times 2^b \ln(2)}{(2^b - 1)^2},
\]

where \( f_{C_{\mathcal{PU},i}}(b) \) is the pdf of Shannon capacity \( C_{\mathcal{PU},i} \), \( b \) and \( Z \) are random variables, \( F_{C_{\mathcal{PU},i}} \) is the CDF of Shannon capacity \( C_{\mathcal{PU},i} \). Based on the definition of CDF, \( F_{C_{\mathcal{PU},i}}(b) = P(C_{\mathcal{PU},i} \leq b) \) and the random variable \( C_{\mathcal{PU},i} \) takes on a value less than or equal to \( b \).

**Proof:** See Appendix C.2.

### 5.5.2 Distribution of Secrecy Rate

In this section, the achievable secrecy rate between a node and its \( i \)th neighbour is analysed. Similar to the pdf of \( C_{\mathcal{PU},i} \), the pdf of \( C_{\mathcal{SU},i} \) (the achievable secrecy rate between a secondary user and its \( i \)th closest neighbour that is a secondary receiver) can be derived [169].

Considering the proposed system model and using the secrecy capacity formula (5.5), the achievable secrecy rate between a node and its \( i \)th closest neighbour can be shown as:
5.5 Achievable Secrecy Rate: Analysis and Discussions

\( C_{SU,i} = \max\{C_{PU,i} - C_{Eve}, 0\} \) where, as mentioned earlier, \( C_{PU,i} = \log_2 \left( 1 + \frac{\sum_{k=0}^{N} P_k I_{f_k}}{N d_{AW}} \right) \) and \( C_{Eve} = \log_2 \left( 1 + \frac{\sum_{k=0}^{N} P_k I_{f_k}}{N g_{r}^{2\alpha}} \right) \).

The random variable \( C_{PU,i} \) is a transformation of the random variable \( X_i \triangleq R_{2i}^2 \) through the monotonic function \( k(x) = \log_2(1 + P_{x} b W) \). Therefore its pdf is followed by the rule

\[
f_{C_{PU,i}}(b) = \ln(2) \left( \frac{\pi \lambda_{PU}}{(i - 1)!} \right)^{\frac{i}{\alpha}} \exp \left( -\frac{\pi \lambda_{PU}}{2b - 1} \right) , \quad b \geq 0.
\]

By applying the above-mentioned rule, \( f_{C_{PU,i}}(b) \) can be presented as:

\[
f_{C_{PU,i}}(b) = \ln(2) \left( \frac{\pi \lambda_{PU}}{(i - 1)!} \right)^{\frac{i}{\alpha}} \exp \left( -\frac{\pi \lambda_{PU}}{2b - 1} \right) , \quad b \geq 0.
\]

From (5.14), by putting \( \lambda_{EVE} \) instead of \( \lambda_{PU} \) and setting \( i = 1 \), the pdf of \( C_{Eve} \) can be shown as:
5.5 Achievable Secrecy Rate: Analysis and Discussions

\[ f_{CEve}(b) = \ln(2) \left( \frac{\pi \lambda_{EVE}}{\alpha} \right) \left( \sum_{k=0}^{N} \mathcal{P}_k I_{fk} \right)^{\frac{1}{\alpha}} \times \frac{2^b}{(2^b - 1)^{1 + \frac{1}{\alpha}}} \times \exp \left( -\pi \lambda_{EVE} \left( \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{fk}}{2^b - 1} \right)^{\frac{1}{\alpha}} \right), \quad b \geq 0. \] (5.15)

Based on the homogeneous Poisson process’ properties, sequences \( \{d_i\}_{i=0}^{\infty} \) and \( \{g_i\}_{i=0}^{\infty} \) are mutually independent. As a result of this, random variables \( C_{PU,i} \) and \( C_{Eve} \) are also independent. From this, it can be understood that the pdf of \( f_{CSU,i} \) can result from the convolution of \( f_{C_{PU,i}}(b) \) and \( f_{C_{Eve}}(b) \) as follows:

\[
f_{CSU,i}(b) = \begin{cases} 
  f_{C_{PU,i}}(b) * f_{C_{Eve}}(b) + \mathcal{P}_{0,i} \delta(b), & b \geq 0 \\
  0, & b < 0,
\end{cases} \quad (5.16)
\]

where \( \delta(.) \) is the Dirac delta function and \( \mathcal{P}_{0,i} \) denotes the probability of zero secrecy capacity such that \( \mathcal{P}_{0,i} = \mathbb{P}\{C_{PU,i} - C_E < 0\} = \mathbb{P}\{\sum_{k=0}^{N} \mathcal{P}_k I_{fk} - \sum_{k=0}^{N} \mathcal{P}_{k,e} < 0\} \). Where \( e \) is the eavesdropper with the strongest received signal from the transmitter and \( \mathcal{P}_{k,e} \) is its \( k^{th} \) received power. \( \mathcal{I}, \mathcal{I}_E, \mathcal{I}_k \) and \( \mathcal{P}_{k,e} \) are independent and they are all related to the primary and secondary receivers. Moreover, \( \mathcal{N} \) is not related to the position of the users in the system. Now, we can replace (5.14) and (5.15) into (5.16) and therefore the pdf \( f_{CSU,i}(b) \) is obtained as below:

\[
f_{CSU,i}(b) = \begin{cases} 
  \Omega \times \left( \sum_{k=0}^{N} \mathcal{P}_k I_{fk} \right)^{\frac{(i+1)}{\alpha}} \mathcal{K}_i(b), & b \geq 0, \\
  \mathcal{P}_{0,i} \delta(b), & b = 0, \\
  0, & b < 0,
\end{cases} \quad (5.17)
\]
where \( \Omega = \frac{(\ln 2)^2 \pi^{i+1} \lambda_{PU}^{i} \lambda_{EVE}}{(i-1)! i^2} \) and \( \mathcal{A}_i(b) \) denotes the convolution integral that can be shown as:

\[
\mathcal{A}_i(b) = \int_b^\infty \left( \frac{2(2\tau-b)}{(2\tau-1)^{1+\frac{i}{\alpha}} (2\tau-b-1)^{1+\frac{i}{\alpha}}} \times \exp\left(-\pi \lambda_{PU} \left( \frac{\sum_{k=0}^N |\mathcal{P}_i(k)|}{2\tau-1} \right)^{\frac{1}{\alpha}} - \pi \lambda_{EVE} \left( \frac{\sum_{k=0}^N |\mathcal{P}_i(k)|}{2(2\tau-b-1)} \right)^{\frac{1}{\alpha}} \right) \right) d\tau. 
\]

(5.18)

### 5.5.3 Outage Probability of Secrecy Capacity

Now, from the above analysis and the formula for \( \mathcal{P}_{0,i} \) formula, the outage of the capacity can be determined. In the mean time, the existence of secrecy capacity between a user (as a network node) and its \( i^{th} \) closest neighbour can also be determined. We know that:

\[
\mathcal{P}_{0,i} = \int_0^\infty f_{C_E}(u) du \int_0^u f_{C_{PU},i}(x) dx. 
\]

(5.19)

Moreover, the probability of existence of a non-zero secrecy capacity is \( \mathcal{P}_{\text{exist},i} = \mathbb{P}\{C_{SU,i} > 0\} \) that can be written as:

\[
\mathcal{P}_{\text{exist},i} = 1 - \mathcal{P}_{0,i} = \int_0^\infty f_{C_{Eve}}(u) du \int_0^u f_{C_{PU},i}(t) dt. 
\]

(5.20)

Since the outage occurs if the signal power drops below the noise power level, the probability of signal outage can be computed if the probability distribution of the interfering signals (fading) is known. From the above discussion, the probability of an outage in secrecy capacity is:

\[
\mathcal{P}_{\text{out},i}(d_s) = \mathbb{P}\{C_{SU,i} < d_s\} = 1 - \mathbb{P}\{C_{SU,i} \geq d_s\} = 1 - \int_{d_s}^\infty f_{C_{SU,i}}(b) db, 
\]

(5.21)
where $d_s$ indicates an argument of the outage probability function that is $d_s > 0$. We know the probability of outage is the CDF of the random variable $C_{SU}$ that is evaluated at $d_s$ and by integrating of the corresponding pdf in (5.16). Thus, we obtain the following finding:

**Lemma 5.5.3.** The outage probability of secrecy capacity is found as:

\[
Pr_{out,i}(d_s) = 1 - \int_{d_s}^{\infty} \int_{b}^{\infty} f_{C_{SU},i}(\frac{\sum_{k=0}^{N} P_k I_{f_k}}{2^b - 1}) \times \frac{\sum_{k=0}^{N} P_k I_{f_k}}{(2^b - 1)^2} \times 2^b \ln(2) \times f_{C_{Eve}}(\frac{\sum_{k=0}^{N} P_k I_{f_k}}{2(\tau - b) - 1}) \times \frac{\sum_{k=0}^{N} P_k I_{f_k}}{(2(\tau - b) - 1)^2} \times 2^{(\tau - b)} \ln(2) d\tau db,
\]

where we have: $f_{C_{SU},i}(b) = f_{C_{PU},i}(b) \ast f_{C_{Eve}}(b) = \int_{b}^{\infty} f_{C_{PU},i}(\tau) f_{C_{Eve}}(\tau - b) d\tau$.

**Proof:** See Appendix C.3.

### 5.6 Simulation Results

In this section, we present the simulation results between our discussions and analysis. We also exhibit the relationships between outage probability, secrecy capacity and the densities of secondary users and the eavesdroppers. Such investigations are carried out in a cognitive radio system whereby primary users transmit with multiple levels of power. In this chapter, we assume the outage probability of secrecy capacity as the probability that the secrecy capacity is lower than a pre-defined threshold $d_S$. In fact, we evaluate the outage probability of secrecy capacity to indicate an insecure channel (communication link) between the users (legitimate users) if it is high.

Subsequently, we use the simulation results to test the performance results for the secrecy capacity and the outage probability between a node and the other nodes in its neighbourhood. In the following, we set the densities of SUs and eavesdroppers are $\lambda_{SU} = \lambda_{EVE} = 0.1$. The primary user density is $\lambda_{PU} = 1$ while the secondary users’ transmission power is...
$\mathcal{P}_{SU} = 1$ Watts. Additionally, noise power is assumed to $\mathcal{N} = 1$ Watts, the primary transmitter power, $\mathcal{P}_{pu} = \{5, 10, 15, 20\}$ Watts and the threshold rate $d_S = 1$.

Figure 5.2 illustrates the comparison of different values of secrecy capacity outage probability versus the eavesdropper density $\lambda_{EVE}$ in the presence of secondary users. In fact, it shows the relationship between the malicious user density $\lambda_{EVE}$ and secrecy outage probability in different positions of the primary users’ neighbours. It can be seen that in Fig. 5.2 where $i$ increases, the outage probability of secrecy capacity is higher. Moreover, the outage probability of secrecy capacity will increase as the density of malicious users (i.e. $\lambda_{EVE}$) grow. The increase in the outage probability of secrecy capacity with the growth of eavesdroppers’ density means that when malicious user density increases, they have higher probabilities of being closer to the secondary users. Therefore, the interference power of
eavesdroppers grows while the information leaked to malicious nodes. This then results in lower higher secrecy capacities and outage probabilities. The same goes for other values of \( i \). In addition, it can be seen that where \( i \) increases, the outage probability of secrecy capacity is higher. Such results are due to the fact that when primary receivers are farther from the primary transmitters, the secrecy capacity between the primary transmitter and the primary receiver is smaller.

![Figure 5.3](image_url)

Figure 5.3 The relationship between outage probability of secrecy capacity and the secondary users density.

Figure 5.3 shows the relationship between outage probability of secrecy capacity and the secondary users density for various values of \( i \). As can be seen in this figure, when \( i \) increases, the secrecy outage probability increases. This means, when secondary users are closer to the primary user, the interference powers at the primary user will be larger. In addition to this, when the density of secondary users \( \lambda_{SU} \) grows, the outage probability of secrecy capacity
also becomes larger. This is because when secondary users are closer and denser, they will destroy the quality of the primary’s link rather than that of the malicious users’.

Figure 5.4 CDF of the ASR between a network’s node and its $i^{th}$ closest neighbour.

Figure 5.4 displays the CDF of the achievable secrecy rate between a network node and its $i^{th}$ closest neighbour as it is formulated in (5.17). Fig. 5.4 indicates that the $F_{CSU,i}(b)$ and the outage probability are relatively small when $i$ is small. Furthermore, the CDF of the achievable secrecy rate raises slowly as the distance to the $i^{th}$ closer neighbour node increases with $i$. The mentioned point insinuates that the achievable secrecy rate has a greater likelihood of high secrecy capacity when $i$ is small. When $i$ grows, the achievable secrecy rate becomes smaller due to the distance between the primary transmitter and the primary receiver becoming larger.

Figure 5.5 shows the CDF of the achievable secrecy rate between the primary user and its $i^{th}$ closest neighbour according to (5.14). In this case, due to the dependence of the outage probability on the CDF, as can be seen in Fig. 5.5, in the case with small $i$’s, the CDF takes lower values. So, when the CDF is small, the outage probability is small by comparison.
5.6 Simulation Results

Figure 5.5 CDF of the ASR between the primary user and its $i^{th}$ closest neighbour. Increasing $i$ results in an increase in the distance between the PU and its $i^{th}$ closest neighbour which in the case of this figure, leads to the achievable secrecy rate decreasing gradually. This implies that for small values of $i$, the achievable secrecy rate is likely to be greater.

Figure 5.6 displays the pdf of the achievable secrecy rate between a node and its $i^{th}$ closest neighbour for various values of $i$. It shows that there is a higher probability that the achievable secrecy rate has a smaller value when the distance to the $i^{th}$ closest neighbour increases with $i$.

Figure 5.7 illustrates the CDF of the achievable secrecy rate between the eavesdropper and its closest neighbour. According to this figure, when the CDF is small, the outage probability is also small. However, as the CDF increases, the outage probability increases accordingly.
5.6 Simulation Results

Figure 5.6 pdf of the achievable secrecy rate between a node and its $i^{th}$ closest neighbour.

Figure 5.7 CDF of the achievable secrecy rate between a developer and its closest neighbour.
5.6 Simulation Results

Figure 5.8 pdf of the achievable secrecy rate between an eyedropper and it’s closest neighbour.

Figure 5.8 illustrates the pdf of the achievable secrecy rate between the eavesdropper and its closest neighbour. As can be seen, when the pdf grows the achievable secrecy also decreases. The safest mode for the network is when the pdf is at its smallest value. We can use these mentioned facts from figures (5.7 and 5.8) to obtain the effects of the presence of the malicious node on its neighbouring nodes. More importantly, it means that when the closest neighbour is the primary user, then protecting it becomes one of the most vital objectives of the cognitive radio network. In fact, we aim to minimise interference on the primary users.

Figure 5.9 presents the pdf of the primary user’s achievable secrecy rate versus the number of power levels. It indicates that increasing the number of power levels causes the outage probability of secrecy capacity to decrease gradually. Through comparison with the conventional methods such as the binary power level “0” or “1”, the suggested multi-power level system shows improvements in securing the fundamental requirements of a cognitive
radio system. In the rest of this section, we concentrate on the change in achievable capacity rate relating to the loss exponent of medium ($\alpha$).
Figure 5.10 shows the variation of the achievable secrecy rate for various values of \( N \) versus loss exponent of medium. As can be seen, as \( \alpha \) increases so does the achievable secrecy rate. This is while, the achievable secrecy rate decreases moderately when \( N \) increases. Moreover, the maximum achievable secrecy rate in the lowest impact of noise is 0.7 and in the highest impact of noise is 0.6. The optimal value of \( \alpha \) is 0.656 in the medium impact of noise.

![Figure 5.10](image)

**Figure 5.10** Achievable secrecy rate for various values of \( \alpha \) versus loss exponent of medium.

Figure 5.11 illustrates that by increasing the interference powers from the primary user, the achievable secrecy rate drops. In fact, this reduction is faster by increasing interference powers from the primary user. This case is more important in the secondary user’s power allocation when the secondary user needs to choose its transmission power without having any interference with the primary user.

![Figure 5.11](image)

**Figure 5.11** Achievable secrecy rate for various values of the interference powers from the PU vs loss exponent of medium.
5.7 Conclusion

Firstly, in this chapter, we presented a cognitive radio model with legitimate and secondary users in the presence of a malicious node. This is to analyse the impact of the stochastic interference on the fundamental limits of secure communications in the physical layer of a cognitive radio system. We further considered multiple levels of transmission power for the primary user when the cognitive and malicious nodes followed Poisson processes. In addition, from an information-theoretic perspective, we assumed a secure communication graph and analysed the interference from cognitive users to legitimate and malicious users. We derived an expression for the achievable secrecy rate between the primary transmitter and the primary receiver. Finally, by taking into account the performance results, we discussed how the Poisson processes of the primary user and eavesdroppers affected the achievable secrecy rate and outage probability between a network node and its neighbour. Characterizing the outage probability and the achievable secrecy rate can tell us how much information is transmitted through this cognitive system securely. To recognize how secure a cognitive radio system is, the achievable secrecy rate analysis is useful. The extracted information from the proposed scheme is necessary to find some countermeasures against any potential attacks on a cognitive radio system.
Chapter 6

Conclusions and Future Work

6.1 Concluding Remarks

Maintaining acceptable user satisfaction is becoming more challenging for network and spectrum license providers in the wireless communication arena. Following the unprecedented explosion in the number of mobile devices and the amount of data traffic, higher performance capabilities are expected of the wireless systems. One of the most important performance metrics is guaranteeing an even more stringent QoS. Providing such QoS guarantees for an increasing volume of users requires effective interference management. In order to achieve the goal, more advanced technologies capable of satisfying such user requirements have been employed. Future generations of wireless networks will be equipped with technologies that are remarkably more efficient and reliable in meeting higher network throughputs and user QoS needs.

Cognitive radio technology plays an undeniable and integral role in realizing the vision of future wireless networks. It is aimed at addressing the challenges associated with spectrum scarcity and inefficiency through dynamic spectrum access methods. The main idea is to allow secondary users to opportunistically use primary bands when not occupied. In order to keep pace with the projected trend of technological advancement in spectrum-
aware communications, cognitive radio networks need to compound some functionalities. These include capabilities such as spectrum sensing, spectrum decision, spectrum sharing and spectrum mobility. Therefore, innovative schemes and suitable security protocols are required for cognitive radio networks at different layers of the protocol stack. This necessity is even more prominent at the lower layers where both requirement compliance and the provision of secure and reliable communication links exist. The prime objective and contribution of this Ph.D. thesis has been the study of the existing conventional technologies and the proposition of new and innovative design alternatives for enhanced cognitive radio network throughput.

To accomplish this, through a background study into the existing literature, research gaps were investigated and identified. System simulations in addition to mathematical modelling were carried out to analyse the practicality and feasibility of the proposed methods. Upon verification, obtained results were evaluated and benchmarked against available conventional schemes and algorithms in favour of the justifications and hypotheses analysed. Research phases presented were conducted in pursuit of practical and realistic assumptions in an effort to improve currently existing strategies and guidelines. This thesis focuses on the throughput analysis of cognitive radio networks. The pivotal impetus of this research is to investigate significant challenges related to the cognitive radio systems. This is carried out through an emphasis on developing novel algorithms to enhance cognitive radio systems’ performance.

In addition, great attention has also been steered towards the security analysis of the cognitive radio technology as it is identified as an essential and challenging design issue. Moreover, further to making original and realistic contributions, the technical content in this thesis are directed at providing interesting insights for future research. Since all chapters of the thesis addressed related research issues, the main contributions of the thesis are summarized as follows providing an scope of the overall research conducted.

In chapter 3, with the aim of discovering more spectrum opportunities, a combined spectrum sensing strategy was proposed. In fact, the proposed spectrum technique combines
OSA and SS methods to further improve spectrum management. Since for an ideal detection scheme, $P_d$ should be as high as possible while $P_{fa}$ maximised and minimised, respectively, these two performance probabilities were considered in our proposed spectrum sensing strategy. To this end, the proposed scheme is required to avoid the sensing throughput trade-off problem occurring due to the use of the conventional frame structure with two separated slots. Adopting SST allows the system to avoid the trade-off between the sensing and transmission times. This in turn leads to the enhancement of system throughput due to the simultaneous maximization of both the spectrum sensing and data transmission times under this strategy. Therefore, the detection of weak signals transferred from PUs is made possible due to the increased sensing time. In this case, the probability of mis-detection decreases which leads to an enhanced detection probability. Obviously, a higher detection probability means a better protection of primary user QoS and a lower probability of $P_{fa}$. All the above-mentioned factors will make spectrum utilisation more efficient that help to achieve higher throughput levels. In addition, the continuous spectrum sensing in SST leads to higher QoS levels for PUs as one of the main goals of cognitive radio systems. Moreover, due to the continuity in data transmission, data transmission time will be prolonged. This results in an increased achievable throughput of the proposed cognitive radio scheme. Additionally, energy detection can be utilized as a local spectrum sensing technique. It can provide robustness against channel fading through the identification of spectrum holes at low complexity. In addition, energy detection is easy to implement and does not need any prior knowledge about the primary signal. In MSST on the other hand, the SU performs two tasks during sensing. It firstly determines whether or not the PU is active. Secondly, It detects the power level at which the PU is transmitting. In fact, the SU can take advantage of assuming multiple levels of transmission power. It can adjust its transmission power accordingly to achieve a higher throughput. In short, MSST is rewarded with adjustable power allocation. This results in protecting the PU’s transmission whilst increasing the SU’s throughput. This gives
the SU more opportunity to transmit than the OSA is used. The SU can access the licensed frequency bands only when the bands are not being used by PUs. More specially, the average achievable throughput of MSST under a single high detection probability constraint was studied. It can be evidently seen that MSST can achieve significantly improved throughput compared to the respective conventional cognitive radio strategy. The findings indicate that $P_d$ of MSST increases as the number of samples increase. Further, more number of samples for detection results in better detection of the PU and decreases the probability of harmful interference. In addition, compare to the TSST scheme, the MSST shows higher achievable throughput. This is while a decrease in the average achievable throughput as the $P_d$ receives higher values is small. This means that the MSST can better protection for the PUs on one hand, while achieving an increased throughput for SUs on the other. This is applicable even for very large values of $P_d$ and for very weak signals from the PUs.

In chapter 4, a multi-level power allocation strategy for the SUs is proposed. The mentioned multi-level power allocation not only uses a single slot frame structure to improve the overall throughput of the system, but also the SUs are able to choose from different power levels according to their receiving energy from the PUs. Having reached this point, the need to address the problem of SU power level selection is more prominently visible. Since the two-level power allocation approaches impose low performance in the secondary side of the cognitive radio systems, assuming a multi-level power allocation avoids degradation in the SUs’ performance. Moreover, as we discussed in detail earlier in this thesis, the binary modelling does not present a proper scenario in the spectrum scarcity. In this regard, the proposed multi-level power allocation strategy is studied for the scenario whereby the PU transmits with multiple levels of power. A higher transmission rate and a better sensing accuracy are the main results achieved due to increasing both sensing and transmission times. Since the SU requires to recognize the actual power of the PU, a better design of transmission strategy at the SU side was assumed. In addition, using the single-slot frame structure in
6.1 Concluding Remarks

the proposed multi-level power allocation scheme shows throughput improvements on the secondary user side whilst simultaneously avoiding the sensing trade-off problem. With the aim of obtaining a reliable scheme for the allocation power in the SUs’ side, the average achievable rate at the SUs was maximized. This was carried out under the SU average transmit power and the PU average interference power level constraints. Considering these limitations results in keeping the long-term power budget of the SUs. It also allows for a better protection of the PUs from harmful interferences which in turn leads to the provision of higher PU QoS. Since in the optimization problem, the value of power allocations for the SUs and the optimal domains for the PUs’ transmission powers were desired, jointly optimizing clusters and assignments proves difficult. To solve this problem and to optimize the clusters, K-means methods were considered. Since recognizing the PUs’ power level in a short period of time is necessary, Elkan’s algorithm was considered to avoid complexity of calculations when assigning powers to the domains. Finally, power allocation versus receiving energy under the multi-power level scheme and the conventional strategies was studied. This comparison in the power allocation policies were carried out under the same assumption where the single-slot frame structure is used for all schemes. In all conventional strategies, a constant power for different receiving energy levels is allocated when the receiving energy is low. As a result, the proposed multi-level power allocation scheme allocates more power than the conventional strategies. However, when the receiving energy is large, the proposed algorithm allocates less power which leads to better protection of the PUs. In addition, increasing the number of levels leads to larger allocations in small amounts of received energy which shows better recognition of the PU’s power. Moreover, the study of the average achievable rate of the SU versus the average transmit power shows the proposed multi-level and the conventional power allocations have all the same values of rate in the low average transmit power region. Nevertheless, the proposed multi-power level allocation reaches higher throughput margins in higher average transmit powers. Since the
rate is decided by the average interference power constraint, the average achievable rates of all schemes remain constant when the average transmit power increases. In addition, one of the most impressive advantage of the proposed multi-level power allocation appears when the number of levels increase. When this happens, the throughput witnesses an upper limit and does not experience trade-off complexity and performance. Therefore, choosing an optimal number of power levels is not necessary and introduces low calculation complexity. The achievable rate levels were also compared between the proposed multi-level scheme and the conventional power allocation in the low average transmit range. The former scheme was considered to be equipped with a single-slot and the latter supporting a classic frame structure. The results indicated a equal rate level for both schemes. However, due to an increase in the transmission time as a result of applying a single-slot frame structure, the proposed system proved to achieve higher average achievable rate levels. This was the case for when the average transmit power was high. In addition, results related to the study of the average achievable rate versus the permitted average interference power for the two schemes were also analysed. Conclusions were made to suggest that there is not a significant difference between the two schemes. On the other hand, when the interference power reaches a threshold (maximum value), the throughput tends towards a constant value. However, the maximum value of the permitted average interference power in the proposed scheme is greater than the maximum permitted interference power in the conventional system. Due to this, the proposed scheme shows a higher tolerance and demonstrates a higher capability to provide better protection of PU QoS.

In chapter 5, the security parameters of the proposed multi-power level cognitive radio scheme is investigated. Due to the importance of and the challenging nature of the issue of security in cognitive radio networks, the ASR, as one of the most important PHY security parameters is studied. In this chapter, we assume multiple levels of power for the PUs’ transmission for exploiting a potential of this kind of networks in security issues. Con-
6.1 Concluding Remarks

Considering more than one power level for the PUs’ transmission can be used as a potential countermeasure for the purpose of security. In this case, detecting the exact PUs’ power level will be difficult for malicious users which will consequently make extracting information very difficult. Practically, malicious users require to access the PUs’ transmission power in a short time to allocate a corresponding power maliciously. However, in the multi-power level case, it becomes more difficult for malicious users to recognise the exact PUs’ power level. Additionally, in the recognition process, time has a key role and the process has to be completed during a limited time so as not to miss the PUs’ transmission power. Moreover, in our proposed scheme, due to assuming a multi-power level scheme for the PUs, the received power at the primary receiver is potentially high. Also, the value of the fading coefficient does not pose an issue. Since in proposed model, only a statistical description of the nodes’ location was available, a stochastic spatial model was used. Moreover, since there was no a priori information about the nodes’ positions, they were treated as randomly distributed according to a homogeneous Poisson point process. In addition, in our proposed scheme, nodes were scattered randomly and a S-graph is established as an appropriate geometrical representation of the information of secure links. In this chapter, the interference impact of the SUs in our proposed cognitive radio scheme was investigated. The total interference of the SUs on the PU and malicious users was obtained. In addition, the pdf of the interference power on the PU and the SUs was analysed. Since unlike conventional strategies, the SUs are considered inside the studied region, the expression of the PUs’ ASR is derived in the presence of the SUs. Moreover, the influence of the stochastic distribution of legitimate users and eavesdroppers on the ASR was studied. Additionally, the cdf of the ASR and the outage probability of the PUs’ secrecy capacity was analysed. For a Poisson field of PUs, SUs and eavesdroppers, expressions were driven for the existence and outage probabilities of the secrecy capacity. Illustrated from the analysis results, the outage probability of the secrecy capacity increases in the following cases. Firstly, this happens when the distance between
the PU and its neighbour increases. Secondly, as eavesdropper density increases, they have higher probabilities of being closer to the receivers. Therefore, the interference power of malicious users grows while the information leaked to eavesdropper nodes drops. Moreover, when PRs are farther from PTs, the secrecy capacity between the PT and the PR is lower. The same scenario exists for increasing the density of the SUs. However, when the SUs are closer and denser, the outage probability of secrecy capacity is not as high as the case of increasing the density of eavesdroppers. This is because they do not degrade the quality of the PU’s link as much as the malicious users do. In addition, the cdf of the ASR between a network node and its $i^{th}$ closest neighbour as well as the cdf of the ASR between the PU and their corresponding $i^{th}$ closest neighbour is analysed. For the first case (i.e. network node), the CDF of ASR rises slowly as the distance with the $i^{th}$ closest neighbouring node increases with “$i$”. The mentioned point insinuates that the ASR has a greater likelihood of high secrecy capacity when nodes are farther. For the second case (i.e. PU) when the distance between the PU and its neighbour increases, this leads to the ASR decreasing gradually. This implies that the ASR is greater when the PUs are closer to the receivers. Moreover, the analysis of the pdf of the ASR between a node and its neighbour shows there is a higher probability that the ASR has a smaller value when the nodes are closer. In addition, the analysis of the cdf of the ASR between eavesdropper and its closest neighbour shows when the cdf is small, the outage probability is also small accordingly. However, in a similar situation, when the pdf of the ASR grows, the ASR decreases. The case in which the PDF is at its lowest value is known as the network’s safest mode. The main goal is minimizing the interference level on the PUs. When the eavesdropper’s closest neighbour is the PU, protecting it becomes one of the most vital objectives of the proposed cognitive radio scheme. Additionally, increasing the number of power levels causes the outage probability of secrecy capacity to decrease gradually. This shows that our proposed multi-power level scheme is effective in introducing improvements in meeting the fundamental requirements of the cognitive radio networks.
6.2 Future Works

Finally, the analysis of the ASR for various values of noise versus the loss exponent of medium shows that as the loss exponent of medium increases, the ASR grows. Moreover, increasing the interference powers from the PU when the loss exponent of medium grows results in ASR reductions. This case is more noticeable in the SUs’ power allocation where the SUs require to choose their transmission power without having any interference on the PUs.

6.2 Future Works

Having an exploratory review in the literature is indicative of the fact that existing research on cognitive radio technology is far from thorough. This dissertation covers some of the most significant issues in cognitive radio networks and provides frames (foundations) for future research. However, a number of challenges and issues still remain that need to be addressed. To this end, some future research directions are described in this section. The multi-power level transmission in this thesis was crucially investigated in all chapters. In chapter 3, the system model for the proposed cognitive radio scheme was designed in the non-cooperative case for cognitive users. It is interesting to investigate the system throughput by performing data transmission and spectrum sensing simultaneously. This analysis can be carried out assuming cooperative spectrum sensing for SUs’ side of the cognitive radio system. An accompanying consideration can be when the PU transmits with more than one power level. More specifically, the average achievable throughput of the mentioned cognitive radio system could be studied under a single high-target detection probability constraint. In addition, the problem of maximizing the ergodic throughput under joint average transmit and interference power constraints could be investigated. Further, an algorithm could be proposed that acquires the optimal detection probability and the power allocation strategy. Both of these parameters enable the system to achieve maximum throughput in the mentioned cognitive radio strategy.
In the context of multi-level power allocation in cognitive radio systems, the challenge of being impractical for a huge number of power domains needs to be addressed. This can be seen in the proposed algorithm presented in chapter 4. This can be achieved through establishing proper strategies and robust algorithm able to handle large data amounts with minimal execution time and low calculation complexity.

Security parameters analysis with a multi-power level transmission was conducted in chapter 5. However, the study was limited to eavesdroppers’ efforts in sensing the PUs’ transmission power in order to take advantage of this in cognitive radio networks. Given the impact of an attack (i.e. primary user emulation attack) on the throughput of the proposed system, it would be interesting to formulate the detection problem as a joint optimization problem. The objective of such a formulation would therefore be to increase the total achievable throughput regarding a band on the total interference imposed on the PUs’ side. The cooperative spectrum sensing is known as a functional method to improve the detection performance and accuracy. Therefore, it could be employed in making decisions on the status of the PUs. The strategy could also be used in recognizing PUs’ transmission power levels in order to carry out power allocation on the SUs’ side.
Bibliography


Appendix A

Appendices to Chapter 3

A.1 Proof of Lemma. 3.4.1

From \( f(P_{p,i}|E) = \frac{f(E|P_{p,i})P(P_{p,i})}{f(E)} \) and (3.5), the following equation can be written:

\[
f(P_{p,i}|E_2) = E(M - 1) e^{\left(-\frac{E}{2N_0 + 2X_1 P_{p,i}}\right)} \times \frac{P(P_{p,i})}{f(E_2)} = f(P_{p,i}|E_1) \times \frac{f(E_1)}{f(E_2)} e^{\left(-\frac{E_2 - E_1}{2N_0 + 2X_1 P_{p,i}}\right)} \quad (A.1)
\]

Consequently, We have:

\[
f(P_{p,j}|E_2) = f(P_{p,j}|E_1) \times \frac{f(E_1)}{f(E_2)} e^{\left(-\frac{E_2 - E_1}{2N_0 + 2X_1 P_{p,j}}\right)}. \quad (A.2)
\]

Since, \( i < j, P_{p,i} < P_{p,j} \) holds. Assuming \( f(P_{p,j}|E_1) < f(P_{p,j}|E_1) \) and from A.1, we can have \( f(P_{p,i}|E_2) < f(P_{p,j}|E_2) \).
Appendix B

Appendices to Chapter 4

B.1 Proof of Lemma. 4.5.1

We define the following function: \( L_{i,j}(W) = R_{ov}(W, \mathcal{P}_{s,i}) - R_{ov}(W, \mathcal{P}_{s,j}) \), which can be derived as follows:

\[
L_{i,j}(W) = \frac{W \cdot \Gamma(M/2)}{\Gamma(M/2)(2N_0)^{M/2}} \times \Pr(H_0) \times \left[ \log_2 \left( 1 + \frac{\beta \mathcal{P}_{s,i}}{2N_0} \right) - \log_2 \left( 1 + \frac{\beta \mathcal{P}_{s,k}}{2N_0} \right) \right] - \\
\mu \alpha \Pr(H_1) \frac{W \cdot \Gamma(M/2)}{\Gamma(M/2)(2N_0 + 2X_1 \mathcal{P}_{p,i})^{M/2}} \times (\mathcal{P}_{s,i} - \mathcal{P}_{s,k}) + \Pr(H_1) \times \\
\frac{W \cdot \Gamma(M/2)}{\Gamma(M/2)(2N_0 + 2X_1 \mathcal{P}_{p,i})^{M/2}} \times \left[ \log_2 \left( 1 + \frac{\mathcal{P}_{s,i} \beta}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) - \log_2 \left( 1 + \frac{\mathcal{P}_{s,k} \beta}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) \right] - \\
\lambda \times \left[ \Pr(H_1) \frac{W \cdot \Gamma(M/2)}{\Gamma(M/2)(2N_0)^{M/2}} \times \left( \log_2 \left( 1 + \frac{\beta \mathcal{P}_{s,i}}{2N_0} \right) - \log_2 \left( 1 + \frac{\beta \mathcal{P}_{s,k}}{2N_0} \right) \right) \right] \\
eq \frac{W \cdot \Gamma(M/2)}{\Gamma(M/2)} \times \frac{1}{(2N_0)^{M/2}} \Pr(H_0) \times \left( \log_2 \left( 1 + \frac{\beta \mathcal{P}_{s,i}}{2N_0} \right) - \log_2 \left( 1 + \frac{\beta \mathcal{P}_{s,k}}{2N_0} \right) \right) \\
- \mu \alpha \Pr(H_1) \frac{W \cdot \Gamma(M/2)}{\Gamma(M/2)(2N_0 + 2X_1 \mathcal{P}_{p,i})^{M/2}} \times (\mathcal{P}_{s,i} - \mathcal{P}_{s,k}) + \Pr(H_1) \frac{W \cdot \Gamma(M/2)}{\Gamma(M/2)(2N_0 + 2X_1 \mathcal{P}_{p,i})^{M/2}} \times \left( \log_2 \left( 1 + \frac{\mathcal{P}_{s,i} \beta}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) - \log_2 \left( 1 + \frac{\mathcal{P}_{s,k} \beta}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) \right) - \\
\lambda \frac{\Pr(H_0)}{(2N_0)^{M/2}} \\
\times \left( \log_2 \left( 1 + \frac{\mathcal{P}_{s,i} \beta}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) - \log_2 \left( 1 + \frac{\mathcal{P}_{s,k} \beta}{2N_0 + X_2 \mathcal{P}_{p,i}} \right) \right) - \lambda \frac{\Pr(H_0)}{(2N_0)^{M/2}}
\]
B.1 Proof of Lemma. 4.5.1

\[ + \Pr(H_1) \frac{e^{2N_0(X_1 P_{p,j})}}{(2N_0 + 2X_1 P_{p,j})^M} (P_{s,i} - P_{s,k}) \]

\[ = \frac{\gamma^{\frac{M}{2}} - 1}{\Gamma(\frac{M}{2})} \times \frac{\Pr(H_0)}{N_0^\frac{M}{2}} \left[ \log_2 \left( 1 + \frac{\beta P_{s,i}}{2N_0} \right) - \log_2 \left( 1 + \frac{\beta P_{s,k}}{2N_0} \right) - \lambda (P_{s,i} - P_{s,k}) \right] \]

\[ + \frac{\gamma^{\frac{M}{2}} - 1}{\Gamma(\frac{M}{2})(2N_0)^\frac{M}{2}} \Pr(H_1) \times (-\mu \alpha (P_{s,i} - P_{s,k}) + \log_2 \left( 1 + \frac{P_{s,k} \beta}{2N_0 + X_2 P_{p,i}} \right) - \log_2 \left( 1 + \frac{P_{s,k} \beta}{2N_0 + X_2 P_{p,i}} \right) - \lambda (P_{s,i} - P_{s,k}) \right] \]

To simplify (B.1), we can define:

\[ B_{i,k} = -\mu \alpha (P_{s,i} - P_{s,k}) + \left( \log_2 \left( 1 + \frac{P_{s,i} \beta}{2N_0 + X_2 P_{p,i}} \right) - \log_2 \left( 1 + \frac{P_{s,k} \beta}{2N_0 + X_2 P_{p,i}} \right) \right) - \lambda (P_{s,i} - P_{s,k}). \]  

(B.2)

and

\[ C_{i,k} = \log_2 \left( 1 + \frac{P_{s,i} \beta}{2N_0} \right) - \log_2 \left( 1 + \frac{P_{s,k} \beta}{2N_0} \right) - \lambda (P_{s,i} - P_{s,k}). \]  

(B.3)

From (B.2) and (B.3), the below function can be written:

\[ F(X) = \frac{B_{i,k}}{(2N_0 + 2X_2 P_{p,i})^\frac{M}{2}} e^{\frac{X_2 P_{p,i}}{2N_0(N_0 + X_2 P_{p,i})}} + \frac{C_{i,k}}{N_0^\frac{M}{2}} \]  

(B.4)

To determine the sign of the function, the derivative of the function is calculated as:

\[ \frac{\partial F}{\partial X} = \frac{B_{i,k}}{(2N_0 + 2X_2 P_{p,i})^\frac{M}{2}} \times \frac{X_2 P_{p,i}}{2N_0(N_0 + X_2 P_{p,i})} e^{\frac{X_2 P_{p,i}}{2N_0(N_0 + X_2 P_{p,i})}}. \]  

(B.5)
We can assume:

\[ C_1 = \frac{B_{i,k}}{(2N_0 + 2X_2 P_{p,i})^M} \]  

(B.6)

From (B.5) and (B.6), if \( C_1 > 0 \) then \( \frac{\partial F}{\partial X} > 0 \) otherwise if \( C_1 < 0 \) then \( \frac{\partial F}{\partial X} < 0 \). In addition the second derivative of function \( F(X) \) is calculated as:

\[ \frac{\partial^2 F}{\partial X} = \frac{B_{i,k}}{(2N_0 + 2X_2 P_{p,i})^M} \times \left( \frac{X_2 P_{p,i}}{2N_0(N_0 + X_2 P_{p,i})} \right)^2 \times e^{\frac{X_2 P_{p,j}}{2N_0(N_0 + X_2 P_{p,i})}}. \]  

(B.7)

From the above equation (B.7), if \( \frac{B_{i,k}}{(2N_0 + 2X_2 P_{p,i})^M} = C_1 > 0 \) then \( \frac{\partial^2 F}{\partial X} > 0 \) or if \( C_1 < 0 \) then \( \frac{\partial^2 F}{\partial X} < 0 \). We have \( \mathcal{W}_1 \in \mathcal{D}_i, \mathcal{W}_2 \in \mathcal{D}_j \) and from (4.16), it can be known that \( \mathcal{L}_{i,j}(\mathcal{W}_1) > 0 \) and \( \mathcal{L}_{i,j}(\mathcal{W}_2) < 0 \). In (4.16), the sign of \( \mathcal{L}_{i,j}(\mathcal{W}) \) is decided by (B.5) and (B.7) which is strictly monotonic function. Therefore, for any \( \mathcal{W}_3 > \mathcal{W}_2 \), there are \( \mathcal{L}_{i,j}(\mathcal{W}_3) < 0 \) and \( \mathcal{W}_3 \notin \mathcal{D}_i \).
Appendix C

Appendices to Chapter 5

C.1 Proof of Lemma. 5.5.1

The characteristic function of random variable $\mathcal{X}$, i.e., $\phi_{\mathcal{X}}(\mathcal{W}) = E(e^{i\mathcal{W}\mathcal{X}})$ can be calculated as:

$$\phi_{\mathcal{X}}(\mathcal{W}) = \exp \left( -\lambda_{SU} \pi \Gamma \left( 1 - \frac{2}{\alpha} \right) e^{\left( \frac{-\pi}{\alpha} \right) \mathcal{W} \left( \frac{2}{\alpha} \right)} \right), \quad (C.1)$$

where $\mathcal{W} \in \mathbb{R}$ is the argument of characteristic function, $\alpha$ is the path loss exponent and $\Gamma$ denotes the gamma function. The inverse Fourier transform [187] can be taken as follows.

By putting $\beta = \frac{2}{\alpha}$ in (C.1), we have:

$$\phi_{\mathcal{X}}(\mathcal{W}) = \exp \left( -\lambda_{SU} \pi \Gamma(1 - \beta) e^{\left( \frac{-\pi\beta}{\pi} \right) \mathcal{W} \beta} \right),$$

when $\alpha = 4$ then $\beta = \frac{1}{2}$, so we can write:

$$\phi_{\mathcal{X}}(\mathcal{W}) = \exp \left( -\frac{\sqrt{\lambda_{SU}}}{\mathcal{X}} \pi \Gamma(1 - \beta) \lambda_{SU} \sqrt{\mathcal{W}} \right).$$

In the probability theory, the above characteristic function follows the stable characteristic functions in the form $e^{-\frac{\pi\beta}{\pi^2}}$, where $\beta$ is a constant. Since, $0 < \beta < 1$ i.e. $\beta = \frac{1}{2}$, the mentioned
above equation is an inverse Gaussian distribution, so the pdf of interference formula follows:

\[ f_Z(z) = -\frac{\pi}{2} \lambda_{SU} z^{-\frac{3}{2}} e^{-\frac{\pi^2 \lambda_{SU}^2}{2z}}. \] \ (C.2)

The cumulative distribution function can be obtained:

\[ F_Y(y; \beta = \frac{1}{2}) = \text{erfc} \left( \frac{\pi \lambda_{SU}^2}{2\sqrt{z}} \right), \] \ (C.3)

where \( \text{erfc}(x) = 1 - \text{erf}(x) \) that \( \text{erf}(x) \) is the error function and can also be written as: \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} \, dt \). Generally, when \( q = \pi \lambda_{SU} \Gamma(1 - \beta) \), the pdf of interference can be written as:

\[ f_Z(z; \alpha) = \frac{1}{\pi z} \sum_{k=1}^{\infty} \frac{\Gamma(\beta(k + 1))}{k!} \left( \frac{q}{z^{\beta}} \right)^k \sin \left( k\pi(1 - \beta) \right). \] \ (C.4)

Correspondingly, we have:

\[ F_Z(z; \alpha) = \frac{1}{\pi z} \sum_{k=1}^{\infty} \frac{\Gamma(\beta k)}{k!} \left( \frac{q}{z^{\beta}} \right)^k \sin \left( k\pi(1 - \beta) \right). \] \ (C.5)

By putting \( \beta = \frac{1}{2} \) in expressions (C.4) and (C.5):

\[ f_Z(z; \alpha) = \frac{1}{\pi z} \sum_{k=1}^{\infty} \frac{\Gamma(\frac{1}{2}(k + 1))}{k!} \left( \frac{q}{z^{\frac{1}{2}}} \right)^k \sin \left( \frac{k\pi}{2} \right) = \frac{\pi}{2} \lambda_{SU} z^{-\frac{3}{2}} e^{-\frac{\pi^2 \lambda_{SU}^2}{2z}}. \] \ (C.6)

**C.2 Proof of Lemma. 5.5.2**

We know that:

\[ C_{\mathcal{W}, i} = \log_2 \left( 1 + \frac{P_{ra}}{a_i^4 (\mathcal{J}_p + \mathcal{I}_p)} \right), \] \ (C.7)

where \( a_i^4 \) denotes the distance between the PT and its \( i^{th} \) closest neighbour node. Therefore, the pdf of all random variables (i.e. \( \mathcal{I}_p, \mathcal{J}_p \) and \( a_i^4 \)) in equation (C.7) should be calculated.
The pdf of the interference from all of the secondary users to primary receiver (i.e. $I_P$) is obtained earlier in (C.6). Now, from the equations (5.3), (5.6) and (C.7) when $\alpha = 4$, the pdf of $C_{PU,i}$ can be derived as follows. First, we assume that $I_P = Z$, $X = a_4^i$, $V = a_2^i$, $N = N_p$, $Y = N + Z$ and $U = X \times Y$. We take: $Z = Y - N$ and we have $f_Y(y) = f_Z(Y - N)$. Then the pdf of $Y = N + Z$ from (C.6) can be calculated as:

$$f_Y(y) = \frac{\pi}{2} \lambda_{SU} (Y - N)^{\frac{1}{2}} e^{-\frac{\pi^2 \lambda_{SU}}{2}(Y - N)}, \text{ for } Y > N.$$ \hspace{1cm} (C.8)

Second, we determine the pdf of the variable $X$. We have $X = a_4^i = V^2$ and from [169], $V = a_2^i$ depicts Poisson arrival times on the line with the constant arrival rate $\pi \lambda_{SU}$ and $V$ has an Erlang distribution of order $i$ and rate $\pi \lambda_{SU}$. The pdf of $V$ can be written as:

$$f_V(v) = \begin{cases} \frac{(\pi \lambda_{SU})^{i-1} e^{-\pi \lambda_{SU} v}}{(i-1)!}, & v \geq 0 \\ 0, & v > 0. \end{cases} \hspace{1cm} (C.9)$$

We have $X = V^2$, let $X = g(v) = V^2$, therefore derivative of $g(v)$ is:

$$g'(v) = 2v = 2\sqrt{X}. \hspace{1cm} (C.10)$$

Also, we can write:

$$f_{X}(x) = \frac{1}{|g'(v)|} f_V(g^{-1}(x)), \hspace{1cm} (C.11)$$

where $g^{-1}(x) = \sqrt{x}$, so:

$$f_Y(g^{-1}(x)) = f_Y(\sqrt{x}). \hspace{1cm} (C.12)$$

From (C.10), (C.11) and (C.12), we have:

$$f_X(x) = \frac{1}{2\sqrt{x}} f_Y(\sqrt{x}). \hspace{1cm} (C.13)$$
The pdf of the $X^i = a_1^i$ can be obtained from the combination of (C.9) and (C.13):

$$f_{X^i}(x) = \frac{1}{2\sqrt{x}} \times (\pi \lambda_{SU})^i \left(\sqrt{x}\right)^{i-1} e^{-\pi \lambda_{SU} \sqrt{x}} \frac{1}{(i-1)!}.$$  \hfill (C.14)

Next, to determine the pdf of the product of $X^i$ and $Y$ (i.e. $U = X^i Y$), we need to obtain the CDF of $U$. As mentioned earlier, $Y = N + X^i$, so the CDF can be calculated as follows:

$$F_U(u) = P_r\{U \leq u\} = P_r\{X^i Y \leq u\} = P_r\{Y \leq \frac{u}{X^i}\}.  \hfill (C.15)$$

In addition, we can write:

$$0 \leq X^i < \infty$$

$$N \leq N + X^i < \infty$$

$$N \leq Y < \infty,$$  \hfill (C.16)

also, clearly $X^i = a_1^i$ is positive. Then:

$$X^i > 0$$

$$Y \leq \frac{U}{X^i} \Rightarrow X^i \leq \frac{U}{Y}$$

$$0 \leq X^i \leq \frac{U}{Y}.$$  \hfill (C.17)

Since the random variables, $X^i$ and $Y$, are independent and using equations (C.16) and (C.17), the CDF equation can be re-written as:

$$F_U(u) = P_r\{Y \leq \frac{u}{X^i}\} = P_r\{N < Y < \infty\} \cdot P_r\{0 \leq X^i \leq \frac{u}{Y}\},$$  \hfill (C.18)
where,

\[
\mathcal{P}_r\{N < Y < \infty\} = \int_N^\infty f_Y(y)dy,
\]  
(C.19)

and

\[
\mathcal{P}_r\{0 \leq \mathcal{X} < \frac{u}{X}\} = \int_0^{\frac{u}{X}} f_X(x)dx.
\]  
(C.20)

Substituting (C.19) and (C.20) in equation (C.18), the CDF of \(U\) is:

\[
F_U(u) = \int_N^\infty f_Y(y)dy \times \int_0^{\frac{u}{X}} f_X(x)dx = \int_N^\infty \frac{\pi}{2} \lambda_{SU} (Y - N) \frac{3}{2} e^{-\lambda_{SU} \frac{3}{2} (Y - N)} dy + \int_0^{\frac{u}{X}} \frac{1}{2\sqrt{x}} \times \frac{(\pi \lambda_{SU})^i (\sqrt{x})^{i-1} e^{-\lambda_{SU} \sqrt{x}}}{(i-1)!} dx.
\]  
(C.21)

Now, the pdf of \(u\) is the derivative of \(F_U(u)\) that can be calculated as below:

\[
(F_U(u))' = f_U(u),
\]

\[
f_U(u) = \int_N^\infty f_Y(y)dy \times \left(\int_0^{\frac{u}{X}} f_X(x)dx\right)' = \int_N^\infty \frac{1}{y} f_Y(y)dy \times f_X\left(\frac{u}{y}\right) = \int_N^\infty \frac{1}{y} \times \frac{\pi}{2} \times f_X\left(\frac{u}{y}\right) \lambda_{SU} (Y - N) \frac{3}{2} e^{-\lambda_{SU} \frac{3}{2} (Y - N)} dy.
\]  
(C.22)

Since we know the pdf of random variables, \(Y\) and \(X\), the pdf of the Shannon capacity of the PU (i.e. \(C_{PU}\)) can be obtained. First, using the definition of CDF, the CDF of \(C_{PU,i}\) is:

\[
\mathcal{F}_{C_{PU,i}}(b) = \Pr\{C_{PU,i} \leq b\},
\]  
(C.23)
where $b$ is an arbitrary constant. By putting $\mathcal{Y} = \mathcal{N} + \mathcal{Z}$ and $\mathcal{U} = \mathcal{X} \times \mathcal{Y}$ in (5.3), we can write:

$$C_{\mathcal{P} \mathcal{U}, i} = \log_2 \left( 1 + \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{\mathcal{X} (\mathcal{N} + \mathcal{Z})} \right) = \log_2 \left( 1 + \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{\mathcal{U}} \right). \quad (C.24)$$

In addition, it can be written that:

$$1 + \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{\mathcal{U}} = 2^b \quad \text{and} \quad \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{\mathcal{U}} = 2^b - 1 \quad (C.25)$$

So, the CDF of $C_{\mathcal{P} \mathcal{U}, i}$ is:

$$\mathcal{F}_{C_{\mathcal{P} \mathcal{U}, i}}(b) = \Pr \left\{ C_{\mathcal{P}} \leq b \right\} = \Pr \left\{ \log_2 \left( 1 + \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{\mathcal{U}} \right) \leq b \right\}$$

$$= \Pr \left\{ \left( \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{\mathcal{U}} \right) \leq 2^b - 1 \right\} = \Pr \left\{ \mathcal{U} \geq \left( \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{2^b - 1} \right) \right\}$$

$$= 1 - \Pr \left\{ \mathcal{U} < \left( \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{2^b - 1} \right) \right\}, \quad b > 0. \quad (C.26)$$

Next, the above equation can be written in shortened form as:

$$\mathcal{F}_{C_{\mathcal{P} \mathcal{U}, i}}(b) = 1 - \mathcal{F}_{\mathcal{U}} \left( \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{2^b - 1} \right), \quad b > 0. \quad (C.27)$$
Second the pdf of $C_{PU,i}$ is:

$$f_{C_{PU,i}}(b) = (F_{C_{PU,i}}(b))' = -F_{C_{PU,i}} \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) \times \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) \times \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) \times \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) \times \sum_{k=0}^{N} \mathcal{P} I_{f_k} \times 2^b \ln(2) \times \frac{2^b ln(2)}{(2^b - 1)^2}. \quad (C.28)$$

\section*{C.3 Proof of Lemma. 5.5.3}

We have:

$$f_{C_{SU,i}}(b) = f_{C_{PU,i}}(b) \times f_E(b) = \int_{b}^{\infty} f_{C_{PU,i}}(\tau) f_E(\tau - b) d\tau. \quad (C.29)$$

In addition:

$$f_{C_{PU,i}}(b) = f_{C_{PU,i}} \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) \times \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) \times 2^b \ln(2), \quad (C.30)$$

where $f_{C_{PU,i}} \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) = f_{\mathcal{U}}(u)$ and

$$f_E(b) = f_E \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) \times \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) \times 2^b \ln(2), \quad (C.31)$$

where $f_E \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) = f_{\mathcal{U}}(u)$ when we put $\lambda_{EVE}$ instead of $\lambda_{PU}$ and set $i = 1$ in (5.14).

By substituting (C.30) and (C.31) in (C.29), we can rewrite (C.29) as:

$$f_{C_{SU,i}}(b) = \int_{b}^{\infty} f_{C_{PU,i}} \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) \times \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^b - 1} \right) \times 2^b \ln(2) \times f_E \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^{(\tau - b)} - 1} \right) \times \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^{(\tau - b)} - 1} \right) \times \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^{(\tau - b)} - 1} \right) \times \left( \frac{\sum_{k=0}^{N} \mathcal{P} I_{f_k}}{2^{(\tau - b)} - 1} \right) \times 2^{(\tau - b)} \ln(2) d\tau. \quad (C.32)$$
Now, from (5.21) and (C.32), the probability of an outage in secrecy capacity can be formulated as follow:

\[
\mathcal{P}_{\text{out},i}(d_S) = 1 - \int_{d_s}^{\infty} \int_{b}^{\infty} f_{C_{PU,i}} \left( \sum_{k=0}^{N} \mathcal{P}_k I_{f_k} \right) \times \sum_{k=0}^{N} \mathcal{P}_k I_{f_k} \frac{2^b}{(2^b - 1)^2} \times 2^b \ln(2) \times f_E \left( \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{2^{(\tau-b) - 1}} \right) \times \frac{\sum_{k=0}^{N} \mathcal{P}_k I_{f_k}}{(2^{(\tau-b) - 1})^2} \times 2^{(\tau-b)} \ln(2) d\tau db. \tag{C.33}
\]