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Root-flipped multiband pulses with inherently aligned echoes

Samy Abo Seada¹, Joseph V Hajnal¹, and Shaihan J Malik¹

¹Division of Imaging Sciences and Biomedical Engineering, King’s College London, London, United Kingdom

Synopsis

Root-flipped multiband pulses have peculiar spin-echo behaviour due to their non-linear phase profile. In spin-echo sequences, different slices will typically have different relaxation weightings. This work investigates the typical time-delays for such pulses, and proposes a novel root-flipping method to minimize differences in relaxation weighting.

Purpose

Root-flipping is a recently proposed method for designing multiband band refocusing pulses (1). It can produce shorter pulses than Phase-optimized (2) and Time-shifted (3) methods because RF energy is more evenly distributed across the duration of the pulse. Consequently, the spin-echo formation in different slices develops differently. There are two quantities of interest, firstly the time of arrival ($T_{arr}$) of the echo in each slice and secondly the time that magnetization in each slice spends in the transverse plane ($T_{xy}$) (see Figure 1). For a conventional sequence both of these terms are equal to the echo time, but this is not the case for root-flipped designs. Here we use $\Delta T_{xy}$ and $\Delta T_{arr}$ to describe the spread in these parameters across the set of simultaneously excited slices.

Previous work (3,1) has identified three terms to classify this effect:

1) Aligned-echo excitation: Matching the bandwidth of the excitation pulse to that of the refocusing pulse. Echoes arrive simultaneously but with different $T_2$-weighting. At readout, $\Delta T_{arr} = 0$ and $\Delta T_{xy} \neq 0$

2) Align-TE excitation: Matching the duration of the excitation pulse to that of the refocusing pulse. Echoes arrive at different times with equal $T_2$-weighting. At readout, $\Delta T_{arr} \neq 0$ and $\Delta T_{xy} = 0$

3) Minimum-duration excitation: Matching the peak amplitude of the excitation pulse to that of the refocusing pulse. Echoes arrive at different times with different $T_2$-weighting. At readout, $\Delta T_{arr} \neq 0$ and $\Delta T_{xy} \neq 0$

In this work, we propose a novel root-flipping algorithm that aligns the echoes such that $\Delta T_{arr} \approx 0$ and $\Delta T_{xy} \approx 0$.

Theory

In the Root-flipping method the multi-slice frequency profile is represented as roots along the unit circle on the complex plane. The stopband frequencies have roots on the unit circle and the passband frequencies have roots away from this. Flipping roots inside or outside the unit circle redistributes the pulse energy in the time-domain. In the original method from Sharma et al. (1), the root-pattern across each passband was unconstrained. Having an unequal number of roots inside
and outside the unit circle across a passband leads to temporal displacement, which leads to the misalignment in spin-echo development.

Ensuring an equal number of roots inside and outside the unit circle for each passband restricts temporal displacement, which results in $\Delta T_{arr} \approx 0$ regardless of the excitation method. Using this in conjunction with the “align-TE” excitation method also gives the desired property, $\Delta T_{xy} \approx 0$. We refer to this as the “align-all” approach.

**Methods**
The proposed root-flip pattern requires pulses with an even number of passband roots. This is related to the time-bandwidth product (TBP) of the target design. For example, TBP=4 gives two roots per passband; all pulses in this work were designed this way. The root-flipping method searches through root-flipping patterns to obtain the solution with the lowest peak RF amplitude. In this work the Genetic Algorithm as implemented in Matlab was used to perform this search. The original flipping pattern was used (code made available by Sharma et al. http://www.vuiis.vanderbilt.edu/~grissowa/) alongside a modified version which selects an even number of roots per passband closest to the passband-center and flips them evenly across the unit circle (Figure 2).

The signal evolution for transverse magnetization in the presence of dephasing was simulated using Bloch simulations with a secondary frequency axis. The predicted signal was estimated by Lorentzian weighted-averaging along this axis for $T_2^* = 45\,ms$. $T_2$ was 80ms, which corresponded to white matter values at 3T (4). For each slice, 7 spatial points were simulated across 30% of the slice-width. The nominal echo time of this sequence was 90ms.

**Results**
Figure 3 shows example simulated spin-echo experiments. As expected, either $T_{xy}$ or $T_{arr}$ differs through the group of five simultaneously excited slices for the existing methods. The proposed approach results in nearly complete alignment of the echoes. Figure 4 shows that the result is stable across a range of designs. Figure 5 shows that on average the align-all root-flipped pulses are 19.6% longer in duration than those from the original method. This is expected, since they are more constrained designs.

**Discussion and Conclusion**
Root-flipping for reducing the peak amplitude of RF refocusing pulses results in a spread of echo arrival time ($\Delta T_{arr}$) and/or ‘transverse time’ ($\Delta T_{xy}$) of the magnetization in different slices, leading to different $T_2^*$ and/or $T_2$ weighting. We propose a constrained root-flipping approach that minimises both issues, which comes at a cost in slightly increased pulse duration (19.6%). A limitation is that the approach requires an even number of roots per passband, constraining the time-bandwidth product. It was empirically found that TBP designs between 3.40 and 4.2 lead to two roots per passband, but an analytic relationship has not been identified.

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**References**


Figure 1 – Top: RF excitation and refocusing pulses (root-flipped design with 5 slices) for a nominal TE of 90ms. Bottom: Transverse magnetization as a function of time for the five slices. $T_{xy}$ is the time from excitation to the moment the slice fully refocuses. $T_{txy}$ is the time from the center of the excitation pulse to the moment the slice fully refocuses. For conventional spin-echo sequences these two values are equal to the nominal echo-time.
Figure 2:
Top row - Root-plots on the complex plane for a normal root-flipped pulse (middle) and an align-all root-flip pulse. The arrows indicate passbands with an unmatched number of roots about the unit circle which lead to temporal displacement of RF energy in the time-domain (bottom row). Align-all pulses have even number of passband roots for all passbands and consequently excite and refocus the center of the slices simultaneously.
Figure 3: Example of Spin-echo simulations for multiband 5 pulses with different excitation methods for a nominal echo time of 90ms. The five slices were simulated with $T_2$ and $T_2^*$ relaxation. Subfigures a, b and c all use the same refocusing pulse with Align-TE, Minimum-duration and aligned-echo excitation respectively. a) and b) only differ in a small amount of $T_2$ weighting across the slices. c) shows aligned echoes with different $T_2$ weighting. d) shows an example of an align-all simulation, where the echoes arrive almost simultaneously with almost equal $T_2$ weighting.
Figure 4 – Top row: Difference in echo arrival time ($\Delta T_{arr}$) for a range of multiband factors and slice separations. The aligned-echo has perfect time-alignment, where-as the Align-TE and minimum duration have a few milli-seconds of mismatch. This will lead to a difference in $T_2^*$-weighting. Align-all comes very close to aligning the echoes, which will happen regardless of the excitation method used. Bottom row: Difference between the longest and shortest $T_{xy}$. This value is near zero for the Align-TE method and similar for the Align-all method. Outliers show that Align-all solutions can also be found without the constraint.

Figure 5 – Duration comparison between phase-optimized, normal root-flipped and align-all root-flipped pulses. For refocusing pulses, the average duration increase is 19.6%.