A new approach to stability and stabilization analysis for continuous-time Takagi-Sugeno fuzzy systems with time delay

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Abstract—Till now, there are lots of stability and stabilization results about T-S (Takagi-Sugeno) fuzzy systems with time delay, but most of them are independent of the analysis of membership functions. Since the membership functions are an essential component to make a fuzzy system different from others, the conditions without its information are conservative. In this brief paper, a new Lyapunov-Krasovskii functional is designed to investigate the stability and stabilization of continuous-time T-S fuzzy systems with time delay. Different from the existing results in the literature, the integrand of the Lyapunov-Krasovskii functional in this paper depends not only on the integral variable but also on the membership functions, and thus, the information of the time-derivative of membership functions can also be used to reduce the conservativeness of finding the maximum delay bounds. Utilizing the information of the time-derivative of membership, a bunch of controllers are designed according to their sign, and then a switching idea is applied to stabilize the fuzzy system. In the end, two examples are given to illustrate the feasibility and validity of the design and analysis.

Keywords: Takagi–Sugeno’s fuzzy model, Time delay, Parallel distributed compensation law, Membership dependent Lyapunov-Krasovskii functional.

I. INTRODUCTION

In the recent decades, the stability and stabilization analysis for fuzzy systems ([24]-[30]) especially for Takagi-Sugeno fuzzy model [1] with time delay has been a hot topic (see [2]-[13], [22], [23] and the references therein) and the focus is on how to maximize the tolerance of time delay. All kinds of Lyapunov-Krasovskii functional have been developed to reduce the conservativeness of finding the maximum delay bounds, for example, an augmented Lyapunov-Krasovskii functional is proposed in [5] and an improved Jensen’s inequality which is a more general and tighter bounding technology is used to deal with the cross product terms. The results in [5] are improved by [8] where a fuzzy weighting-dependent Lyapunov-Krasovskii functional is designed for uncertain T-S fuzzy systems with time-varying delays. The results in [8] are less conservative than [5] because of an input-output approach and the fuzzy weighting-dependent Lyapunov-Krasovskii functional. A discretized Lyapunov-Krasovskii functional is proposed in [2] where the number of free variables increases as the discretization level increases. These variables help to reduce the conservativeness but increase the computing burden heavily. For the case of constant time delay, the results in [5], [8] and [2] are further improved by [11] where new simple and effective stabilization conditions are proposed by applying the Wirtinger inequality [14] and fuzzy line-integral Lyapunov functional [15]. The results in [11] are improved by [17] where a new augmented Lyapunov-Krasovskii functional is constructed and some new cross terms are included. For the first time, a distributed fuzzy optimal control law relied on actual physical meaning by adaptive dynamic programming algorithm is proposed in [20] which gives a new thought for the optimization of multi-agent system. However, all of the results mentioned above are independent of the analysis of membership functions which willlead to conservativeness because the membership functions are an essential component to make a fuzzy system different from other systems. More recently, a membership function dependent Lyapunov-Krasovskii functional is designed in [18] and some less conservative results are obtained by analyzing the time derivative of the membership function, however the drawback is that the local stabilization region becomes smaller as the delay increases.

Based on the above discussions, the contributions of this paper are as follows: 1), A new membership-function dependent Lyapunov-Krasovskii functional is designed in this paper which is different from the existing ones such as [11], [17] where the integrands are independent of the membership functions. 2), A new method is used to deal with the time derivative of the Lyapunov-Krasovskii functional. Since the integrand of the Lyapunov-Krasovskii functional depends not only on the integral variable but also on the membership functions, a switching idea is applied to ensure that the time derivative of the Lyapunov-Krasovskii functional is negative. Due to the above contributions, the obtained stability and stabilization criteria improve the existing results. In the end, two examples are provided for verification. One example is for stability and the other one is for stabilization. Both examples show that less conservative results can be obtained in this paper than the existing ones in the literature.
II. PRELIMINARIES AND BACKGROUND

Consider the following nonlinear model

\[
\dot{x}(t) = f_1(z(t)) x(t) + f_2(z(t)) x(t - \tau) + f_3(z(t)) t, \quad t \in [-\tau, 0]
\]

where \(x(t) \in \mathbb{R}^n\) is the state, \(u(t) \in \mathbb{R}^m\) is the input, \(z(t) \in \mathbb{R}^p\) is an unknown premise variables, \(f_1(\cdot)\), \(f_2(\cdot)\) and \(f_3(\cdot)\) are nonlinear functions or matrix functions with proper dimensions, \(\phi(t)\) is the initial condition and the time delay \(\tau\) is assumed to be constant. Applying the sector nonlinearity method or local approximation method in [16], one has the following well known time delay T-S fuzzy model:

\[
\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) A_i x(t) + \sum_{i=1}^{r} h_i(z(t)) A_{r_i} x(t - \tau) + \sum_{i=1}^{r} h_i(z(t)) B_i u(t),
\]

\[
x(t) = \phi(t), \quad t \in [-\tau, 0],
\]

where \(A_i \in \mathbb{R}^{n \times n}\), \(A_{r_i} \in \mathbb{R}^{n \times n}\), \(B_i \in \mathbb{R}^{n \times m}\) are known matrices and \(h_i(z(t))\) are membership functions.

To lighten the notation, we will drop the time \(t\), for instance, we will use \(x\) instead of \(x(t)\). For simplicity, single sums are written as \(X_h = \sum h_i X_i\). For any matrix \(X\), \(H e(X) = X + X^T\).

In this paper, the following controller (3) is used to stabilize the fuzzy system

\[
u = K_h x(t) + K_{rh} x(t - \tau).
\]

The following Lemma 1 is useful in this paper.

**Lemma 1:** (see [19]) For a function \(F(t) = \int_{\alpha(t)}^\alpha f(s,t) ds\), if \(\alpha_1(\cdot), \alpha_2(\cdot)\) are differentiable for \(t\) and \(f(\cdot)\) is continuous for \(s\) and partially derivitive for \(t\), we have

\[
d\frac{\alpha_2(t)}{dt} f(\alpha_2(t), t) - \alpha_1(t) f(\alpha_1(t), t) + \int_{\alpha_1(t)}^{\alpha_2(t)} \frac{\partial f(s,t)}{\partial t} ds.
\]

In the following, we will discuss how to ensure \(\dot{X}_h \leq 0\), \(\dot{Y}_h \leq 0\) and \(\dot{Z}_h \leq 0\) where \(X_i > 0, Y_i > 0\) and \(Z_i > 0\). Note

\[
\dot{X}_h = \sum_{i=1}^{r-1} h_i X_i = \sum_{k=1}^{r-1} \dot{h}_k (X_k - X_r),
\]

\[
\dot{Y}_h = \sum_{i=1}^{r} h_i Y_i = \sum_{k=1}^{r} \dot{h}_k (Y_k - Y_r),
\]

\[
\dot{Z}_h = \sum_{i=1}^{r} h_i Z_i = \sum_{k=1}^{r-1} \dot{h}_k (Z_k - Z_r),
\]

where \(\dot{h}_k\) are the time-derivative of membership functions and are negative or positive as time goes by. Since \(X_i, Y_i\) and \(Z_i\) are variables to be designed, we can use a switching idea to ensure \(\dot{X}_h \leq 0\), \(\dot{Y}_h \leq 0\) and \(\dot{Z}_h \leq 0\) as follows:

\[
\begin{cases}
\text{if } \dot{h}_k \leq 0, \text{ then } X_k - X_r \geq 0, Y_k - Y_r \geq 0, Z_k - Z_r \geq 0, \\
\text{if } \dot{h}_k > 0, \text{ then } X_k - X_r < 0, Y_k - Y_r < 0, Z_k - Z_r < 0.
\end{cases}
\]

There are \(2^{r-1}\) possible cases in (8). Let \(H_i, l = 1, 2, \cdots, 2^{r-1}\) be the set that contains the possible permutations of \(\dot{h}_k\) and \(C_l\) be the set that contains the constraints of \(X_i, Y_i\) and \(Z_l\), (8) can be presented as

\[
\begin{cases}
\text{if } H_i, \text{ then } C_l.
\end{cases}
\]

For example, if \(r = 3\), we have

\[
\begin{align*}
\dot{X}_h &= \dot{h}_1 (X_1 - X_3) + \dot{h}_2 (X_2 - X_3), \\
\dot{Y}_h &= \dot{h}_1 (Y_1 - Y_3) + \dot{h}_2 (Y_2 - Y_3), \\
\dot{Z}_h &= \dot{h}_1 (Z_1 - Z_3) + \dot{h}_2 (Z_2 - Z_3).
\end{align*}
\]

There are \(2^2\) constraints \(C_l, l = 1, 2, 3, 4\) to ensure \(\dot{X}_h \leq 0, \dot{Y}_h \leq 0, \dot{Z}_h \leq 0\) and (9) is expressed as following

\[
\begin{align*}
&\text{If } H_1, \text{ then } C_1; \quad \text{If } H_2, \text{ then } C_2; \\
&\text{If } H_3, \text{ then } C_3; \quad \text{If } H_4, \text{ then } C_4,
\end{align*}
\]

where

\[
\begin{align*}
H_1 : & \quad \dot{h}_1 \leq 0, \dot{h}_2 \leq 0; \\
H_2 : & \quad \dot{h}_1 \leq 0, \dot{h}_2 > 0; \\
H_3 : & \quad \dot{h}_1 > 0, \dot{h}_2 \leq 0; \\
H_4 : & \quad \dot{h}_1 > 0, \dot{h}_2 > 0.
\end{align*}
\]

Based on the above discussion, we get the following lemma.

**Lemma 2:** For some membership function dependent matrices \(X_h, Y_h\) and \(Z_h\) where \(X_i > 0, Y_i > 0\) and \(Z_i > 0\) are free variables, we have \(\dot{X}_h \leq 0, \dot{Y}_h \leq 0, \dot{Z}_h \leq 0\), if the switching rules (9) are satisfied where \(l = 1, 2, \cdots, 2^{r-1}\).

**Lemma 2** is used to get some stabilization conditions, for different \(C_l\) and \(H_i\), \(l = 1, 2, \cdots, 2^{r-1}\), the corresponding controller is

\[
u_l = K_{l,h} x(t) + K_{l,\tau} x(t - \tau),
\]

\[
K_{l,h} = \sum_{i=1}^{r} h_i K_{l,i}, K_{l,\tau} = \sum_{i=1}^{r} h_i K_{l,\tau,i}.
\]

The final controller (3) becomes a switching controller below

\[
u : \begin{cases}
u_1 = K_{1,h} x(t) + K_{1,\tau} x(t - \tau) \quad \text{for } H_1, \\
\vdots \\
u_l = K_{l,h} x(t) + K_{l,\tau} x(t - \tau) \quad \text{for } H_l.
\end{cases}
\]
III. MAIN RESULTS

In this section, the stability and stabilization problem for T-S fuzzy system with time delay is considered by utilizing new Lyapunov-Krasovskii functional and a switching control method.

A. Stability

Theorem 1: For a given scalar $\tau > 0$, the closed-loop T-S fuzzy system (2) is globally asymptotically stable if there exist matrices $P_{1i}$, $P_{2i}$, $Q_i > 0, Z_i > 0, M_{1i}, M_{2i}, M_{3i}, M_{4i}$, such that the following inequalities hold for $i, j = 1, \ldots, r$,

$$
\dot{Q}_i \leq 0, \quad \dot{Z}_i \leq 0, \quad \dot{\hat{P}}_i \leq 0,
$$

\begin{equation}
\mathcal{P}_i = \begin{bmatrix}
P_{11i} & P_{12i} \\
* & \ P_{22i}
\end{bmatrix}
\end{equation}

\begin{equation}
\Theta_{ij} + \Theta_{ji} \leq 0,
\end{equation}

$$
\Theta_{ij} =
\begin{bmatrix}
\Theta_{11ij} & \Theta_{12ij} & \Theta_{13ij} & \Theta_{14ij} \\
* & \Theta_{22ij} & \Theta_{23ij} & \Theta_{24ij} \\
* & * & -12\tau^{-2}Z_i & \Theta_{34ij} \\
* & * & * & \Theta_{44ij}
\end{bmatrix},
$$

$$
\Theta_{11ij} = \text{He} (P_{12i} + M_{1i}A_j) + Q_i - 4Z_i,
\Theta_{12ij} = -P_{21i} - 2Z_i + M_{1i}A_j + A_j^T M_{2i},
\Theta_{13ij} = P_{22i} + 6\tau^{-1}Z_i + A_j^T M_{4i},
\Theta_{14ij} = P_{11i} - M_{1i} + A_j^T M_{2i},
\Theta_{22ij} = -Q_i - 4Z_i + \text{He} (M_{3i}A_j),
\Theta_{23ij} = -P_{22i} + 6\tau^{-1}Z_i + A_j^T M_{4i},
\Theta_{24ij} = A_j^T M_{2i} - M_{3i},
\Theta_{34ij} = P_{22i} - M_{4i}, \Theta_{44ij} = \tau^2 Z_i - \text{He} (M_{2i}).
$$

Proof: Choose the Lyapunov-Krasovskii functional as

$$
V(x) = V_1(x_t) + V_2(x_t) + V_3(x_t),
$$

\begin{equation}
V_1(x_t) = \rho^T \mathcal{P}_i \rho, \quad \rho^T = \begin{bmatrix}
x(t)^T \\
\int_{t-\tau}^t x(s)^T ds
\end{bmatrix},
\end{equation}

$$
V_2(x_t) = \int_{t-\tau}^t x(s)^T Q_i x(s) ds,
$$

$$
V_3(x_t) = \tau \int_{-\tau}^{0} \int_{t-\tau}^{t} \dot{x}(s)^T Z_i \dot{x}(s) ds d\theta.
$$

It follows that

$$
\dot{V}_1(x_t) = 2\rho^T \dot{\mathcal{P}}_i \rho + \rho^T \dot{\mathcal{P}}_i \rho.
$$

Applying Lemma 1, we have

$$
\dot{V}_2(x_t) = x^T Q_i x - x(t-\tau)^T Q_i x(t-\tau)
+ \int_{t-\tau}^t x(s)^T \dot{Q}_i x(s) ds,
$$

$$
\dot{V}_3(x_t) = \tau^2 \dot{x}^T Z_i \dot{x} - \tau \int_{t-\tau}^t \dot{x}(s)^T Z_i \dot{x}(s) ds + \tau \int_{-\tau}^{0} \int_{t-\tau}^{t} \ddot{x}(s)^T Z_i \ddot{x}(s) ds d\theta.
$$

Considering the constraints in (12), we have

$$
\dot{V}(x_t) \leq 2\rho^T \mathcal{P}_i \begin{bmatrix}
\dot{x}(t) \\
x(t) - x(t-\tau)
\end{bmatrix}
+ x(t)^T Q_i x(t) - x(t)^T Q_i x(t-\tau)
+ \tau^2 \dot{x}(t)^T Z_i \dot{x}(t) - \tau \sum_{i=1}^{r} h_i \int_{t-\tau}^t \dot{x}(s)^T Z_i \dot{x}(s) ds.
$$

Using the zero equation

$$
2M \times (A_h x + A_{rh} x(t-\tau) - \dot{x}) = 0,
$$

$\mathcal{M} = x^T M_{1h} + \dot{x}^T M_{2h} + x(t-\tau)^T M_{3h} + \int_{t-\tau}^t x(s)^T ds M_{4h},$

and the results in [14] to deal with the term

$$
-\tau \int_{t-\tau}^t \dot{x}(s)^T Z_i \dot{x}(s) ds,
$$

we have $\dot{V}(x_t) < \eta^T \Omega \eta$ which can be ensured by (13), where

$$
\eta^T = \begin{bmatrix}
x(t)^T \\
x(t)^T \int_{t-\tau}^t x(s)^T ds \\
\int_{t-\tau}^t \dot{x}(s)^T ds
\end{bmatrix},
\Omega =
\begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\
* & \Omega_{22} & \Omega_{23} & \Omega_{24} \\
* & * & -12\tau^{-2}Z_i & \Omega_{34} \\
* & * & * & \Omega_{44}
\end{bmatrix},
$$

$$
\Omega_{11} = \text{He} (P_{12h} + M_{1h}A_{rh}) + Q_h - 4Z_h,
\Omega_{12} = -P_{21h} - 2Z_h + M_{1h}A_{rh} + A_{rh}^T M_{2h},
\Omega_{13} = P_{22h} + 6\tau^{-1}Z_h + A_{rh}^T M_{4h},
\Omega_{14} = P_{11h} - M_{1h} + A_{rh}^T M_{2h},
\Omega_{22} = -Q_h - 4Z_h + \text{He} (M_{3h}A_{rh}),
\Omega_{23} = -P_{22h} + 6\tau^{-1}Z_h + A_{rh}^T M_{4h},
\Omega_{24} = A_{rh}^T M_{2h} - M_{3h},
\Omega_{34} = P_{22h} - M_{4h}, \Omega_{44} = \tau^2 Z_h - \text{He} (M_{2h}).
$$

Remark 1: Note, the Lyapunov-Krasovskii functional used in Theorem 1 is different from the one used in the literature such as [11], [17], [18]. In $V_2(x_t)$ and $V_3(x_t)$ of this paper, $Q_h$ and $Z_h$ are independent of the integral variable $s$ but on the time $t$, so the information of the time-derivative of the membership function can be used to obtain new less conservative stability conditions, at the same time, there comes a problem that how to deal with the integral part $\int_{t-\tau}^t x(s)^T Q_h x(s) ds$ and $\tau \int_{-\tau}^{0} \int_{t-\tau}^{t} \ddot{x}(s)^T Z_i \ddot{x}(s) ds d\theta$. A simple and direct method is letting $Q_h \leq 0, \dot{Z}_h \leq 0$ and $\dot{\mathcal{P}}_i \leq 0$. 

0. Since $\dot{Q}_h = \sum_{k=1}^{r-1} \dot{h}_k (Q_k - Q_r)$, $\ddot{Z}_h = \sum_{k=1}^{r-1} \dot{h}_k (Z_k - Z_r)$,$$
abla_h = \sum_{k=1}^{r-1} \dot{h}_k (\nabla_k - \nabla_r)$, then $\dot{Q}_h \leq 0$, $\ddot{Z}_h \leq 0$ and $\nabla_h \leq 0$ can be ensured by the switching rules (9). Then, based on Lemma 2, applying (13) with each $C_l$, we obtain a maximum delay denoted as $\tau_l$, and the final delay obtained by Theorem 1 is $\tau = \min_{l=1, \ldots, r-1} (\tau_l)$. Since Theorem 1 does not depend on the initial conditions of the fuzzy systems, the switching method is straightforward. In order to avoid too many parameters, let $\tilde{M}$ denote a maximum delay denoted as $\tau$, and the final controller is (11). For any initial states $\phi (0)$, it can be driven to the origin by a sequence of controller $u_l$ activated by $H_l$. Note, there are $2r-1$ constraints in $C_l$ and we need compute $2r-1$ times to get the maximum delay bound. For each time, the number of LMI is $3 \times 2^{r-1} + 3 + (r+1) / 2$ and the computation burden may increase as the number of rules increase. The number of decision variables of Theorem 1 is $n(3n+2)r + 4n^2r$, and Theorem 2 is $n(3n+2)r + 4n^2r + 2mn$.  

Remark 3: Comparing with the existing results in the literature, the merit of this paper is that the Lyapunov-Krasovskii functional is dependent on the membership functions and the time derivatives of the membership functions are also considered. If we let $Q_h$, $Z_h$ and $P_h$ be membership function independent as $Q$, $Z$, $P$ and the slack variables be $M_{ih} = 0$, $M_{ih} = 0$, Theorem 1 and Theorem 2 in this paper will become the results in [11], so this brief paper contains [11] as a special case. Another possible choice is letting $V_2 (x_l) = \int_{-\tau}^{t} x(s)^T Q_{h(s)} x(s) ds$ where $Q_{h(s)} = \sum_{l=1}^{n} \hat{h}_l (z(s)) Q_l$ is dependent on the integral variable $s$, but it has been shown in [18] that this choice could not help reduce the conservativeness.  

Remark 4: The differences between this paper and [18] are as follows: 1), The chosen Lyapunov-Krasovskii functional is different. In this paper, the integrand of $V_2 (x_l)$ and $V_3 (x_l)$ depends not only on the integral variable $s$ but also on the membership functions, while in [18], $V_2 (x_l)$ and $V_3 (x_l)$ are independent of the membership function. 2), The method to deal with the time derivatives of the membership function is completely different. In this paper, a novel switching idea is applied to ensure the time-derivative of the Lyapunov-Krasovskii functional is negative, while in [18], the upper bounds on the time-derivative of the membership functions defined a priori by using the $\mod$ function and floor function $\lfloor \cdot \rfloor$. The applicable scope is different. In this paper, Theorem 1 is global stability and Theorem 2 is global stabilization, while in [18], the results are only local and the obtained local stabilization region becomes smaller as the delay increases.  

IV. NUMERICAL EXAMPLES  

In this section, two examples are presented to demonstrate the effectiveness of the proposed method. Example 1 is for stability and Example 2 is for stabilization.  

**Example 1**: Consider the following two-rule fuzzy system that has been studied in [2], [5], [8], [11], [17].

\[
A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix},
\]

\[
A_{r_1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \quad A_{r_2} = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}.
\]

This open loop fuzzy system has been studied extensively in the literature and the goal is to compute the maximum delay $\tau$ under which the fuzzy system is still stable. Table 1 shows the maximum delay $\tau$ obtained by different methods. From this table, we can see the best result is obtained by applying...
the method proposed in this paper. In this paper, \( \lambda_1 \) and \( \lambda_2 \) are searched by using the most common brute-force algorithm and the searching scope is \([-1, 3]\) with step 0.01. For example, the largest delay \( \tau = 2.5932 \) is obtained by using the latest method in [17], while applying Theorem 1 with constraints \( C_1 : \{ Q_1 \geq Q_2, Z_1 \geq Z_2, P_1 \geq P_2 \} \), we get \( \tau_1 = 3.6610 \) and with constraint \( C_2 : \{ Q_1 < Q_2, Z_1 < Z_2, P_1 < P_2 \} \), we get \( \tau_2 = 3.4859 \), so the final maximal delay is \( \tau = \min_{1 \leq i \leq 2} (\tau_i) = 3.4859 \). Obviously, the method in this paper is much less conservative than the existing ones.

In order to show what leads to the less conservative results, we consider several cases: (I) means \( M_{1h} = M_1, M_{2h} = M_2, M_{3h} = M_3, M_{4h} = M_4 \); (II) means \( M_{1h} = M_1, M_{2h} = M_2, M_{3h} = 0, M_{4h} = 0 \); (III) means \( M_{1h} = M_1, M_{2h} = M_2, M_{3h} = 0, M_{4h} = 0, P_h = \emptyset, Q_h = Q \) and \( Z_h = Z \).

For (I), applying Theorem 1 with constraint \( C_1 \) and \( C_2 \), we get \( \tau_1 = 3.6384 \) and \( \tau_2 = 3.4859 \) respectively, so the final maximal delay is \( \tau = \min_{1 \leq i \leq 2} (\tau_i) = 3.4859 \) which shows that whether the variables \( M_{1h}, M_{2h}, M_{3h}, M_{4h} \) are dependent on the membership function has no effect on the results; for (II), we get \( \tau = 2.9709 \) which shows \( M_{3h}, M_{4h} \) can help reduce the conservativeness; for (III), we only get \( \tau = 2.0183 \) which is more conservative than the existing results and shows that \( P_h, Q_h, Z_h \) are very important in getting the less conservative results.

### Example 2: Consider the two-rule fuzzy system that has been studied in [2], [10], [11], [17], [18] where the system and input matrices are as follows:

\[
A_1 = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},
\]
\[
A_{\tau 1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, \quad A_{\tau 2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix}.
\]

This two-rule fuzzy system has been studied extensively in the literature in the past several years and the goal is to compute the maximum delay \( \tau \) under which the fuzzy system can be stabilized by the designed controller. Using different methods to compute the maximum delay \( \tau \) we get Table 2 which shows that better results can be obtained by using the method proposed in this paper than the ones in the literatures. For example, the delay \( \tau = 1.4257 \) (\( \varepsilon = 1.26 \)) is obtained by using the method in [11], while applying Theorem 2 with constraints \( C_1 \), and searching \( \lambda_1 = 2.8, \lambda_2 = 1.1 \) we get \( \tau_1 = 1.6499 \) and with constraint \( C_2, \lambda_1 = 2.8, \lambda_2 = 1.1 \), we also get \( \tau_2 = 1.6499 \), so the final maximum delay is \( \tau = \min_{1 \leq i \leq 2} (\tau_i) = 1.6499 \) which is much less conservative than the existing ones. The result in [18] \( (\tau = 1.6421) \) is comparable to the result obtained in this paper, but it is only a local stabilization and the stabilization region is very small as \( \tau = 1.6421 \).

<table>
<thead>
<tr>
<th>Method</th>
<th>Max ( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[18]</td>
<td>1.6421</td>
</tr>
<tr>
<td>[17]</td>
<td>1.4959</td>
</tr>
</tbody>
</table>

Similar to Example 1, we consider two cases: (I) means \( \lambda_2 = 0; \) (II) means \( \lambda_2 = 0, P_h = \emptyset, Q_h = Q \) and \( Z_h = Z \).

For (I), applying Theorem 2 with constraint \( C_1, \lambda_1 = 1.2 \), we get \( \tau_1 = 1.5854 \) and applying Theorem 2 with \( C_2, \lambda_1 = 1.98 \), we get \( \tau_2 = 1.6386 \), so \( \tau = \min_{1 \leq i \leq 2} (\tau_i) = 1.5854 \) which shows \( \lambda_2 \) can improve the result but the improvement is not obvious. Similarly, for (II), applying Theorem 2 with \( \lambda_1 = 1.28 \) we get \( \tau_1 = 1.2431 \) which shows that \( P_h, Q_h, Z_h \) are very important for Theorem 2.

Table II: The maximum delay \( \tau \) obtained by different methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Max ( \tau )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[18]</td>
<td>1.6421</td>
</tr>
<tr>
<td>[17]</td>
<td>1.4959</td>
</tr>
</tbody>
</table>

Suppose the membership functions are \( h_1 = \frac{1}{1 + e^{-2x_1}} \) and \( h_2 = \frac{e^{-2x_1}}{1 + e^{-2x_1}} \). Applying Theorem 2 with \( C_1 \) and \( C_2 \), we get the maximum delay \( \tau = 1.6499 \) and two controllers \( u_1 \) and \( u_2 \) which correspond to \( H_1 : h_1 \leq 0 \) and \( H_2 : h_2 > 0 \) respectively:

\[
\begin{align*}
u_1 &= K_{1,1} x(t) + K_{1,\tau 1} x(t - \tau) \quad \text{for } H_1, \\
u_2 &= K_{2,1} x(t) + K_{2,\tau 1} x(t - \tau) \quad \text{for } H_2,
\end{align*}
\]
Given $\phi(0) = [1 \ 1]^T$ and choosing $u = u_1$, we get $\dot{h}_1(0) = -5.7434$ which satisfies $H_1$. Figure 1 shows the evolution of $\dot{h}_1$ and control input $u$. The time of point A is $t = 1.6499$ which is the maximal delay obtained by Theorem 2 and $h_1'(1.6499) = -0.0021613$, so at this point, the controller is still $u = u_1$. The time of point B is $t = 1.82$ and $h_1(1.82) = 1.16476 - 0.005 > 0$, so at this point the controller is switched to $u_2$. The time of point C is $t = 4.66$ and at this point the controller is switched to $u_1$ again. Figure 2 shows the trajectories of the system states which are stabilized by the switching controller.

V. Conclusions

In this paper, we have studied the stability and stabilization for continuous-time T-S fuzzy systems with time-delay. A new membership dependent Lyapunov-Krasovskii functional has been designed to deal with the time delay and a switching control strategy is applied to stabilize the fuzzy systems. Comparing with the recent work, less conservative results can be obtained by using the new method proposed in this paper. In the end, the effectiveness of the proposed results are demonstrated by two examples. The method is simple but effective and can be used to deal with different problems such as filtering design, observer design and descriptor systems[21].

REFERENCES


