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Asynchronous Piecewise Output-Feedback Control for Large-Scale Fuzzy Systems via Distributed Event-Triggering Schemes

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Abstract—This paper examines the problem of distributed event-triggering output-feedback control for discrete-time large-scale fuzzy systems in the network scenario. A novel distributed event-triggering control scheme is proposed, where each subsystem transmits its output information and premise variables only when its local output error exceeds a specified threshold. The resulting closed-loop fuzzy control system subject to nonsynchronous grades of membership, is explored under such circumstances. In virtue of a piecewise Lyapunov function together with some matrix inequality convexification techniques, the solution to the distributed event-triggering output-feedback control problem is derived in the form of linear matrix inequalities. It will be shown that the closed-loop control system is asymptotically stable while significantly reducing the communications via networks. Finally, a direct current (DC) microgrid with solar photovoltaic arrays is utilized to illustrate the validity and practicability of the proposed results.

Index Terms—Large-scale fuzzy systems, distributed event-triggering control, asynchronous piecewise design.

I. INTRODUCTION

In real-life industrial applications, the strong nonlinearities of physical plants may give rise to severe difficulties for analysis and synthesis of systems. Fuzzy logic control (FLC) has been successfully applied to control many complex nonlinear systems or even nonanalytic systems [1]–[3]. Among all the existing fuzzy models, the Takagi-Sugeno (T-S) model has received sustained attention as it can be regarded as a competitive candidate for the control of complex nonlinear systems [4]–[7]. Using the T-S fuzzy models can approximate smoothly nonlinear systems in an arbitrary precision. Moreover, based

on the T-S fuzzy model, some fruitful linear control methods are valid to resolve its control problems. During the past few decades, a large number of theoretical findings have been published around the stability analysis and control synthesis for T-S fuzzy systems in the existing literature [8]–[16].

As is well known, the communication network in control systems brings many benefits, such as simple installation and low maintenance, thus they have been widely used in the various areas of control disciplines instead of point-to-point connections [17]–[19]. Because of the communication over a shared wire/wireless network, its resources and/or energy sources are generally limited [20], [21]. In recent years, we have witnessed rapidly increasing interest in the study of event-based control to avoid the occurrence of collision/larger delays/channel congestion in the communication network. A threshold is given *a priori*, which determines that the control signals can be transmitted or not, see, e.g., [22]–[27]. For instance, the observer-based controllers for linear systems were studied in [22] based on a two-channel event-triggering scheme. With the aid of perturbed linear and piecewise linear system approaches, some sufficient conditions for stability and performance of the resulting closed-loop systems were derived in terms of LMIs. In [23], a two-channel event-triggered control scheme for linear systems was proposed by using an impulsive model approach, and the stability and performance for event-triggered controllers with larger minimum inter-event times than the existing results in the literature were guaranteed. The work in [24] focused on event-triggered control for networked linear systems. The effect of the network transmission delay and the properties of the event-triggering scheme were taken into account. The co-design for both the controller and the parameters of triggers was derived in the form of LMIs. To date, it has been named a variety of similar terms including the event-triggering control [22]–[24], interrupt-based control [25], self-triggering control [26], state-triggering control [27]. More recently, some relevant work has considered the characteristics of membership functions for fuzzy systems [28]–[33]. In [28], the degree of overlap between the membership functions was taken into account, and the relaxed LMI conditions to the PDC controller design were proposed for fuzzy systems. This work in [29] assumed that knowledge of membership function shape was available *a priori* in the form of polynomial constraints, relax stability and performance conditions for fuzzy systems with the PDC controller were derived. When requiring priori asynchronous knowledge, the work in [30] derived the LMI-based design

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results on fuzzy controller. In [31], further extensions to uncertain grades of membership were presented, and the less conservative results were derived by relaxing some restrictive conditions. In [32], the asynchronous grades of membership were considered as disturbances, and the design of fuzzy state-feedback controller to synchronize the chaotic systems was derived in the form of LMIs. The work in [33] reconstructed the synchronous grades of membership between the controller and the plant under communication network, two relaxing criteria to stability analysis and controller synthesis were derived.

On the other hand, researchers have paid more attention to the control issue of large-scale systems because of its extensive application in many practical scenarios, such as multiagent systems, transportation networks, power plants, and industrial processes [34]. Multiagent systems, such as unmanned aerial vehicles, mobile robots, and distributed sensor networks, have been a very active research subject [35]. Such systems could be regarded as a class of large-scale systems without interconnections, and their structures might dictate a technique for the stability analysis and controller design problems including coordinated control [36]. In this way, the analysis and design subproblems are directly associated with the subsystems in addition to a coordinator that achieves the overall job. For the large-scale systems with coupling, considering that the centralized control requires faster computers with larger memory to implement excessive information processing, currently the decentralized control has attracted considerable interest in the field of large-scale systems. Instead of developing a single controller, a cluster of controllers, which are mutually independent, can be designed to execute the overall control task [37]. Unfortunately, the decentralized control policy expresses the weaker stabilizing ability, particularly when the degree of interconnections among subsystems is very strong [38]. In the distributed control approach, the extra feedbacks including the interconnection information are offered to strengthen the stabilizing ability and to enhance the performance. Therefore, those weaknesses appeared in both centralized control and decentralized control can be avoided by using the distributed control method.

Recently, some updated results have been presented with the aid of fuzzy/piecewise Lyapunov functional, and it has been verified that the inherent conservatism induced by the common Lyapunov function approach can be relaxed as considering a fuzzy/piecewise Lyapunov functional [39]–[42]. Furthermore, the T-S fuzzy-model-based approach has been developed for large-scale nonlinear systems [46]–[49]. For instance, some design results have been derived for decentralized state-feedback control of large-scale systems in [46]–[48]. The work in [49] studied the decentralized $\mathcal{H}_\infty$ filtering problem for large-scale nonlinear systems with time-varying delays. The work in [50] investigated the decentralized event-triggering control scheme for large-scale networked T-S fuzzy systems. The results on $\mathcal{H}_\infty$ decentralized fuzzy observer-based controller were derived with the aid of the decoupled method in [51]. To design the $\mathcal{H}_\infty$ fuzzy controller with reduced number of linear matrix inequalities (LMIs), [52], [53] investigated the Lyapunov-based method combined with S-

procedure. It is worth noting that, few results on the distributed event-triggering output-feedback control have been reported for large-scale fuzzy systems based on the piecewise Lyapunov functional approach, which motivates us for this study.

This paper is concerned with the distributed event-triggering output feedback control problem for a class of discrete-time large-scale nonlinear systems in the network environment. The presented large-scale system consists of a group of nonlinear subsystems with interconnections. Each nonlinear subsystem is described using a T-S model, and exchanges its output information over communication networks. To effectively reduce data transmission in communication channels, a novel event-triggering control scheme is proposed, in which each subsystem transmits its output information and premise variables only when its output error exceeds a specified threshold. Also, the asynchronous feature of premise variables contained in the system and the proposed controller is considered. Based on the piecewise Lyapunov functional approach together with some matrix inequality convexification techniques, sufficient conditions ensuring the existence of distributed event-triggering output feedback controller will be derived in the form of LMIs. A DC microgrid with photovoltaic (PV) arrays is utilized to check the feasibility of the obtained theoretical results.

For a class of large-scale networked fuzzy systems in the discrete-time domain, the main contribution of this paper is to present a design scheme of distributed event-triggering output feedback controller at the first attempt. More precisely, a novel distributed event-triggering control scheme will be proposed, where each subsystem transmits its output information and premise variables just when its local output error exceeds a specified threshold, and the piecewise Lyapunov functional approach is employed to guarantee the convergence of the resulting closed-loop systems with nonsynchronous grades of membership. The sufficient conditions with less conservatism are obtained in the form of LMIs for the desired controller design.

Notations: In this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n \times m}$ are the n -dimensional Euclidean space and the $n \times m$ dimensional real matrices, respectively. $P > 0$ (≥ 0) means that matrix P is positive definite (positive semidefinite). $\text{Sym}\{A\}$ denotes $A + A^T$. $\mathbf{I}_n$ and $0_{m \times n}$ represent the identity matrix and zero matrix, respectively. The subscripts n and $m \times n$ are omitted when their sizes are irrelevant or can be determined from the context. For the matrix A , A^{-1} and A^T represent its inverse and transpose, respectively. $\text{diag}\{\dots\}$ denotes a block-diagonal matrix. The notation $\|\cdot\|$ represents the Euclidean vector norm, and $\|\cdot\|_2$ is the $l_2[0, \infty)$ norm. The notation $\star$ represents the symmetric terms.

II. PROBLEM FORMULATION AND PRELIMINARIES

This paper considers a large-scale system composed of N discrete-time nonlinear subsystems. Firstly, based on T-S model representation, the i -th subsystem can be described by

Plant Rule $\mathcal{R}_i^l$: IF $\zeta_{i1}(t)$ is $\mathcal{F}_{i1}^l$, $\zeta_{i2}(t)$ is $\mathcal{F}_{i2}^l$, $\dots$, and

$\zeta_{ig}(t)$ is $\mathcal{F}_{ig}^l$, **THEN**

$$\begin{cases} x_i(t+1) = (A_{il} + \Delta A_{il})x_i(t) + B_{il}u_i(t) \\ \quad + \sum_{\substack{f=1 \\ f \neq i}}^N A_{ifl}x_f(t), \\ y_i(t) = C_i x_i(t), \end{cases} \quad (1)$$

where $l \in \mathcal{L}_i := \{1, 2, \dots, r_i\}$, $i \in \mathcal{N} := \{1, 2, \dots, N\}$, $\mathcal{R}_i^l$ denotes the l -th fuzzy inference rule; r_i denotes the number of inference rules; $\mathcal{F}_{i\phi}^l$ ($\phi = 1, 2, \dots, g$) denote fuzzy sets; $x_i(t) \in \mathbb{R}^{n_{xi}}$, $u_i(t) \in \mathbb{R}^{n_{ui}}$, and $y_i(t) \in \mathbb{R}^{n_{yi}}$ denote the system state, the control input, and the system output, respectively; $\zeta_i(t) := [\zeta_{i1}(t), \zeta_{i2}(t), \dots, \zeta_{ig}(t)]$ are the measurable variables; $\{A_{il}, B_{il}, C_i\}$ is real constant matrices for the l -th local model, and A_{ifl} represents the coupling of the l -th local model between the i -th and f -th subsystems, and ΔA_{il} denotes the uncertainty term of the l -th local model, which satisfies that

$$\Delta A_{il} = W_{il}\Delta_{il}(t)E_{il}, l \in \mathcal{L}_i \quad (2)$$

where W_{il} and E_{il} are known matrices with appropriate dimensions. $\Delta_{il}(t) \in \mathbb{R}^{s_1 \times s_2}$ are the unknown time-varying matrix functions with Lebesgue measurable elements satisfying

$$\Delta_{il}^T(t)\Delta_{il}(t) \leq \mathbf{I}_{s_2}, l \in \mathcal{L}_i. \quad (3)$$

Remark 2.1. To avoid complicated notations, this paper only considers a common output matrix C_i . Nevertheless, the results proposed in this paper can be extended to the presented system with the fuzzy matrix C_{il} .

After using fuzzy blending, we can directly obtain the i -th global T-S fuzzy model as below:

$$\begin{cases} x_i(t+1) = \sum_{l=1}^{r_i} \mu_{il}[\zeta_i(t)] (A_{il} + \Delta A_{il})x_i(t) \\ \quad + \sum_{l=1}^{r_i} \mu_{il}[\zeta_i(t)] B_{il}u_i(t) \\ \quad + \sum_{\substack{f=1 \\ f \neq i}}^N \sum_{l=1}^{r_i} \mu_{il}[\zeta_i(t)] A_{ifl}x_f(t), \\ y_i(t) = C_i x_i(t), \end{cases} \quad (4)$$

where

$$\begin{aligned} A_i(\mu_i) &:= \sum_{l=1}^{r_i} \mu_{il} A_{il}, \Delta A_i(\mu_i) := \sum_{l=1}^{r_i} \mu_{il} \Delta A_{il}, \\ B_i(\mu_i) &:= \sum_{l=1}^{r_i} \mu_{il} B_{il}, A_{if}(\mu_i) := \sum_{l=1}^{r_i} \mu_{il} A_{ifl}, \\ \mu_{il}[\zeta_i(t)] &:= \frac{\prod_{\phi=1}^g \mu_{i\phi}[\zeta_{i\phi}(t)]}{\sum_{\zeta=1}^{r_i} \prod_{\phi=1}^g \mu_{i\zeta\phi}[\zeta_{i\phi}(t)]} \geq 0, \\ \sum_{l=1}^{r_i} \mu_{il}[\zeta_i(t)] &= 1. \end{aligned} \quad (5)$$

Remark 2.2. The works in [48], [49], [54], [55] studied the large-scale fuzzy systems with linear coupling A_{if} instead of a general class of large-scale fuzzy systems with nonlinear coupling A_{ifl} in (1). Note that the control problem for large-scale fuzzy systems with nonlinear coupling in (1) becomes

more challenging than those with linear one available in [48], [49], [54], [55].

Remark 2.3. For simplicity, the uncertainty terms ΔA_{il} are only considered in (1), $l \in \mathcal{L}_i, i \in \mathcal{N}$. Note that in (1), a trivial extension on the methods presented in this paper can be done to that case where the uncertain terms subject to (2) and (3) could also appear in the system matrices B_{il} and A_{ifl} , $l \in \mathcal{L}_i, i \in \mathcal{N}$.

In order to relax the inherent conservatism induced by a common Lyapunov function, a piecewise Lyapunov functional approach is introduced in this paper to address the problem of distributed event-triggering control for the considered system in (4). For each fuzzy subsystem $i \in \mathcal{N}$, the idea proposed in [56] is referred here, where the premise variable space is split into two different classes of regions: crisp regions and fuzzy regions. Specifically, the region subject to $0 < \mu_{il}[\zeta_i(t)] < 1$ is named the fuzzy region, where a convex combination of several local models is used to represent the system dynamics. At the same time, the crisp region is that the region with $\mu_{il}[\zeta_i(t)] = 1$ for some rules l , and the rest of membership functions equals to zero. In the crisp region the l -th local model represents the system dynamics.

In terms of the partition policy of premise variables, we further define $\{\mathcal{S}_{ij}\}_{j \in \mathcal{J}_i}$ as the partition space and $\mathcal{J}_i$ as the set of region indices. Thus, the i -th global T-S fuzzy system in (4) is rewritten as

$$\begin{cases} x_i(t+1) = (A_{ij} + \Delta A_{ij})x_i(t) + B_{ij}u_i(t) \\ \quad + \sum_{\substack{f=1 \\ f \neq i}}^N A_{ifj}x_f(t), \\ y_i(t) = C_i x_i(t), \zeta_i(t) \in \mathcal{S}_{ij}, j \in \mathcal{J}_i, i \in \mathcal{N} \end{cases} \quad (6)$$

with

$$\begin{aligned} A_{ij} &:= \sum_{m \in \mathcal{I}_i(j)} \mu_{im} A_{im}, \Delta A_{ij} := \sum_{m \in \mathcal{I}_i(j)} \mu_{im} \Delta A_{im}, \\ B_{ij} &:= \sum_{m \in \mathcal{I}_i(j)} \mu_{im} B_{im}, A_{ifj} := \sum_{m \in \mathcal{I}_i(j)} \mu_{im} A_{ifm}, \\ 0 < \mu_{im}[\zeta_i(t)] < 1, \sum_{m \in \mathcal{I}_i(j)} \mu_{im}[\zeta_i(t)] &= 1, \end{aligned} \quad (7)$$

where the set $\mathcal{I}_i(j)$ contains some indices of system matrices for each fuzzy region, but the set $\mathcal{I}_i(j)$ contains only a single index for each crisp region.

In the context of networked control systems, the standard time-triggering implementation is unachievable because of the limited communication bandwidth. Inspired by [22], an event-triggering mechanism (ETM), which makes a decision on the transmission of system outputs, is proposed here to save the communication resources and to guarantee the desired performance. To implement the proposed solution, the ETM occurred in the sensor system is expressed as

$$\text{ETM: } y_i(t) \text{ is sent} \Leftrightarrow \|y_i(t) - \hat{y}_i(t-1)\| > \sigma_i \|y_i(t)\|, \quad (8)$$

where $\hat{y}_i(t)$ denotes that the system output is transmitted over the communication network.

Considering the operation in (8), an event-triggering strategy is given by

$$\hat{y}_i(t) = \begin{cases} y_i(t), & \text{when } \|y_i(t) - \hat{y}_i(t-1)\| > \sigma_i \|y_i(t)\|, \\ \hat{y}_i(t-1), & \text{when } \|y_i(t) - \hat{y}_i(t-1)\| \leq \sigma_i \|y_i(t)\|, \end{cases} \quad (9)$$

and

$$\hat{\zeta}_i(t) = \begin{cases} \zeta_i(t), & \text{when } y_i(t) \text{ is sent,} \\ \hat{\zeta}_i(t-1), & \text{when } y_i(t) \text{ is not sent.} \end{cases} \quad (10)$$

For simplicity, we introduce a new set Ω_i as below:

$$\Omega_i := \{(j, s, k) \mid \zeta_i(t) \in \mathcal{S}_{ij}, \zeta_i(t+1) \in \mathcal{S}_{is}, \hat{\zeta}_i(t) \in \mathcal{S}_{ik}, (j, s, k) \in \mathcal{J}_i\} \quad (11)$$

where $\{s \equiv j, k \equiv j\}$ represents that $\zeta_i(t), \zeta_i(t+1)$ and $\hat{\zeta}_i(t)$ stay at the same region S_{ij} , otherwise the different regions happen among $\zeta_i(t), \zeta_i(t+1)$ and $\hat{\zeta}_i(t)$.

Now, given each partition of premise variable space $\hat{\zeta}_i(t)$ in (11), a distributed event-triggering output feedback piecewise controller is proposed as below:

$$u_i(t) = K_{ik}\hat{y}_i(t) + \sum_{\substack{f=1 \\ f \neq i}}^N K_{ifk}\hat{y}_f(t), \hat{\varsigma}_i(t) \in \mathcal{S}_{ik}, \quad (12)$$

where $\{K_{ik}, K_{ifk}\} \in \mathbb{R}^{n_u \times n_y}, k \in \mathcal{I}_i, i \in \mathcal{N}$ are controller gains to be designed; $\hat{y}_i(t), i \in \mathcal{N}$ stands for the system output applied to the controller.

Fig. 1. A distributed event-triggering control scheme

The distributed event-triggering control scheme is shown in Fig. 1, where ETM and BF denote the event-triggering mechanism and the buffer, respectively; $\hat{y}_i(t)$ denotes the latest measurement output transmitted over the communication network. A sensor system includes a BF that is to reserve $\hat{y}_i(t)$ and an ETM that is to determine whether transmitting the measurement output $y_i(t)$ and the premise variables $\zeta_i(t)$ or not.

Remark 2.4. Note that the work in [43] proposes a PDC fuzzy filter with event-triggering in which the premise variables between the system and filter are synchronous. Such a filter may be impractical because of the existence of networked imperfections. Based on the event-triggering control scheme, a more reasonable assumption with the nonsynchronous premise variables is considered in [44], [45]. However, it less effectively makes use of the limited network bandwidth because

the premise variables are transmitted under the conventional time-triggering approach.

Remark 2.5. It is worth noting that when $K_{ifk} = 0$ for any $k \in \mathcal{J}_i, i \in \mathcal{N}$, the distributed controller proposed in (12) degenerates the decentralized one that is widely studied in [47]–[49]. The simulation study will show that the supplemental feedbacks containing the interconnected information are provided in the proposed distributed controller (12) for the local controllers to ensure the better stabilization margins than the decentralized one.

Remark 2.6. In particular, the premise variables of the distributed piecewise controller in (12) undergo event-triggering control, such that the asynchronous premise variable spaces between $\zeta_i(t)$ and $\hat{\zeta}_i(t)$ are closer to the actuals. It has been shown in [31], when the information between $\zeta_i(t)$ and $\hat{\zeta}_i(t)$ is unavailable, this condition tends to generate a linear controller, which lowers the stabilization ability of the desired controller.

The closed-loop control system consisting of (6) and (12) is represented as

$$x_i(t+1) = (\mathcal{A}_{ij} + \Delta \mathcal{A}_{ij}) x_i(t) + \mathcal{B}_{ij} K_{ik} \hat{y}_i(t) + \mathcal{B}_{ij} \sum_{\substack{f=1 \\ f \neq i}}^N K_{ifk} \hat{y}_f(t) + \sum_{\substack{f=1 \\ f \neq i}}^N \mathcal{A}_{ifj} x_f(t), \quad (13)$$

where $\zeta_i(t) \in \mathcal{S}_{ij}, \hat{\zeta}_i(t) \in \mathcal{S}_{ik}, \{j, k\} \in \mathcal{J}_i, i \in \mathcal{N}$.

Here, the idea proposed in [22] is borrowed here to consider the event-triggering counterpart as a disturbance, that is

$$w_i(t) = \hat{y}_i(t) - y_i(t). \quad (14)$$

By submitting (14) into (13), the closed-loop control system is remodeled as

$$\begin{aligned}
x_i(t+1) = & \bar{\mathcal{A}}_{ij}x_j(t) + \mathcal{B}_{ij}K_{ik}w_i(t) + \sum_{\substack{f=1 \\ f \neq i}}^N \bar{\mathcal{A}}_{if}x_f(t) \\
& + \sum_{\substack{f=1 \\ f \neq i}}^N \mathcal{B}_{ij}K_{ifk}w_f(t), \tag{15}
\end{aligned}$$

where $\zeta_i(t) \in \mathcal{S}_{ij}, \hat{\zeta}_i(t) \in \mathcal{S}_{ik}, \{j, k\} \in \mathcal{J}_i, i \in \mathcal{N}$, and $\bar{A}_{ij} = A_{ij} + B_{ij}K_{ik}C_i + \Delta A_{ij}, \bar{A}_{ifj} = A_{ifj} + B_{ij}K_{ifk}C_f$.

III. ANALYSIS AND DESIGN WITH DISTRIBUTED EVENT-TRIGGERING

In this section, by using the piecewise Lyapunov functional approach together with some matrix inequality convexification techniques, the stability analysis and controller design on distributed event-triggering output feedback will be conducted respectively for the large-scale fuzzy system in (4).

A. Stability Analysis

Here, the stability analysis for the closed-loop fuzzy control system in (15) is first given as below:

Lemma 3.1. Considering the large-scale fuzzy system in (4), and the distributed event-triggering piecewise controller in

(12), then the resulting closed-loop control system is asymptotically stable, if there exist the positive-definite symmetric matrices $\{P_{ij}, H_{if}\} \in \mathbb{R}^{n_{xi} \times n_{xi}}, \{M_{if}, Q_i\} \in \mathbb{R}^{n_{yi} \times n_{yi}}$; and matrices $\{K_{ik}, K_{ifk}\} \in \mathbb{R}^{n_{ui} \times n_{yi}}$, matrix multiplier $\mathcal{G}_i \in \mathbb{R}^{(2n_{xi}+n_{yi}) \times n_{xi}}$, and the positive scalars σ_i , such that for all $(j, s, k) \in \Omega_i, j \in \mathcal{J}_i, i \in \mathcal{N}$ the following matrix inequalities hold:

$$\begin{bmatrix} \Pi_{ijsk} & \mathbb{A}_{ifj} & \mathbb{K}_{ifjk} \\ \star & -\mathbb{H}_{if} & 0 \\ \star & \star & -\mathbb{M}_{if} \end{bmatrix} < 0, \quad (16)$$

where

$$\begin{aligned} \Pi_{ijsk} &= \Theta_{ijs} + \text{Sym} \left\{ \mathcal{G}_i \begin{bmatrix} -\mathbf{I} & \bar{\mathcal{A}}_{ij} & \mathcal{B}_{ij}K_{ik} \end{bmatrix} \right\}, \\ \Theta_{ijs} &= \begin{bmatrix} P_{is} & 0 & 0 \\ \star & \Theta_{ij(1)} & 0 \\ \star & \star & \sum_{\substack{f=1 \\ f \neq i}}^N M_{fi} - Q_i \end{bmatrix}, \\ \mathbb{H}_{if} &= \text{diag} \left\{ \underbrace{H_{i1} \cdots H_{if, f \neq i} \cdots H_{iN}}_{N-1} \right\}, \\ \mathbb{M}_{if} &= \text{diag} \left\{ \underbrace{M_{i1} \cdots M_{if, f \neq i} \cdots M_{iN}}_{N-1} \right\}, \\ \mathbb{A}_{ifj} &= \left[\underbrace{\mathcal{G}_i \bar{\mathcal{A}}_{i1j} \cdots \mathcal{G}_i \bar{\mathcal{A}}_{ifj, f \neq i} \cdots \mathcal{G}_i \bar{\mathcal{A}}_{iNj}}_{N-1} \right], \\ \mathbb{K}_{ifjk} &= \left[\underbrace{\mathcal{G}_i \mathcal{B}_{ij} K_{i1k} \cdots \mathcal{G}_i \mathcal{B}_{ij} K_{ifk, f \neq i} \cdots \mathcal{G}_i \mathcal{B}_{ij} K_{iNk}}_{N-1} \right], \\ \Theta_{ij(1)} &= -P_{ij} + \sum_{\substack{f=1 \\ f \neq i}}^N H_{fi} + \sigma_i^2 C_i^T Q_i C_i. \end{aligned} \quad (17)$$

Proof. Consider the piecewise Lyapunov function as below:

$$V(t) = \sum_{i=1}^N V_i(t) = \sum_{i=1}^N x_i^T(t) P_{ij} x_i(t), \zeta_i(t) \in \mathcal{S}_{ij}, j \in \mathcal{J}_i \quad (18)$$

where $0 < P_{ij} = P_{ij}^T \in \mathbb{R}^{n_{xi} \times n_{xi}}$.

Defining $\Delta V_i(t) = V_i(t+1) - V_i(t)$, one gets

$$\Delta V(t) = \sum_{i=1}^N \{x_i^T(t+1) P_{is} x_i(t+1) - x_i^T(t) P_{ij} x_i(t)\}, \quad (19)$$

with $(j, s) \in \Omega_i, \Omega_i := \{(j, s) \mid \zeta_i(t) \in \mathcal{S}_{ij}, \zeta_i(t+1) \in \mathcal{S}_{is}, (j, s) \in \mathcal{J}_i\}$.

In addition, defining $\chi_i(t) = \begin{bmatrix} x_i^T(t+1) & x_i^T(t) \\ w_i^T(t) \end{bmatrix}^T$, and the matrix multiplier $\mathcal{G}_i \in \mathbb{R}^{(2n_{xi}+n_{yi}) \times n_{xi}}$, it follows from the trajectory of the system (15) that

$$\begin{aligned} 0 &= 2 \sum_{i=1}^N \chi_i^T(t) \mathcal{G}_i \left[-x_i(t+1) + \bar{\mathcal{A}}_{ij} x_i(t) + \mathcal{B}_{ij} K_{ik} w_i(t) \right] \\ &+ 2 \sum_{i=1}^N \chi_i^T(t) \mathcal{G}_i \left[\sum_{\substack{f=1 \\ f \neq i}}^N \bar{\mathcal{A}}_{ifj} x_f(t) + \sum_{\substack{f=1 \\ f \neq i}}^N \mathcal{B}_{ij} K_{ifk} w_f(t) \right]. \end{aligned} \quad (20)$$

Note that

$$2\bar{x}^T \bar{y} \leq \kappa^{-1} \bar{x}^T Q^{-1} \bar{x} + \kappa \bar{y}^T Q \bar{y}, \quad (21)$$

where $Q = Q^T > 0$. Now, we define the symmetric positive-definite matrices $H_{if} \in \mathbb{R}^{n_{xi} \times n_{xi}}$ and $M_{if} \in \mathbb{R}^{n_{yi} \times n_{yi}}$, and by virtue of (21), it yields

$$\begin{aligned} &2 \sum_{i=1}^N \chi_i^T(t) \mathcal{G}_i \sum_{\substack{f=1 \\ f \neq i}}^N \bar{\mathcal{A}}_{ifj} x_f(t) \\ &\leq \sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N \chi_i^T(t) \mathcal{G}_i \bar{\mathcal{A}}_{ifj} H_{if}^{-1} \bar{\mathcal{A}}_{ifj}^T \mathcal{G}_i^T \chi_i(t) \\ &+ \sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N x_f^T(t) H_{if} x_f(t), \end{aligned} \quad (22)$$

and

$$\begin{aligned} &2 \sum_{i=1}^N \chi_i^T(t) \mathcal{G}_i \sum_{\substack{f=1 \\ f \neq i}}^N \mathcal{B}_{ij} K_{ifk} w_f(t) \\ &\leq \sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N \chi_i^T(t) \mathcal{G}_i \mathcal{B}_{ij} K_{ifk} M_{if}^{-1} K_{ifk}^T \mathcal{B}_{ij}^T \mathcal{G}_i^T \chi_i(t) \\ &+ \sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N w_f^T(t) M_{if} w_f(t). \end{aligned} \quad (23)$$

Note that

$$\sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N x_f^T(t) H_{if} x_f(t) = \sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N x_i^T(t) H_{fi} x_i(t), \quad (24)$$

and

$$\sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N w_f^T(t) M_{if} w_f(t) = \sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N w_i^T(t) M_{fi} w_i(t). \quad (25)$$

It follows from (9) that

$$\begin{aligned} \|w_i(t)\| &= \|\hat{y}_i(t) - y_i(t)\| \\ &\leq \sigma_i \|y_i(t)\|, i \in \mathcal{N}. \end{aligned} \quad (26)$$

Define the matrix $0 < Q_i = Q_i^T \in \mathbb{R}^{n_{yi} \times n_{yi}}$. It is easy to see from (26) that

$$\sigma_i^2 y_i^T(t) Q_i y_i(t) - w_i^T(t) Q_i w_i(t) \geq 0. \quad (27)$$

Then, it gets from (19)-(27) that

$$\begin{aligned} \Delta V(t) &= \sum_{i=1}^N \{x_i^T(t+1) P_{is} x_i(t+1) - x_i^T(t) P_{ij} x_i(t)\} \\ &- 2 \sum_{i=1}^N \chi_i^T(t) \mathcal{G}_i x_i(t+1) \\ &+ 2 \sum_{i=1}^N \chi_i^T(t) \mathcal{G}_i [\bar{\mathcal{A}}_{ij} x_i(t) + \mathcal{B}_{ij} K_{ik} w_i(t)] \end{aligned}$$

$$\begin{aligned}
& + 2 \sum_{i=1}^N \chi_i^T(t) \mathcal{G}_i \sum_{\substack{f=1 \\ f \neq i}}^N \bar{\mathcal{A}}_{ifj} x_f(t) \\
& + 2 \sum_{i=1}^N \chi_i^T(t) \mathcal{G}_i \sum_{\substack{f=1 \\ f \neq i}}^N \mathcal{B}_{ij} K_{ifk} w_f(t) \\
& \leq \sum_{i=1}^N \{ x_i^T(t+1) P_{is} x_i(t+1) - x_i^T(t) P_{ij} x_i(t) \} \\
& - 2 \sum_{i=1}^N \chi_i^T(t) \mathcal{G}_i x_i(t+1) \\
& + 2 \sum_{i=1}^N \chi_i^T(t) \mathcal{G}_i [\bar{\mathcal{A}}_{ij} x_i(t) + \mathcal{B}_{ij} K_{ik} w_i(t)] \\
& + \sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N \chi_i^T(t) \mathcal{G}_i \bar{\mathcal{A}}_{ifj} H_{if}^{-1} \bar{\mathcal{A}}_{ifj}^T \mathcal{G}_i^T \chi_i(t) \\
& + \sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N x_i^T(t) H_{fi} x_i(t) \\
& + \sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N \chi_i^T(t) \mathcal{G}_i \mathcal{B}_{ij} K_{ifk} M_{if}^{-1} K_{ifk}^T \mathcal{B}_{ij}^T \mathcal{G}_i^T \chi_i(t) \\
& + \sum_{i=1}^N \sum_{\substack{f=1 \\ f \neq i}}^N w_i^T(t) M_{fi} w_i(t) \\
& + \sum_{i=1}^N \{ \sigma_i^2 y_i^T(t) Q_i y_i(t) - w_i^T(t) Q_i w_i(t) \} \\
& = \sum_{i=1}^N \chi_i^T(t) \left\{ \Pi_{ijsk} + \sum_{\substack{f=1 \\ f \neq i}}^N \mathcal{G}_i \bar{\mathcal{A}}_{ifj} H_{if}^{-1} \bar{\mathcal{A}}_{ifj}^T \mathcal{G}_i^T \right. \\
& \quad \left. + \sum_{\substack{f=1 \\ f \neq i}}^N \mathcal{G}_i \mathcal{B}_{ij} K_{ifk} M_{if}^{-1} K_{ifk}^T \mathcal{B}_{ij}^T \mathcal{G}_i^T \right\} \chi_i(t), \tag{28}
\end{aligned}$$

where

$$\begin{aligned}
\Pi_{ijsk} &= \Theta_{ijs} + \text{Sym} \{ \mathcal{G}_i [-\mathbf{I} \quad \bar{\mathcal{A}}_{ij} \quad \mathcal{B}_{ij} K_{ik}] \}, \\
\Theta_{ijs} &= \begin{bmatrix} P_{is} & 0 & 0 \\ \star & \Theta_{ij(1)} & 0 \\ \star & \star & \sum_{\substack{f=1 \\ f \neq i}}^N M_{fi} - Q_i \end{bmatrix}, \\
\Theta_{ij(1)} &= -P_{ij} + \sum_{\substack{f=1 \\ f \neq i}}^N H_{fi} + \sigma_i^2 C_i^T Q_i C_i. \tag{29}
\end{aligned}$$

Now, by Schur complement lemma, it is not difficult to see from (28) that the inequality in (16) implies $\Delta V(t) < 0$, which completes the proof.

Remark 3.1. Note that Lemma 3.1 is derived by the utilization of a piecewise Lyapunov function defined in (18). It is straightforward to see that if $P_{ij} \equiv P_i$ for any $j \in \mathcal{J}_i, i \in \mathcal{N}$, the expression in (18) degrades into the common Lyapunov function proposed in [46]. The design results obtained via the piecewise Lyapunov function are less conservative than those using the common one, which will be verified in Section IV.

Remark 3.2. It is also obvious that the bounding inequalities in (22) and (23) are used by introducing the matrices $\{H_{if}, M_{if}\}$. If considering the matrices $\{H_{ifj}, M_{ifj}\}$, more slack matrix variables will be inserted, and the obtained results may be less conservative to some extent.

B. Distributed Event-Triggering Controller Design

Considering the system (4), this section focuses on designing the distributed event-triggering controller with piecewise form. Before starting it, the following assumptions are given:

Assumption 3.1. The premise variables $\zeta_i(t+1)$, $\zeta_i(t)$, and $\hat{\zeta}_i(t)$ stay at the same region, or both $\zeta_i(t+1)$ and $\hat{\zeta}_i(t)$ stay near $\zeta_i(t)$, and the partition of premise variable space $\{\mathcal{S}_{ij}\}_{j \in \mathcal{J}_i}$ satisfies $j > 3$.

Assumption 3.2. The output matrix $C_i, i \in \mathcal{N}$ is of full row rank, then there exist nonsingular transformation matrix $T_i \in \mathbb{R}^{n_{xi} \times n_{xi}}, i \in \mathcal{N}$ satisfying [58]

$$C_i T_i = [\mathbf{I}_{n_{yi}} \quad 0_{n_{yi} \times (n_{xi} - n_{yi})}], i \in \mathcal{N}. \tag{30}$$

Based on Assumptions 3.1 and 3.2, as well as some matrix inequality convexification techniques, the sufficient condition ensuring the existence of the desired distributed event-triggering piecewise controller is given as below:

Theorem 3.1. Consider the large-scale fuzzy system in (4), and a distributed event-triggering piecewise controller in (12), the closed-loop fuzzy control system with the Assumptions 3.1 and 3.2 is asymptotically stable, if there exist the positive-definite symmetric matrices $\{\bar{P}_{ij}, \bar{H}_{fi}\} \in \mathbb{R}^{n_{xi} \times n_{xi}}, \{\bar{M}_{fi}, Z_i\} \in \mathbb{R}^{n_{yi} \times n_{yi}}, \bar{H}_{0i} \leq \bar{H}_{fi}, \bar{M}_{0i} \leq \bar{M}_{fi}$; and matrices $\{\bar{K}_{ik}, \bar{K}_{ifk}\} \in \mathbb{R}^{n_{ui} \times n_{yi}}, \{G_{ik}, G_{ifk}\} \in \mathbb{R}^{n_{xi} \times n_{xi}}, \{X_{ik}, X_{ifk}\} \in \mathbb{R}^{n_{yi} \times n_{yi}}, i \in \mathcal{N}$, and the positive scalars $\{\epsilon_i, \sigma_i\}$, such that for all $(j, s, k) \in \Omega_i, m \in \mathcal{I}_i(j), j \in \mathcal{J}_i, i \in \mathcal{N}$ the following LMIs hold:

$$\begin{bmatrix} \tilde{\Theta}_{ijkm} & \bar{\mathcal{A}}_{ifkm} & \bar{\mathcal{K}}_{ifkm} & \bar{\mathcal{R}}_i & \epsilon_i \mathcal{W}_{im} & \mathcal{E}_{ikm}^T \\ \star & \bar{\mathcal{H}}_{ifk} & 0 & 0 & 0 & 0 \\ \star & \star & \bar{\mathcal{M}}_{ifk} & 0 & 0 & 0 \\ \star & \star & \star & -\bar{\mathcal{P}}_{is} & 0 & 0 \\ \star & \star & \star & \star & -\epsilon_i \mathbf{I} & 0 \\ \star & \star & \star & \star & \star & -\epsilon_i \mathbf{I} \end{bmatrix} < 0, \tag{31}$$

where

$$\begin{aligned}
\tilde{\Theta}_{ijkm} &= \begin{bmatrix} 0 & 0 & 0 \\ \star & \bar{P}_{ij} - G_{ik} - G_{ik}^T & 0 \\ \star & \star & Z_i - X_{ik} - X_{ik}^T \end{bmatrix} \\
&+ \text{Sym} \{ R_1 [-G_{ik} \quad A_{im} G_{ik} + B_{im} \mathcal{K}_{ik} \quad B_{im} \bar{K}_{ik}] \}, \\
\bar{\mathcal{H}}_{ifk} &= \bar{\mathcal{H}}_{if} - G_{ik} - G_{ik}^T, \bar{\mathcal{M}}_{ifk} = \bar{\mathcal{M}}_{if} - X_{ik} - X_{ik}^T, \\
\bar{\mathcal{R}}_i &= [\bar{R}_1 \quad \bar{R}_2 \quad \bar{R}_3 \quad \bar{R}_4], \mathcal{K}_{ik} = [\bar{K}_{ik} \quad 0],
\end{aligned}$$

$$\begin{aligned}
\bar{\mathbb{P}}_{is} &= \text{diag} \left\{ \bar{P}_{is} \quad \frac{\bar{H}_{0i}}{N-1} \quad \frac{\bar{M}_{0i}}{N-1} \quad W_i \right\}, \\
G_{ik} &= T_{ci} \begin{bmatrix} X_{ik} & 0 \\ G_{ik(1)} & G_{ik(2)} \end{bmatrix}, \\
G_{ifk} &= T_{cf} \begin{bmatrix} X_{ifk} & 0 \\ G_{ifk(1)} & G_{ifk(2)} \end{bmatrix}, \mathcal{W}_{im} = \begin{bmatrix} W_{im} \\ 0 \\ 0 \end{bmatrix}, \\
\bar{R}_1 &= \begin{bmatrix} G_{ik}^T \\ 0 \\ 0 \end{bmatrix}, \bar{R}_2 = \begin{bmatrix} 0 \\ G_{ik}^T \\ 0 \end{bmatrix}, \\
\bar{R}_3 &= \begin{bmatrix} 0 \\ 0 \\ X_{ik}^T \end{bmatrix}, \bar{R}_4 = \begin{bmatrix} 0 \\ \sigma_i G_{ik}^T C_i^T \\ 0 \end{bmatrix}, \\
\mathcal{E}_{ikm} &= \begin{bmatrix} 0 & E_{im} G_{ik} & 0 \end{bmatrix}, \\
\bar{\mathbb{H}}_{if} &= \text{diag} \{ \bar{H}_{i1} \cdots \bar{H}_{if, f \neq i} \cdots \bar{H}_{iN} \}, \\
\mathbb{G}_{ifk} &= \text{diag} \{ G_{i1k} \cdots G_{ifk, f \neq i} \cdots G_{iNk} \}, \\
\bar{\mathbb{M}}_{if} &= \text{diag} \{ \bar{M}_{i1} \cdots \bar{M}_{if, f \neq i} \cdots \bar{M}_{iN} \}, \\
\mathbb{X}_{ifk} &= \text{diag} \{ X_{i1k} \cdots X_{ifk, f \neq i} \cdots X_{iNk} \}, \\
\bar{\mathbb{A}}_{ifkm} &= \begin{bmatrix} \bar{A}_{i1km} \cdots \bar{A}_{ifkm, f \neq i} \cdots \bar{A}_{iNkm} \end{bmatrix}, \\
\bar{A}_{ifkm} &= R_1 A_{ifm} G_{ifk} + R_1 B_{im} \mathcal{K}_{ifk}, \mathcal{K}_{ifk} = \begin{bmatrix} \bar{K}_{ifk} & 0 \end{bmatrix}, \\
\bar{\mathbb{K}}_{ifkm} &= \begin{bmatrix} R_1 B_{im} \bar{K}_{i1k} \cdots R_1 B_{im} \bar{K}_{ifk, f \neq i} \cdots R_1 B_{im} \bar{K}_{iNk} \end{bmatrix}.
\end{aligned} \tag{32}$$

Furthermore, the distributed event-triggering piecewise controller gains can be calculated by

$$K_{ik} = \bar{K}_{ik} X_{ik}^{-1}, K_{ifk} = \bar{K}_{ifk} X_{ifk}^{-1}, k \in \mathcal{J}_i, f \neq i, \{i, f\} \in \mathcal{N}. \tag{33}$$

Proof. For simplicity, we define $\bar{P}_{ij} = P_{ij}^{-1}$, $\bar{H}_{if} = H_{if}^{-1}$, $\bar{H}_{0i} \leq \bar{H}_{fi}$, $\bar{M}_{if} = M_{if}^{-1}$, $\bar{M}_{0i} \leq \bar{M}_{fi}$, $Z_i = Q_i^{-1}$, $f \neq i, j \in \mathcal{J}_i$, $(i, f) \in \mathcal{N}$, and by using Schur complement lemma, then the following inequality implies (16) is satisfied,

$$\begin{bmatrix} \bar{\Pi}_{ijk} & \bar{\mathbb{A}}_{ifjk} & \bar{\mathbb{K}}_{ifjk} & \bar{\mathbb{R}}_i \\ \star & -\bar{\mathbb{H}}_{if}^{-1} & 0 & 0 \\ \star & \star & -\bar{\mathbb{M}}_{if}^{-1} & 0 \\ \star & \star & \star & -\bar{\mathbb{P}}_{is} \end{bmatrix} < 0, \tag{34}$$

where

$$\begin{aligned}
\bar{\Pi}_{ijk} &= \bar{\Theta}_{ij} + \text{Sym} \{ \mathcal{G}_i \begin{bmatrix} -\mathbf{I} & \bar{\mathcal{A}}_{ij} & \mathcal{B}_{ij} K_{ik} \end{bmatrix} \}, \\
\bar{\mathbb{A}}_{ifjk} &= \begin{bmatrix} \mathcal{G}_i \bar{\mathcal{A}}_{i1j} \cdots \mathcal{G}_i \bar{\mathcal{A}}_{ifj, f \neq i} \cdots \mathcal{G}_i \bar{\mathcal{A}}_{iNj} \end{bmatrix}, \\
\bar{\mathbb{H}}_{if} &= \text{diag} \{ \bar{H}_{i1} \cdots \bar{H}_{if, f \neq i} \cdots \bar{H}_{iN} \}, \\
\bar{\mathbb{K}}_{ifjk} &= \begin{bmatrix} \mathcal{G}_i \mathcal{B}_{ij} K_{i1k} \cdots \mathcal{G}_i \mathcal{B}_{ij} K_{ifk, f \neq i} \cdots \mathcal{G}_i \mathcal{B}_{ij} K_{iNk} \end{bmatrix}, \\
\bar{\mathbb{R}}_i &= \begin{bmatrix} R_1 & R_2 & R_3 & R_4 \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\bar{\mathbb{P}}_{is} &= \text{diag} \left\{ \bar{P}_{is} \quad \frac{\bar{H}_{0i}}{N-1} \quad \frac{\bar{M}_{0i}}{N-1} \quad Z_i \right\}, \\
\bar{\mathbb{M}}_{if} &= \text{diag} \{ \bar{M}_{i1} \cdots \bar{M}_{if, f \neq i} \cdots \bar{M}_{iN} \}, \\
\bar{\Theta}_{ij} &= \begin{bmatrix} 0 & 0 & 0 \\ \star & -\bar{P}_{ij}^{-1} & 0 \\ \star & \star & -W_i^{-1} \end{bmatrix}, \\
R_1 &= \begin{bmatrix} \mathbf{I} \\ 0 \\ 0 \end{bmatrix}, R_2 = \begin{bmatrix} 0 \\ \mathbf{I} \\ 0 \end{bmatrix}, \\
R_3 &= \begin{bmatrix} 0 \\ 0 \\ \mathbf{I} \end{bmatrix}, R_4 = \begin{bmatrix} 0 \\ \sigma_i C_i^T \\ 0 \end{bmatrix}.
\end{aligned} \tag{35}$$

Note that

$$X_i - Y_i - Y_i^T + Y_i^T X_i^{-1} Y_i = (X_i - Y_i)^T X_i^{-1} (X_i - Y_i) \geq 0, \tag{36}$$

where $X_i = X_i^T > 0$, which implies that

$$-Y_i^T X_i^{-1} Y_i \leq X_i - Y_i - Y_i^T. \tag{37}$$

For the purpose of matrix inequality linearization, we further denote

$$\begin{aligned}
\mathcal{G}_{ik} &= \begin{bmatrix} G_{ik}^T \\ 0 \\ 0 \end{bmatrix}, \mathbb{G}_{ifk} = \text{diag} \{ G_{i1k} \cdots G_{ifk, f \neq i} \cdots G_{iNk} \}, \\
\mathbb{X}_{ifk} &= \text{diag} \{ X_{i1k} \cdots X_{ifk, f \neq i} \cdots X_{iNk} \}, \\
\Gamma_{ik} &:= \text{diag} \{ G_{ik}, G_{ik}, X_{ik}, \mathbb{G}_{ik}, \mathbb{X}_{ik}, \mathbf{I}, \mathbf{I}, \mathbf{I}, \mathbf{I} \},
\end{aligned} \tag{38}$$

where $\{G_{ik}, G_{ifk}\} \in \mathbb{R}^{n_{xi} \times n_{xi}}$, $\{X_{ik}, X_{ifk}\} \in \mathbb{R}^{n_{yi} \times n_{yi}}$, $k \in \mathcal{J}_i$, $i \in \mathcal{N}$.

Here, by performing a congruence transformation to (34) by Γ_i , and take the relation in (37), the following inequality brings out the condition (16),

$$\begin{bmatrix} \hat{\Pi}_{ijk} & \bar{\mathbb{A}}_{ifjk} & \bar{\mathbb{K}}_{ifjk} & \bar{\mathbb{R}}_i \\ \star & \bar{\mathbb{H}}_{if} - \mathbb{G}_{ifk} - \mathbb{G}_{ifk}^T & 0 & 0 \\ \star & \star & \bar{\mathbb{M}}_{if} - \mathbb{X}_{ifk} - \mathbb{X}_{ifk}^T & 0 \\ \star & \star & \star & -\bar{\mathbb{P}}_{is} \end{bmatrix} < 0, \tag{39}$$

where

$$\begin{aligned}
\hat{\Pi}_{ijk} &= \hat{\Theta}_{ijk} + \text{Sym} \{ R_1 \begin{bmatrix} -G_{ik} & \bar{\mathcal{A}}_{ij} G_{ik} & \mathcal{B}_{ij} K_{ik} X_{ik} \end{bmatrix} \}, \\
\bar{\mathbb{R}}_i &= \begin{bmatrix} \bar{R}_1 & \bar{R}_2 & \bar{R}_3 & \bar{R}_4 \end{bmatrix}, \\
\bar{\mathcal{A}}_{ij} &= \mathcal{A}_{ij} + \mathcal{B}_{ij} K_{ik} C_i + \Delta \mathcal{A}_{ij}, \\
\bar{\mathbb{P}}_{is} &= \text{diag} \left\{ \bar{P}_{is} \quad \frac{\bar{H}_{0i}}{N-1} \quad \frac{\bar{M}_{0i}}{N-1} \quad Z_i \right\}, \\
\hat{\Theta}_{ijk} &= \begin{bmatrix} 0 & 0 & 0 \\ \star & \bar{P}_{ij} - G_{ik} - G_{ik}^T & 0 \\ \star & \star & W_i - X_{ik} - X_{ik}^T \end{bmatrix}, \\
\bar{R}_1 &= \begin{bmatrix} G_{ik}^T \\ 0 \\ 0 \end{bmatrix}, \bar{R}_2 = \begin{bmatrix} 0 \\ G_{ik}^T \\ 0 \end{bmatrix}, \\
\bar{R}_3 &= \begin{bmatrix} 0 \\ 0 \\ X_{ik}^T \end{bmatrix}, \bar{R}_4 = \begin{bmatrix} 0 \\ \sigma_i G_{ik}^T C_i^T \\ 0 \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\bar{\mathbb{H}}_{if} &= \text{diag}\{\underbrace{\bar{H}_{i1} \cdots \bar{H}_{if,f \neq i} \cdots \bar{H}_{iN}}_{N-1}\}, \\
\mathbb{G}_{ifk} &= \text{diag}\{\underbrace{G_{i1k} \cdots G_{ifk,f \neq i} \cdots G_{iNk}}_{N-1}\}, \\
\bar{\mathbb{M}}_{if} &= \text{diag}\{\underbrace{\bar{M}_{i1} \cdots \bar{M}_{if,f \neq i} \cdots \bar{M}_{iN}}_{N-1}\}, \\
\mathbb{X}_{ifk} &= \text{diag}\{\underbrace{X_{i1k} \cdots X_{ifk,f \neq i} \cdots X_{iNk}}_{N-1}\}, \\
\bar{\mathbb{A}}_{ifjk} &= \underbrace{[\bar{A}_{i1j} G_{i1k} \cdots \bar{A}_{ifj,f \neq i} G_{ifk,f \neq i} \cdots \bar{A}_{iNj} G_{iNk}]}_{N-1}, \\
\bar{\mathcal{A}}_{ifjk} &= R_1 \mathcal{A}_{ifj} + R_1 \mathcal{B}_{ij} K_{ifk} C_f, \\
\mathbb{K}_{ifk} &= \underbrace{[\bar{\mathcal{B}}_{i1k} \cdots \bar{\mathcal{B}}_{ifk,f \neq i} \cdots \bar{\mathcal{B}}_{iNk}]}_{N-1}, \\
\bar{\mathcal{B}}_{ifk} &= R_1 \mathcal{B}_{ij} K_{ifk,f \neq i} X_{ifk,f \neq i}.
\end{aligned} \tag{40}$$

To derive an LMI-based result, we further specify G_{ik} as

$$\begin{aligned}
G_{ik} &= T_i \begin{bmatrix} X_{ik} & 0 \\ G_{ik(1)} & G_{ik(2)} \end{bmatrix}, \\
G_{ifk} &= T_f \begin{bmatrix} X_{ifk} & 0 \\ G_{ifk(1)} & G_{ifk(2)} \end{bmatrix}, i \in \mathcal{N}
\end{aligned} \tag{41}$$

where $\{G_{ik(1)}, G_{ifk(1)}\} \in \mathbb{R}^{(n_{xi}-n_{yi}) \times n_{yi}}$, $\{G_{ik(2)}, G_{ifk(2)}\} \in \mathbb{R}^{(n_{xi}-n_{yi}) \times (n_{xi}-n_{yi})}$, $\{X_{ik}, X_{ifk}\} \in \mathbb{R}^{n_{yi} \times n_{yi}}$, $i \in \mathcal{N}$.

It follows from (30) and (41) that

$$\begin{aligned}
K_{ik} C_i G_{ik} &= K_{ik} \begin{bmatrix} \mathbf{I} & 0 \end{bmatrix} \begin{bmatrix} X_{ik} & 0 \\ G_{ik(1)} & G_{ik(2)} \end{bmatrix} \\
&= \begin{bmatrix} K_{ik} X_{ik} & 0 \end{bmatrix} \\
&= \begin{bmatrix} \bar{K}_{ik} & 0 \end{bmatrix} \\
&= \mathcal{K}_{ik}, k \in \mathcal{L}_i,
\end{aligned} \tag{42}$$

and

$$\begin{aligned}
K_{ifk} C_f G_{ifk} &= K_{ifk} \begin{bmatrix} \mathbf{I} & 0 \end{bmatrix} \begin{bmatrix} X_{ifk} & 0 \\ G_{ifk(1)} & G_{ifk(2)} \end{bmatrix} \\
&= \begin{bmatrix} K_{ifk} X_{ifk} & 0 \end{bmatrix} \\
&= \begin{bmatrix} \bar{K}_{ifk} & 0 \end{bmatrix} \\
&= \mathcal{K}_{ifk}, k \in \mathcal{L}_i, i \in \mathcal{N}.
\end{aligned} \tag{43}$$

By substituting (42) and (43) into (39), and extracting the membership functions, then the inequality in (39) can be reformulated as

$$\sum_{m \in \mathcal{I}_i(j)} \mu_{im} \Sigma_{ijksm} < 0, (j, s, k) \in \Omega_i, \tag{44}$$

where $m \in \mathcal{I}_i(j)$, $j \in \mathcal{J}_i$, $i \in \mathcal{N}$, and

$$\begin{aligned}
\Sigma_{ijkm} &= \begin{bmatrix} \tilde{\Pi}_{ijksm} & \bar{\mathbb{A}}_{ijkm} & \bar{\mathbb{K}}_{ikm} & \bar{\mathbb{R}}_i \\ \star & \bar{\mathbb{H}}_{if} & 0 & 0 \\ \star & \star & \bar{\mathbb{M}}_{if} & 0 \\ \star & \star & \star & -\bar{\mathbb{P}}_{is} \end{bmatrix}, \\
\tilde{\Pi}_{ijksm} &= \tilde{\Theta}_{ijk} + \text{Sym}\{\mathcal{W}_{ikm} \Delta_{im}(t) \mathcal{E}_{im}\}, \\
\bar{\mathbb{R}}_i &= \begin{bmatrix} \bar{R}_1 & \bar{R}_2 & \bar{R}_3 & \bar{R}_4 \end{bmatrix},
\end{aligned}$$

$$\bar{\mathbb{P}}_{is} = \text{diag} \left\{ \bar{P}_{is} \quad \frac{H_{0i}}{N-1} \quad \frac{M_{0i}}{N-1} \quad Z_i \right\}, \tag{45}$$

and all notations are defined in (32).

By introducing the positive scalars ϵ_i , and using Lemma A2 in Appendix, the inequality in (31) is obtained, which completes the proof.

Remark 3.3. It is noted that, when considering a fuzzy matrix C_{iv} , $v \in \mathcal{J}_i$, the corresponding LMI-based design can be obtained by specifying G_{ikv} and G_{ifkv} as

$$\begin{aligned}
G_{ikv} &= T_{iv} \begin{bmatrix} X_{ik} & 0 \\ G_{ikv(1)} & G_{ikv(2)} \end{bmatrix}, \\
G_{ifkv} &= T_{fv} \begin{bmatrix} X_{ifk} & 0 \\ G_{ifkv(1)} & G_{ifkv(2)} \end{bmatrix}, i \in \mathcal{N}
\end{aligned}$$

where $\{G_{ikv(1)}, G_{ifkv(1)}\} \in \mathbb{R}^{(n_{xi}-n_{yi}) \times n_{yi}}$, $\{G_{ikv(2)}, G_{ifkv(2)}\} \in \mathbb{R}^{(n_{xi}-n_{yi}) \times (n_{xi}-n_{yi})}$, $\{X_{ik}, X_{ifk}\} \in \mathbb{R}^{n_{yi} \times n_{yi}}$, $i \in \mathcal{N}$.

Remark 3.4. Assumed that the input matrix B_{im} , $i \in \mathcal{N}$ is of full column rank, there exist nonsingular transformation matrix $T_{im} \in \mathbb{R}^{n_{xi} \times n_{xi}}$ satisfying [58]

$$T_{im} B_{im} = \begin{bmatrix} \mathbf{I}_{n_{ui}} \\ 0_{(n_{xi}-n_{ui}) \times n_{ui}} \end{bmatrix}, i \in \mathcal{N}.$$

Similarly, we can specify

$$\mathcal{G}_i = \begin{bmatrix} G_{ikm} \\ 0 \\ 0 \end{bmatrix}, G_{ikm} = \begin{bmatrix} G_{ik(1)} & G_{ik(2)} \\ 0 & G_{ik(3)} \end{bmatrix} T_{im}.$$

Based on the above relations, it is easy to derive an LMI-based result similar to Theorem 3.1. Due to the consideration of page length, the corresponding results are omitted here.

Remark 3.5. It is also worth pointing out that the descriptor approach developed in [59] can be used to derive the LMI-based result on the SOF controller design. Based on the idea, the closed-loop fuzzy control system consisting of (6) and (12) can be re-expressed as

$$\begin{aligned}
E_i \bar{x}_i(t+1) &= \bar{\mathcal{A}}_{ijk} \bar{x}_i(t) + \bar{\mathcal{W}}_{ik} w_i(t) \\
&+ \sum_{\substack{f=1 \\ f \neq i}}^N \bar{\mathcal{A}}_{ifjk} \bar{x}_f(t) + \sum_{\substack{f=1 \\ f \neq i}}^N \bar{\mathcal{W}}_{ifk} \bar{w}_f(t),
\end{aligned}$$

where

$$\begin{aligned}
\bar{x}_i(t) &= \begin{bmatrix} x_i^T(t) & u_i^T(t) \end{bmatrix}^T, E_i = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & 0 \end{bmatrix}, \\
\bar{\mathcal{A}}_{ijk} &= \begin{bmatrix} \mathcal{A}_{ij} + \Delta \mathcal{A}_{ij} & \mathcal{B}_{ij} \\ K_{ik} C_i & -\mathbf{I} \end{bmatrix}, \bar{\mathcal{W}}_{ik} = \begin{bmatrix} 0 \\ K_{ik} \end{bmatrix}, \\
\bar{\mathcal{A}}_{ifjk} &= \begin{bmatrix} \mathcal{A}_{ifj} & 0 \\ K_{ifk} C_f & 0 \end{bmatrix}, \bar{\mathcal{W}}_{ifk} = \begin{bmatrix} 0 \\ K_{ifk} \end{bmatrix}.
\end{aligned}$$

It is clear to see that $\mathcal{A}_{ifj}$ and $K_{ifk} C_f$ are separated into different blocks. Specifically, the supplemental feedbacks, containing the interconnected information provided in the proposed distributed controller (12), do not work in that case. Thus, the descriptor approach developed in [59] may fail to solve the proposed distributed event-triggering output feedback control problem in this paper.

As the asynchronous information between $\zeta_i(t)$ and $\hat{\zeta}_i(t)$ are unavailable, the corresponding results on designing the distributed event-triggering linear controller are summarized as below:

Corollary 3.1. Considering the large-scale fuzzy system in (4), and a distributed event-triggering piecewise controller in (12), the closed-loop fuzzy control system with the Assumption 3.2 is asymptotically stable, if there exist the positive-definite symmetric matrices $\{\bar{P}_i, \bar{H}_{fi}\} \in \mathbb{R}^{n_{xi} \times n_{xi}}$, $\{\bar{M}_{fi}, W_i\} \in \mathbb{R}^{n_{yi} \times n_{yi}}$, $\bar{H}_{0i} \leq \bar{H}_{fi}$, $\bar{M}_{0i} \leq \bar{M}_{fi}$, matrices $\{\bar{K}_i, \bar{K}_{if}\} \in \mathbb{R}^{n_{ui} \times n_{yi}}$, $\{G_i, G_{if}\} \in \mathbb{R}^{n_{xi} \times n_{xi}}$, $\{X_i, X_{if}\} \in \mathbb{R}^{n_{yi} \times n_{yi}}$, $i \in \mathcal{N}$, and the positive scalars $\{\epsilon_i, \sigma_i\}$, such that for all $(l, s) \in \mathcal{L}_i$, $i \in \mathcal{N}$ the following LMIs hold:

$$\begin{bmatrix} \tilde{\Theta}_{il} & \bar{\mathbb{A}}_{il} & \bar{\mathbb{K}}_{ifl} & \bar{\mathbb{R}}_i & \epsilon_i \mathcal{W}_{il} & \mathcal{E}_{il}^T \\ \star & \bar{\mathbb{H}}_{if} & 0 & 0 & 0 & 0 \\ \star & \star & \bar{\mathbb{M}}_{if} & 0 & 0 & 0 \\ \star & \star & \star & -\bar{\mathbb{P}}_{is} & 0 & 0 \\ \star & \star & \star & \star & -\epsilon_i \mathbf{I} & 0 \\ \star & \star & \star & \star & \star & -\epsilon_i \mathbf{I} \end{bmatrix} < 0, \quad (46)$$

and

$$\begin{aligned} \tilde{\Theta}_{il} &= \begin{bmatrix} 0 & 0 & 0 \\ \star & \bar{P}_i - G_i - G_i^T & 0 \\ \star & \star & Z_i - X_i - X_i^T \end{bmatrix} \\ &+ \text{Sym} \{ R_1 [-G_i \quad A_{il}G_i + B_{il}\mathcal{K}_i \quad B_{il}\bar{K}_i] \}, \\ \bar{\mathbb{H}}_{if} &= \bar{H}_{if} - G_i - G_i^T, \bar{\mathbb{M}}_{if} = \bar{M}_{if} - X_{if} - X_{if}^T, \\ \bar{\mathbb{R}}_i &= [\bar{R}_1 \quad \bar{R}_2 \quad \bar{R}_3 \quad \bar{R}_4], \mathcal{K}_i = [\bar{K}_i \quad 0] \\ \bar{\mathbb{P}}_{is} &= \text{diag} \left\{ \bar{P}_{is} \quad \frac{\bar{H}_{0i}}{N-1} \quad \frac{\bar{M}_{0i}}{N-1} \quad Z_i \right\}, \\ \mathcal{W}_{il} &= \begin{bmatrix} W_{il} \\ 0 \\ 0 \end{bmatrix}, \mathcal{E}_{il} = [0 \quad E_{il}G_i \quad 0], \\ \bar{R}_1 &= \begin{bmatrix} G_i^T \\ 0 \\ 0 \end{bmatrix}, \bar{R}_2 = \begin{bmatrix} 0 \\ G_i^T \\ 0 \end{bmatrix}, \\ \bar{R}_3 &= \begin{bmatrix} 0 \\ 0 \\ X_i^T \end{bmatrix}, \bar{R}_4 = \begin{bmatrix} 0 \\ \sigma_i G_i^T C_i^T \\ 0 \end{bmatrix}, \\ \bar{\mathbb{H}}_{if} &= \text{diag} \{ \underbrace{\bar{H}_{i1} \cdots \bar{H}_{if, f \neq i} \cdots \bar{H}_{iN}}_{N-1} \}, \\ \mathbb{G}_{if} &= \text{diag} \{ \underbrace{G_{i1} \cdots G_{if, f \neq i} \cdots G_{iN}}_{N-1} \}, \\ \bar{\mathbb{M}}_{if} &= \text{diag} \{ \underbrace{\bar{M}_{i1} \cdots \bar{M}_{if, f \neq i} \cdots \bar{M}_{iN}}_{N-1} \}, \\ \mathbb{X}_{if} &= \text{diag} \{ \underbrace{X_{i1} \cdots X_{if, f \neq i} \cdots X_{iN}}_{N-1} \}, \\ \bar{\mathbb{A}}_{ifl} &= [\underbrace{\bar{A}_{i1l} \cdots \bar{A}_{ifl, f \neq i} \cdots \bar{A}_{iNl}}_{N-1}], \\ \bar{\mathbb{A}}_{ifl} &= R_1 (A_{ifl}G_{if} + B_{il}\mathcal{K}_{if}), \mathcal{K}_{if} = [\bar{K}_{if} \quad 0] \\ \bar{\mathbb{K}}_{ifl} &= [\underbrace{R_1 B_{il} \bar{K}_{i1} \cdots R_1 B_{il} \bar{K}_{if, f \neq i} \cdots R_1 B_{il} \bar{K}_{iN}}_{N-1}]. \quad (47) \end{aligned}$$

Furthermore, the distributed event-triggering linear con-

troller gains can be calculated by

$$K_i = \bar{K}_i X_i^{-1}, K_{if} = \bar{K}_{if} X_{if}^{-1}, f \neq i, \{i, f\} \in \mathcal{N}. \quad (48)$$

Remark 3.5. The premise variables no longer need to transmit via networks as designing an event-triggering linear controller in (48). Compared with the event-triggering piecewise controller in (12), the requirements of hardware and software can be reduced when designing the linear one, while it raises the design conservatism.

IV. NUMERICAL EXAMPLES

In this section, the effectiveness and practicability of proposed control design method are validated by two numerical examples.

Example 4.1. Consider a discrete-time large-scale system in the form of (1) with three T-S fuzzy subsystems:

Plant Rule $\mathcal{R}_i^l$: IF $x_{i1}(t)$ is $\mathcal{F}_{i1}^l$, THEN

$$\begin{cases} x_i(t+1) = A_{il}x_i(t) + B_{il}u_i(t) + \sum_{\substack{f=1 \\ f \neq i}}^N A_{ifl}x_f(t), \\ y_i(t) = C_i x_i(t), \end{cases}$$

where $l \in \{1, 2, \dots, 10\}$, $i \in \{1, 2, 3\}$, and all the system parameters are listed in Appendix B.

Fig. 2. Membership functions

The normalized membership functions are shown in Fig. 2. In accordance with the partition policy in the premise variable spaces, there exist seven subspaces for each fuzzy subsystem,

$$\begin{aligned} \mathcal{S}_{i1} &= \{x_{i1}(t) | r_{i1} \leq x_{i1}(t) \leq r_{i2}\}, \\ \mathcal{S}_{i2} &= \{x_{i1}(t) | r_{i2} < x_{i1}(t) \leq r_{i3}\}, \\ \mathcal{S}_{i3} &= \{x_{i1}(t) | r_{i3} < x_{i1}(t) \leq r_{i4}\}, \\ \mathcal{S}_{i4} &= \{x_{i1}(t) | r_{i4} < x_{i1}(t) \leq r_{i5}\}, \\ \mathcal{S}_{i5} &= \{x_{i1}(t) | r_{i5} < x_{i1}(t) \leq r_{i6}\}, \\ \mathcal{S}_{i6} &= \{x_{i1}(t) | r_{i6} < x_{i1}(t) \leq r_{i7}\}, \\ \mathcal{S}_{i7} &= \{x_{i1}(t) | r_{i7} < x_{i1}(t) \leq r_{i8}\}. \end{aligned}$$

It is straightforward to see from Fig. 2 that $\{\mathcal{S}_{i1}, \mathcal{S}_{i3}, \mathcal{S}_{i5}, \mathcal{S}_{i7}\}$ are crisp regions, and $\{\mathcal{S}_{i2}, \mathcal{S}_{i4}, \mathcal{S}_{i6}\}$ are fuzzy regions, and the

index set is $\mathcal{J}_i = \{1, 2, \dots, 10\}$, $\mathcal{I}_i(1) = \{i1\}$, $\mathcal{I}_i(2) = \{i2, i3\}$, $\mathcal{I}_i(3) = \{i4\}$, $\mathcal{I}_i(4) = \{i5, i6\}$, $\mathcal{I}_i(5) = \{i7\}$, $\mathcal{I}_i(6) = \{i8, i9\}$, $\mathcal{I}_i(7) = \{i10\}$.

It should be noted that the open-loop interconnected system is unstable. The objective here is to design a distributed event-triggering output-feedback controller in the form of (12) such that the resulting closed-loop system is asymptotically stable while the data communications are transmitted through the network as little as possible. Here, we consider that the closed-loop fuzzy control system satisfies the Assumption 3.1, and assumed that $\sigma_i = 0.03$, no feasible solutions can be obtained via the common Lyapunov function method proposed in [46], and the decentralized controller proposed in [50]. Nevertheless, by using Theorem 3.1, the controller gains can be calculated as shown in Appendix C.

Example 4.2. Consider a DC microgrid with direct current/direct current (DC/DC) converter. The model representation is given by [60]

$$\begin{cases} \dot{v}_{PV} = \frac{1}{C_{PV}} (\phi_{PV} - \phi_L u), \\ \dot{\phi}_L = \frac{1}{L} (R_0 (\phi_0 - \phi_L) - R_L \phi_L - v_0) \\ \quad + \frac{1}{L} (V_D + v_{PV} - R_M \phi_L) u - \frac{V_D}{L}, \\ \dot{v}_0 = \frac{1}{C_0} (\phi_L - \phi_0), \end{cases}$$

where in the solar PV, V_D denotes the forward voltage of the power diode; R_0, R_L , and R_M denote the resistance on the capacitance C_0 , the inductance L , and the MOSFET, respectively; v_{PV}, ϕ_L , and v_0 denote the PV array voltage, the current at the inductance L , and the voltage at the capacitance C_0 , respectively; ϕ_0 is the measurable load current.

Fig. 3. A DC microgrid in Example 4.2

Here, Fig. 3 shows a DC microgrid with three solar PVs,

and its dynamic model is described as

$$\begin{cases} \dot{v}_{PV(i)} = \frac{1}{C_{PV(i)}} \phi_{PV(i)} v_{PV(i)} - \frac{1}{C_{PV(i)}} \phi_L(i) u(i), \\ \dot{\phi}_L(i) = \left(-\frac{1}{L(i)} R_0(i) - \frac{1}{L(i)} R_L(i) - \frac{V_D(i)}{L(i) \phi_L(i)} \right) \phi_L(i) \\ \quad + \left(\frac{R_0(i)}{L(i) (R_{line(i)} + R_{load})} - \frac{1}{L(i)} \right) v_0(i) \\ \quad - \sum_{j=1, j \neq i}^N \frac{R_0(i) R_{load}}{L(i) (R_{line(i)} + R_{load})} \phi_0(j) \\ \quad + \frac{1}{L(i)} (V_D(i) + v_{PV(i)} - R_M(i) \phi_L(i)) u(i), \\ \dot{v}_0(i) = \frac{1}{C_0(i)} \phi_L(i) - \frac{1}{C_0(i) (R_{line(i)} + R_{load})} v_0(i) \\ \quad + \sum_{j=1, j \neq i}^N \frac{R_{load}}{C_0(i) (R_{line(i)} + R_{load})} \phi_0(j), i = \{1, 2, 3\} \end{cases}$$

In the simulation, the values of the plant parameters are $V_{D(1)} = 12$ V, $C_{PV(1)} = 0.00921$ F, $C_{0(1)} = 0.00962$ F, $L_{(1)} = 0.01116$ H, $R_{L(1)} = 1.7$ Ω , $R_{0(1)} = 3.2$ Ω , $R_{M(1)} = 0.8$ Ω ; $V_{D(2)} = 12$ V, $C_{PV(2)} = 0.00785$ F, $C_{0(2)} = 0.00895$ F, $L_{(2)} = 0.01563$ H, $R_{L(2)} = 1.4$ Ω , $R_{0(2)} = 3.8$ Ω , $R_{M(2)} = 0.7$ Ω ; $V_{D(3)} = 12$ V, $C_{PV(3)} = 0.00813$ F, $C_{0(3)} = 0.00898$ F, $L_{(3)} = 0.01982$ H, $R_{L(3)} = 1.3$ Ω , $R_{0(3)} = 2.7$ Ω , $R_{M(3)} = 0.7$ Ω . For simplicity, assume that $v_{PV(j)} = 0.5 R_{load} \phi_0(j)$, $v_{PV(1)} = 18.3$ V, $\phi_{L(1)} = 0.65$ A, $v_{PV(2)} = 21.7$ V, $\phi_{L(2)} = 0.72$ A, $v_{PV(3)} = 16.5$ V, $\phi_{L(3)} = 0.78$ A, and by discretizing the interconnected T-S fuzzy system at the sampling period $T = 0.1$ s, the corresponding discrete-time T-S fuzzy interconnected system is expressed as:

Plant Rule $\mathcal{R}_i^l$: IF $\phi_{PV(i)}$ is (0.38 A, 0.42 A, 0.46 A, 0.50 A, 0.54 A, 0.58 A, 0.62 A, 0.66 A), **THEN**

$$\begin{cases} x_i(t+1) = A_{il} x_i(t) + B_{il} u_i(t) + \sum_{j=1, j \neq i}^3 \bar{F}_{ij} x_j(t), \\ y_i(t) = C_i x_i(t), \end{cases}$$

where $l = \{1, 2, \dots, 8\}$, $i = \{1, 2, 3\}$.

Fig. 4. Membership functions

The normalized membership functions are shown in Fig. 4. In terms of the partition policy in the premise variable space, there exist five subspaces for each fuzzy subsystem,

$$\mathcal{S}_{i1} = \{\phi_{PV(i)} | r_{i1} \leq x_{i1}(t) \leq r_{i2}\}$$

$$\begin{aligned}
\mathcal{S}_{i2} &= \{\phi_{PV(i)} | r_{i2} < x_{i1}(t) \leq r_{i3}\} \\
\mathcal{S}_{i3} &= \{\phi_{PV(i)} | r_{i3} < x_{i1}(t) \leq r_{i4}\} \\
\mathcal{S}_{i4} &= \{\phi_{PV(i)} | r_{i4} < x_{i1}(t) \leq r_{i5}\} \\
\mathcal{S}_{i5} &= \{\phi_{PV(i)} | r_{i5} < x_{i1}(t) \leq r_{i6}\}.
\end{aligned}$$

It is clear to see from Fig. 4 that $\{\mathcal{S}_{i2}, \mathcal{S}_{i4}\}$ are crisp regions, and $\{\mathcal{S}_{i1}, \mathcal{S}_{i3}, \mathcal{S}_{i5}\}$ are fuzzy regions. Then, for the region index term j in the fuzzy system, the index set is $\mathcal{J}_i = \{1, 2, \dots, 8\}$, $\mathcal{I}_i(1) = \{i1, i2\}$, $\mathcal{I}_i(2) = i3$, $\mathcal{I}_i(3) = \{i4, i5\}$, $\mathcal{I}_i(4) = i6$, $\mathcal{I}_i(5) = \{i7, i8\}$. Here, assume that the closed-loop fuzzy control system satisfies the Assumption 3.1, and $\sigma_i = 0.25$, by applying Theorem 3.1 the controller gains can be calculated as listed in Appendix D.

Given the initial conditions $x_1(0) = [5.9, 0, -2.3]^T$, $x_2(0) = [5.3, -2.5, 0]^T$, and $x_3(0) = [5.6, 0, 4.6]^T$. As shown in Figs. 5-7, the closed-loop system with the distributed event-triggering piecewise controllers is asymptotically stable, and the event-triggering condition is shown in Fig. 8.

Fig. 5. State responses for subsystem 1

Fig. 6. State responses for subsystem 2

Fig. 7. State responses for subsystem 3

Fig. 8. Event-triggering condition

V. CONCLUSIONS

This paper investigated the distributed event-triggering control problem for a class of discrete-time large-scale fuzzy systems. A piecewise Lyapunov functional approach was proposed, together with some matrix inequality convexification techniques, the sufficient conditions for solving the distributed event-triggering control problem were derived in the form of LMIs. It has been shown that the closed-loop fuzzy control system is asymptotically stable while reducing the communication information via networks. Finally, a class of practical large-scale nonlinear system in the area of power systems has been employed to demonstrate the effectiveness of the proposed control approaches. One interesting future research topic is the extension of the proposed methods to adaptive policy learning algorithm as shown in [62] for the large-scale fuzzy systems with event-triggering control.

APPENDIX A

Lemma A1. [63] Given positive semidefinite symmetric matrix $W \in \mathbb{R}^{n \times n}$, two positive integers n_2 and n_1 satisfying

$n_2 \geq n_1 \geq 1$, the following inequality holds

$$\left(\sum_{k=n_1}^{n_2} x(k) \right)^T W \left(\sum_{k=n_1}^{n_2} x(k) \right) \leq (n_2 - n_1 + 1) \sum_{k=n_1}^{n_2} x^T(k) W x(k).$$

Lemma A2. [64] Given matrices $M = M^T, S, N$, and $\Delta(t)$; then the following inequality holds,

$$M + \text{Sym} \{S\Delta(t)N\} < 0$$

for all $\Delta^T(t)\Delta(t) \leq \mathbf{I}$, if and only if for the scalar $\varepsilon > 0$ such that

$$M + \varepsilon SS^T + \varepsilon^{-1} N^T N < 0$$

for the third subsystem.

APPENDIX B

$$\begin{bmatrix} A_{11} & B_{11} & A_{12} & B_{12} \\ A_{13} & B_{13} & A_{14} & B_{14} \\ A_{15} & B_{15} & A_{16} & B_{16} \\ A_{17} & B_{17} & A_{18} & B_{18} \\ A_{19} & B_{19} & A_{110} & B_{110} \end{bmatrix} = \begin{bmatrix} 1.02 & 0.45 & 2.9 & 1.02 & 0.66 & 3.2 \\ 0 & 0.34 & 0.8 & 0 & 0.52 & 1.1 \\ 1.02 & 0.38 & 2.8 & 1.02 & 0.72 & 2.4 \\ 0 & 0.63 & 0.9 & 0 & 0.59 & 1.3 \\ 1.02 & 0.65 & 3.3 & 1.02 & 0.59 & 2.7 \\ 0 & 0.57 & 1.4 & 0 & 0.97 & 0.9 \\ 1.02 & 0.70 & 3.7 & 1.02 & 0.49 & 2.9 \\ 0 & 0.75 & 1.5 & 0 & 0.44 & 0.9 \\ 1.02 & 0.32 & 2.7 & 1.02 & 0.56 & 2.5 \\ 0 & 0.44 & 1.1 & 0 & 0.44 & 0.7 \end{bmatrix},$$

$$A_{121} = A_{122} = A_{124} = A_{134} = A_{135} = A_{136}$$

$$= A_{137} = A_{138} = A_{139} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix},$$

$$A_{123} = A_{125} = A_{126} = A_{127}$$

$$= A_{129} = A_{132} = A_{133} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_{128} = A_{1210} = A_{131} = A_{1310} = \begin{bmatrix} 0.01 & 0.01 \\ 0 & 0 \end{bmatrix},$$

$$W_{11} = W_{12} = W_{13} = W_{14} = W_{15} = W_{16}$$

$$= W_{17} = W_{18} = W_{19} = W_{110} = \begin{bmatrix} 0.01 & 0 \end{bmatrix},$$

$$E_{11} = E_{14} = E_{18} = \begin{bmatrix} 0.002 \\ 0 \end{bmatrix},$$

$$E_{12} = E_{13} = E_{15} = E_{16} = E_{19} = \begin{bmatrix} 0.001 \\ 0 \end{bmatrix},$$

$$E_{17} = E_{110} = \begin{bmatrix} 0.003 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

for the first subsystem, and

$$\begin{bmatrix} A_{21} & B_{21} & A_{22} & B_{22} \\ A_{23} & B_{23} & A_{24} & B_{24} \\ A_{25} & B_{25} & A_{26} & B_{26} \\ A_{27} & B_{27} & A_{28} & B_{28} \\ A_{29} & B_{29} & A_{210} & B_{210} \end{bmatrix} = \begin{bmatrix} 1.32 & 0.55 & 2.6 & 1.21 & 0.46 & 3.5 \\ 0 & 0.47 & 1.1 & 0 & 0.49 & 1.4 \\ 1.21 & 0.65 & 3.3 & 1.18 & 0.62 & 2.9 \\ 0 & 0.44 & 1.8 & 0 & 0.44 & 0.7 \\ 1.27 & 0.61 & 2.8 & 1.02 & 0.45 & 3.2 \\ 0 & 0.44 & 0.9 & 0 & 0.44 & 1.1 \\ 1.02 & 0.45 & 2.9 & 1.02 & 0.45 & 3.2 \\ 0 & 0.44 & 0.9 & 0 & 0.44 & 1.0 \\ 1.02 & 0.45 & 2.8 & 1.02 & 0.45 & 2.9 \\ 0 & 0.44 & 0.9 & 0 & 0.44 & 1.1 \end{bmatrix},$$

$$A_{211} = A_{212} = A_{213} = A_{216} = A_{217} = A_{218}$$

$$= A_{219} = A_{2110} = A_{232} = A_{233} = A_{234}$$

$$= A_{235} = A_{233} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_{214} = A_{215} = A_{231} = \begin{bmatrix} 0.01 & 0.01 \\ 0 & 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$W_{21} = W_{22} = W_{23} = W_{24} = W_{25} = W_{26} = W_{27}$$

$$= W_{28} = W_{29} = W_{210} = \begin{bmatrix} 0.01 & 0 \end{bmatrix},$$

$$E_{21} = E_{23} = E_{24} = E_{26} = E_{27} = E_{29} = \begin{bmatrix} 0.001 \\ 0 \end{bmatrix},$$

$$E_{22} = E_{210} = \begin{bmatrix} 0.003 \\ 0 \end{bmatrix},$$

$$E_{28} = \begin{bmatrix} 0.004 \\ 0 \end{bmatrix}, E_{25} = \begin{bmatrix} 0.002 \\ 0 \end{bmatrix}$$

for the second subsystem, and

$$\begin{bmatrix} A_{31} & B_{31} & A_{32} & B_{32} \\ A_{33} & B_{33} & A_{34} & B_{34} \\ A_{35} & B_{35} & A_{36} & B_{36} \\ A_{37} & B_{37} & A_{38} & B_{38} \\ A_{39} & B_{39} & A_{310} & B_{310} \end{bmatrix} = \begin{bmatrix} 1.18 & 0.48 & 3.5 & 1.17 & 0.64 & 2.8 \\ 0 & 0.44 & 1.5 & 0 & 0.44 & 0.9 \\ 1.23 & 0.65 & 2.5 & 1.14 & 0.52 & 2.8 \\ 0 & 0.44 & 0.7 & 0 & 0.44 & 0.8 \\ 1.24 & 0.43 & 3.4 & 1.21 & 0.38 & 3.2 \\ 0 & 0.44 & 1.5 & 0 & 0.44 & 1.4 \\ 1.25 & 0.53 & 3.3 & 1.16 & 0.54 & 3.1 \\ 0 & 0.44 & 1.7 & 0 & 0.44 & 0.8 \\ 1.26 & 0.58 & 3.1 & 1.26 & 0.58 & 2.6 \\ 0 & 0.44 & 0.8 & 0 & 0.44 & 0.8 \end{bmatrix},$$

$$A_{311} = A_{323} = A_{327} = A_{329} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix},$$

$$A_{313} = A_{315} = A_{316} = A_{317} = A_{318} = A_{319}$$

$$= A_{3110} = A_{321} = A_{324} = A_{326} = A_{328} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_{312} = A_{314} = A_{322} = A_{325} = A_{3210} = \begin{bmatrix} 0.01 & 0.01 \\ 0 & 0 \end{bmatrix},$$

$$W_{31} = W_{32} = W_{33} = W_{34} = W_{35} = W_{36} \\ = W_{37} = W_{38} = W_{39} = W_{310} = \begin{bmatrix} 0.01 & 0 \end{bmatrix},$$

$$E_{31} = E_{37} = E_{39} = \begin{bmatrix} 0.002 \\ 0 \end{bmatrix},$$

$$E_{32} = E_{33} = E_{35} = E_{36} = E_{38} = \begin{bmatrix} 0.001 \\ 0 \end{bmatrix},$$

$$E_{34} = E_{310} = \begin{bmatrix} 0.003 \\ 0 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

for the third subsystem.

APPENDIX C

$$\begin{bmatrix} K_{11} & K_{121} & K_{131} \\ K_{12} & K_{122} & K_{132} \\ K_{13} & K_{123} & K_{133} \\ K_{14} & K_{124} & K_{134} \\ K_{15} & K_{125} & K_{135} \\ K_{16} & K_{126} & K_{136} \\ K_{17} & K_{127} & K_{137} \end{bmatrix} \\ = \begin{bmatrix} -0.1479 & -0.0016 & -0.0011 \\ -0.0933 & -0.0019 & -0.0007 \\ -0.0706 & -0.003 & -0.0003 \\ -0.0707 & -0.003 & -0.0003 \\ -0.0701 & -0.003 & -0.0003 \\ -0.0853 & -0.007 & -0.0006 \\ -0.1490 & -0.0013 & -0.0011 \end{bmatrix}$$

for the first subsystem, and

$$\begin{bmatrix} K_{21} & K_{211} & K_{231} \\ K_{22} & K_{212} & K_{232} \\ K_{23} & K_{213} & K_{233} \\ K_{24} & K_{214} & K_{234} \\ K_{25} & K_{215} & K_{235} \\ K_{26} & K_{216} & K_{236} \\ K_{27} & K_{217} & K_{237} \end{bmatrix} \\ = \begin{bmatrix} -0.2576 & -0.0011 & -0.0016 \\ -0.2573 & -0.0011 & -0.0015 \\ -0.2407 & -0.0014 & -0.0010 \\ -0.2659 & -0.0019 & -0.0012 \\ -0.2596 & -0.0017 & -0.0012 \\ -0.2361 & -0.0011 & -0.0011 \\ -0.2371 & -0.0012 & -0.0012 \end{bmatrix}$$

for the second subsystem, and

$$\begin{bmatrix} K_{31} & K_{311} & K_{321} \\ K_{32} & K_{312} & K_{322} \\ K_{33} & K_{313} & K_{323} \\ K_{34} & K_{314} & K_{324} \\ K_{35} & K_{315} & K_{325} \\ K_{36} & K_{316} & K_{326} \\ K_{37} & K_{317} & K_{327} \end{bmatrix}$$

$$\begin{bmatrix} -0.2786 & -0.0016 & -0.0013 \\ -0.2785 & -0.0017 & -0.0013 \\ -0.2879 & -0.0014 & -0.0015 \\ -0.2613 & -0.0012 & -0.0013 \\ -0.2678 & -0.0011 & -0.0013 \\ -0.2792 & -0.0011 & -0.0017 \\ -0.3067 & -0.0012 & -0.0016 \end{bmatrix}$$

APPENDIX D

$$\begin{bmatrix} K_{11} & K_{121} & K_{131} \\ K_{12} & K_{122} & K_{132} \\ K_{13} & K_{123} & K_{133} \\ K_{14} & K_{124} & K_{134} \\ K_{15} & K_{125} & K_{135} \end{bmatrix} \\ = \begin{bmatrix} 0.1094 & -5.5231 \times 10^{-4} & -5.5244 \times 10^{-4} \\ 0.1124 & -3.8606 \times 10^{-4} & -3.8615 \times 10^{-4} \\ 0.1160 & -2.4752 \times 10^{-4} & -2.4758 \times 10^{-4} \\ 0.1218 & 2.7316 \times 10^{-4} & 2.7322 \times 10^{-5} \\ 0.1231 & 6.4303 \times 10^{-4} & 6.4318 \times 10^{-5} \end{bmatrix}$$

for the first subsystem, and

$$\begin{bmatrix} K_{21} & K_{211} & K_{231} \\ K_{22} & K_{212} & K_{232} \\ K_{23} & K_{213} & K_{233} \\ K_{24} & K_{214} & K_{234} \\ K_{25} & K_{215} & K_{235} \end{bmatrix} \\ = \begin{bmatrix} 0.0858 & -3.1564 \times 10^{-4} & -4.8591 \times 10^{-4} \\ 0.0880 & -2.2428 \times 10^{-4} & -3.7897 \times 10^{-4} \\ 0.0907 & -2.4475 \times 10^{-4} & -2.9979 \times 10^{-4} \\ 0.0945 & -1.4419 \times 10^{-4} & -1.8010 \times 10^{-4} \\ 0.0959 & -9.4065 \times 10^{-5} & -1.1746 \times 10^{-4} \end{bmatrix}$$

for the second subsystem, and

$$\begin{bmatrix} K_{31} & K_{311} & K_{321} \\ K_{32} & K_{312} & K_{322} \\ K_{33} & K_{313} & K_{323} \\ K_{34} & K_{314} & K_{324} \\ K_{35} & K_{315} & K_{325} \end{bmatrix} \\ = \begin{bmatrix} 0.0951 & -1.5077 \times 10^{-5} & -1.6251 \times 10^{-5} \\ 0.0967 & -1.2703 \times 10^{-5} & -1.3683 \times 10^{-5} \\ 0.0975 & -2.3247 \times 10^{-5} & -2.4984 \times 10^{-6} \\ 0.1066 & -2.7242 \times 10^{-6} & -2.9195 \times 10^{-6} \\ 0.1069 & -2.3271 \times 10^{-6} & -2.5058 \times 10^{-6} \end{bmatrix}$$

for the third subsystem.

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