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A Novel Approach to Reliable Output Feedback Control of Fuzzy-Affine Systems with Time-Delays and Sensor Faults

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Abstract—This paper proposes a novel system-augmentation approach to the delay-dependent reliable piecewise-affine $\mathcal{H}_\infty$ static output feedback (SOF) control for nonlinear systems with time-varying delay and sensor faults in the piecewise-Markovian-Lyapunov-functional-based framework. The nonlinear plant is described by a continuous-time Takagi-Sugeno (T-S) fuzzy-affine model with parametric uncertainties, and the sensor faults are characterized by a Markov process. Specifically, by applying a state-input augmentation technique, the original closed-loop system is firstly reformulated into a descriptor fuzzy-affine system. Based on a new piecewise-Markovian Lyapunov-Krasovskii functional (LKF), combined with a Wirtinger-based integral inequality, improved reciprocally convex inequality and S-procedure, a novel bounded real lemma is then derived for the underlying closed-loop system. Furthermore, by taking advantage of the redundancy of descriptor system formulation, together with a linearisation procedure, the piecewise-affine controller synthesis is carried out. It is shown that the desired piecewise-affine controller gains can be attained by solving a linear matrix inequality (LMI)-based optimization problem. Finally, simulation examples are performed to confirm the effectiveness and less conservatism of the presented approach.

Index Terms—T-S fuzzy-affine systems, Output feedback control, Piecewise-Markovian Lyapunov functional, Time-varying delay.

I. INTRODUCTION

During the past decades, fuzzy logic control (FLC) has attracted extensive interests both in academic research and industrial applications due to their substantial applications in many disciplines, ranging from power systems, telecommunications, mechanical/robotic systems, automobile, industrial/chemical processes, and so on [1]–[3]. Essentially, the FLC algorithm composes of a family of heuristic control rules, where fuzzy sets and fuzzy logic are employed for the description of linguistic terms and the evaluation of the

rules, respectively [4]–[11]. Among various FLC schemes, the approach based on T-S fuzzy models, also called fuzzy-dynamic models, is particularly suitable for the model-based nonlinear control. Basically, this model employs a set of fuzzy rules to characterize a global nonlinear system in terms of a group of local linear models which are smoothly connected by fuzzy membership functions [12], [13], and a great number of theoretical results on stability analysis, model approximation, and controller design have been proposed for T-S fuzzy models during the past years [14]–[18].

Additionally, it is known that time-delays exist in many dynamic systems, and the appearance of time-delays incurs nonminimum phase, performance degradation, even instability. Therefore, in recent years, the studies on analysis and synthesis of fuzzy systems with time-delays have been gradually launched, and the central issue is to reduce the conservatism of the stability analysis criteria of time-delay systems [19]–[21]. Among different methodologies, the direct Lyapunov function approach has been recognized as a powerful tool to deal with time-delay. In fact, this approach mainly relies on the construction of LKFs and the employment of tight techniques for manipulating the time-derivative (or difference) of the LKF. More specifically, a key feature of this approach is firstly to construct LKFs, which contain more useful information on time-delays and aim at the reduction of the inherent conservatism of this approach. Several attempts have been pursued concerning the structure of the functional by extending state-based LKFs [20], [21], discretized Lyapunov functions or discontinuous Lyapunov functions [22], [23]. Then by virtue of some more or less tight techniques to bound the crossing terms of the derivative of the LKF, the tractable stability analysis criteria were established. For example, by employing the free-weighting matrix approach, the authors in [24] addressed the problem of delay-dependent stabilization for T-S fuzzy delayed systems; In [25], by adopting the reciprocally convex inequality, the authors studied the reliable $\mathcal{H}_\infty$ control problem for fuzzy systems with infinite-distributed delay. It has been shown that these choices of LKFs and over-bounding techniques inevitably induce some degree of conservatism. Therefore, the approaches developed to the controller design of time-delay fuzzy systems still have room for further improvement in terms of conservatism reduction. Moreover, it should be noticed that in the continuous-time framework, most of the existing works on the analysis and design of fuzzy systems were developed based on a common quadratic Lyapunov function (CQLF), i.e.,

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it is assumed that there exists a common Lyapunov matrix for all subsystems. However, the method based on CQLF has been shown to be conservative since many fuzzy systems are stable but do not admit the existence of such CQLF. Comparatively speaking, piecewise quadratic Lyapunov functions (PQLFs) are recognized to be a much richer class of Lyapunov function candidates with less conservatism than CQLF candidates. It is thus of great significance, by a much conservatism reduction points of view, to further study the PQLFs-based analysis and design of fuzzy-affine systems with time-varying delay, which is the partial motivation of our research.

On another active research frontier, reliable control strategies have been receiving considerable attention due to the growing demands on system reliability. The aim of reliable control is to synthesize a desired controller such that the closed-loop system is stable with satisfactory performance, not only when all the control components are active, but also in the case of some admissible component failures [26], [27]. To ensure a higher reliability level and better control performance, reliable control systems on the basis of various control strategies have been presented to achieve these critical requirements, i.e., the pole region assignment scheme [27], the coprime factorization scheme [28], the algebraic Riccati equation (ARE)-based scheme [29], Hamilton-Jacobi inequality (HJI)-based scheme [30], and the LMI-based scheme [31], [32], for instance. It is noted that although the reliable control design problems for fuzzy dynamic systems with linear local models have received increasing interest, little attention has been dedicated to developing numerically testable reliable SOF controller synthesis conditions. In particular, SOF control is of extreme importance and more realistic since the full state measurement is not always possible for most of practical complicated nonlinear systems [15], [31]. In addition, it has been validated that a T-S fuzzy model with offset terms is intrinsically with substantial improvement of function approximation capabilities [33], [34]. The analysis and synthesis of T-S fuzzy-affine systems are basically more complicated than T-S fuzzy linear systems. These cases indicate that the reliable output feedback controller synthesis problem for continuous-time delayed fuzzy-affine systems with sensor faults has not been fully solved and remains to be open and challenging, which is another motivation for this research.

Encouraged by the issues mentioned above, in this paper, we will deal with the problem of delay-dependent reliable piecewise-affine $\mathcal{H}_\infty$ SOF controller synthesis for continuous-time T-S fuzzy-affine systems with time-varying delay and Markovian jumping sensor faults. Specifically, by a novel system-augmentation approach, the original system will be firstly reformulated into a descriptor fuzzy-affine system, and then based on a new piecewise-Markovian LKF, combined with a Wirtinger-based integral inequality, reciprocally convex inequality and S-procedure, a novel bounded real lemma will be proposed for the underlying closed-loop system. Furthermore, by taking advantage of the redundancy of descriptor system formulation, together with a linearisation procedure, the controller synthesis will be performed. It will be shown that the desired piecewise-affine controller gains can be computed by a convex optimization scheme. Finally, simulation results are

presented to confirm the effectiveness and less conservatism of the developed approach.

Notations. $\mathbb{S}^n$ represents the set of $n \times n$ real symmetric and positive definite matrices; $\text{Sym}\{A\}$ refers to the shorthand notation for $A + A^T$; $\mathbb{E}[\cdot]$ stands for the mathematical expectation; signals that are square integrable over $[0, \infty)$ is specified by $L_2[0, \infty)$ with the norm $\|\cdot\|$.

II. MODEL DESCRIPTION

The T-S fuzzy-dynamic model is characterized by fuzzy IF-THEN rules, whose relationship exhibits the approximation of the nonlinear systems. Along the idea of [34], a continuous-time T-S fuzzy-affine dynamic model can be expressed as follows,

Plant Rule $\mathcal{F}^l$: IF $\xi_1(x(t))$ is $\mathcal{F}_1^l$ and $\xi_2(x(t))$ is $\mathcal{F}_2^l$ and $\dots$ and $\xi_g(x(t))$ is $\mathcal{F}_g^l$, THEN

$$\begin{cases} \dot{x}(t) = (A_l + \Delta A_l)x(t) + (A_{dl} + \Delta A_{dl})x(t - d(t)) \\ \quad + a_l + \Delta a_l + (B_l + \Delta B_l)u(t) + D_{l1}w(t) \\ y(t) = (C_l + \Delta C_l)x(t) + C_{dl}x(t - d(t)) + D_{l2}w(t) \\ z(t) = L_lx(t) + L_{dl}x(t - d(t)) \\ x(t) = \phi(t), \quad -d_2 \leq t \leq 0, \quad l \in \mathcal{L} := \{1, 2, \dots, M\} \end{cases} \quad (1)$$

where $\mathcal{F}^l$ refers to the l -th fuzzy inference rule; M represents the number of inference rules; $\mathcal{F}_\nu^l (\nu = 1, 2, \dots, g)$ are fuzzy sets; $x(t) \in \mathbb{R}^{n_x}$ denotes the system state; $u(t) \in \mathbb{R}^{n_u}$ refers to the control input; $y(t) \in \mathbb{R}^{n_y}$ is the system measurement output; $z(t) \in \mathbb{R}^{n_z}$ is the controlled output; $w(t) \in \mathbb{R}^{n_w}$ denotes the exogenous disturbance input belonging to $L_2[0, \infty)$; $\xi(x(t)) := [\xi_1(x(t)), \xi_2(x(t)), \dots, \xi_g(x(t))]$ represent some measurable variables of the system; $\phi(t)$ is a real-valued initial condition sequence on $[-d_2, 0]$. $d(t)$ is a time-varying delay subject to

$$0 \leq d_1 \leq d(t) \leq d_2 \leq \infty, \quad \dot{d}(t) \leq \mu < \infty \quad (2)$$

where d_1 , d_2 and μ are real constant scalars. We denotes $(A_l, A_{dl}, a_l, B_l, C_l, C_{dl}, D_{l1}, D_{l2}, L_l, L_{dl})$ as the l -th local model of the system, and $(\Delta A_l, \Delta A_{dl}, \Delta a_l, \Delta B_l, \Delta C_l)$ as the uncertainty terms of the l -th local model with the form

$$\begin{aligned} & \begin{bmatrix} \Delta A_l & \Delta A_{dl} & \Delta B_l & \Delta a_l \end{bmatrix} \\ & = U_{l1} \Delta_l(t) \begin{bmatrix} V_{l1} & V_{l2} & V_{l3} & V_{l4} \end{bmatrix}, \\ & \Delta C_l = U_{l2} \Delta_l(t) V_{l1}, \quad l \in \mathcal{L} \end{aligned} \quad (3)$$

with U_{l1} , U_{l2} , V_{l1} , V_{l2} , V_{l3} , and V_{l4} being known and real constant matrices, and Lebesgue measurable elements $\Delta_l(t) : \mathbb{Z}^+ \rightarrow \mathbb{R}^{k_1 \times k_2}$, $l \in \mathcal{L}$ subject to

$$\Delta_l^T(t) \Delta_l(t) \leq \mathbf{I}_{k_2}, \quad l \in \mathcal{L}. \quad (4)$$

Let $\mu_l[x(t)]$ be the normalized fuzzy-basis function of $\mathcal{F}^l$ where $\mathcal{F}^l := \prod_{\nu=1}^g \mathcal{F}_\nu^l$ and

$$\mu_l[x(t)] := \frac{\prod_{\nu=1}^g \mu_{l\nu}[\xi_\nu(x(t))]}{\sum_{p=1}^M \prod_{\nu=1}^g \mu_{p\nu}[\xi_\nu(x(t))]} \geq 0, \quad \sum_{l=1}^M \mu_l[x(t)] = 1 \quad (5)$$

where $\mu_{l\nu}[\xi_\nu(x(t))]$ is the grade of membership of $\xi_\nu(x(t))$ in $\mathcal{F}_\nu^l$. In the sequel, for the simplicity of notations, we specify $\mu_l[x(t)]$ as μ_l .

Applying a singleton fuzzifier, product fuzzy inference and center-average defuzzifier, provides the subsequent global T-S fuzzy dynamic model,

$$\begin{cases} \dot{x}(t) = (A(\mu) + \Delta A(\mu))x(t) + a(\mu) + \Delta a(\mu) \\ \quad + (A_d(\mu) + \Delta A_d(\mu))x(t-d(t)) \\ \quad + (B(\mu) + \Delta B(\mu))u(t) + D_1(\mu)w(t) \\ y(t) = (C(\mu) + \Delta C(\mu))x(t) + C_d(\mu)x(t-d(t)) \\ \quad + D_2(\mu)w(t) \\ z(t) = L(\mu)x(t) + L_d(\mu)x(t-d(t)) \end{cases} \quad (6)$$

where

$$\chi(\mu) := \sum_{l=1}^M \mu_l \chi_l, \quad \chi \in \{A, \Delta A, A_d, \Delta A_d, a, \Delta a, B, \Delta B, C, \Delta C, C_d, D_1, D_2, L, L_d\}. \quad (7)$$

In this paper, we address the delay-dependent reliable $\mathcal{H}_\infty$ SOF control problem for the fuzzy-affine dynamic model (6) in the piecewise-Lyapunov-functionals framework. Along the lines in [33] and [34], we divide the state space into crisp subspaces (for certain $\mu_l[x(t)] = 1$) and fuzzy subspaces ($0 < \mu_l[x(t)] < 1$). In this respect, we can further rewrite the system (6) as a piecewise-fuzzy-affine system with the form,

$$\begin{cases} \dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-d(t)) \\ \quad + a_i + \Delta a_i + (B_i + \Delta B_i)u(t) + D_{i1}w(t) \\ y(t) = (C_i + \Delta C_i)x(t) + C_{di}x(t-d(t)) + D_{i2}w(t) \\ z(t) = \mathcal{L}_i x(t) + \mathcal{L}_{di}x(t-d(t)) \\ x(t) = \phi(t), \quad -d_2 \leq t \leq 0, \quad x(t) \in \mathcal{S}_i, \quad i \in \mathcal{I} \end{cases} \quad (8)$$

where we view fuzzy systems as operating-subspace-based models, and exploit the close connection between T-S fuzzy systems and piecewise-affine systems, and $\{\mathcal{S}_i\}_{i \in \mathcal{I}}$ and $\mathcal{I}$ standing for the state-space partition and the set of subspace indices, respectively, and

$$\begin{cases} A_i := \sum_{s \in \mathcal{S}(i)} \mu_s A_s, \quad \Delta A_i = \sum_{s \in \mathcal{S}(i)} \mu_s \Delta A_s, \\ A_{di} := \sum_{s \in \mathcal{S}(i)} \mu_s A_{ds}, \quad \Delta A_{di} = \sum_{s \in \mathcal{S}(i)} \mu_s \Delta A_{ds}, \\ a_i := \sum_{s \in \mathcal{S}(i)} \mu_s a_s, \quad \Delta a_i = \sum_{s \in \mathcal{S}(i)} \mu_s \Delta a_s, \\ B_i := \sum_{s \in \mathcal{S}(i)} \mu_s B_s, \quad \Delta B_i = \sum_{s \in \mathcal{S}(i)} \mu_s \Delta B_s, \\ C_i := \sum_{s \in \mathcal{S}(i)} \mu_s C_s, \quad \Delta C_i := \sum_{s \in \mathcal{S}(i)} \mu_s \Delta C_s, \\ C_{di} := \sum_{s \in \mathcal{S}(i)} \mu_s C_{ds}, \quad D_{i1} := \sum_{s \in \mathcal{S}(i)} \mu_s D_{s1}, \\ D_{i2} := \sum_{s \in \mathcal{S}(i)} \mu_s D_{s2}, \quad \mathcal{L}_i := \sum_{s \in \mathcal{S}(i)} \mu_s L_s, \\ \mathcal{L}_{di} := \sum_{s \in \mathcal{S}(i)} \mu_s L_{ds} \end{cases} \quad (9)$$

with $0 < \mu_s[x(t)] \leq 1$, $\sum_{s \in \mathcal{S}(i)} \mu_s[x(t)] = 1$. For each region $\mathcal{S}_i$, $i \in \mathcal{I}$, the set $\mathcal{S}(i)$ involves the indices for the local system modes used in the interpolation. Evidently, $\mathcal{S}(i)$ in a crisp subspace refers to only one element.

Denote the indices of subspaces $\mathcal{I} = \mathcal{I}_0 \cup \mathcal{I}_1$, where $\mathcal{I}_0$ represents the index set of subspaces, which cover the origin, and $\mathcal{I}_1$ refers to the index set of subspaces otherwise.

To formulate the reliable control problem of the T-S fuzzy-affine dynamic system in (8) with sensor faults, we characterize the sensor faults with a stochastic process in the following

form,

$$y(t, r(t)) = \Sigma(r(t))y(t), \quad (10)$$

where $\Sigma(r(t)) := \text{diag}\{\sigma_1(r(t)), \sigma_2(r(t)), \dots, \sigma_{n_y}(r(t))\}$, $0 \leq \sigma_\iota(r(t)) \leq 1$, $\iota = 1, 2, \dots, n_y$, and $\{r(t), t \geq 0\}$ is a Markov process describing the sensor-fault mode and taking values in a finite set $\mathcal{I} := \{1, 2, \dots, N\}$ with the mode transition rate (TR) matrix $\mathbb{T} := [\lambda_{mn}]_{N \times N}$ described by

$$\Pr\{r(t+h) = n | r(t) = m\} = \begin{cases} \lambda_{mn}h + o(h), & m \neq n \\ 1 + \lambda_{mm}h + o(h), & m = n \end{cases}$$

where $h > 0$, $\lim_{h \rightarrow 0} (o(h)/h) = 0$, and $\lambda_{mn} \geq 0$, for $n \neq m$, denotes the TR from mode m at time t to mode n at time $t+h$, and $\lambda_{mm} = -\sum_{n=1, n \neq m}^N \lambda_{mn}$. For $r(t) = m$, $m \in \mathcal{I}$, the system sensor-fault coefficient matrix of the m -th mode is specified by Σ_m . In particular, $\sigma_\iota(r(t)) = 0$, $0 < \sigma_\iota(r(t)) < 1$, and $\sigma_\iota(r(t)) = 1$ refer to the complete fault, partial fault, and without fault cases of the ι -th sensor, respectively.

In accordance with the T-S fuzzy-affine dynamic system (8), we seek a *piecewise-affine* SOF controller as,

$$u(t) = K_i \Sigma(r(t))y(t) + k_i, \quad i \in \mathcal{I} \quad (11)$$

where $K_i \in \mathbb{R}^{n_u \times n_y}$, $i \in \mathcal{I}$, $k_i \in \mathbb{R}^{n_u \times 1}$, $i \in \mathcal{I}_1$, are the controller gain matrices to be synthesized. It is noted that when $k_i \equiv 0$, $i \in \mathcal{I}$, (11) reduces to a *piecewise-linear* controller. It is evident that the piecewise-affine controller in (11) is more consistent with the piecewise-fuzzy-affine dynamical system in (8) and will result in better performances compared with a piecewise-linear one.

It is also worth mentioning that when deriving the LMIs-based conditions for SOF controller synthesis of systems, one generally necessitates the computation of the inverse of the Lyapunov matrices. However, for the PQLFs-based controller design of fuzzy-affine systems, due to the application of S-procedure with the term $\bar{G}_i^T W_i \bar{G}_i$ [38], the inverse of piecewise-Lyapunov matrices $P_i := \bar{F}_i^T T \bar{F}_i$ will be involved in the terms $P_i^{-1} \bar{G}_i^T W_i \bar{G}_i P_i^{-1}$. It is thus difficult to design a piecewise-affine SOF controller (11) based on the approaches proposed in [15], [18] for fuzzy-affine systems. On the other hand, due to the simultaneous existence of parameter uncertainties ΔB_i and ΔC_i in the input and output matrices, it is also difficult to employ the celebrated Projection lemma [21] for SOF controller design. Especially, the measurement output is also disturbed by external disturbance $w(t)$ and stochastic sensor faults, which bring much more difficulty on the numerical tractability of SOF controller design conditions for the underlying fuzzy dynamic systems. To circumvent these problems, we shall resort to a novel system-augmentation approach to piecewise-affine SOF controller design. To this end, we rewrite the resulting closed-loop system as

$$\begin{cases} \dot{\bar{x}} = 0 \\ \dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t-d(t)) \\ \quad + a_i + \Delta a_i + (B_i + \Delta B_i)u(t) + D_{i1}w(t) \\ 0 \cdot \dot{u}(t) = K_i \Sigma_m ((C_i + \Delta C_i)x(t) + C_{di}x(t-d(t)) \\ \quad + D_{i2}w(t)) + k_i - u(t) \\ z(t) = \mathcal{L}_i x(t) + \mathcal{L}_{di}x(t-d(t)), \quad i \in \mathcal{I}, \quad m \in \mathcal{I} \end{cases} \quad (12)$$

which can be further re-organized as

$$\begin{cases} E\dot{\bar{x}}(t) = \bar{A}_{i,m}\bar{x}(t) + \bar{A}_{di,m}H\bar{x}(t-d(t)) + \bar{D}_{i,m}w(t) \\ z(t) = \bar{L}_i\bar{x}(t) + \bar{L}_{di}H\bar{x}(t-d(t)), \quad i \in \mathcal{I}, \quad m \in \mathcal{I} \end{cases} \quad (13)$$

where

$$\left\{ \begin{array}{l} \bar{x}(t) = \begin{bmatrix} x^T(t) & u^T(t) \end{bmatrix}^T, \\ E := \text{diag}\{\mathbf{I}_{n_x}, \mathbf{0}_{n_u \times n_u}\}, \\ \bar{A}_{i,m} := \begin{bmatrix} \mathcal{A}_i + \Delta\mathcal{A}_i & \mathcal{B}_i + \Delta\mathcal{B}_i \\ K_i\Sigma_m(\mathcal{C}_i + \Delta\mathcal{C}_i) & -\mathbf{I} \end{bmatrix}, \\ \bar{A}_{di,m} := \begin{bmatrix} \mathcal{A}_{di}^T + \Delta\mathcal{A}_{di}^T & \mathcal{C}_{di}^T\Sigma_m^T K_i^T \end{bmatrix}^T, \\ \bar{D}_{i,m} := \begin{bmatrix} \mathcal{D}_{i1}^T & \mathcal{D}_{i2}^T\Sigma_m^T K_i^T \end{bmatrix}^T, \\ \bar{L}_i := \begin{bmatrix} \mathcal{L}_i & \mathbf{0}_{n_z \times n_u} \end{bmatrix}, \quad H := \begin{bmatrix} \mathbf{I} & \mathbf{0}_{n_x \times n_u} \end{bmatrix}, \end{array} \right\}, \quad \text{if } i \in \mathcal{I}_0, \\ \left\{ \begin{array}{l} \bar{x}(t) = \begin{bmatrix} 1 & x^T(t) & u^T(t) \end{bmatrix}^T, \\ E := \text{diag}\{\mathbf{I}_{(n_x+1)}, \mathbf{0}_{n_u \times n_u}\}, \\ \bar{A}_{i,m} := \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ a_i + \Delta a_i & \mathcal{A}_i + \Delta\mathcal{A}_i & \mathcal{B}_i + \Delta\mathcal{B}_i \\ k_i & K_i\Sigma_m(\mathcal{C}_i + \Delta\mathcal{C}_i) & -\mathbf{I} \end{bmatrix}, \\ \bar{A}_{di,m} := \begin{bmatrix} \mathbf{0} & \mathcal{A}_{di}^T + \Delta\mathcal{A}_{di}^T & \mathcal{C}_{di}^T\Sigma_m^T K_i^T \end{bmatrix}^T, \\ \bar{D}_{i,m} := \begin{bmatrix} \mathbf{0} & \mathcal{D}_{i1}^T & \mathcal{D}_{i2}^T\Sigma_m^T K_i^T \end{bmatrix}^T, \\ \bar{L}_i := \begin{bmatrix} \mathbf{0}_{n_z \times 1} & \mathcal{L}_i & \mathbf{0}_{n_z \times n_u} \end{bmatrix}, \\ H := \begin{bmatrix} \mathbf{0}_{n_x \times 1} & \mathbf{I} & \mathbf{0}_{n_x \times n_u} \end{bmatrix}, \end{array} \right\}, \quad \text{if } i \in \mathcal{I}_1. \quad (14)$$

Remark 2.1. In light of a system-augmentation technique, the original closed-loop system is reformulated as a descriptor system. A particularity of this descriptor redundancy is that the appearance of crossing terms between the system matrices and the controller gains is reduced, which makes more tractable to achieve the LMI formulation of piecewise-affine SOF controller synthesis conditions. More details on the benefit of this descriptor formulation will be emphasized in the next section.

III. DELAY-DEPENDENT RELIABLE PIECEWISE-AFFINE $\mathcal{H}_\infty$ SOF CONTROLLER ANALYSIS AND SYNTHESIS

In this section, by constructing a new piecewise-Markovian LKF, together with the Wirtinger-based integral inequality, reciprocally convex inequality and S-procedure, we will firstly present a novel bounded real lemma for the closed-loop system (13). Then by adopting a linearisation technique, the piecewise-affine SOF controller synthesis will be carried out.

A. Boundary Continuity Matrices Set-Up

Following the ideas in [33] and [35], to search for the piecewise Lyapunov functionals continuous across the subspace boundaries, it is required to construct the following continuity matrices $\bar{F}_i = \begin{bmatrix} f_i & F_i \end{bmatrix}$, $i \in \mathcal{I}$ with $f_i = 0$ for $i \in \mathcal{I}_0$, which are employed to describe the boundary among the subspaces, and satisfy the boundary conditions for each subsystem $\mathcal{S}_i$ as follows,

$$\bar{F}_i\hat{x}(t) = \bar{F}_j\hat{x}(t), \quad x(t) \in \mathcal{S}_i \cap \mathcal{S}_j, \quad i, j \in \mathcal{I} \quad (15)$$

where $\hat{x}(t) := \begin{bmatrix} 1 & x^T(t) \end{bmatrix}^T$, $f_i \in \mathbb{R}^{n_f \times 1}$, $F_i \in \mathbb{R}^{n_f \times n_x}$. Notice that, per construction, the matrices F_i are of full

column rank without loss of generality [33]. In this case, we basically have $n_f \geq n_x$.

Additionally, for the conservatism reduction of the stability analysis criteria, with the S-procedure technique [33], we also introduce the matrices $\bar{G}_i = \begin{bmatrix} g_i & G_i \end{bmatrix}$, $i \in \mathcal{I}$ with $g_i = 0$ for $i \in \mathcal{I}_0$ such that

$$\begin{bmatrix} g_i & G_i \end{bmatrix} \begin{bmatrix} 1 \\ x(t) \end{bmatrix} \succeq 0, \quad x(t) \in \mathcal{S}_i, \quad i \in \mathcal{I} \quad (16)$$

where $g_i \in \mathbb{R}^{n_g \times 1}$ and $G_i \in \mathbb{R}^{n_g \times n_x}$.

Notice that the aforementioned notation $\succeq$ infers that each entry of the vector is nonnegative. For convenient notations, we also signify

$$\begin{bmatrix} \bar{F}_i & \bar{G}_i \end{bmatrix} = \begin{cases} \begin{bmatrix} F_i & G_i \\ f_i & F_i \end{bmatrix}, & \text{if } i \in \mathcal{I}_0, \\ \begin{bmatrix} g_i & G_i \end{bmatrix}, & \text{if } i \in \mathcal{I}_1. \end{cases} \quad (17)$$

Remark 3.1. It is worth mentioning that the continuity matrices $\bar{F}_i$ and subspace boundary matrices $\bar{G}_i$, $i \in \mathcal{I}$, can be calculated systematically from a prescribed partition of the state-space, and more details about the construction procedure of $\bar{F}_i$ and $\bar{G}_i$, $i \in \mathcal{I}$, please kindly refer to [33] and [35].

B. Novel Bounded Real Lemma Based on Piecewise-Markovian Lyapunov Functionals

Theorem 3.1. The system (13) is stochastically stable (SS) with a guaranteed $\mathcal{H}_\infty$ performance γ , if there exist matrices $P_{m1} = P_{m1}^T \in \mathbb{R}^{(2n_x+n_f) \times (2n_x+n_f)}$, $P_{i2} \in \mathbb{R}^{n_u \times (2n_x+n_f)}$, $P_{i3} \in \mathbb{R}^{n_u \times n_u}$, $\{R_1, R_2, R_3, Z_1, Z_2\} \in \mathbb{S}^{n_x}$, $X \in \mathbb{R}^{2n_x \times 2n_x}$, and $W_i \in \mathbb{R}^{n_g \times n_g}$, $i \in \mathcal{I}$, such that the following matrix inequalities hold,

$$\begin{aligned} & \text{Sym}\{\Lambda_{i,m1}\bar{F}_i^T\mathbb{P}_{i,m}\bar{F}_i\Lambda_2^T(d(t))\} + \Lambda_3\mathcal{R}\Lambda_3^T - \Lambda_4Z_1\Lambda_4^T \\ & + \Lambda_2(d(t))\left(\tilde{E}\sum_{n=1}^N\lambda_{mn}\bar{F}_i^T\mathbb{P}_{i,n}\bar{F}_i\right)\Lambda_2^T(d(t)) \\ & - \Lambda_5Z_2\Lambda_5^T + \mathcal{A}_{i,m}^TH^TZ_0H\mathcal{A}_{i,m} \\ & + e_1\bar{G}_i^TW_i\bar{G}_ie_1^T + \mathcal{L}_i^T\mathcal{L}_i - \gamma^2e_8e_8^T < 0, \quad (18) \end{aligned}$$

$$\bar{\mathbb{P}}_{i,m} := \text{diag}\{\mathbf{I}_{n_x}, \mathbf{I}_{n_x}, \bar{F}_i^T\}P_{m1}\text{diag}\{\mathbf{I}_{n_x}, \mathbf{I}_{n_x}, \bar{F}_i\} > 0, \quad (19)$$

where $i \in \mathcal{I}$, $m \in \mathcal{I}$, and

$$\begin{cases} \Lambda_{i,m1} := \begin{bmatrix} e_1H^T - e_3 & e_3 - e_4 & \mathcal{A}_{i,m}^T \end{bmatrix}, \\ \Lambda_2(d(t)) := \begin{bmatrix} d_1e_5 & (d(t) - d_1)e_6 + (d_2 - d(t))e_7 & e_1 \end{bmatrix}, \\ \Lambda_3 := \begin{bmatrix} e_1 & e_2 & e_3 & e_4 \end{bmatrix}, \\ \Lambda_4 := \begin{bmatrix} e_1H^T - e_3 & e_1H^T + e_3 - 2e_5 \end{bmatrix}, \\ \Lambda_5 := \begin{bmatrix} e_3 - e_2 & e_3 + e_2 - 2e_6 & e_2 - e_4 \\ e_2 + e_4 - 2e_7 \end{bmatrix}, \\ \mathcal{R} := \text{diag}\{H^T(R_1 + R_2 + R_3)H, -(1 - \mu)R_3, -R_1, -R_2\}, \\ \tilde{E} := \text{diag}\{\mathbf{I}_{n_x}, \mathbf{I}_{n_x}, E\}, \quad \tilde{F}_i := \text{diag}\{\mathbf{I}_{n_x}, \mathbf{I}_{n_x}, \bar{F}_i, \mathbf{I}_{n_u}\}, \\ Z_0 := d_1^2Z_1 + d^2Z_2, \quad Z_1 := \text{diag}\{Z_1, 3Z_1\}, \\ Z_2 := \begin{bmatrix} \text{diag}\{Z_2, 3Z_2\} & * \\ X^T & \text{diag}\{Z_2, 3Z_2\} \end{bmatrix}, \\ \mathbb{P}_{i,m} := \begin{bmatrix} P_{m1} & \mathbf{0} \\ P_{i2} & P_{i3} \end{bmatrix}, \end{cases}$$

$$\left\{ \begin{array}{l} \mathcal{A}_{i,m} := \begin{bmatrix} \bar{\mathcal{A}}_{i,m} & \bar{\mathcal{A}}_{di,m} & \mathbf{0}_{(n_x+n_u) \times 5n_x} & \bar{\mathcal{D}}_{i,m} \end{bmatrix}, \\ e_\kappa := \begin{bmatrix} \mathbf{0}_{\kappa-1} & \mathbf{I}_{\sigma(\kappa)} & \mathbf{0}_{8-\kappa} \end{bmatrix}^T \\ \in \mathbb{R}^{(7n_x+n_u+n_w) \times \sigma(\kappa)}, \kappa = 1, \dots, 8, \\ \sigma(1) := n_x + n_u, \\ \mathcal{A}_{i,m} := \begin{bmatrix} \bar{\mathcal{A}}_{i,m} & \bar{\mathcal{A}}_{di,m} & \mathbf{0}_{(n_x+n_u+1) \times 5n_x} & \bar{\mathcal{D}}_{i,m} \end{bmatrix}, \\ e_\kappa := \begin{bmatrix} \mathbf{0}_{\kappa-1} & \mathbf{I}_{\sigma(\kappa)} & \mathbf{0}_{8-\kappa} \end{bmatrix}^T \\ \in \mathbb{R}^{(7n_x+n_u+n_w+1) \times \sigma(\kappa)}, \kappa = 1, \dots, 8, \\ \sigma(1) := n_x + n_u + 1, \\ \mathcal{L}_i := \begin{bmatrix} \bar{\mathcal{L}}_i & \mathcal{L}_{di} & \mathbf{0}_{n_z \times (5n_x+n_w)} \end{bmatrix}, \end{array} \right\}, \quad \text{if } i \in \mathcal{S}_0, \quad (20)$$

with

$$\sigma(\kappa) := \begin{cases} n_x, & \kappa = 2, \dots, 7, \\ n_w, & \kappa = 8. \end{cases} \quad (21)$$

Proof. It is worth mentioning that here we merely provide the proof of condition (19) for $i \in \mathcal{S}_1$. Because the condition (19) for $i \in \mathcal{S}_0$ can be tackled as a special case of (19) for $i \in \mathcal{S}_1$ in the context of $a_l + \Delta a_l = 0$ and without involvement of the S-procedure.

First, let $\mathbb{C}[-d_2, 0]$ be the space of continuous functions on the interval $[-d_2, 0]$, and define $\bar{x}_\tau(t) := \bar{x}(t + \tau)$, $\bar{x}_\tau(t) \in \mathbb{C}[-d_2, 0]$, $\tau \in [-d_2, 0]$. Then $\{(\bar{x}_\tau(t), r(t)), t \geq 0\}$ is a Markov process with initial state $(\bar{\phi}_t, r(0))$, where $\bar{\phi}_t = \begin{bmatrix} 1 & \phi_t^T & \mathbf{0} \end{bmatrix}^T$.

Construct the following piecewise-Markovian LKF for the system (13),

$$V(\bar{x}(t), r(t), t) := \sum_{c=1}^3 V_c(\bar{x}(t), r(t), t), \quad (22)$$

with

$$\left\{ \begin{array}{l} V_1(\bar{x}(t), r(t), t) := \tilde{x}^T(t) \tilde{E} \tilde{F}_i^T \mathbb{P}_i(r(t)) \tilde{F}_i \tilde{x}(t), \\ V_2(\bar{x}(t), r(t), t) := \int_{t-d_1}^t \bar{x}^T(s) H^T R_1 H \bar{x}(s) ds \\ \quad + \int_{t-d_2}^t \bar{x}^T(s) H^T R_2 H \bar{x}(s) ds \\ \quad + \int_{t-d(t)}^t \bar{x}^T(s) H^T R_3 H \bar{x}(s) ds, \\ V_3(\bar{x}(t), r(t), t) := d_1 \int_{-d_1}^0 \int_{t+s}^t \dot{\tilde{x}}^T(\tau) H^T Z_1 H \dot{\tilde{x}}(\tau) d\tau ds \\ \quad + d \int_{-d_2}^{-d_1} \int_{t+s}^t \dot{\tilde{x}}^T(\tau) H^T Z_2 H \dot{\tilde{x}}(\tau) d\tau ds, \end{array} \right. \quad (23)$$

where $\tilde{x}(t) := \begin{bmatrix} \int_{t-d_1}^t x^T(s) ds & \int_{t-d_2}^t x^T(s) ds & \bar{x}^T(t) \end{bmatrix}^T$, $\{R_1, R_2, R_3, Z_1, Z_2\} \in \mathbb{S}^{n_x}$, and

$$\left\{ \begin{array}{l} \mathbb{P}_{i,m} := \begin{bmatrix} P_{m1} & \mathbf{0} \\ P_{i2} & P_{i3} \end{bmatrix}, \quad m = r(t), \\ \tilde{F}_i := \text{diag}\{\mathbf{I}_{n_x}, \mathbf{I}_{n_x}, \bar{F}_i, \mathbf{I}_{n_u}\}, \\ \tilde{E} := \text{diag}\{\mathbf{I}_{n_x}, \mathbf{I}_{n_x}, E\}, \quad d = d_2 - d_1 \end{array} \right. \quad (24)$$

with matrices $P_{m1} = P_{m1}^T \in \mathbb{R}^{(2n_x+n_f) \times (2n_x+n_f)}$ satisfying $\mathbb{P}_{i,m} := \text{diag}\{\mathbf{I}_{n_x}, \mathbf{I}_{n_x}, \bar{F}_i^T\} P_{m1} \text{diag}\{\mathbf{I}_{n_x}, \mathbf{I}_{n_x}, \bar{F}_i\} > 0$, $P_{i2} \in \mathbb{R}^{n_u \times (2n_x+n_f)}$ and $P_{i3} \in \mathbb{R}^{n_u \times n_u}$.

Then, considering the piecewise-Markovian LKF defined in (22), the system (13) is SS with an $\mathcal{H}_\infty$ performance γ under zero initial conditions and any nonzero $w(t) \in L_2[0, \infty)$, if

we can show

$$\mathcal{D}[V(\bar{x}(t), r(t), t)] + \mathbb{E}\{\|z(t)\|_2^2\} - \gamma^2 \|w(t)\|_2^2 < 0, \quad (25)$$

where $\mathcal{D}$ refers to the weak infinitesimal generator of the Markov process $\{\bar{x}(t), r(t), t \geq 0\}$. In view of the definition of $\mathcal{D}$ [31], one has

$$\begin{aligned} \mathcal{D}[V(\bar{x}(t), r(t), t)] \\ := \lim_{\delta \rightarrow 0} \frac{1}{\delta} [\mathbb{E}\{V(\bar{x}(t+\delta), r(t+\delta), t+\delta) | \bar{x}(t), \\ r(t) = m\} - V(\bar{x}(t), r(t), t)]. \end{aligned} \quad (26)$$

In the sequel, we introduce the following augmented vector to express an upper bound of $\mathcal{D}[V(\bar{x}(t), r(t), t)]$,

$$\left\{ \begin{array}{l} \zeta(t) := \begin{bmatrix} \bar{x}^T(t) & x^T(t-d(t)) & x^T(t-d_1) \\ & x^T(t-d_2) & \vartheta_1^T(t) & \vartheta_2^T(t) & \vartheta_3^T(t) & w^T(t) \end{bmatrix}, \\ \vartheta_1(t) := \frac{1}{d_1} \int_{t-d_1}^t x(s) ds, \\ \vartheta_2(t) := \frac{1}{d(t)-d_1} \int_{t-d(t)}^{t-d_1} x(s) ds, \\ \vartheta_3(t) := \frac{1}{d_2-d(t)} \int_{t-d_2}^{t-d(t)} x(s) ds. \end{array} \right. \quad (27)$$

Define $e_\kappa := \begin{bmatrix} \mathbf{0}_{\kappa-1} & \mathbf{I}_{\sigma(\kappa)} & \mathbf{0}_{8-\kappa} \end{bmatrix}^T \in \mathbb{R}^{(7n_x+n_u+n_w+1) \times \sigma(\kappa)}$, $1 \leq \kappa \leq 8$, where

$$\sigma(\kappa) := \begin{cases} n_x + n_u + 1, & \kappa = 1, \\ n_x, & \kappa = 2, \dots, 7, \\ n_w, & \kappa = 8. \end{cases} \quad (28)$$

It is obvious that $\bar{x}(t) = e_1^T \zeta(t)$, $x(t-d(t)) = e_2^T \zeta(t)$, and $w(t) = e_8^T \zeta(t)$.

Performing the time-derivative of $V(\bar{x}(t), r(t), t)$ along the trajectory of closed-loop system (13), yields

$$\begin{aligned} \mathcal{D}[V_1(\bar{x}(t), r(t), t)] \\ = \lim_{\delta \rightarrow 0} \frac{1}{\delta} \left\{ \tilde{x}^T(t+\delta) \tilde{E} \left[\sum_{n=1}^N \Pr\{r(t+\delta) = n | r(t) = m\} \right. \right. \\ \left. \left. \times \tilde{F}_i^T \mathbb{P}_{i,n} \tilde{F}_i \right] \tilde{x}(t+\delta) - \tilde{x}^T(t) \tilde{E} \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \tilde{x}(t) \right\} \\ = \lim_{\delta \rightarrow 0} \frac{1}{\delta} \left\{ \tilde{x}^T(t+\delta) \tilde{E} \left[\sum_{n=1}^N (\lambda_{mn} \delta + o(\delta)) \tilde{F}_i^T \mathbb{P}_{i,n} \tilde{F}_i \right] \right. \\ \left. \times \tilde{x}(t+\delta) + [\tilde{x}(t+\delta) - \tilde{x}(t)]^T \tilde{E} \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \tilde{x}(t+\delta) \right. \\ \left. + \tilde{x}^T(t+\delta) \tilde{F}_i^T \mathbb{P}_{i,m}^T \tilde{F}_i \tilde{E} [\tilde{x}(t+\delta) - \tilde{x}(t)] \right\} \\ = 2 \begin{bmatrix} H(\bar{x}(t) - \bar{x}(t-d_1)) \\ H(\bar{x}(t-d_1) - \bar{x}(t-d_2)) \\ E \dot{\tilde{x}}(t) \end{bmatrix}^T \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \tilde{x}(t) \\ + \tilde{x}^T(t) \left(\tilde{E} \sum_{n=1}^N \lambda_{mn} \tilde{F}_i^T \mathbb{P}_{i,n} \tilde{F}_i \right) \tilde{x}(t) \\ = \zeta^T(t) \left(\text{Sym}\{\Lambda_{i,m1} \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \Lambda_2^T(d(t))\} \right. \\ \left. + \Lambda_2(d(t)) \left(\tilde{E} \sum_{n=1}^N \lambda_{mn} \tilde{F}_i^T \mathbb{P}_{i,n} \tilde{F}_i \right) \Lambda_2^T(d(t)) \right) \zeta(t), \end{aligned} \quad (29)$$

where $\Lambda_{i,m1}$ and $\Lambda_2(d(t))$ are defined in (20), and $\zeta(t)$ is defined in (27).

Similarly, we have

$$\begin{aligned} & \mathcal{D}[V_2(\bar{x}(t), r(t), t)] \\ &= \bar{x}^T(t) H^T (R_1 + R_2 + R_3) H \bar{x}(t) \\ & \quad - x^T(t-d_1) R_1 x(t-d_1) - x^T(t-d_2) R_2 x(t-d_2) \\ & \quad - (1-\mu) x^T(t-d(t)) R_3 x(t-d(t)) \\ &= \zeta^T(t) \Lambda_3 \mathcal{R} \Lambda_3^T \zeta(t), \end{aligned} \quad (30)$$

$$\begin{aligned} & \mathcal{D}[V_3(\bar{x}(t), r(t), t)] \\ &= \dot{\bar{x}}^T(t) H^T (d_1^2 Z_1 + d^2 Z_2) H \dot{\bar{x}}(t) \\ & \quad - d_1 \int_{t-d_1}^t \dot{\bar{x}}^T(s) H^T Z_1 H \dot{\bar{x}}(s) ds \\ & \quad - d \int_{t-d(t)}^{t-d_1} \dot{\bar{x}}^T(s) H^T Z_2 H \dot{\bar{x}}(s) ds \\ & \quad - d \int_{t-d_2}^{t-d(t)} \dot{\bar{x}}^T(s) H^T Z_2 H \dot{\bar{x}}(s) ds, \end{aligned} \quad (31)$$

where Λ_3 and $\mathcal{R}$ are defined in (20).

Using Lemma A1 (shown in the Appendix) to the second term in the right-hand side (RHS) of equation (31), we have

$$\begin{aligned} & d_1 \int_{t-d_1}^t \dot{\bar{x}}^T(s) H^T Z_1 H \dot{\bar{x}}(s) ds \\ & \geq \begin{bmatrix} H(\bar{x}(t) - \bar{x}(t-d_1)) \\ H(\bar{x}(t) + \bar{x}(t-d_1) - \frac{2}{d_1} \vartheta_1(t)) \end{bmatrix}^T \text{diag}\{Z_1, 3Z_1\} (*) \\ &= \zeta^T(t) \Lambda_4 \mathcal{Z}_1 \Lambda_4^T \zeta(t), \end{aligned} \quad (32)$$

where Λ_4 and $\mathcal{Z}_1$ are defined in (20).

Analogously, by virtue of Lemma A1, the third and fourth terms in the RHS of equation (31) can be lower-bounded respectively by

$$\begin{aligned} & d \int_{t-d(t)}^{t-d_1} \dot{\bar{x}}^T(s) H^T Z_2 H \dot{\bar{x}}(s) ds \\ & \geq \frac{1}{\alpha(t)} \begin{bmatrix} x(t-d_1) - x(t-d(t)) \\ x(t-d_1) + x(t-d(t)) - \frac{2}{d(t)-d_1} \vartheta_2(t) \end{bmatrix}^T \\ & \quad \text{diag}\{Z_2, 3Z_2\} (*), \quad (33) \\ & d \int_{t-d_2}^{t-d(t)} \dot{\bar{x}}^T(s) H^T Z_2 H \dot{\bar{x}}(s) ds \\ & \geq \frac{1}{\beta(t)} \begin{bmatrix} x(t-d(t)) - x(t-d_2) \\ x(t-d(t)) + x(t-d_2) - \frac{2}{d_2-d(t)} \vartheta_3(t) \end{bmatrix}^T \\ & \quad \text{diag}\{Z_2, 3Z_2\} (*), \quad (34) \end{aligned}$$

where $\alpha(t) := \frac{d(t)-d_1}{d}$ and $\beta(t) := 1 - \alpha(t)$.

It is noted that $d_1 \leq d(t) \leq d_2$, and thus $\alpha(t)$ satisfies $0 < \alpha(t) < 1$. Then by Lemma A2, the inequality (35) holds for a suitable matrix $X \in \mathbb{R}^{2n_x \times 2n_x}$,

$$\begin{aligned} & d \int_{t-d(t)}^{t-d_1} \dot{\bar{x}}^T(s) H^T Z_2 H \dot{\bar{x}}(s) ds \\ & \quad + d \int_{t-d_2}^{t-d(t)} \dot{\bar{x}}^T(s) H^T Z_2 H \dot{\bar{x}}(s) ds \\ & \geq \begin{bmatrix} H(\bar{x}(t-d_1) - \bar{x}(t-d(t))) \\ H(\bar{x}(t-d_1) + \bar{x}(t-d(t)) - \frac{2}{d(t)-d_1} \vartheta_2(t)) \\ H(\bar{x}(t-d(t)) - \bar{x}(t-d_2)) \\ H(\bar{x}(t-d(t)) + \bar{x}(t-d_2) - \frac{2}{d_2-d(t)} \vartheta_3(t)) \end{bmatrix}^T \\ & \quad \times \begin{bmatrix} \text{diag}\{Z_2, 3Z_2\} & * \\ X^T & \text{diag}\{Z_2, 3Z_2\} \end{bmatrix} (*) \\ &= \zeta^T(t) \Lambda_5 \mathcal{Z}_2 \Lambda_5^T \zeta(t), \end{aligned} \quad (35)$$

where Λ_5 and $\mathcal{Z}_2$ are defined in (20).

Furthermore, considering the structural partitioning information in (16) and utilizing S-procedure, together with (29), (30), (31), (32) and (35), we have

$$\begin{aligned} \text{LHS}(25) & \leq \zeta^T(t) \left(\text{Sym}\{\Lambda_{i,m1} \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \Lambda_2^T(d(t))\} \right. \\ & \quad + \Lambda_2(d(t)) \left(\tilde{E} \sum_{n=1}^N \lambda_{mn} \tilde{F}_i^T \mathbb{P}_{i,n} \tilde{F}_i \right) \Lambda_2^T(d(t)) \\ & \quad + \Lambda_3 \mathcal{R} \Lambda_3^T - \Lambda_4 \mathcal{Z}_1 \Lambda_4^T - \Lambda_5 \mathcal{Z}_2 \Lambda_5^T \\ & \quad + \mathcal{A}_{i,m}^T H^T Z_0 H \mathcal{A}_{i,m} + e_1 \bar{G}_i^T W_i \bar{G}_i e_1 \\ & \quad \left. + \mathcal{L}_i^T \mathcal{L}_i - \gamma^2 e_8 e_8^T \right) \zeta(t) \end{aligned} \quad (36)$$

where e_1 , Z_0 , $\mathcal{A}_{i,m}$ and $\mathcal{L}_i$ are defined in (20). The latter clearly renders that the condition (19) leads to $\text{LHS}(25) < 0$, which means that the system (13) is SS with an $\mathcal{H}_\infty$ performance index according to the Lyapunov stability theory. This completes the proof. ■

Remark 3.2. On the basis of the piecewise-Markovian LKF (22), together with the Wirtinger-based integral inequality, improved reciprocally convex inequality and S-procedure, a novel bounded real lemma for the fuzzy-affine system (13) is presented in Theorem 3.1. It is worth mentioning that based upon the Wirtinger-based integral inequality, the stability analysis problem has been investigated for linear systems with time-delays in the literature [39], where the construction of common Lyapunov matrix P readily leads to the delay-dependent stability analysis condition in a convex optimization framework. However, for the stability analysis of fuzzy-affine systems within the piecewise-Markovian-Lyapunov-functional framework, the introduction of piecewise-Markovian Lyapunov matrices $\tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i$ in (22) induces that the performance analysis conditions (19) are affine with respect to the time-varying and nonlinear delay term $d^2(t)$, i.e., the quadratic term $\Lambda_2(d(t)) \left(\tilde{E} \sum_{n=1}^N \lambda_{mn} \tilde{F}_i^T \mathbb{P}_{i,n} \tilde{F}_i \right) \Lambda_2^T(d(t))$ in (19), which will incur some obstacles in the numerical tractability of performance analysis problem. However, due to the *non-positive property* of TRs λ_{mm} , $m \in \mathcal{I}$, it is unfortunate that the Schur complement cannot be applied to perform this decoupling. Alternatively, the powerful Projection lemma (see the Appendix) will be utilized. Thus, in the sequel, a decoupling between the nonlinear time-varying delay term will be initiated by introducing a free matrix variable. This decoupling technique enables one to acquire a more readily tractable condition for performance analysis.

Theorem 3.2. The system in (13) is SS with a guaranteed $\mathcal{H}_\infty$ performance γ , if there exist matrices $P_{m1} = P_{m1}^T \in \mathbb{R}^{(2n_x+n_f) \times (2n_x+n_f)}$, $P_{i2} \in \mathbb{R}^{n_u \times (2n_x+n_f)}$, $P_{i3} \in \mathbb{R}^{n_u \times n_u}$, $\{R_1, R_2, R_3, Z_1, Z_2\} \in \mathbb{S}^{n_x}$, $\mathcal{J}_0 \in \mathbb{R}^{3n_x \times (7n_x+n_u+n_w)}$, $\mathcal{J}_1 \in \mathbb{R}^{(3n_x+1) \times (7n_x+n_u+n_w+1)}$, $X \in \mathbb{R}^{2n_x \times 2n_x}$, and $0 \preceq W_i \in \mathbb{R}^{n_g \times n_g}$, $i \in \mathcal{I}$, such that the condition (19) and the following matrix inequalities hold,

$$\begin{bmatrix} \Xi_{i,m}^{(\ell)} & * & * \\ -\mathcal{J} & \lambda_{mm} \bar{\mathbb{P}}_{i,m} & * \\ \mathcal{P}_{i,m} \Pi_m^T \bar{\Lambda}_2^{(\ell)T} & \mathbf{0} & -\mathcal{P}_{i,m} \end{bmatrix} < 0,$$

$$i \in \mathcal{I}, m \in \mathcal{I}, \ell = 1, 2, \quad (37)$$

where

$$\left\{ \begin{aligned} \Xi_{i,m}^{(\ell)} &:= \text{Sym}\{\Lambda_{i,m1} \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \Lambda_2^{(\ell)T} + \bar{\Lambda}_2^{(\ell)} \mathcal{J}\} \\ &\quad + \Lambda_3 \mathcal{R} \Lambda_3^T - \Lambda_4 \mathcal{Z}_1 \Lambda_4^T - \Lambda_5 \mathcal{Z}_2 \Lambda_5^T \\ &\quad + \mathcal{A}_{i,m}^T H^T \mathcal{Z}_0 H \mathcal{A}_{i,m} + e_1 \bar{G}_i^T W_i \bar{G}_i e_1^T \\ &\quad + \mathcal{L}_i^T \mathcal{L}_i - \gamma^2 e_8 e_8^T, \\ \Lambda_2^{(1)} &:= \begin{bmatrix} d_1 e_5 & d e_6 & e_1 \end{bmatrix}, \\ \Lambda_2^{(2)} &:= \begin{bmatrix} d_1 e_5 & d e_7 & e_1 \end{bmatrix}, \\ \bar{\Lambda}_2^{(1)} &:= \begin{bmatrix} d_1 e_5 & d e_6 & e_1 \bar{H}_1^T \end{bmatrix}, \\ \bar{\Lambda}_2^{(2)} &:= \begin{bmatrix} d_1 e_5 & d e_7 & e_1 \bar{H}_1^T \end{bmatrix}, \\ \mathcal{P}_{i,m} &:= \text{diag}\{\bar{\mathbb{P}}_{i,1}, \dots, \bar{\mathbb{P}}_{i,m-1}, \bar{\mathbb{P}}_{i,m+1}, \dots, \bar{\mathbb{P}}_{i,N}\}, \\ \Pi_m &:= \begin{bmatrix} \sqrt{\lambda_{m1}} \mathbf{I} & \dots & \sqrt{\lambda_{m,m-1}} \mathbf{I} \\ & \sqrt{\lambda_{m,m+1}} \mathbf{I} & \dots & \sqrt{\lambda_{mN}} \mathbf{I} \end{bmatrix}, \\ \bar{H}_1 &:= \begin{bmatrix} \mathbf{I} & \mathbf{0}_{(n_x+1) \times n_u} \end{bmatrix}, \\ \mathcal{J} &:= \begin{cases} \mathcal{J}_0, & \text{if } i \in \mathcal{I}_0, \\ \mathcal{J}_1, & \text{if } i \in \mathcal{I}_1, \end{cases} \end{aligned} \right. \quad (38)$$

with the other notations defined the same as in (20).

Proof. On the basis of Theorem 3.1, the system (13) is SS with an $\mathcal{H}_\infty$ performance γ if the conditions (19) and (19) hold.

Firstly, rewrite (19) as

$$\Gamma(d(t)) \Theta_{i,m} \Gamma^T(d(t)) < 0, \quad (39)$$

where

$$\left\{ \begin{aligned} \Gamma(d(t)) &:= \begin{bmatrix} \mathbf{I}_{(7n_x+n_u+n_w+1)} & \bar{\Lambda}_2(d(t)) \end{bmatrix}, \\ \Theta_{i,m} &:= \text{diag}\{\Xi_{i,m}, \lambda_{mm} \bar{\mathbb{P}}_{i,m}\}, \\ \Xi_{i,m} &:= \text{Sym}\{\Lambda_{i,m1} \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \Lambda_2^T(d(t))\} \\ &\quad + \Lambda_2(d(t)) \left(\tilde{E} \sum_{n=1, n \neq m}^N \lambda_{mn} \tilde{F}_i^T \mathbb{P}_{i,n} \tilde{F}_i \right) \Lambda_2^T(d(t)) \\ &\quad + \Lambda_3 \mathcal{R} \Lambda_3^T - \Lambda_4 \mathcal{Z}_1 \Lambda_4^T - \Lambda_5 \mathcal{Z}_2 \Lambda_5^T \\ &\quad + \mathcal{A}_{i,m}^T H^T \mathcal{Z}_0 H \mathcal{A}_{i,m} + e_1 \bar{G}_i^T W_i \bar{G}_i e_1^T \\ &\quad + \mathcal{L}_i^T \mathcal{L}_i - \gamma^2 e_8 e_8^T, \\ \bar{\Lambda}_2(d(t)) &:= \begin{bmatrix} d_1 e_5 & (d(t) - d_1) e_6 + (d_2 - d(t)) e_7 \\ & e_1 \bar{H}_1^T \end{bmatrix}. \end{aligned} \right. \quad (40)$$

By implicit null space calculation, we have,

$$\begin{bmatrix} \bar{\Lambda}_2(d(t)) \\ -\mathbf{I}_{(3n_x+1)} \end{bmatrix}_\perp = \begin{bmatrix} \mathbf{I}_{(7n_x+n_u+n_w+1)} & \bar{\Lambda}_2(d(t)) \end{bmatrix}. \quad (41)$$

Then, assigning

$$\left\{ \begin{aligned} \Psi &\leftarrow \Theta_{i,m}, \\ \mathcal{M} &\leftarrow \mathbf{I}, \\ \mathcal{M}_\perp &\leftarrow \mathbf{0}, \\ \mathcal{N} &\leftarrow \Gamma_\perp(d(t)), \\ \mathcal{N}_\perp &\leftarrow \Gamma(d(t)), \\ \mathcal{G} &\leftarrow \tilde{\mathcal{J}} \in \mathbb{R}^{(3n_x+1) \times (10n_x+n_u+n_w+2)}, \end{aligned} \right. \quad (42)$$

and applying Projection lemma to (39) based on (41) and (42), the following inequality implies (39),

$$\Theta_{i,m} + \text{Sym}\{\Gamma_\perp(d(t)) \tilde{\mathcal{J}}\} < 0. \quad (43)$$

Now, it follows from (43) that by introducing the free matrix $\tilde{\mathcal{J}}$, the nonlinear coupling of time-delay $d(t)$ in the quadratic term $\Lambda_2(d(t)) \lambda_{mm} \tilde{E} \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \Lambda_2^T(d(t))$ has

been eliminated. It is worth mentioning that, however, the matrix $\tilde{\mathcal{J}}$ is intrinsically with a high dimension, which will incur heavy computational burden. To balance the computational complexity and conservatism, here we specify $\tilde{\mathcal{J}} := \begin{bmatrix} \mathcal{J}_1 & \mathbf{0}_{(3n_x+1) \times (3n_x+1)} \end{bmatrix}$, where $\mathcal{J}_1 \in \mathbb{R}^{(3n_x+1) \times (7n_x+n_u+n_w+1)}$. Furthermore, with respect to Schur complement, we can rewrite the condition (43) as,

$$\begin{bmatrix} \Xi_{i,m}(d(t)) & * & * \\ -\mathcal{J}_1 & \lambda_{mm} \bar{\mathbb{P}}_{i,m} & * \\ \mathcal{P}_{i,m} \Pi_m^T \bar{\Lambda}_2^T(d(t)) & \mathbf{0} & -\mathcal{P}_{i,m} \end{bmatrix} < 0, \quad i \in \mathcal{I}_1, \quad (44)$$

where

$$\begin{aligned} \Xi_{i,m}(d(t)) &:= \text{Sym}\{\Lambda_{i,m1} \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \Lambda_2^T(d(t)) \\ &\quad + \bar{\Lambda}_2(d(t)) \mathcal{J}_1\} + \Lambda_3 \mathcal{R} \Lambda_3^T - \Lambda_4 \mathcal{Z}_1 \Lambda_4^T \\ &\quad - \Lambda_5 \mathcal{Z}_2 \Lambda_5^T + \mathcal{A}_{i,m}^T H^T \mathcal{Z}_0 H \mathcal{A}_{i,m} \\ &\quad + e_1 \bar{G}_i^T W_i \bar{G}_i e_1^T + \mathcal{L}_i^T \mathcal{L}_i - \gamma^2 e_8 e_8^T. \end{aligned} \quad (45)$$

Notice that the condition in (44) is affine with respect to the time-varying delay $d(t)$, which satisfies

$$d_1 \leq d(t) \leq d_2. \quad (46)$$

The latter implies that $d(t)$ may take any value in $[d_1, d_2]$. Then $d(t)$ can be further expressed as a convex combination in the following form,

$$d(t) = \eta d_1 + (1 - \eta) d_2, \quad (47)$$

where $0 \leq \eta \leq 1$. Since $d(t)$ in (47) varies with respect to η linearly, it is only required that (44) holds with $\eta = 0$ and $\eta = 1$, respectively, which leads to (37) directly. Similarly, it can also be shown that (37) implies (19) in the context of $i \in \mathcal{I}_0$.

Hence, one can immediately conclude that the closed-loop system (13) is SS with an $\mathcal{H}_\infty$ performance γ if (19) and (37) hold. The proof is completed. ■

In the following subsection, on the basis of the new delay-dependent bounded real lemma presented in Theorem 3.2, we will give the controller design procedure.

C. Robust and Reliable Piecewise-Affine $\mathcal{H}_\infty$ Controller Design

The aim of this subsection is to derive the reliable piecewise-affine SOF controller synthesis conditions based upon Theorem 3.2, which are implementable via a convex optimization scheme.

Theorem 3.3. Consider the fuzzy-affine system (1). If there exist matrices $\{P_{m1(1)}, P_{m1(4)}, R_1, R_2, R_3, Z_1, Z_2\} \in \mathbb{S}^{n_x}$, $P_{m1(2)} \in \mathbb{R}^{n_x \times n_x}$, $\{P_{m1(3)}, P_{m1(5)}\} \in \mathbb{R}^{n_x \times n_f}$, $P_{m1(6)} \in \mathbb{R}^{n_u \times n_f}$, $P_{i2(1)} \in \mathbb{R}^{n_u \times n_u}$, $X \in \mathbb{R}^{2n_x \times 2n_x}$, $\mathcal{J}_0 \in \mathbb{R}^{3n_x \times (7n_x+n_u+n_w)}$, $\mathcal{J}_1 \in \mathbb{R}^{(3n_x+1) \times (7n_x+n_u+n_w+1)}$, $\bar{K}_i \in \mathbb{R}^{n_u \times n_y}$, $0 \preceq W_i \in \mathbb{R}^{n_g \times n_g}$, and scalars $\epsilon_i > 0$, $i \in \mathcal{I}$, and $\bar{k}_i \in \mathbb{R}^{n_u \times 1}$, $i \in \mathcal{I}_1$, such that the condition (19) and the following LMIs hold,

$$\begin{bmatrix} \Xi_{is,m}^{(\ell)} & * & * \\ \Upsilon_{is,m1} & \Upsilon_{i,m2} & * \\ \mathbb{U}_{is,m} & \tilde{\mathcal{Z}}_0 & -\epsilon_i \mathbf{I} \end{bmatrix} < 0, \quad s \in \mathcal{S}(i), \quad i \in \mathcal{I}, \quad m \in \mathcal{I}, \quad \ell = 1, 2, \quad (48)$$

where

$$\left\{ \begin{aligned}
 \Xi_{is,m}^{(\ell)} &:= \text{Sym}\{\bar{\Lambda}_1 \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \Lambda_2^{(\ell)T} + \Lambda_2^{(\ell)} (\mathcal{P}_{i,m} \mathbb{A}_s \\
 &\quad + \mathcal{H}_i \mathbb{K}_{is,m}) + \bar{\Lambda}_2^{(\ell)} \mathcal{J}\} + \Lambda_3 \mathcal{R} \Lambda_3^T - \Lambda_4 \mathcal{Z}_1 \Lambda_4^T \\
 &\quad - \Lambda_5 \mathcal{Z}_2 \Lambda_5^T + e_1 \bar{G}_i^T W_i \bar{G}_i e_1^T - \gamma^2 e_8 e_8^T + \epsilon_i \mathbb{V}_s^T \mathbb{V}_s, \\
 \bar{\Lambda}_1 &:= \begin{bmatrix} e_1 H^T - e_3 & e_3 - e_4 & \mathbf{0} \end{bmatrix}, \\
 \Upsilon_{is,m1} &:= \begin{bmatrix} \mathcal{J}^T & \bar{\Lambda}_2^{(\ell)} \Pi_m \mathcal{P}_{i,m} & \mathbb{A}_s^T \mathcal{Z}_0 & \mathbb{L}_s^T \end{bmatrix}^T, \\
 \Upsilon_{i,m2} &:= \text{diag}\{\lambda_{mm} \mathbb{P}_{i,m}, -\mathcal{P}_{i,m}, -\mathcal{Z}_0, -\mathbf{I}\}, \\
 \mathcal{P}_{i,m} &:= \begin{bmatrix} F_i^T P_{m1(3)}^T & F_i^T P_{m1(5)}^T & F_i^T P_{m1(6)}^T \bar{F}_i & \mathbf{0} \end{bmatrix}^T, \\
 \mathcal{H}_i &:= \begin{bmatrix} \rho_3 \bar{H}_2 & \rho_2 \bar{H}_2 & \bar{H}_3 \bar{F}_i & \rho_1 \mathbf{I} \end{bmatrix}^T, \\
 \bar{\mathcal{Z}}_0 &:= \begin{bmatrix} \mathbf{0}_{2k_1 \times N(3n_x+1)} & \begin{bmatrix} \mathcal{Z}_0 U_{s1} & \mathbf{0}_{n_x \times k_1} \end{bmatrix}^T \\ & \mathbf{0}_{2k_1 \times n_z} \end{bmatrix}, \\
 \mathbb{P}_{i,m} &:= \begin{bmatrix} P_{m1} & & \\ \begin{bmatrix} \rho_3 P_{i2(1)} \bar{H}_2 & \rho_2 P_{i2(1)} \bar{H}_2 & P_{i2(1)} \bar{H}_3 \\ & \mathbf{0} & \rho_1 P_{i2(1)} \end{bmatrix} \end{bmatrix}, \\
 P_{m1} &:= \begin{bmatrix} P_{m1(1)} & P_{m1(2)} & P_{m1(3)} \\ P_{m1(2)}^T & P_{m1(4)} & P_{m1(5)} \\ P_{m1(3)}^T & P_{m1(5)}^T & P_{m1(6)} \end{bmatrix}, \\
 \bar{H}_2 &:= \begin{bmatrix} \mathbf{I} & \mathbf{0}_{n_u \times (n_x - n_u)} \end{bmatrix}, \\
 \bar{H}_3 &:= \begin{bmatrix} \mathbf{I} & \mathbf{0}_{n_u \times (n_f - n_u)} \end{bmatrix}, \\
 \mathbb{A}_s &:= \begin{bmatrix} A_s & B_s & A_{ds} & \mathbf{0}_{n_x \times 5n_x} & D_{s1} \end{bmatrix}, \\
 \mathbb{K}_{is,m} &:= \begin{bmatrix} \bar{K}_i \Sigma_m C_s & -P_{i2(1)}^T & \bar{K}_i \Sigma_m C_{ds} \\ & \mathbf{0}_{n_u \times 5n_x} & \bar{K}_i \Sigma_m D_{s2} \end{bmatrix}^T, \\
 \mathbb{U}_{is,m} &:= \begin{bmatrix} \bar{\Lambda}_2^{(\ell)} \mathcal{P}_{i,m} U_{s1} & \bar{\Lambda}_2^{(\ell)} \mathcal{H}_i \bar{K}_i \Sigma_m U_{s2} \end{bmatrix}^T, \\
 \mathbb{L}_s &:= \begin{bmatrix} L_s & \mathbf{0}_{n_z \times n_u} & L_{ds} & \mathbf{0}_{n_z \times (5n_x + n_w)} \end{bmatrix}, \\
 \mathbb{V}_s &:= \begin{bmatrix} V_{1s} & V_{3s} & V_{2s} & \mathbf{0}_{k_1 \times (5n_x + n_w)} \\ V_{1s} & \mathbf{0}_{k_1 \times n_u} & \mathbf{0}_{k_1 \times n_x} & \mathbf{0}_{k_1 \times (5n_x + n_w)} \end{bmatrix}, \\
 &\quad \text{if } i \in \mathcal{J}_0, \\
 \mathbb{A}_s &:= \begin{bmatrix} a_s & A_s & B_s & A_{ds} & \mathbf{0}_{n_x \times 5n_x} & D_{s1} \end{bmatrix}, \\
 \mathbb{K}_{is,m} &:= \begin{bmatrix} \bar{k}_i & \bar{K}_i \Sigma_m C_s & -P_{i2(1)}^T & \bar{K}_i \Sigma_m C_{ds} \\ & \mathbf{0}_{n_u \times 5n_x} & \bar{K}_i \Sigma_m D_{s2} \end{bmatrix}, \\
 \mathbb{U}_{is,m} &:= \begin{bmatrix} \mathbf{0}_{(7n_x + n_u + n_w + 1) \times 1} & \bar{\Lambda}_2^{(\ell)} \mathcal{P}_{i,m} U_{s1} \\ & \bar{\Lambda}_2^{(\ell)} \mathcal{H}_i \bar{K}_i \Sigma_m U_{s2} \end{bmatrix}^T, \\
 \mathbb{L}_s &:= \begin{bmatrix} \mathbf{0}_{n_z \times 1} & L_s & \mathbf{0}_{n_z \times n_u} & L_{ds} \\ & & \mathbf{0}_{n_z \times (5n_x + n_w)} \end{bmatrix}, \\
 \mathbb{V}_s &:= \begin{bmatrix} V_{s4} & V_{s1} & V_{s3} & V_{s2} \\ \mathbf{0}_{k_1 \times 1} & V_{s1} & \mathbf{0}_{k_1 \times n_u} & \mathbf{0}_{k_1 \times n_x} \\ & & \mathbf{0}_{k_1 \times (5n_x + n_w)} \\ & & \mathbf{0}_{k_1 \times (5n_x + n_w)} \end{bmatrix}, \\
 &\quad \text{if } i \in \mathcal{J}_1,
 \end{aligned} \right\}, \quad (49)$$

with the other notations defined the same as in (20) and (38), respectively, then the closed-loop system in (13) is SS with a guaranteed $\mathcal{H}_\infty$ performance γ . Specifically, the desired controller gains can be computed as,

$$K_i = P_{i2(1)}^{-T} \bar{K}_i, \quad i \in \mathcal{J}, \quad k_i = P_{i2(1)}^{-T} \bar{k}_i, \quad i \in \mathcal{J}_1. \quad (50)$$

Proof. Please see the Appendix.

Remark 3.3. Theorem 3.3 proposes sufficient conditions on the solvability of delay-dependent reliable piecewise-affine $\mathcal{H}_\infty$ SOF controller design problem for T-S fuzzy-affine systems with time-varying delay and sensor faults. In comparison

to most of existing methods to SOF control for continuous-time fuzzy dynamic systems, Theorem 3.3 is characterized with some advantages in terms of less conservatism and much broader applications. Firstly, the SOF controller design procedure for continuous-time fuzzy-affine systems with time-delay in Theorem 3.3 is developed in the *piecewise-Markovian-Lyapunov-functionals* framework, which leads to the conservatism reduction of the analysis and synthesis conditions. While in [36] and [37], a common Lyapunov functional is adopted for the SOF controller design. Secondly, due to the fact that control input and measurement output matrices are simultaneously with parametric uncertainties ΔB_l and ΔC_l , $l \in \mathcal{L}$, especially the consideration of measurement noise $w(t)$ and existence of sensor faults, it is quite difficult to employ the commonly utilized approach as in [38], for reliable SOF controller synthesis. However, by a process of descriptor model transformation, the original closed-loop system is reformulated into a descriptor system, which makes all the controller gain variables K_i and k_i locate in the left of every closed-loop matrices in the $\bar{\mathcal{A}}_{i,m}$, $\bar{\mathcal{A}}_{di,m}$, and $\bar{\mathcal{D}}_{i,m}$. In this case, taking the linearisation procedure in (62)-(63), one can readily synthesize the piecewise-affine SOF controller without calculating the inverse of piecewise-Markovian Lyapunov matrices $\tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i$, $i \in \mathcal{J}$, $m \in \mathcal{I}$, which facilitates the realization of output feedback controller synthesis for fuzzy-affine systems. On the other hand, even if one may follow the similar way in [38] (namely, not a descriptor system-based approach) to design a piecewise-affine controller for delayed fuzzy-affine systems with perfect sensor and an uncertainty and noisy-free measurement outputs, i.e., $\Sigma_m = \mathbf{I}_{n_y}$, $\mathcal{I} = \{1\}$, $\Delta C_l = 0$ and $D_{l2} = 0$, some additional restrictions have to be made to reformulate the SOF controller synthesis procedure into a convex optimization problem. Especially, in [38], either the input matrices B_l or output matrices C_l are supposed to be of full-column/row rank, which is a prerequisite for SOF controller design of fuzzy systems. Then, due to the introduction of a congruent transformation with the inverse of piecewise Lyapunov functions $\bar{F}_i^{-1} P_{1(6)}^{-1} \bar{F}_i^{-T}$, it is also required that the continuity matrices $\bar{F}_i$ are invertible. To guarantee the latter, a stable virtual system usually needs to be constructed. Meantime, to facilitate the SOF controller synthesis, another constraint $\bar{G}_i = \bar{F}_i$ is necessarily imposed. Furthermore, a new term $\hat{W}_i := P_{1(6)}^{-1} W_i P_{1(6)}^{-1}$ arises after the congruent manipulation by $\bar{F}_i^{-1} P_{1(6)}^{-1} \bar{F}_i^{-T}$, and bear in mind that the original constraint on $W_i \succeq 0$ still exists. In this case, $P_{1(6)}^{-1} \in \mathbb{S}^{n_f}$ should be an M-matrix, which is nonsingular square with all off-diagonal entries being non-positive and all principal minors positive, to necessitate that $P_{1(6)} = \hat{P}_{1(6)}^{-1}$ is merely with nonnegative entries, i.e., $P_{1(6)} = P_{1(6)}^T > 0$ and $P_{1(6)} \succeq 0$. In this setting, $\hat{W}_i \succeq 0$ implies that $W_i = P_{1(6)} \hat{W}_i P_{1(6)} \succeq 0$. However, in this paper, with the application of system-augmentation, these structural constraints on the control input matrices and measurement output matrices are no more required. There is also no necessity to compute the inverse of the Lyapunov matrices $\bar{F}_i^{-1} P_{1(6)}^{-1} \bar{F}_i^{-T}$, and thus, the requirement of $\bar{G}_i = \bar{F}_i$, the imposition on $P_{1(6)}^{-1}$ being an M-matrix, and the construction of the virtual system are all eliminated. It is thus reasonable

to conclude that by the employment of system-augmentation, the decoupling technique and algebraic computation applied above follow in a much simpler and more elegant way than most traditional methods [15], [36]–[38], where substantial restrictions are imposed on the system matrices and Lyapunov functional variables.

Remark 3.4. Note that in Theorem 3.3, when specifying the continuity matrices $[f_i \ F_i] \equiv [0_{n_x \times 1} \ \mathbf{I}]$, one could immediately achieve another controller design results on the basis of following CQLF:

$$V_1(x(t), r(t), t) = \hat{x}^T(t) \tilde{E} \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \hat{x}(t) = \hat{x}^T(t) \hat{E} \hat{\mathbb{P}}_m \hat{x}(t), \quad (51)$$

where

$$\hat{x}(t) := \left[\int_{t-d_1}^t x^T(s) ds \quad \int_{t-d_2}^{t-d_1} x^T(s) ds \quad x^T(t) \quad u^T(t) \right]^T$$

and $\hat{E} := \text{diag}\{\mathbf{I}_{3n_x}, 0_{n_u \times n_u}\}$, and $V_2(x(t), r(t), t)$ and $V_3(x(t), r(t), t)$ are defined the same as in (23). Hence, one can readily claim that the CQLF is a special case of the more general PQLFs. In the subsequent section, it will also be shown the PQLFs-based SOF controller synthesis conditions are basically less conservative than the CQLF-based ones.

Remark 3.5. It is also noted that the performance analysis and synthesis conditions in Theorems 3.1–3.3 are developed for the fuzzy-affine systems subject to sensor faults Σ_m , $m \in \mathcal{I}$. In fact, the current results can be readily extended to the underlying system with ideal sensor, i.e., the sensor is perfectly in operation with $\Sigma_m = \mathbf{I}_{n_y}$, $\mathcal{I} = \{1\}$.

Remark 3.6. It is also worth pointing out that the conditions developed in Theorem 3.3 rely on the derivative of time-varying delay with $\dot{d}(t) \leq \mu < \infty$. Nevertheless, it can be readily extended for the case of delay-derivative-independent results by imposing $R_3 = 0$ in Theorem 3.3 for the underlying systems.

IV. SIMULATION RESULTS

To illustrate the effectiveness and less conservativeness of the presented conditions, two examples are introduced. The first one starts from a nonlinear model and shows the effectiveness of the proposed design procedure, whereas the second one is numerical and demonstrates the less conservatism of reliable piecewise-affine controller in the piecewise-Lyapunov-functionals framework over CQLF-based reliable piecewise-affine/linear one in Theorem 3.3.

Example 4.1. As characterized in Fig. 1, the purpose of this example is to synthesize a controller that drives a cart on the x - y plane to move along the path $y = 0$ by a constant velocity $v_0 = 1\text{m/s}$. The cart's motion trajectory is dominated by the torque u about the z -axis with respect to the subsequent dynamical equations [16]:

$$\begin{bmatrix} \dot{\theta}(t) \\ \dot{\psi}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{k}{T} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \theta(t) \\ \psi(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ v_0 \sin(\theta(t)) \end{bmatrix} + \begin{bmatrix} 0 & 0.05 & 0 \\ 0 & 0 & 0 \\ \frac{k}{T} & 0 & -\frac{k}{T} \end{bmatrix} \begin{bmatrix} \theta(t-d(t)) \\ \psi(t-d(t)) \\ y(t-d(t)) \end{bmatrix}$$

Fig. 1. Autonomous land vehicle in Example 4.1

$$+ \begin{bmatrix} \frac{1}{T} \\ \frac{1}{T} \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 0.5 \\ 0.1 \\ 0 \end{bmatrix} w(t), \quad (52)$$

where θ refers to the heading angle with time derivative ψ ; $T = 1 \text{ (kg} \cdot \text{m}^2\text{)}$ refers to the moment of inertia of the cart with regard to the center of mass; $k = 0.1$ represents the damping coefficient; u stands for the control torque; and w represents the exogenous disturbance input. We also assume that the origin of the trajectories can take any possible angle in the range $\theta \in [-\frac{3\pi}{5}, \frac{3\pi}{5}]$ and any initial distance from the line.

Define the system states as $x = [\theta^T(t) \ \psi^T(t) \ y^T(t)]^T$. It is noted that the orientation measurement of the vehicle induces the nonlinear term $\sin(\theta)$. By the linearizing process, the nonlinear dynamic model (52) can be approximately represented as a set of linear affine models corresponding to different operating points. Specifically, the linearisation of the nonlinear plant is carried out at the origin and $(\pm \frac{\pi}{15}, 0)$, respectively, and the error between the linearized model and the original nonlinear plant can be considered as the uncertainties. Finally, we end up with the following continuous-time T-S fuzzy-affine model with time-delay,

Plant Rule $\mathcal{F}^l$: IF $x_1(t)$ is $\mathcal{F}^l$, THEN

$$\begin{cases} \dot{x}(t) = (A_l + \Delta A_l)x(t) + (A_{dl} + \Delta A_{dl})x(t-d(t)) \\ \quad + a_l + \Delta a_l + (B_l + \Delta B_l)u(t) + D_{l1}w(t) \\ y(t) = C_l x(t) + C_{dl}x(t-d(t)) + D_{l2}w(t) \\ z(t) = L_l x(t), \quad l = 1, 2, 3 \end{cases} \quad (53)$$

where the system parameters are given in (54) and the parametric uncertainties are followed with

$$\begin{bmatrix} \Delta A_l & \Delta A_{dl} & \Delta B_l & \Delta a_l \end{bmatrix} = U_{l1} \Delta_l(t) \begin{bmatrix} V_{l1} & V_{l2} & V_{l3} & V_{l4} \end{bmatrix}, \quad l = 1, 2, 3$$

where

$$\begin{bmatrix} U_{l1}^T & | & V_{l1} & | & V_{l2} & | & V_{l3} & | & V_{l4} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0.2 & | & 0.2 & 0 & 0 & | & 0.2 & 0 & 0 & | & 0.1 & | & 0.1 \end{bmatrix}.$$

The measurement output and controlled output are specified,

$$\begin{bmatrix} A_1 & A_{d1} & B_1 & D_{11} & a_1 \\ A_2 & A_{d2} & B_2 & D_{21} & a_2 \\ A_3 & A_{d3} & B_3 & D_{31} & a_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0.05 & 0 & 1 & 0.5 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 1 & 0.1 & 0 \\ 1.3090 & 0 & 0 & 0.1 & 0 & -0.1 & 0 & 0 & 0.5627 \\ 0 & 1 & 0 & 0 & 0.05 & 0 & 1 & 0.5 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 1 & 0.1 & 0 \\ 1 & 0 & 0 & 0.1 & 0 & -0.1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0.05 & 0 & 1 & 0.5 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 1 & 0.1 & 0 \\ 1.3090 & 0 & 0 & 0.1 & 0 & -0.1 & 0 & 0 & -0.5627 \end{bmatrix} \quad (54)$$

Fig. 2. Membership functions in Examples 4.1 and 4.2 respectively, by

$$\begin{cases} y(t) = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0.2 & 0.1 & 0 \end{bmatrix} x(t-d(t)) + 0.5w(t) \\ z(t) = \begin{bmatrix} 0.5 & 0 & 0 \end{bmatrix} x(t) \end{cases}$$

The normalized membership functions are depicted in Fig. 2. In view of the space partition introduced in Section II, we attain three subspaces $\mathcal{S}_1 := \{x \in \mathbb{R}^2 | -\varphi_2 \leq x_1 \leq -\varphi_1\}$, $\mathcal{S}_2 := \{x \in \mathbb{R}^2 | -\varphi_1 \leq x_1 \leq \varphi_1\}$, $\mathcal{S}_3 := \{x \in \mathbb{R}^2 | \varphi_1 \leq x_1 \leq \varphi_2\}$, where φ_1 and φ_2 are chosen to be $\varphi_1 = \frac{\pi}{15}$ and $\varphi_2 = \frac{3\pi}{5}$, respectively. With regard to the information implicated in the fuzzy rule base, the constraint matrices F 's and G 's for the PQLFs can be computed in (55) with $\vartheta = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ and $n_x = 3$,

By the subspaces depicted in Fig. 2, one has $\mathcal{I} = \{1, 2, 3\}$, $\mathcal{S}(1) = \{1, 2\}$, $\mathcal{S}(2) = \{2\}$, $\mathcal{S}(3) = \{2, 3\}$. Obviously, $\mathcal{S}_2$ is a crisp subspace, and $\mathcal{S}_1$ and $\mathcal{S}_3$ are fuzzy subspaces. In this example, the time-varying delay $d(t) = 0.5 + 0.4 \sin 0.5t$, and thus the lower and upper bounds of time-delay are 0.1 and 0.9, respectively, and $\mu = 0.2$. Assume that there may exist sensor faults with 50% reduction in signal strength. Then, it is obvious that $\mathcal{I} = \{1, 2\}$, $\Sigma_1 = 1$, and $\Sigma_2 = 0.5$. The sensor failure rates are assumed to be $\lambda_{11} = -2$, $\lambda_{12} = 2$, $\lambda_{21} = 0.5$, and $\lambda_{22} = -0.5$.

The purpose is to synthesize a piecewise-affine SOF controller (11) such that the resulting closed-loop system is SS with an $\mathcal{H}_\infty$ performance $\gamma_{\min}$. Unfortunately, it is validated that there is no feasible solution by utilizing the delay-dependent approaches proposed in [18], [23], [24]. While

Fig. 3. One possible fault mode and time-varying delay in Example 4.1

Fig. 4. State responses of the closed-loop system in Example 4.1

applying Theorem 3.3 with $\rho_1 = \rho_2 = 0$ and $\rho_3 = 0.2$, it indeed leads to the $\mathcal{H}_\infty$ performance $\gamma_{\min} = 1.1255$ for the desired controller. The controller gains are characterized by (50) with the parameters,

$$\begin{bmatrix} K_1 & k_1 \end{bmatrix} = \begin{bmatrix} -5.0302 & 31.0563 \end{bmatrix}, \\ K_2 = -4.7797, \\ \begin{bmatrix} K_3 & k_3 \end{bmatrix} = \begin{bmatrix} -5.0305 & -31.0562 \end{bmatrix}.$$

To further verify the design method and the effectiveness

$$\begin{aligned}
\begin{bmatrix} F_1 & F_2 & F_3 & f_1 & f_2 & f_3 \end{bmatrix} &:= \begin{bmatrix} -\vartheta & \mathbf{0}_{1 \times n_x} & \mathbf{0}_{1 \times n_x} & -\varphi_1 & 0 & 0 \\ \mathbf{0}_{1 \times n_x} & \mathbf{0}_{1 \times n_x} & \vartheta & 0 & 0 & -\varphi_1 \\ \mathbf{I} & \mathbf{I} & \mathbf{I} & \mathbf{0}_{n_x \times 1} & \mathbf{0}_{n_x \times 1} & \mathbf{0}_{n_x \times 1} \end{bmatrix}, \\
\begin{bmatrix} G_1 & G_2 & G_3 & g_1 & g_2 & g_3 \end{bmatrix} &:= \begin{bmatrix} \vartheta & \mathbf{0}_{1 \times n_x} & \vartheta & \varphi_2 & 0 & -\varphi_1 \\ -\vartheta & \mathbf{0}_{1 \times n_x} & -\vartheta & -\varphi_1 & 0 & \varphi_2 \end{bmatrix}
\end{aligned} \quad (55)$$

Fig. 5. Response of the ratio $\sqrt{\int_0^{T_f} z^T(t)z(t)dt}/\sqrt{\int_0^{T_f} w^T(t)w(t)dt}$ in Example 4.1

of the presented results, simulations are performed with the initial condition $x(t) = [-3 \sin 0.1t \quad 2.5e^{0.01t} \quad -e^{0.01t}]^T$, $t \in [-0.9, 0]$. The time-varying delay $d(t) = 0.5 + 0.4 \sin 0.5t$ and one possible Markovian jumping sensor failure mode information are depicted in Fig. 3. The external disturbance is given by $w(t) = 0.5e^{-0.01t} \sin(t)$. In addition, in the simulation, to characterize practical circumstance, it is further assumed that the measurement output is simultaneously involved with external disturbance $w(t)$ and Gaussian white noise. Fig. 4 describes the state responses of the closed-loop systems, and Fig. 5 characterizes the time response of the ratio $\sqrt{\int_0^{T_f} z^T(t)z(t)dt}/\sqrt{\int_0^{T_f} w^T(t)w(t)dt}$ under zero initial condition. It can be inspected from Fig. 5 that the time response of ratio $\sqrt{\int_0^{T_f} z^T(t)z(t)dt}/\sqrt{\int_0^{T_f} w^T(t)w(t)dt}$ is less than 0.8, which is obviously less than the calculated disturbance attenuation level 1.1255, and thus the designed SOF controller achieves satisfactory performance in spite of the sensor faults existing as in Fig. 3.

To further confirm the advantages of PQLFs-based piecewise-affine controller synthesis conditions over CQLF-based piecewise-affine/linear ones on the basis of Theorem 3.3, in the sequel we perform another simulation example.

Example 4.2 Consider a continuous-time T-S fuzzy-affine system in (1) with time-varying delay, which includes three local models with parameters in (IV) and the uncertainties

$$\begin{aligned}
&\begin{bmatrix} U_{l1}^T & U_{l2} & V_{l1} & V_{l2} & V_{l3} & V_{l4} \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0.2 & 0.1 & 0.2 & 0 & 0.1 & 0.1 \\ & & & 0.1 & 0.2 \end{bmatrix}, \quad l = 1, 2, 3.
\end{aligned}$$

The Markovian jumping properties for the sensor faults are

chosen the same as in Example 4.1.

The membership functions are borrowed from Fig. 2 with $\xi(t) = x_1(t)$. According to the partition scheme described in Section II, we divide the state space into three subspaces: $\mathcal{S}_1 := \{x \in \mathbb{R}^2 | -\varphi_2 \leq x_1 \leq -\varphi_1\}$, $\mathcal{S}_2 := \{x \in \mathbb{R}^2 | -\varphi_1 \leq x_1 \leq \varphi_1\}$, $\mathcal{S}_3 := \{x \in \mathbb{R}^2 | \varphi_1 \leq x_1 \leq \varphi_2\}$, where φ_1 and φ_2 are selected to be $\varphi_1 = 5$ and $\varphi_2 = 30$, respectively. Similarly, the constraint matrices F 's and G 's can be computed as (55) with $\vartheta = [1 \quad 0]$ and $n_x = 2$.

The purpose here is to synthesize a piecewise-affine controller (11) for the above system with an $\mathcal{H}_\infty$ performance γ . Before carrying out the numerical tests, we first provide some illustrations on the structure of controller (11). First, when $k_i \equiv 0$, the piecewise-affine controller (11) reduces to a piecewise-linear one. Second, when the matrices $\Sigma_m \equiv 1$ and $\mathcal{I} = \{1\}$, then the sensor is fault-free.

To this end, by applying Theorem 3.3, together with the consideration of Remarks 3.4, 3.5 and 3.6, a detailed comparison of the attained minimum $\mathcal{H}_\infty$ performance indices $\gamma_{\min}$ by the above-mentioned piecewise affine/linear controllers in the context of faulty/fault-free sensors and with fixed lower bound of time-delay $d_1 = 0.1$, and different upper bounds of time-delay d_2 and delay-derivatives is shown in Table I, where as mentioned in Remark 3.6, unknown μ indicates the corresponding delay-derivative-independent controller design results.

By inspection of Table I, one can clearly observe that: (i) The PQLFs-based controller synthesis approaches are obviously with less conservatism than the CQLF-based ones; (ii) The obtained $\mathcal{H}_\infty$ performance indices $\gamma_{\min}$ based upon piecewise-affine controllers are generally better than those based upon piecewise-linear ones. Hence, the superiority of the PQLFs-based piecewise-affine controller synthesis strategy is apparent.

V. CONCLUSIONS

In this paper, the delay-dependent reliable piecewise-affine $\mathcal{H}_\infty$ SOF control problem for continuous-time T-S fuzzy-affine systems with time-varying delay and sensor faults has been investigated in a descriptor system framework. By constructing a piecewise-Markovian LKF, combined with the Wirtinger-based integral inequality, improved reciprocally convex inequality and S-procedure, a new bounded real lemma has been firstly presented for the underlying fuzzy-affine system, and then the piecewise-affine controller synthesis has been developed. It has been shown that the desired controller gains can be computed by solving a set of strict LMIs. Finally, simulation examples have been performed to state the effectiveness and less conservatism of the developed approach.

$$\begin{aligned}
\begin{bmatrix} A_1 & A_{d1} & B_1 & D_{11} & a_1 \\ C_1 & C_{d1} & & D_{21} & \\ L_1 & L_{d1} & & & \end{bmatrix} &= \begin{bmatrix} 0.6 & 1.2 & 0.3 & -0.2 & 1 & 0.6 & 0 \\ 0 & -0.6 & 0 & 0.1 & 1 & 0.3 & 0.5 \\ 1 & 0.5 & 0.3 & 0.2 & & -0.3 & \\ 1 & -0.2 & 0 & 0.1 & & & \\ -1 & 2 & 0.2 & 0.1 & 1 & 0.9 & 0 \\ 0 & 1 & -0.1 & -0.2 & 1 & 0.3 & 0 \\ 1 & 0.5 & 0.3 & 0.2 & & -0.3 & \\ 1 & -0.2 & 0 & 0.1 & & & \end{bmatrix}, \\
\begin{bmatrix} A_2 & A_{d2} & B_2 & D_{12} & a_2 \\ C_2 & C_{d2} & & D_{22} & \\ L_2 & L_{d2} & & & \end{bmatrix} &= \begin{bmatrix} 0 & 0.5 & -0.01 & -0.01 & 1 & 0.6 & 0 \\ 0 & 0 & 0 & -0.02 & 1 & 0.3 & -0.5 \\ 1 & 0.5 & 0.3 & 0.2 & & -0.3 & \\ 1 & -0.2 & 0 & 0.1 & & & \end{bmatrix}, \\
\begin{bmatrix} A_3 & A_{d3} & B_3 & D_{13} & a_3 \\ C_3 & C_{d3} & & D_{23} & \\ L_3 & L_{d3} & & & \end{bmatrix} &= \begin{bmatrix} 0 & 0.5 & -0.01 & -0.01 & 1 & 0.6 & 0 \\ 0 & 0 & 0 & -0.02 & 1 & 0.3 & -0.5 \\ 1 & 0.5 & 0.3 & 0.2 & & -0.3 & \\ 1 & -0.2 & 0 & 0.1 & & & \end{bmatrix},
\end{aligned} \tag{56}$$

TABLE I
COMPARISON OF MINIMUM $\mathcal{H}_\infty$ PERFORMANCE FOR PIECEWISE AFFINE AND LINEAR CONTROLLERS IN EXAMPLE 4.2

Methods	Controllers	$\mu = 0.2$		$\mu = 0.5$		$\mu = 0.9$		unknown μ	
		$d_2 = 0.5$	$d_2 = 0.8$	$d_2 = 0.5$	$d_2 = 0.8$	$d_2 = 0.5$	$d_2 = 0.8$	$d_2 = 0.5$	$d_2 = 0.8$
PQLFs-based	Case-I	0.8213	1.7437	0.8225	1.7502	0.8243	1.7719	0.8277	1.7741
	Case-II	0.8941	1.8513	0.8953	1.8600	0.8971	1.8793	0.8997	1.8972
	Case-III	0.4101	0.4510	0.4103	0.4513	0.4311	0.4515	0.4315	0.4518
	Case-IV	0.4487	0.4833	0.4490	0.4838	0.4492	0.4840	0.4495	0.4853
CQLF-based	Case-I	0.9537	2.0360	0.9538	2.0631	0.9625	2.0704	0.9668	2.0718
	Case-II	1.1388	2.1883	1.1432	2.1930	1.1486	2.2064	1.1501	2.2175
	Case-III	0.9062	1.2049	0.9064	1.2865	0.9068	1.2871	0.9069	1.2881
	Case-IV	0.9460	1.2494	0.9714	1.3394	0.9750	1.3399	0.9788	1.3409

Case-I: piecewise-affine controller in the context of sensor faults

Case-II: piecewise-linear controller in the context of sensor faults

Case-III: piecewise-affine controller in the context of sensor fault-free, i.e., $\Sigma_m = \mathbf{I}_{n_y}$, $\mathcal{I} = \{1\}$

Case-IV: piecewise-linear controller in the context of sensor fault-free, i.e., $\Sigma_m = \mathbf{I}_{n_y}$, $\mathcal{I} = \{1\}$

It is noted that the extensions of the proposed method to the controller design for continuous-time fuzzy-affine dynamic systems with integrated communication delays in the networked circumstance, deserve further investigation. In addition, applications of the proposed theoretical results to some real-world complex nonlinear systems such as the active queue management (AQM) systems [35], continuous stirred-tank reactor (CSTR) systems [36], three-tank system [37] etc., are also part of our future works.

APPENDIX

A. Useful Lemmas

Lemma A1. [39] Consider a given matrix $Z \in \mathbb{S}^n$. Then, for all continuous function $\omega \in [a, b] \rightarrow \mathbb{R}^n$, the following inequality holds:

$$\int_a^b \omega^T(\tau) Z \omega(\tau) d\tau \geq \frac{1}{b-a} \left(\int_a^b \omega(\tau) d\tau \right)^T Z \left(\int_a^b \omega(\tau) d\tau \right) + \frac{3}{b-a} \varsigma^T Z \varsigma, \tag{57}$$

where $\varsigma := \int_a^b \omega(\tau) d\tau - \frac{2}{b-a} \int_a^b \int_a^s \omega(\tau) d\tau ds$.

Remark A1. Due to the fact that $Z > 0$, the second term in the RHS of (57) is definitely positive. It is thus obvious that the inequality proposed in Lemma A1 covers the celebrated Jensen's inequality

$$\begin{aligned}
(b-a) \int_a^b \omega^T(\tau) Z \omega(\tau) d\tau \\
\geq \left(\int_a^b \omega(\tau) d\tau \right)^T Z \left(\int_a^b \omega(\tau) d\tau \right).
\end{aligned} \tag{58}$$

In such situation, Lemma A1 is generally more accurate and thus less conservative than the Jensen's inequality (58).

Lemma A2 (Reciprocally convex inequality). [40] Given two matrices $Z_1 \in \mathbb{S}^{n_1}$ and $Z_2 \in \mathbb{S}^{n_2}$, the improved reciprocally convex combination guarantees that, if there exists a matrix $X \in \mathbb{R}^{n_1 \times n_2}$ such that $\begin{bmatrix} Z_1 & X \\ X^T & Z_2 \end{bmatrix} \geq 0$, then the inequality

$$\begin{bmatrix} \frac{1}{\alpha} Z_1 & \mathbf{0} \\ \mathbf{0} & \frac{1}{1-\alpha} Z_2 \end{bmatrix} \geq \begin{bmatrix} Z_1 & X \\ X^T & Z_2 \end{bmatrix},$$

holds for any scalar $\alpha \in (0, 1)$.

Lemma A3 (Projection lemma). [41] Considering matrices $\Psi = \Psi^T \in \mathbb{R}^{n \times n}$, $\mathcal{M} \in \mathbb{R}^{m_1 \times n}$, $\mathcal{N} \in \mathbb{R}^{m_2 \times n}$, the linear matrix inequality $\Psi + \mathcal{M}^T \mathcal{G}^T \mathcal{N} + \mathcal{N}^T \mathcal{G} \mathcal{M} < 0$ holds, if and only if

$$\begin{cases} \mathcal{M}_\perp^T \Psi \mathcal{M}_\perp < 0, & \text{if } \mathcal{N}_\perp = \mathbf{0}, \mathcal{M}_\perp \neq \mathbf{0} \\ \mathcal{N}_\perp^T \Psi \mathcal{N}_\perp < 0, & \text{if } \mathcal{M}_\perp = \mathbf{0}, \mathcal{N}_\perp \neq \mathbf{0} \\ \mathcal{M}_\perp^T \Psi \mathcal{M}_\perp < 0, \mathcal{N}_\perp^T \Psi \mathcal{N}_\perp < 0, & \text{if } \mathcal{N}_\perp \neq \mathbf{0}, \mathcal{M}_\perp \neq \mathbf{0} \end{cases}$$

where $\mathcal{M}_\perp$ and $\mathcal{N}_\perp$ represent the right null spaces of $\mathcal{M}$ and $\mathcal{N}$, respectively.

Lemma A4. [41] With appropriately dimensioned real matrices $\mathbb{W} = \mathbb{W}^T$, $\mathbb{U}$, $\mathbb{V}$, and $\Delta(t)$, the condition $\mathbb{W} + \text{Sym}\{\mathbb{U}\Delta(t)\mathbb{V}\} < 0$ is feasible for all admissible $\Delta(t)$ satisfying $\Delta^T(t)\Delta(t) \leq \mathbf{I}$, if and only if $\mathbb{W} + \epsilon \mathbb{U}\mathbb{U}^T + \epsilon^{-1} \mathbb{V}^T \mathbb{V} < 0$ for some positive scalar ϵ .

B. Proof of Theorem 3.3

The claimed result follows from the condition (37) in Theorem 3.2. Based on (9), the inequality (37) can be explicitly

reformulated in the following form:

$$\sum_{s \in \mathcal{S}(i)} \mu_s \begin{bmatrix} \hat{\Xi}_{is,m}^{(\ell)} & * & * \\ \mathcal{J} & \lambda_{mm} \bar{\mathbb{P}}_{i,m} & * \\ \mathcal{P}_{i,m} \Pi_m^T \bar{\Lambda}_2^{(\ell)T} & \mathbf{0} & -\mathcal{P}_{i,m} \end{bmatrix} < 0, \quad i \in \mathcal{I}_1, \quad (59)$$

where

$$\begin{cases} \hat{\Xi}_{is,m}^{(\ell)} := \text{Sym}\{\hat{\Lambda}_{is,m1} \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \Lambda_2^{(\ell)T} + \bar{\Lambda}_2^{(\ell)} \mathcal{J}\} \\ \quad + \Lambda_3 \mathcal{R} \Lambda_3^T - \Lambda_4 \mathcal{Z}_1 \Lambda_4^T - \Lambda_5 \mathcal{Z}_1 \Lambda_5^T \\ \quad + \mathcal{A}_{is,m}^T H^T \mathcal{Z}_0 H \mathcal{A}_{is,m} + e_1 \bar{G}_i^T W_i \bar{G}_i e_1^T \\ \quad + \mathbb{L}_s^T \mathbb{L}_s - \gamma^2 e_8 e_8^T, \\ \hat{\Lambda}_{is,m1} := \begin{bmatrix} e_1 H^T - e_3 & e_3 - e_4 & \mathcal{A}_{is,m}^T \end{bmatrix}, \\ \mathcal{A}_{is,m} := \begin{bmatrix} \hat{\Lambda}_{is,m1(1)} & \hat{\Lambda}_{is,m1(2)} & \hat{\Lambda}_{is,m1(3)} \end{bmatrix}, \\ \hat{\Lambda}_{is,m1(1)} := \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ a_s + \Delta a_s & A_s + \Delta A_s & B_s + \Delta B_s \\ k_i & K_i \Sigma_m (C_s + \Delta C_s) & -\mathbf{I} \end{bmatrix}, \\ \hat{\Lambda}_{is,m1(2)} := \begin{bmatrix} \mathbf{0} \\ A_{ds} + \Delta A_{ds} \\ K_i \Sigma_m C_{ds} \end{bmatrix}, \\ \hat{\Lambda}_{is,m1(3)} := \begin{bmatrix} \mathbf{0} \\ D_{s1} \\ K_i \Sigma_m D_{s2} \end{bmatrix}, \\ \mathbb{L}_s := \begin{bmatrix} \mathbf{0}_{n_z \times 1} & L_s & \mathbf{0}_{n_z \times n_u} & L_{ds} & \mathbf{0}_{n_z \times (5n_x + n_w)} \end{bmatrix}. \end{cases} \quad (60)$$

Considering the inherently nonnegative characteristic of the fuzzy-basis functions, together with Schur complement, one then readily arrives at the following inequality implying (59):

$$\begin{bmatrix} \hat{\Xi}_{is,m}^{(\ell)} & * & * \\ \mathcal{J} & \lambda_{mm} \bar{\mathbb{P}}_{i,m} & * \\ \mathcal{P}_{i,m} \Pi_m^T \bar{\Lambda}_2^{(\ell)T} & \mathbf{0} & -\mathcal{P}_{i,m} \end{bmatrix} < 0, \quad s \in \mathcal{S}(i), \quad i \in \mathcal{I}_1, \quad m \in \mathcal{I}, \quad \ell = 1, 2. \quad (61)$$

For tractability and conservatism reduction of the controller synthesis procedure, and by inspection of the inner structure of system matrices in (14), where we can observe that the controller gains do not appear in the first two-rows of the closed-loop system matrices $\bar{\mathcal{A}}_{i,m}$, $\bar{\mathcal{A}}_{di,m}$ and $\bar{\mathcal{D}}_{i,m}$, we can thus prescribe the Lyapunov matrices P_{m1} , P_{i2} and P_{i3} implicitly in (61) as

$$\begin{cases} P_{m1} := \begin{bmatrix} P_{m1(1)} & P_{m1(2)} & P_{m1(3)} \\ P_{m1(2)}^T & P_{m1(4)} & P_{m1(5)} \\ P_{m1(3)}^T & P_{m1(5)}^T & P_{m1(6)} \end{bmatrix}, \\ P_{i2} := [\rho_3 P_{i2(1)} \bar{H}_2 \quad \rho_2 P_{i2(1)} \bar{H}_2 \quad P_{i2(1)} \bar{H}_3], \\ P_{i3} := \rho_1 P_{i2(1)}, \quad i \in \mathcal{I}_1, \end{cases} \quad (62)$$

where $\{P_{m1(1)}, P_{m1(4)}\} \in \mathbb{S}^{n_x}$, $P_{m1(2)} \in \mathbb{R}^{n_x \times n_x}$, $\{P_{m1(3)}, P_{m1(5)}\} \in \mathbb{R}^{n_x \times n_f}$, $P_{m1(6)} \in \mathbb{R}^{n_f \times n_f}$, $P_{i2(1)} \in \mathbb{R}^{n_u \times n_u}$, $\bar{H}_2 := [\mathbf{I} \quad \mathbf{0}_{n_u \times (n_x - n_u)}]$, $\bar{H}_3 := [\mathbf{I} \quad \mathbf{0}_{n_u \times (n_f - n_u)}]$.

It is noted that in this way the Lyapunov variable $P_{i2(1)}$ can be absorbed by the controller gain variables K_i and k_i by

introducing

$$\bar{K}_i := P_{i2(1)}^T K_i, \quad \bar{k}_i := P_{i2(1)}^T k_i. \quad (63)$$

Then, with the substitution of matrices P_{m1} , P_{i2} and P_{i3} defined in (62) into (61) and application of Schur complement, a slight technical effort with reference to uncertainty relationships characterized in (3) and (4) leads to the following condition

$$\begin{bmatrix} \hat{\Xi}_{is,m}^{(\ell)} & * \\ \Upsilon_{is,m1} & \Upsilon_{i,m2} \end{bmatrix} + \tilde{\mathbb{U}}_{is,m}^T \begin{bmatrix} \Delta_s(t) & \\ & \Delta_s(t) \end{bmatrix} \tilde{\mathbb{V}}_s < 0, \quad s \in \mathcal{S}(i), \quad i \in \mathcal{I}_1, \quad m \in \mathcal{I}, \quad \ell = 1, 2, \quad (64)$$

which is reformulated from (61), and

$$\begin{cases} \tilde{\Xi}_{is,m}^{(\ell)} := \text{Sym}\{\bar{\Lambda}_1 \tilde{F}_i^T \mathbb{P}_{i,m} \tilde{F}_i \Lambda_2^{(\ell)T} + \Lambda_2^{(\ell)} (\mathcal{P}_{i,m} \mathbb{A}_s \\ \quad + \mathcal{H}_i \mathbb{K}_{is,m}) + \bar{\Lambda}_2^{(\ell)} \mathcal{J}\} + \Lambda_3 \mathcal{R} \Lambda_3^T - \Lambda_4 \mathcal{Z}_1 \Lambda_4^T \\ \quad - \Lambda_5 \mathcal{Z}_2 \Lambda_5^T + e_1 \bar{G}_i^T W_i \bar{G}_i e_1^T - \gamma^2 e_8 e_8^T, \\ \tilde{\mathbb{U}}_{is,m} := [\mathbb{U}_{is,m} \quad \bar{\mathcal{Z}}_0], \\ \tilde{\mathbb{V}}_s := [\mathbb{V}_s \quad \mathbf{0}_{2k_2 \times (N(3n_x+1)+n_x+n_z)}] \end{cases} \quad (65)$$

with $\bar{\Lambda}_1$, $\Lambda_2^{(\ell)}$, $\bar{\Lambda}_2^{(\ell)}$, $\mathbb{A}_s$, $\mathbb{K}_{is,m}$, $\mathcal{P}_{i,m}$, $\mathcal{H}_i$, $\bar{\mathcal{Z}}_0$, $\Upsilon_{is,m1}$, $\Upsilon_{i,m2}$, $\mathbb{V}_s$ and $\mathbb{U}_{is,m}$ defined in (49). Hence, according to Lemma A4, with the introduction of a set of scalars $\epsilon_i > 0$, $i \in \mathcal{I}_1$, and by Schur complement, it can be readily inspected that (48) leads to (61).

Additionally, the conditions in (48) result in $-P_{i2(1)} - P_{i2(1)}^T < 0$, which means that $P_{i2(1)}$ is invertible. Then, the controller gains can be calculated by (50). This completes the proof. ■

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