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Strongly γ-Deformed $\mathcal{N}=4$ Supersymmetric Yang-Mills Theory as an Integrable Conformal Field Theory

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We demonstrate by explicit multiloop calculation that γ-deformed planar $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM) theory, supplemented with a set of double-trace counterterms, has two nontrivial fixed points in the recently proposed double scaling limit, combining vanishing 't Hooft coupling and large imaginary deformation parameter. We provide evidence that, at the fixed points, the theory is described by an integrable nonunitary four-dimensional conformal field theory. We find a closed expression for the four-point correlation function of the simplest protected operators and use it to compute the exact conformal data of operators with arbitrary Lorentz spin. We conjecture that both conformal symmetry and integrability should survive in γ-deformed planar $\mathcal{N}=4$ SYM theory for arbitrary values of the deformation parameters.

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Introduction.—The most general theory which admits an AdS_5 dual description in terms of a string σ-model [1,2] is believed to be γ-deformed $\mathcal{N}=4$ supersymmetric Yang-Mills (SYM theory) [3,4]. At the classical level, this σ model is integrable and conformal. At the quantum level, it admits a solution in terms of the γ-deformed quantum spectral curve (QSC_γ) [5–9]. It is not obvious, however, that this solution yields the correct description of γ-deformed planar $\mathcal{N}=4$ SYM theory at any 't Hooft coupling $g^2=g_{\mathrm{YM}}^2 N_c$, since it automatically implies conformal symmetry and integrability of the theory. Both properties are highly debated, especially due to the loss of supersymmetry for the general deformation parameters $\gamma_1,\gamma_2,\gamma_3$, breaking the R symmetry $SU(4)\to U(1)^3$.

The main danger for both conformality and integrability in this theory comes from the fact that γ-deformed $\mathcal{N}=4$ SYM theory is not complete at the quantum level [10–12]. Namely, in order to preserve renormalizability, it has to be supplemented with new double-trace counterterms of the kind $\mathrm{tr}(\phi_j\phi_k^\dagger)\mathrm{tr}(\phi_k\phi_j^\dagger)$ and $\mathrm{tr}(\phi_j\phi_k)\mathrm{tr}(\phi_j^\dagger\phi_k^\dagger)$, with $\phi_{j=1,2,3}$ being a complex scalar field [10,13,14]. The corresponding coupling constants run with the scale, thus breaking the conformal symmetry. For example, for the double-trace interaction term $\alpha_{jj}^2\mathrm{tr}(\phi_j\phi_j)\mathrm{tr}(\phi_j^\dagger\phi_j^\dagger)$ the one-loop β function is given by [11]

$$\beta_{\alpha_{jj}^2}=\frac{g^4}{\pi^2}\sin^2\gamma_j^+\sin^2\gamma_j^-+\frac{\alpha_{jj}^4}{4\pi^2}, \tag{1}$$

where $\gamma_1^\pm=\mp\frac{1}{2}(\gamma_2\pm\gamma_3)$, $\gamma_2^\pm=\mp\frac{1}{2}(\gamma_3\pm\gamma_1)$, and $\gamma_3^\pm=\mp\frac{1}{2}(\gamma_1\pm\gamma_2)$. However, at weak coupling, the β function has two fixed points $\beta_{\alpha_{jj}^2}(\alpha_{jj})=0$,

$$\alpha_{jj}^2=\pm 2ig^2\sin\gamma_j^+\sin\gamma_j^-+O(g^4). \tag{2}$$

At these fixed points, which should persist at arbitrary values of g^2 and arbitrary N_c, γ-deformed $\mathcal{N}=4$ SYM theory should be a genuine nonsupersymmetric conformal field theory (CFT). In addition, it is natural to conjecture that the QSC_γ formalism gives the integrability description of this theory in the planar limit precisely at the fixed points.

To elucidate the role of the double-trace couplings we examine the scaling dimensions of the operators $\mathrm{tr}(\phi_j^J)$. Such operators are protected in the undeformed theory but receive quantum corrections for nonzero deformation parameters γ_i. For $J\geq 3$ the contribution of the double-trace terms to their anomalous dimensions γ_J is suppressed in the planar limit [15]. However, this is not the case for $J=2$ for which $\gamma_{J=2}$ would diverge without the double-trace coupling contribution. At the fixed points (2), we get a finite but complex anomalous dimension

$$\gamma_{J=2}(g) = \mp \frac{ig^2}{2\pi^2} \sin\gamma_j^+ \sin\gamma_j^- + O(g^4). \tag{3}$$

This means that we deal with a nonunitary CFT.

In this Letter we confirm explicitly, in the double scaling (DS) limit introduced in [15], that γ-deformed planar $\mathcal{N}=4$ SYM theory does have a conformal fixed point parameterized by g^2 and the three deformation parameters γ_1, γ_2, γ_3. The existence of this fixed point was first discussed in [12] in the DS limit.

The double scaling limit of γ-deformed $\mathcal{N}=4$ SYM theory [15] combines the $g \to 0$ limit and large imaginary twists $e^{-i\gamma_j/2} \to \infty$, so that $\xi_j = g e^{-i\gamma_j/2}$ remain finite. In particular, for $\xi_1 = \xi_2 = 0$ and $\xi_3 \equiv 4\pi\xi \neq 0$, one obtains a nonunitary, biscalar theory [15]

$$\mathcal{L}_\phi = N_c \mathrm{tr}\left(\sum_{i=1,2} \partial^\mu \phi_i^\dagger \partial_\mu \phi_i + (4\pi)^2 \xi^2 \phi_1^\dagger \phi_2^\dagger \phi_1 \phi_2\right). \tag{4}$$

In this limit the double-trace counterterms become

$$\mathcal{L}_{\mathrm{dt}}/(4\pi)^2 = \alpha_1^2 \sum_{i=1}^{2} \mathrm{tr}(\phi_i\phi_i)\mathrm{tr}(\phi_i^\dagger\phi_i^\dagger) + -\alpha_2^2 \mathrm{tr}(\phi_1\phi_2)\mathrm{tr}(\phi_2^\dagger\phi_1^\dagger)$$
$$-\alpha_3^2 \mathrm{tr}(\phi_1\phi_2^\dagger)\mathrm{tr}(\phi_2\phi_1^\dagger), \tag{5}$$

where $\alpha_1^2 \equiv \alpha_{11}^2/(4\pi)^2 = \alpha_{22}^2/(4\pi)^2$. In the DS limit, relations (2) and (3) simplify as $\alpha_1^2 = \mp \xi^2/2$ and $\gamma_{J=2} = \pm 2i\xi^2$ [12].

In this Letter we compute the β functions for the double-trace couplings at seven loops, confirming that the biscalar theory with Lagrangian $\mathcal{L}_\phi + \mathcal{L}_{\mathrm{dt}}$ given by (4) and (5) is a genuine nonunitary CFT at any coupling ξ. We examine the two-point correlation functions of the operators $\mathrm{tr}(\phi_1\phi_2)$ and $\mathrm{tr}(\phi_1\phi_2^\dagger)$ in this theory and find that they are protected in the planar limit. Moreover, we compute exactly, for any ξ, the four-point function of such protected operators and apply the operator product expansion (OPE) to show that the scaling dimension of the operator $\mathrm{tr}(\phi_1\phi_1)$ satisfies the remarkably simple exact relation

$$(\Delta-4)(\Delta-2)^2\Delta = 16\xi^4. \tag{6}$$

Its solutions define four different functions $\Delta(\xi)$. At weak coupling, the solutions $\Delta = 2 \mp 2i\xi^2 + O(\xi^6)$ describe scaling dimensions of the operator $\mathrm{tr}(\phi_i\phi_i)$ (with $i=1$, 2) at the two fixed points. The two remaining solutions, $\Delta = 4 + \xi^4 + O(\xi^8)$ and $\Delta = -\xi^4 + O(\xi^8)$, describe scaling dimensions of a twist-four operator, carrying the same $U(1)$ charge $J=2$, and its shadow, respectively.

As another manifestation of integrability of the biscalar theory, relation (6) can be reproduced [16] by means of the QSC formalism [5–7] (see [17]).

Perturbative conformality of biscalar theory.—In order to compute the β functions for the double-trace couplings, we consider the following two-point correlation functions of dimension-2 operators:

$$G_1(x) = \langle \mathrm{tr}[\phi_1\phi_1(x)]\mathrm{tr}[\phi_1^\dagger\phi_1^\dagger(0)]\rangle,$$
$$G_2(x) = \langle \mathrm{tr}[\phi_1\phi_2(x)]\mathrm{tr}[\phi_1^\dagger\phi_2^\dagger(0)]\rangle,$$
$$G_3(x) = \langle \mathrm{tr}[\phi_1\phi_2^\dagger(x)]\mathrm{tr}[\phi_1^\dagger\phi_2(0)]\rangle. \tag{7}$$

The reason for this choice is that, in the planar limit, each G_i receives contributions from Feynman diagrams involving double-trace interaction vertices of one kind only. As a consequence, G_i depends only on two coupling constants, ξ and α_i. For arbitrary values of the couplings α_i, the renormalized correlation function $G_i(x)$ satisfies the Callan-Symanzik evolution equation depending on the β function for the coupling α_i.

To compute the correlation functions (7) we employ dimensional regularization with $d = 4-2\epsilon$. We start with G_2 and G_3. In the planar limit, they receive contributions from Feynman diagrams shown in Fig. 1 (left). They consist of a chain of scalar loops joined together through single- or double-trace vertices. In momentum space, their contribution to G_i forms a geometric progression. In configuration space, the bare correlation function is given by

$$G_i/G_i^{(0)} = \sum_{\ell\geq 0} (\xi^2 - \alpha_i^2)^\ell (\pi x^2)^{\ell\epsilon} \times \frac{\Gamma((2-(\ell+2)\epsilon))\Gamma^{\ell+1}(\epsilon)\Gamma^{2\ell}(1-\epsilon)}{\Gamma((\ell+1)\epsilon))\Gamma^{\ell+1}(2-2\epsilon)}, \tag{8}$$

where $i = 2, 3$ and $G_i^{(0)}$ denotes the Born-level contribution. To obtain a finite result for the correlation function, we have to replace bare couplings with their renormalized values

$$\xi^2 \to \mu^{2\epsilon}\xi^2, \qquad \alpha_i^2 \to \mu^{2\epsilon}\alpha_i^2 Z_i, \tag{9}$$

and perform renormalization of the operators in (7) by multiplying G_i by the corresponding Z_{G_i} factor. Requiring $Z_{G_i}G_i$ to be finite for $\epsilon \to 0$ leads to the following expression for Z_i in the minimal subtraction scheme:

$$Z_i = 1 - \frac{(\alpha_i^2 - \xi^2)^2}{\alpha_i^2(\alpha_i^2 - \xi^2 - \epsilon)}, \qquad (i = 2, 3). \tag{10}$$

In the standard manner, we use this relation to find the exact β function for the coupling α_i (for $i = 2$, 3)

$$\beta_i = 2\epsilon\alpha_i^2 + \frac{d\alpha_i^2}{d\ln\mu} = -\alpha_i^2 \frac{d\ln Z_i}{d\ln\mu} = 2(\alpha_i^2 - \xi^2)^2. \tag{11}$$

We deduce from this relation that the β functions vanish for $\alpha_i^2 = \xi^2$, which also implies $Z_i = 1$. As follows from (8), the correlation function at the fixed point is $G_i = G_i^{(0)}$, so that the operators $\mathrm{tr}(\phi_1\phi_2)$ and $\mathrm{tr}(\phi_1\phi_2^\dagger)$ are protected.

The calculation of the correlation function G_1 is more involved. In the planar limit it receives contributions from Feynman diagrams shown in Fig. 1 (right) and those known as "wheel" diagrams [19]. In momentum space, their contribution to G_1 factorizes into a product of Feynman integrals I_L that form a geometric progression,

$$\tilde{G}_1(p) = \int d^4x e^{ipx} G_1(x) = \sum_{\ell\geq 0} \alpha_1^{2\ell} \left[\sum_{L\geq 0} \xi^{4L} I_L(p)\right]^{\ell}. \quad (12)$$

Here the sum over ℓ runs over double-trace vertices and $I_L(p)$ denote $(2L+1)$-loop scalar "wheel" integrals with $2L$ internal vertices shown in Fig. 1 (right).

In dimensional regularization, $I_L(p)$ takes the form $I_L(p^2) = (p^2)^{-(2L+1)\epsilon}/\epsilon^{2L+1}(c_0 + c_1\epsilon + \cdots)$, with L-dependent coefficients c_i. Expressions for I_L at $L = 0$, 1 are known in the literature [20]; we computed I_L for $L = 2$, 3. Using the obtained expressions we determined the expression for the bare correlation function $G_1(x)$ up to seventh order in perturbation theory. Going through the renormalization procedure, we use (9) to express $G_1(x)$ in terms of renormalized coupling constants ξ^2, α_1^2 and require $Z_{G_1} G_1(x)$ to be finite for $\epsilon \to 0$. This fixes the coupling renormalization factor Z_1 and allows us to compute the corresponding β function $\beta_1 = -\alpha_1^2 d \ln Z_1 / d \ln \mu$.

For β_1 we obtained in the minimal subtraction scheme

$$\beta_1 = a(\xi) + \alpha_1^2 b(\xi) + \alpha_1^4 c(\xi), \quad (13)$$

where the functions a, b, and c are given by

$$\begin{aligned} a &= -\xi^4 + \xi^8 - \frac{4}{3}\xi^{12} + O(\xi^{16}), \\ b &= -4\xi^4 + 4\xi^8 - \frac{88}{5}\xi^{12} + O(\xi^{16}), \\ c &= -4 - 4\xi^4 + \frac{4}{3}\xi^8 + O(\xi^{12}). \end{aligned} \quad (14)$$

At weak coupling their expansion runs in powers of ξ^4. Similarly to (11), β_1 is a quadratic polynomial in the double-trace coupling α_1^2. This property follows from the structure of irreducible divergent subgraphs of Feynman diagrams shown in Fig. 1. As a consequence, β_1 has two fixed points,

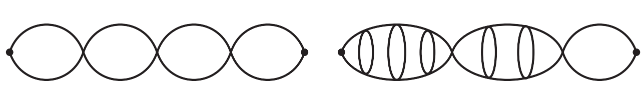

FIG. 1. Feynman diagrams contributing to two-point correlation functions G_2, G_3 (left) and G_1 (right) in the planar limit. Interaction vertices in the left diagram describe either the single-trace coupling ξ^2 or the double-trace coupling α_i^2 (with $i = 2, 3$) depending on the choice of G_i. The right diagram consists of the chain of scalar loops joined together through the double-trace coupling α_1^2. Each internal scalar loop is built using the single-trace coupling ξ^2.

$$\begin{aligned} \alpha_{1,\pm}^2 &= -\frac{1}{2c}\left(b \pm \sqrt{b^2 - 4ac}\right) \\ &= \pm\frac{i\xi^2}{2} - \frac{\xi^4}{2} \mp \frac{3i\xi^6}{4} + \xi^8 \pm \frac{65i\xi^{10}}{48} - \frac{19\xi^{12}}{10} + O(\xi^{14}). \end{aligned} \quad (15)$$

For these values of the double-trace coupling, the correlation function scales as $G_1(x) \sim 1/(x^2)^\Delta$, where the scaling dimension $\Delta_\pm = \Delta(\alpha_{1,\pm}^2)$ is given by [21]

$$\Delta_\pm = 2 \mp 2i\xi^2 \pm i\xi^6 \mp \frac{7i}{4}\xi^{10} + O(\xi^{14}). \quad (16)$$

Notice that, in distinction from (15), the expansion of $\Delta_\pm$ runs in powers of ξ^4. It is straightforward to verify that $\Delta_\pm$ satisfy the exact relation (6).

Curiously, there exists the following relation between the functions (14) and the scaling dimensions at the fixed point:

$$b^2 - 4ac = 4(\Delta_\pm - 2)^2. \quad (17)$$

It can be understood as follows. For generic complex ξ, we find using (13) that for $\mu \to 0$ and $\mu \to \infty$ the coupling $\alpha_1(\mu)$ flows into one of the fixed points $\alpha_\pm$. Then, in the vicinity of a fixed point, for $\mu \to \infty$, the Callan-Symanzik equation fixes the form of renormalized $\tilde{G}_1(p)$

$$\begin{aligned} \tilde{G}_1(p) &\sim e^{4\int_{\alpha_1(p)}^{\alpha_1(\mu)} [d\alpha\alpha\gamma(\alpha)]/\beta_1(\alpha)} \\ &\sim \left(\frac{\alpha_1^2(\mu) - \alpha_\pm^2}{\alpha_1^2(p) - \alpha_\pm^2}\right)^{2\gamma(\alpha_\pm)/\sqrt{b^2-4ac}}, \end{aligned} \quad (18)$$

where $\alpha_1^2(\mu), \alpha_1^2(p)$ are in the vicinity $\alpha_\pm^2$, and $\gamma(\alpha) = \Delta - 2$ is the anomalous dimension. We recall that the bare correlation function $\tilde{G}_1(p)$ is given by the geometric progression (12), so that as a function of α_1^2 it has a simple pole at some α_1^2. After the renormalization procedure, α_1^2 is effectively replaced by a renormalized coupling constant defined at the scale $\mu^2 = p^2$. The requirement for $\tilde{G}_1(p)$ to have a simple pole in $\alpha_1^2(p)$ fixes the exponent on the left-hand side of (18) to be 1, leading to (17).

Thus, we demonstrated by explicit seven-loop calculation that the β functions have two fixed points, $\alpha_2^2 = \alpha_3^2 = -\xi^2$, while $\alpha_{1,\pm}^2(\xi)$ is given by (15). We therefore conclude that, in the planar limit, the biscalar theory (4) and (5) with appropriately tuned double-trace couplings is a genuine nonunitary CFT, at least perturbatively.

Exact correlation function.—We can exploit the conformal symmetry to compute exactly the four-point correlation function

$$G = \langle \mathrm{tr}[\phi_1(x_1)\phi_1(x_2)]\mathrm{tr}[\phi_1^\dagger(x_3)\phi_1^\dagger(x_4)]\rangle = \frac{\mathcal{G}(u,v)}{x_{12}^2 x_{34}^2}, \quad (19)$$

which is obtained from the two-point function $G_1(x)$ defined in (7) by point splitting the scalar fields inside the traces. Here $\mathcal{G}(u,v)$ is a finite function of cross-ratios $u = x_{12}^2 x_{34}^2/(x_{13}^2 x_{24}^2)$ and $v = x_{14}^2 x_{23}^2/(x_{13}^2 x_{24}^2)$, invariant under the exchange of points $x_1 \leftrightarrow x_2$ and $x_3 \leftrightarrow x_4$. It admits the conformal partial wave expansion

$$\mathcal{G}(u,v) = \sum_{\Delta, S/2 \in \mathbb{Z}_+} C_{\Delta,S}^2 u^{(\Delta-S)/2} g_{\Delta,S}(u,v), \qquad (20)$$

where the sum runs over operators with scaling dimensions Δ and even Lorentz spin S. Here $C_{\Delta,S}$ is the corresponding OPE coefficient and $g_{\Delta,S}(u,v)$ is the conformal block [22]. Having computed (19), we can identify the conformal data of the operator $\mathrm{tr}[\phi_1^2(x)]$ by examining the leading asymptotic behavior of G for $x_{12}^2 \to 0$.

In the planar limit G is given by the same set of Feynman diagrams as G_1 (see Fig. 1), with the only difference that two pairs of scalar lines joined at the left- and rightmost vertices are now attached to the points x_1, x_2 and x_3, x_4, respectively. The fact that the contributing Feynman diagrams have a simple iterative form allows us to obtain the following compact representation for G:

$$G = \int \frac{d^4x_{1'} d^4x_{2'}}{x_{1'3}^2 x_{2'4}^2} \langle x_1, x_2 | \frac{1}{1 - \alpha^2 \mathcal{V} - \xi^4 \mathcal{H}} | x_{1'}, x_{2'} \rangle + (x_1 \leftrightarrow x_2). \qquad (21)$$

Here $x_{ij} \equiv x_i - x_j$, $\alpha^2 = \alpha_\pm^2$ is the double-trace coupling at the fixed point, and $\mathcal{V}$, $\mathcal{H}$ are integral operators

$$\mathcal{V}\Phi(x_1,x_2) = \frac{2}{\pi^2} \int \frac{d^4x_{1'} d^4x_{2'}}{x_{11'}^2 x_{22'}^2} \delta^{(4)}(x_{1'2'}) \Phi(x_{1'}, x_{2'}),$$
$$\mathcal{H}\Phi(x_1,x_2) = \frac{1}{\pi^4} \int \frac{d^4x_{1'} d^4x_{2'}}{x_{11'}^2 x_{22'}^2 (x_{1'2'}^2)^2} \Phi(x_{1'}, x_{2'}), \qquad (22)$$

where $\Phi(x_1,x_2)$ is a test function. Expanding (21) in powers of α^2 and ξ^4, we find that the operator $\mathcal{H}$ adds a scalar loop inside the diagram whereas $\mathcal{V}$ inserts a double-trace vertex. The operators $\mathcal{V}$ and $\mathcal{H}$ are not well defined separately; e.g., for an arbitrary $\Phi(x_i)$ the expressions for $\alpha^4 \mathcal{V}^2 \Phi(x_i)$ and $\xi^4 \mathcal{H} \Phi(x_i)$ are given by divergent integrals. However, at the fixed point, their sum is finite by virtue of conformal symmetry.

A remarkable property of the operators $\mathcal{V}$ and $\mathcal{H}$ is that they commute with the generators of the conformal group. This property fixes the form of their eigenstates

$$\Phi_{\Delta,S,n}(x_{10}, x_{20}) = \frac{1}{x_{12}^2} \left(\frac{x_{12}^2}{x_{10}^2 x_{20}^2} \right)^{(\Delta-S)/2} \left(\partial_0 \ln \frac{x_{20}^2}{x_{10}^2} \right)^S, \qquad (23)$$

where $\Delta = 2 + 2i\nu$ and $\partial_0 \equiv (n\partial_{x_0})$, with n being an auxiliary light-cone vector. The state $\Phi_{\Delta,S,n}$ belongs to the principal series of the conformal group and admits a representation in the form of the conformal three-point correlation function

$$\Phi_{\Delta,S,n}(x_{10}, x_{20}) = \langle \mathrm{tr}[\phi_1(x_1)\phi_1(x_2)] O_{\Delta,S,n}(x_0) \rangle, \qquad (24)$$

where the operator $O_{\Delta,S,n}(x_0)$ carries the scaling dimension Δ and Lorentz spin S. The states (23) satisfy the orthogonality condition [23,24]

$$\int \frac{d^4x_1 d^4x_2}{(x_{12}^2)^2} \overline{\Phi_{\Delta',S',n'}}(x_{10'}, x_{20'}) \Phi_{\Delta,S,n}(x_{10}, x_{20})$$
$$= c_1(\nu,S) \delta(\nu - \nu') \delta_{S,S'} \delta^{(4)}(x_{00'}) (nn')^S$$
$$+ c_2(\nu,S) \delta(\nu + \nu') \delta_{S,S'} Y^S(x_{00'})/(x_{00'}^2)^{2-2i\nu-S}, \qquad (25)$$

where $\Delta' = 2 + 2i\nu'$, $Y(x_{00'}) = (n\partial_{x_0})(n'\partial_{x_{0'}}) \ln x_{00'}^2$, and

$$c_1(\nu,S) = \frac{2^{S-1}\pi^7}{(S+1)\nu^2(4\nu^2 + (S+1)^2)},$$
$$c_2(\nu,S) = -\frac{i\pi^5(-1)^S \Gamma^2(S/2 - i\nu + 1)\Gamma(S + 2i\nu + 1)}{\nu(S+1)\Gamma^2(S/2 + i\nu + 1)\Gamma(S - 2i\nu + 2)}. \qquad (26)$$

Calculating the corresponding eigenvalues of the operators (22), we find

$$\mathcal{V}\Phi_{\Delta,S,n}(x_1,x_2) = \delta(\nu)\delta_{S,0}\Phi_{\Delta,S,n}(x_1,x_2),$$
$$\mathcal{H}\Phi_{\Delta,S,n}(x_1,x_2) = h_{\Delta,S}^{-1}\Phi_{\Delta,S,n}(x_1,x_2), \qquad (27)$$

where the function $h(\Delta,S)$ is given by

$$h_{\Delta,S} = \frac{1}{16}(\Delta + S - 2)(\Delta + S)(\Delta - S - 2)(\Delta - S - 4). \qquad (28)$$

Applying Eqs. (25)–(27), we can expand the correlation function (21) over the basis of states (23). This yields the expansion of G over conformal partial waves defined by the operators $O_{\Delta,S}(x_0)$ in the OPE channel $O(x_1)O(x_2)$

$$\mathcal{G}(u,v) = \sum_{S/2 \in \mathbb{Z}_+} \int_{-\infty}^{\infty} d\nu \mu_{\Delta,S} \frac{u^{(\Delta-S)/2} g_{\Delta,S}(u,v)}{h_{\Delta,S} - \xi^4}, \qquad (29)$$

where $\Delta = 2 + 2i\nu$, and $\mu_{\Delta,S} = 1/c_2(\nu,S)$ is related to the norm of the state (25). The fact that the dependence on the double-trace coupling α^2 disappears from (29) can be understood as follows. At weak coupling, expansion of $\mathcal{G}(u,v)$ runs in powers of $\xi^4/h_{\Delta,S}$. Viewed as a function of S, $\xi^4/h_{\Delta,S}$ develops poles at $\nu = \pm iS$ which pinch the integration contour in (29) for $S \to 0$. The contribution of the operator $\mathcal{V}$ is needed to make a perturbative expansion of (29) well defined. For finite ξ^4, these poles provide a vanishing contribution to (29) but generate a branch-cut $\sqrt{-\xi^4}$ singularity of $\mathcal{G}(u,v)$.

At small u, we close the integration contour in (29) to the lower half-plane and pick up residues at the poles located at

$$h_{\Delta,S} = \xi^4 \tag{30}$$

and satisfying $\mathrm{Re}\Delta > S$. The resulting expression for $\mathcal{G}(u,v)$ takes the expected form (20) with the OPE coefficients given by

$$C^2_{\Delta,S} = \frac{S+1}{\pi^4((4-\Delta)\Delta + S(S+2)-2))} \times \frac{\Gamma(S-\Delta+4)\Gamma^2(\frac{1}{2}(S+\Delta))}{\Gamma^2(\frac{1}{2}(S-\Delta+4))\Gamma(S+\Delta-1)}. \tag{31}$$

The relations (30) and (31) define exact conformal data of operators that appear in the OPE of $\mathrm{tr}[\phi_1(x_1)\phi_1(x_2)]$. For $S=0$ the relation (30) leads to (6). At weak coupling, (30) has the two solutions $\Delta = S+2-2\xi^4/[S(S+1)] + O(\xi^8)$ and $\Delta = S+4+2\xi^4/[(S+1)(S+2)] + O(\xi^8)$ describing the operators of twist 2 and 4, respectively. The two remaining solutions of (30) have scaling dimensions $4-\Delta$ and correspond to shadow operators. We verified by explicit calculation that (30) and (31) correctly reproduce the weak coupling expansion of the anomalous dimensions and the OPE coefficients for the operators of twist 2 and 4.

Conclusions.—We demonstrated by explicit multiloop calculation that the strongly γ-deformed planar $\mathcal{N}=4$ SYM has two nontrivial fixed points whose position depends on the properly rescaled 't Hooft coupling. We also provided evidence that, at the fixed points, it is described by an integrable nonunitary four-dimensional conformal field theory. Namely, we found a closed expression for the four-point correlation function of the simplest protected operators and used it to compute the exact conformal data (scaling dimensions and OPE coefficients) of twist-2 and twist-4 operators with arbitrary Lorentz spin. In general, correlation functions in this theory are dominated by fishnet graphs [15,25] which admit a description in terms of integrable noncompact Heisenberg spin chains [26–28]. Following [15,27,29], the integrability can be used to compute these correlation functions and also the amplitudes [30,31].

We conjecture that both conformal symmetry and integrability should survive in γ-deformed planar $\mathcal{N}=4$ SYM theory for arbitrary values of the deformation parameters γ_i. The underlying integrable nonunitary CFT_4 can be studied using the QSC_γ formalism [5–8].

The integrable nonunitary CFTs of the kind considered here also exist in lower or higher dimensions. The known examples include a two-dimensional effective theory describing the high-energy limit of QCD [32], where the two-dimensional fishnet graphs can also be studied [33], the three-dimensional strongly γ-deformed Aharony-Bergman-Jafferis-Maldacena (ABJM) model [29], and a six-dimensional three-scalar model [34] for which the "mother" gauge theory is not known (see also [35]). According to [34], the latter two theories are self-consistent CFTs and do not require adding double-trace counterterms.

It would be interesting to find the dual string description of the biscalar theory. It might be nontrivial due to the tachyon in γ-deformed $\mathrm{AdS}_5 \times S^5$ [37].

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