Abstract—NB-IoT is a new radio access technology targeting a large set of use cases for massive machine-type communications standardized by the 3GPP. Compared to human-oriented technologies, NB-IoT has been enhanced in terms of coverage and power saving capabilities while reducing the complexity at the same time. These features allow connectivity of devices in challenging positions, enabling long battery life. This paper provides an overview of NB-IoT, together with a mathematical model of the network able to predict the maximum performance in a given scenario with a specific configuration of some design parameters. Finally, we present an analysis on how these parameters affect the overall performance and how the optimal configuration may be chosen according to arbitrary criteria.

Index Terms—LPWAN, NB-IoT, Internet of Things

I. INTRODUCTION

In the IoT market for the emerging 5G ecosystem, the effective support of massive machine-type communications [1] is expected to play a key role. [2]. This new set of applications, usually referred to as Low Power Wide Area Networks, enabled an ad-hoc design of wireless technologies able to satisfy the unique requirements in terms of coverage, battery life, and device complexity [3]. One emerging standard among these is NarrowBand-IoT (NB-IoT).

NB-IoT [4] is an access technology defined by the 3rd Generation Partnership Project (3GPP) implementing several mMTC-oriented enhancements compared to other mobile technologies [5]–[7]; examples are: (i) differentiation of User Equipment (UE) performance according to deployment conditions by tuning the behavior of the physical channel and network procedures; (ii) narrow-band transmission and the exploitation of repetitions to reach devices in such challenging conditions; (iii) enhanced power saving mechanisms to improve the battery life; (iv) simplification of procedures and hardware to reduce the UE complexity. Several works and white papers, e.g., [4], [8]–[10], present the main features of NB-IoT and study performance in terms of coverage extension or random access capacity. Information on the standard are currently spread across several technical documents and publications. Also, a method to estimate the overall performance of a whole network and to understand the impact of the many design parameters is still missing.

The aim of this paper is twofold. Firstly, we provide an overview of NB-IoT summarizing all main features and technical information with a particular focus on the uplink. Secondly, this paper presents a mathematical model of the network able to predict the throughput or the success probability in a given scenario and the maximum throughput possible with a certain configuration of the design parameters. We further present an analysis on how these parameters affect the overall performance and how the optimal configuration in terms of coverage classes may be chosen according to arbitrary criteria.

II. NB-IOT TECHNOLOGY

The concept of repeating the transmission of the same packet has been introduced in NB-IoT to improve the coverage by exploiting time diversity and the possibility to combine the replicas. Since repetitions are very time consuming, devices are grouped in three coverage classes which use different amounts of repetitions and other configurations: the higher is the received power, the less repetitions are used by the UE. The coverage classes are named Normal (N), Robust (R) and Extreme (E). To decide in which coverage class to belong to, each UE performs a received power measurement. The outcome is compared to two thresholds, $T_{NB}$ and $T_{RE}$, defining three possible ranges of values. The devices with the lowest received powers (less than $T_{RE}$) will belong to the class Extreme, characterized by a set of parameters meant to overcome the poor coverage conditions; the ones with the highest received powers (more than $T_{NB}$) will belong to the class Normal, characterized by a set of parameters meant to maximize the throughput.

For the sake of coexistence, NB-IoT numerology is inherited mostly from LTE. In both Downlink (DL) and Uplink (UL), the channel is divided into 12 subcarriers of 15 kHz each. The time domain is divided into time slots, each lasting 0.5 ms and consisting of 7 OFDM/SC-FDMA symbols. The smallest time-frequency resolution, named Resource Element (RE), is composed of one subcarrier and one symbol. Time slots are grouped as follows: two time slots form one subframe (1 ms), 10 subframes form one frame (10 ms). To further improve the coverage, a second numerology with 48 subcarriers of 3.75 kHz each, is introduced.

NB-IoT defines the following physical channels for the UL: the Narrowband Physical Random Access Channel (NPRACH), used to initiate the Random Access Procedure (RAP) and the Narrowband Physical Uplink Shared Channel (NPUSCH), used for data transmission from the UE to the evolved Node-B (eNB).
The NPRACH is composed of a contiguous set of either 12, 24, 36 or 48 subcarriers with 3.75 kHz spacing, which are repeated with a periodicity from 0.04 s to 2.56 s. The RAP starts with the transmission of a preamble, with a duration of either 5.6 ms or 6.4 ms (Format 0 and 1, respectively, denoted as \( \tau_p \)) depending on the size of the cell, and can be repeated up to 128 times to improve coverage. A preamble is composed of four symbol groups, each of them transmitted on a different subcarrier. The first subcarrier is chosen randomly, while the following ones are determined according to a deterministic sequence depending on the initial subcarrier. Two UEs selecting the same initial subcarrier, will collide for the entire length of the sequence. Hence, in each NPRACH occurrence there is a number of orthogonal preambles equal to the number of subcarriers allocated to the NPRACH [7].

The number of repetitions \( R_c \), the periodicity \( T_c \) seconds) and the number of subcarriers \( S_c \) are defined for each coverage class \( c \in C \equiv \{ N, R, E \} \). By choosing an appropriate configuration of the aforementioned parameters and different time offsets, it is possible to have a different orthogonal NPRACH, each with its own capacity in terms of accesses per second, for each coverage class. We denote this capacity with \( Z_c = S_c/T_c \).

The initial procedure in the UL is the RAP, which can be triggered as either a response to a paging message or UE-initiated for the purpose of UL data transmission. The RAP includes four messages and starts with the transmission of a preamble (Msg1) on the first available NPRACH opportunity. If multiple UEs choose the same initial subcarrier the preamble sequence will collide but the eNB is not yet aware of it. After the preamble transmission the UE expects to receive the Random Access Response (RAR) message (i.e., Msg2) indicating the preambles identified by the eNB. In this phase, colliding UEs will receive the same RAR without being aware that a collision happened. After Msg2 reception, the UE transmits the Msg3 on the NPUSCH and then expects to receive the Msg4. The Msg4 carries the grant for data transmission and it is also used to resolve the collisions. The Msg3 and Msg4 are transmitted using HARQ. If the RAP fails in any of the aforementioned phases, the UE performs a new attempt after a backoff delay, and up to a maximum of \( A_c \) attempts. Once resources have been granted with the reception of Msg4, the UE starts transmitting its payload on the NPUSCH using HARQ.

The NPUSCH occupies all the UL resources left available after the allocation of the NPRACH. NPUSCH format 1 is used for UL data while NPUSCH format 2 carries UL control information (UCI), which in Release 13 is a DL HARQ ACK. To perform a UL transmission, the eNB allocates a certain amount of resources to the UEs. The minimum amount of resources is called Resource Unit (RU), where the possible RU configurations [7] depend on the UE capabilities and the configured numerology. Given the used Transport Block Size (TBS, up to 1000 bits), the number of required RUs \( N_{\text{RU}} \) depends on the Modulation and Coding Scheme (MCS) used to meet a certain success probability target, where the relationship between MCS, TBS and number of required RUs can be found in [6]. Similarly to the NPRACH, the RUs are repeated a certain number of times to improve the coverage.

### III. System model

#### A. Scenario and coverage

We consider the eNB to be placed in an hexagonal grid with a variable inter-site distance \( d_{\text{IS}} \). As described in [11], in a dense urban scenario like the city of London, \( d_{\text{IS}} \) is 500m, nevertheless the possibility of deploying NB-IoT only in a subset of eNBs is often considered; thus, \( d_{\text{IS}} \) may be larger. We assume the resulting cells to be circular with radius \( R_{\text{cell}} = d_{\text{IS}}/\sqrt{3} \).

Depending on the use case considered, the devices, uniformly spread in the cell, are partitioned among outdoor (O), indoor (I) or deep-indoor (DI) conditions with different proportions. We denote with \( p_c \) the probability for a device to be deployed in condition \( c \), where \( c \in E \equiv \{ O, I, DI \} \).

The received power at distance \( r \) is expressed as \( P_r = P_{r0} - 10\beta \log_{10} r + S - L_{\text{add},e} \) where \( S \) is a random variable with standard deviation \( \sigma_S \) representing the log-normal shadowing, and \( L_{\text{add},e} \) is an additional attenuation representing the penetration loss due to the deployment condition \( e \). Finally, \( \beta \) is the propagation exponent and \( P_{r0} \) represents the received power at 1 km of distance. The values for these, and any other parameter defined afterwards, are reported in Table I.

The UE and the eNB have different transmit powers; thus, the parameter \( P_{r0} \), which accounts for the UEs transmit power, cables losses, antenna gains and channel loss at 1 km, can be \( P_{r0}^{\text{UL}} \) when the UL is considered, and \( P_{r0}^{\text{DL}} \) when the DL is considered.

As a first step we compute the probabilities for a generic device to choose a specific coverage class, as a function of distance, which we denote as \( p_c \left( r \right) \), \( c \in C \).

For the Normal coverage class this probability is expressed in (1) as the weighted average among the three deployment conditions of the probability for the received power to be higher than the threshold \( T_{\text{NR}} \). Following a similar reasoning, (2) and (3) express the probabilities for the other coverage classes.

Then, it is possible to compute the probabilities for any device in the cell to belong to each coverage class by integrating across the circular area the aforementioned equations. (4), (5) and (6) show the resulting expressions assuming the UEs to be deployed uniformly.

One of the metrics we use to evaluate the performance of the system is coverage probability, computed as the fraction of devices configured with a sufficient number of repetitions to be decoded correctly by the eNB. In [12] the authors show the average number of repetitions required to decode the NPUSCH transport block as a function of the Signal-to-Noise Ratio (SNR), in presence or absence of doppler spread \( f_d \), and for different MCS. Since only the average number of repetitions is available, it is not possible to define a minimum requirement. Thus, we assume that the average
correspond to the minimum requirement \((R_{\text{min}})\) with a consequent overestimation of the coverage probability. Considering an effective noise of \(-129\text{dBm}\) [11], and assuming \(f_d = 0\text{Hz}\) and \(MCS_{\text{index}} = 0\), we can approximate the selected curve as \(R_{\text{min}} = 2^{-A}P^{UL-B}\) where \(A = 0.2902\) and \(B = 37.25\).

We can compute the coverage probability at a given distance from the base station, for a given coverage class and deployment condition, as the probability of having a received power such that the required number of repetitions is lower than the actual number of repetitions configured; this leads to (7).

Finally, the coverage probability for a random UE located in the cell and belonging to coverage class \(c\) can be expressed as the average of \(p_{\text{COV}}(r,c,e)\) among the deployment conditions and across the cell area (see (8)).

B. Traffic estimation

In the following analysis we consider all the UEs to implement the same application, characterized by a payload of \(P_L\) bytes and an overall offered traffic, in UL, of \(\lambda\) packets per second. The amount of traffic within each coverage class can be computed as \(\lambda_c = p_c \lambda\).

We assume that the RAPs performed by devices belonging to the same coverage class are handled independently from the other classes. We denote as \(N_{p,c}\) the average number of preambles sent in each occurrence of the NPRACH for coverage class \(c\). Assuming that the distribution of the preamble transmissions is a Poisson Process, and that the probability of choosing a given initial subcarrier is \(1/S\), we can compute the probability of having one or more UEs starting the preamble with a certain subcarrier through (9):

\[
p_{s,c} = 1 - e^{-\frac{N_{p,c}}{s}}
\]

We assume that the transmission of Msg2, Msg3 and Msg4 is always successful once the preamble is received correctly, and that HARQ is not used. Therefore, the number of successful accesses, or the number of Msg2 sent, is equal to the number of subcarriers used at the beginning of the preamble by at least one UE (each group of colliding UEs will behave as a single preamble from the point of view of the eNB and only one will complete the RAP successfully). In a mathematical form \(N_s,c = n_{SC,c}S_c\). Since the NPRACHs have a different periodicity for each coverage class, we can express the same concept referring to a common time unit. \(\lambda_{p,c}\) and \(\lambda_{s,c}\) denote the number of preambles sent and the number of successful accesses per second, respectively. Their relation is shown in (10).

\[
\lambda_{s,c} = Z_c \left(1 - e^{-\frac{\lambda_{p,c}}{Z_c}}\right)
\]

The number of preambles actually sent is equal to the number of devices which start the RAP (\(\lambda_c\)) plus the number of devices which perform another attempt because they failed the previous ones (\(\lambda_{f,c}\)). The latter can be computed as \(\lambda_{f,c} = \lambda_{p,c} - \lambda_{s,c} - \lambda\) where \(\lambda\) denotes the number of devices which failed the RAP after \(A_c\) attempts and can be computed as shown in (11).

\[
\lambda_{f,c} = \lambda_p \left(1 - \frac{Z_c}{\lambda_p} \left(1 - e^{-\frac{\lambda_p}{Z_c}}\right)^{A_c}\right)
\]

Finally, this information can be merged in (12) which can be solved numerically for \(\lambda_{p,c}\).

\[
\psi_{UL} = \frac{2S_{\text{RU}}(3 + N_{\text{RU}}(P_k))}{24000 - 2\sum_{c \in C} Z_c \left(\frac{\tau PR_c}{1/W}\right) \sum_{c \in C} R_c \lambda_{s,c}(\lambda)}
\]
\[ p_{\text{COV}}(r, c, e) = \frac{1}{2} \text{erfc} \left( \frac{10 \beta \log_{10} r - \log_2 R_c - A (P_{UL} - L_{\text{add}, e}) - B}{A \sigma \sqrt{2}} \right) \]  

(7)

\[ p_{\text{COV}, c} = \frac{2}{R_{\text{total}}} \sum_{e \in E} \int_0^{R_{\text{cell}}} p_{\text{COV}}(r, c, e) r dr \]  

(8)

\[ Z_c \left( 1 - e^{-\frac{\lambda p}{Z_c}} \right) + \lambda p_c \left[ 1 - \frac{Z_c}{\lambda p_c} \left( 1 - e^{-\frac{\lambda p}{Z_c}} \right) \right]^\lambda_c = \lambda p_c \]  

(12)

\[ \sum_{c \in C} R_c \lambda_{s,c}(\lambda) > W \]  

(14)

The configuration parameters of interest are nine: the number of repetitions for the three coverage classes, the number of preambles per seconds available in each NPRACH for the three coverage classes, the two thresholds for the coverage class decision and the inter-site distance.

The amount of possible configurations is huge, therefore we decided to estimate the performance metrics choosing randomly a large set of possible inputs with a Monte Carlo approach. In this paper we consider two possible scenarios, one characterized by UEs deployed mostly outdoor (\( \langle p_{o}, p_{1}, p_{2} \rangle = [0.6, 0.3, 0.1] \)) and one by UEs deployed mostly indoor or deep-indoor (\( \langle p_{o}, p_{1}, p_{2} \rangle = [0.1, 0.3, 0.6] \)). The values of the remaining parameters are reported in Table I. The possible values of the model inputs are chosen in the following sets: \( R_c \in \{1, 2, 4, 8, 16, 32, 64, 128 \} \), \( T_{\text{NR}} \), \( T_{\text{RE}} \in [-160, -30] \) dBm, \( Z_c \in [48/0.04, 24/0.04, 12/0.04, 12/0.08, 12/0.16, 12/0.24, 12/0.32, 12/0.64, 12/1.28, 12/2.56] \), \( d_{\text{IS}} \in [0.5, 1, 1.732] \) km. We generated \( 10^5 \) different configurations, then we represented each outcome as a point in a two dimensional plane where the x axis represents \( T_{\text{max}} \) and the y axis represents \( P_s \). (see Fig.1 for the case mostly indoor)

The first aspect we noticed is that the overall performance obtained for the two scenarios is very similar. In fact, it is possible to notice that the coverage probability for the indoor scenario is generally close to 100% and only in very few cases it drops below 90%. In other words the coverage is not a major problem in the context of this work, whereas congestion issues play a more important role. For this reason the results presented afterwards are referred to the indoor scenario only.

We emphasise the importance of studying the impact of these design parameters by noticing that most of the configurations lead to a network with a very poor throughput and success probability, while ideally a good one should be located in the top-right corner of the figure.

An operator may decide to adopt different definitions of best performance based on these results. For instance, the maximum throughput alone is a too simplistic criteria to use, as often the configurations with the highest throughput lead to a very low success probability. We propose to define the best configuration as the one providing the highest \( T_{\text{max}} \) and \( p_s > p_{s,\text{min}} \), with \( p_{s,\text{min}} = 90\% \).

According to this definition, the maximum throughput achievable in the indoor scenario is 45.2 kbps with a success probability of 90.9%, performance obtainable with the configuration reported in Table II. It is possible to observe that \( T_{\text{NR}} \) and \( T_{\text{RE}} \) are such that almost all the UEs choose to belong to coverage class Normal.
TABLE I
SUMMARY OF THE MODEL PARAMETERS AND THEIR VALUES (IF CONSTANT IN THIS WORK) [7], [11].

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{UL}^{0}$</td>
<td>Received power at 1 km in UL</td>
<td>-86.9</td>
<td>dBm</td>
</tr>
<tr>
<td>$P_{DL}^{0}$</td>
<td>Received power at 1 km in DL</td>
<td>-66.9</td>
<td>dBm</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Path Loss exponent</td>
<td>3.76</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>Shadowing standard deviation</td>
<td>9</td>
<td>dB</td>
</tr>
<tr>
<td>$L_{add,e}$</td>
<td>Additional loss $[O, I, DI]$</td>
<td>[0, 20, 40]</td>
<td>dB</td>
</tr>
<tr>
<td>$S_{RU}$</td>
<td>RU number of subcarriers</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$\tau_{RU}$</td>
<td>Required repetitions approximation parameter</td>
<td>8</td>
<td>ms</td>
</tr>
<tr>
<td>$A$</td>
<td>Required repetitions approximation parameter</td>
<td>0.2902</td>
<td>-</td>
</tr>
<tr>
<td>$B$</td>
<td>Required repetitions approximation parameter</td>
<td>37.25</td>
<td>-</td>
</tr>
</tbody>
</table>

NPUSCH

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{RU}$</td>
<td>RU number of subcarriers</td>
<td>1</td>
<td>-</td>
</tr>
<tr>
<td>$\tau_{RU}$</td>
<td>RU duration</td>
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<td>$A$</td>
<td>Required repetitions approximation parameter</td>
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<td>-</td>
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<td>$B$</td>
<td>Required repetitions approximation parameter</td>
<td>37.25</td>
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NPRACh

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<th>Symbol</th>
<th>Description</th>
<th>Value</th>
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<tbody>
<tr>
<td>$\tau_p$</td>
<td>Preamble Repetition Length</td>
<td>5.6</td>
<td>ms</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Maximum number of RAP attempts $[N, R, E]$</td>
<td>[3, 3, 3]</td>
<td>-</td>
</tr>
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</table>

Application

<table>
<thead>
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<th>Symbol</th>
<th>Description</th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>$P_L$</td>
<td>UL data payload</td>
<td>50</td>
<td>B</td>
</tr>
<tr>
<td>$N_{RU}(P_L)$</td>
<td>Number of RUs needed</td>
<td>8</td>
<td>-</td>
</tr>
</tbody>
</table>

In Fig.1 we represent each configuration as a point in a two dimensional plane where on the x axis there is the Success Rate ($P_s$) and on the y axis the highest value, $p_c$, of probability to belong in one of the three coverage classes. With the latter metric we want to express how much the devices are distributed (or not) among the coverage classes; the highest $p_c$ is, the more concentrated are the UEs in a single coverage class.

In Fig.2 we represent each configuration as a point in a two dimensional plane where on the x axis there is the Success Rate ($P_s$) and on the y axis the highest value, $p_c$, of probability to belong in one of the three coverage classes. With the latter metric we want to express how much the devices are distributed (or not) among the coverage classes; the highest $p_c$ is, the more concentrated are the UEs in a single coverage class.

From Fig.2 it is possible to notice that in order to have a success probability above a certain requirement, it is necessary that most of the UEs belong to the same coverage class, normally the one with the largest $Z_c$. The reason for that may be due to the fact that if the coverage is always good, a single coverage class could be able to satisfy all the users without the need of using different number of repetitions. In a scenario like this, an operator may even choose to disable some coverage classes in order to free some NPRACH radio resources useful for the NPUSCH, unless they want to guarantee coverage for the few devices which are in very poor conditions.

In order to understand better the impact of each parameter, we performed the analysis reported in Fig.3. Starting from a configuration characterised by poor performance (the star

Fig. 1. Performance of a NB-IoT network with random generated configurations and indoor deployment ($[p_O, p_{UL}, p_{DL}] = [0.1, 0.3, 0.6]$).

Fig. 2. Relation between the Success Probability and the concentration of devices in a single coverage classes for random configurations.

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Fig. 3. Analysis of the impact of each parameter on the Success Probability.
marker, $T_{\text{max}}^s = 29.8 \text{ kbps}$, $P_s = 39.5\%$), we tried to modify only one parameter, or a triplet of parameters, at a time. By increasing the number of repetitions (the solid line), we noticed that it is possible to increase the success probability with the drawback of a reduced throughput due to the less efficient use of the radio resources in the NPUSCH and vice versa. The small dots represent the performance obtainable by changing only $Z_c$; it is possible to improve both success rate and throughput although a clear scheme is not recognisable. Nevertheless, by changing only these parameters it is not possible to reach a $P_s$ above 55%. In fact, with the starting configuration, only 70% of the devices belong to coverage class Normal. If we change the value of the thresholds (the circular markers) we can change the distribution of the UEs among the coverage classes; in this way we can reach a $P_s$ above 90%. The size of the markers represents the maximum probability of being in a coverage class; as we mentioned beforehand, it is possible to notice that a higher $P_s$ is reachable only when most of the devices are concentrated in a single coverage class.

V. CONCLUSION

This article presented a description of the main features of NB-IoT and of the procedures for data transmission with a focus on the uplink. The paper presented also a mathematical model describing a large network implementing the same uplink oriented application with devices deployed in different connectivity conditions. We provided an analysis in terms of maximum throughput and success probability, considering random configurations of nine design parameters, with the objective of finding the one maximizing the performance. Apart from providing some estimation of the performance which can be achieved in different scenarios, this paper provides an useful methodology to explore the design space of the network efficiently. In fact, the large majority of the random configurations lead to a very poor performance; a smart selection of these parameters can lead to an improvement of an order of magnitude.

Moreover, this analysis shows that in the deployment scenario considered NB-IoT is capable of providing good coverage to all the UEs and that in order to have a high success rate the majority of the devices should be configured to belong to the same coverage class. The impact of the other design parameters is complex and needs to be investigated in further detail. Although the the success probability has an upper bound, it can be varied by changing the number of repetitions improving either the coverage probability or the network throughput. In the same way, the capacity of the three NPRACHs can be optimized in order to guaranteeing the highest throughput without causing a significant drop of the success probability.

VI. ACKNOWLEDGEMENT

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