Adaptive Full-Duplex Communications in Cognitive Radio Networks

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Abstract—In this paper we propose a novel adaptive scheme for full duplex communication of secondary users (SU) in a cognitive radio network. The secondary network operates adaptively in three modes; Cooperative Sensing (CS), Full Duplex Transmit and Sensing (FDTS), and in-band bidirectional Full Duplex Transmit and Receive (FDTR). In the CS mode, the secondary nodes will detect the signal of a primary user (PU) through a novel cooperative MAC protocol and will decide the system’s mode of operation in the subsequent spectrum hole. This adaptive decision is based on dual-threshold detection (DTD) introduced for the first time. When the primary’s signal at SU’s receivers is weak, the system switches to the FDTS mode to avoid higher collisions probability and long or endless collision durations. In the FDTS mode, one of SU’s senses the PU activity continuously whilst transmitting to another node. When the channel conditions allow, the system switches to the FDTR mode, in which the secondary users would communicate bidirectionally in an asynchronous full duplex manner. The novel idea of asynchronous transmission in this mode will result in decreased maximum and average collision durations. Analytical closed forms for probability of collision, average collision duration and cumulative collision duration, as well as throughput of the SU network are derived, and performance of the proposed scheme in terms of the above-mentioned metrics, its effectiveness, and advantages over conventional methods of sensing and transmission are verified via simulations.

Index Terms—Asynchronous cognitive radio, cooperative sensing, full duplex communications.

I. INTRODUCTION

Over the past decade mobile services have evolved from basic voice communication to mobile-broadband multimedia services and sophisticated wireless applications. Any wireless communication relies on the availability of radiofrequency spectrum, which is inherently a finite resource and cannot be produced. This unprecedented growth of wireless devices and services has motivated academia and industry to look for new solutions in utilizing the wireless spectrum more intelligently and efficiently. Cognitive radio (CR) and full duplex (FD) communications are two promising technologies to enhance spectrum utilization and network efficiency, and combination thereof will improve the performance even further.

Cognitive radio is a form of intelligent wireless communication in which secondary users opportunistically operate in licensed channels of primary users (PU) through detection of their activities. This technology has been well studied for more than a decade [1]-[3] and standardized a few years ago [4] for commercial applications. On the other hand, until recently, in-band full duplex communication, i.e. simultaneous transmission and reception over the same frequency band, was not deemed feasible due to strong self-interference from the transmitter to the receiver. However, recent advances in self-interference suppression (SIS) methods [5]-[8] have enabled implementation of this technology in future wireless networks.

In general, a CR network is non time slotted which means primary users can be active or inactive at any time during the whole secondary frame duration. Traditional spectrum sensing schemes, such as listen before talk (LBT) in non time slotted networks, usually do not guarantee good protection for the PUs, as they may become active at any time during SUs’ transmission, and this will cause collision. One solution would be shorter SU transmission frames at the cost of lower SU utility which is not usually considered as a promising solution. Some studies [9]-[12] have proposed full duplex spectrum sensing to alleviate this problem. In such works the inactive or transmitting SUs keep sensing the presence of PUs constantly during their transmission time slot, and upon detection of a PU signal, would cease transmission to avoid collision [9]. Listen and talk (LAT) [10] and TranSensing [11] protocols are two instances of implementing this idea. Although such techniques will noticeably improve PU protection, but will not enhance throughput of the CR network. The authors in [12] presented a threshold-based sensing-transmission scheme in order to maximize SU throughput with minimum collision with PUs in a half duplex (HD) network, which may be applied in full duplex mode either. Their scheme consisted of multiple consecutive sensing or transmission periods, determined according to the SU’s belief about the state of the PU(s). Although this scheme shows enhancement over traditional LBT schemes in terms of throughput and collision probability in HD mode, but it would not be optimal in FD mode. Inspired by this idea, the authors in [13] proposed a similar approach for a full duplex scenario in a CR network with four modes of operation; sensing-only, transmit and sensing, transmit and receive, and channel selection, and developed a mode-selection strategy to maximize the SU throughput for a given PU collision probability. However, the average and maximum collision durations were not optimized, and it could be very long if the primary SNR at secondary receivers is weak.

In [14], we considered an interweave CR/DSA system and introduced the CS and FDTR modes of operation in which secondary users were capable of imperfect SIS. The proposed system exploited cooperative sensing during busy periods of the primary network to guarantee primary’s protection, and bidirectional FD communication of cognitive network within idle periods of the primary network. The asynchronous transmission of secondary nodes could reduce collision...
durations, although the optimum time difference for minimum collision duration was not provided. The system could experience long collision durations in FDTR mode if the primary’s signal was not strong enough to be detected.

In order to alleviate the shortcomings of the system proposed in [14], in this paper we have presented a novel adaptive scheme for full duplex CR networks based on three modes of operation: cooperative sensing (CS), full Duplex transmit and sensing (FDTS), and bidirectional full duplex transmit and receive (FDTR). Our scheme not only enhances the secondary network’s throughput, but will decrease the average and maximum collision durations with primary signal while avoiding long or endless collision duration in varying channel conditions.

The benefits of cooperative sensing in half duplex CR networks have been well studied in numerous works [15], [16] which could be implemented in FD networks with little modifications. In the CS mode when a PU is active, SUs keep sensing the channel over consecutive sensing intervals in a cooperative manner. This will increase detection probability and there would be no collision with primary network during its busy period. In this mode, the system decides on the CR network’s mode of operation for the following idle period of the primary network, based on the existing channel conditions between primary transmitters and secondary users. This mode selection function is implemented through a dual-threshold energy detection scheme, which is introduced here for the first time. If the system decides that the whole network is in such a condition that bidirectional full duplex communication is feasible with tolerable interferences, it will switch to the mode of bidirectional FD communication (FDTR) upon disappearance of PU signal. However, if the network status is not reliable enough for FDTR mode, then the system will switch to the conservative mode of transmit and sensing (FDTS) in which one of the SUs detects the return of a primary signal continuously while transmitting to another node, in full duplex manner.

In the FDTR mode, where both SUs are involved in bidirectional communication, the cognitive system is notified of the return of a primary signal through collision detection. In order to minimize the interferences impinged by SUs on primary network, we have considered asynchronous transmission of SUs as in [14] to decrease collision duration compared to that in the existing methods. We have shown that such alteration in transmission timing in the secondary network, will decrease collision duration to half or less compared to that in the conventional sensing methods, and have provided the optimum time difference for the minimum collision duration.

The main contributions of this paper are as follows. We have presented a novel adaptive scheme for full duplex operation in CR networks which consists of three modes of operation. Mode selection based on dual-threshold detection (DTD) criterion and the cooperative MAC protocol in the CS mode are proposed for the first time. Asynchronous transmission of SUs in the FDTR mode is another novelty which results in shorter average and maximum collision durations and less interference. We have mathematically analyzed the impact of these methods on network metrics such as probability of false alarm and detection, collision probability and duration, and the CR network average throughput. In addition, we have derived analytical closed forms for optimum asynchronous time difference and average throughput of the secondary network under such conditions, and have shown that our scheme outperforms the conventional methods in terms of cognitive network metrics.

The rest of this paper is organized as follows. Section II describes the system model and description of protocols. In section III we derive exact closed form expressions for probability of detection and false alarm in the CS, FDTS and FDTR modes. Also in section III we derive probability of collision, probability density function of collision duration, and the average CR network throughput of the system for synchronous and asynchronous modes of operation, and compare these to the respective counterparts in the traditional LBT scheme. Numerical results are provided in Section IV, and the concluding remarks are presented in Section V. We summarize the used major notations in Table I.

II. SYSTEM MODEL

We consider an interweave cognitive radio network, consisting of a primary transmitter (e.g. PU base station) and multiple primary receivers (as in IEEE 802.22 or IEEE 802.11af standards), and one pair of secondary users, which opportunistically access the licensed PUs’ channel as shown in Fig. 1. We focus on a bidirectional channel, and assume all primary to secondary, and secondary to secondary channels are Rayleigh block fading. Primary transmitter may become active or inactive at any time, modeled as an alternating ON/OFF continuous-time Markov process [17], [18]. The ON and OFF times are exponentially distributed with mean durations of $\mu^{-1}$ and $\lambda^{-1}$ respectively. Secondary users are capable of operating in the in-band full duplex mode i.e. to transmit and receive at the same time over the same frequency band. This requires each secondary device be capable of perfect or partial self-interference suppression. Here we do not consider SIS methods and just quantify this capability by SIS factor $\beta_i$ for the $i$th SU. SIS factor is the ratio between the residual self-interference signal and the original one. In perfect SIS (i.e. no residual self-interference) $\beta_i=0$; and otherwise, only a fraction $1 - \beta_i$ of the original self-interference signal is cancelled. In this paper we assume both SUs have the same SIS factor $\beta$.
The secondary network operates in three modes; cooperative sensing (CS), full duplex transmit and sensing (FDTS), and full duplex transmit and receive (FDTR).

When the primary network is active, the system operates in the CS mode. In this mode, both SUs keep listening to the channel during continuous sensing slots in tandem, and do not transmit. Each sensing slot is of duration $T_s$ seconds which is a fraction of the transmission frame duration $T$. In contrast with conventional sensing methods based on energy detection with one threshold, here we set up two threshold values; $\epsilon_0$ and $\epsilon_1$, where $\epsilon_0 < \epsilon_1$. The lower threshold ($\epsilon_0$) is used for detecting the presence of a primary signal, and the higher threshold ($\epsilon_1$) is employed for selecting the mode of operation of cognitive network in the subsequent OFF period of the primary network. Cooperation between two SUs in sensing the primary’s presence increases the detection probability, which improves protection for the primary network.

In the FDTS mode, one of the SUs acts as the source, and the other one as destination. The source unit transmits to the destination, and at the same time keeps sensing the channel for the return of primary signal. In this mode, communication between SUs is in HD mode, but the source node operates in FD transmit and sensing manner. This simultaneous transmit and sensing can be materialised in LBT or LAT schemes. Here we consider LAT method for the FDTS mode.

In the FDTR mode, assuming there is data on SUs’ buffers provided by higher layers, both secondary units transmit and receive their data bidirectionally in FD manner over the same channel as long as no primary signal has returned to the channel. When it occurs, the primary and secondary signals collide and this will result in errors in secondary network communication which will indirectly inform the SUs of the presence of a PU and will force them to cease transmission instantly and switch back to the CS mode. In this mode of operation, we have no explicit sensing of PUs’ activities.

### A. Dual-Threshold Detection (DTD)

In some cases, where PU signal at SU’s receivers is weak (e.g. when SUs are far from primary transmitter but close to PU receiver, as shown in Fig. 2), the effect of a weak PU signal on SUs’ transmissions is not that much to cause errors in SUs’ transmission. Hence SUs will not notice the return of a PU signal through collision event and may continue full duplex communication and impinge a long and unbearable interference on primary’s signal. To avoid such incidences, the cognitive network will operate in the FDTS mode. For this purpose, we introduce a dual threshold detection scheme. SUs require to measure the energy level of PU signal in the CS mode and compare it with a pre-defined mode selection threshold ($\epsilon_t$) and primary presence threshold ($\epsilon_0$).

In each sensing slot, each SU first compares the detected energy ($\mathcal{E}_d$) with $\epsilon_t$. If $\mathcal{E}_d > \epsilon_t$, it means that PU is present and its power is high enough for the FDTR mode operation in the subsequent OFF period. However if $\mathcal{E}_d < \epsilon_t$, then it is compared with the lower threshold. $\epsilon_0 < \mathcal{E}_d < \epsilon_t$ means that a primary signal with a low energy level is present. Hence, for a conservative system avoiding long collision periods, the CR system should operate in the FDTS mode, during the next upcoming OFF period of primary network. And $\mathcal{E}_d < \epsilon_0$ is interpreted that the primary signal is not present anymore and there is an opportunity for SUs to start communication in the suitable mode decided within previous sensing slots.

### B. Cooperative Sensing and Mode selection MAC

Cooperative sensing and mode selection strategy plays a key role in the proposed scheme. Cooperative sensing is carried out through the following handshaking MAC protocol.

When one SU ascertains the channel is not in use (i.e. $\mathcal{E}_d < \epsilon_0$), it transmits ready-to-send (RTS) signals via a report channel in the next sensing slot and at the same time listens to receive a RTS signal back from the peer SU. RTS signals are very short and assumed to be error free. There are two types of RTS signals; RTS1 and RTS2. RTS1 is declared for operation in the FDTS mode, and RTS2 corresponds to the FDTR mode (see Fig. 3).

As mentioned earlier, the decision to send an RTS1 or RTS2 signal is made in the previous sensing slots through a dual-threshold detection scheme. If both SUs declare RTS1

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**TABLE 1. SIGNIFICANT NOTATIONS.**

| $\mu^{-1}, \lambda^{-1}$ | Average duration of PU’s ON and OFF periods. |
| $\beta$ | SIS factor. |
| $T_s, T$ | Sensing and transmission frames’ length. |
| $\epsilon_0, \epsilon_1$ | Sensing and mode selection thresholds. |
| $p_{f1}, p_{f2}, p_{f3}$ | False alarm probabilities in the CS, FDTS, and FDTR modes. |
| $P_{PER_{FS}}, P_{PER_S}$ | Average PER in SU transmission with and without PU signal. |
| $\bar{\gamma}_S$, $\bar{\gamma}_P$, $\bar{\gamma}_F$ | Average SNR of SU1, SU2, and PU at SU2. |
| $p_{co\text{-}col}, p_{col}$ | Collision probability of our scheme and LBT. |
| $\mu(t)$ | PDF of collision duration. |
| $\tau$, $\theta$ | Average collision and collision free durations. |
| $R_{FDTS}, R_{FDTR}$ | Average throughput of FDTS and FDTR modes. |
| $R_{Async}, R_{Sync.}$ | Average throughput in async. and sync. modes. |

**Fig. 2.** An example of conditions in which the primary’s signal is weak at SUs’ receiver and may not be detected in presence of any strong interferences.

**Fig. 3.** RTS signals declared by SUs.
signals (i.e. both had found $\epsilon_0 < E_d < \epsilon_1$ in the previous sensing slots), they will operate in the FDTS mode and one of the nodes by default will be the source node and the other one would be destination. If both SUs issue RTS2 signals (i.e. both had found $E_d > \epsilon_1$ in the previous sensing slots), then both SUs will switch to the FDTR mode and will start bidirectional communication.

If both SUs issue RTS2 signals (i.e. both had found $E_d > \epsilon_1$ in the previous sensing slots), then these detection events would not be coincident and this in turn would result in shorter average collision durations (see Fig. 6). For the case of equal transmission frames, i.e. $T_1 = T_2 = T$, the optimum timing difference would be $T/2$ for shortest average collision duration.

**Proposition:** For the case of equal transmission frames $T_1 = T_2 = T$, the minimum upper limit of a collision duration happens when timing difference is $L = T/2$, and the minimum average collision duration occurs when timing difference is $L = \frac{1+2\lambda T}{\lambda}W(e^{1+2\lambda T}e^{-1})$, in which $W(\cdot)$ is the Lambert W function.

**Proof.** See Appendix.

Although operating in the FDTR mode enhances the SUs’ throughput, but the SUs will not be able to monitor the PU’s state within a transmission frame. Hence, the probability of colliding with the PU will be higher than in traditional methods of sensing such as Listen Before Talk (LBT). In addition, the PU’s signal may affect SUs differently due to different fading or interference conditions of PU-SU channels.

### C. Asynchronous FD transmission

The sensing method applied in the FDTR mode which is based on collision event usually results in long collision durations and may not be acceptable by the primary network. In order to alleviate this, we propose a change in full duplex synchronization. If both SUs are synchronous and transmit at the same time, upon collision with a PU signal, both NACK signals would be generated at the same time, at the end of the frame. But if SUs transmit asynchronously (or have different transmission durations), then these detection events would not be coincident and this in turn would result in shorter average collision durations (see Fig. 6). For the case of equal transmission frames, i.e. $T_1 = T_2 = T$, the optimum timing difference would be $T/2$ for shortest average collision duration.

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**Fig. 4.** CS, FDTS, and FDTR modes and switching between them through a MAC handshaking protocol.

**Fig. 5.** Mode selection protocol based on DTD scheme for conservative and non conservative systems.

**Fig. 6.** Synchronous (a) and asynchronous (b) transmissions and their effect on collision duration.
It means that the transmitted signal by SU may not be affected by the PU signal, due to deep fading of the channel between the primary transmitter and the secondary receiver, in which case no packet or frame error would be declared although PU is on. This may increase collision probability as well. But such conditions may happen during sensing slots in traditional methods too, and could result in longer collision durations. This method has the drawback that there may be packet or frame errors not caused by collision with a primary signal. Fading, noise or any other interferences may cause such errors which will in turn cause false alarm, interrupt SU’s transmissions, and decrease SU’s throughput. However, our method is a conservative way to avoid any probable collision with primary users, hence provides high primary protection. On the other hand, duration of sensing slots is small compared to the secondary frames, and cooperative signal detection decreases the probability of consecutive false alarms. Therefore, the throughput degradation due to general packet or frame errors is small, as will be seen later in the analysis and numerical results.

III. PERFORMANCE ANALYSIS

A. Sensing Metrics

If prior knowledge of the PU signal is unknown, the energy detection method is optimal for detecting zero-mean constellation signals [19]. In this paper we have considered energy detection method in the CS and FDTS modes. In the CS mode cooperative sensing is performed through consecutive sensing slots of duration $T_s$, sampling frequency $f_s$ and with $N = f_s \cdot T_s$ samples in each slot. For circularly symmetric complex Gaussian primary and noise signals, the probability of false alarm and detection of each node in the CS mode are respectively as follows [19]:

$$P_f(\varepsilon_0, T_s) = Q\left(\frac{\varepsilon_0 - 1}{\sigma_a^2} \sqrt{f_s \cdot T_s}\right),$$

(1)

$$P_d(\varepsilon_0, T_s) = Q\left(\frac{\varepsilon_0}{\sigma_a^2} - 1 \frac{f_s \cdot T_s}{2\gamma + 1}\right),$$

(2)

where $\varepsilon_0$ is the detection threshold, $\sigma_a^2$ is the noise variance, $\gamma$ is primary’s SNR at secondary receiver, and $Q()$ is the complementary distribution function of the standard Gaussian, or complementary error function. For a target probability of detection, $P_d$, the detection threshold $\varepsilon_0$ can be determined by:

$$Q^{-1}(P_d) = \frac{\varepsilon_0}{\sigma_a^2} - 1 \frac{f_s \cdot T_s}{2\gamma + 1}.$$ 

(3)

The value of $\varepsilon_0$ which is used for detection of a PU signal is the same in our scheme and in traditional LBT methods. For a lower probability of collision, we will fuse the results of two sensing units and the hypothesis is that PU is in off state when both SU’s have sensed the channel idle. This will decrease misdetection probability to a very low value but will increase the probability of false alarm, as follows:

$$P_f = 2P_f(\varepsilon_0, T_s) - P_f(\varepsilon_0, T_s)^2, P_{d1} = 2P_d(\varepsilon_0, T_s) - P_d(\varepsilon_0, T_s)^2.$$ 

(4)

$P_{f2}$ and $P_{d2}$ are, respectively, probabilities of false alarm and detection in the CS mode. In FDTS mode, there is no cooperation in sensing, and the residual self-interference signal will affect the sensing probability. The false alarm ($P_{f2}$) and detection ($P_{d2}$) probabilities in the FDTS mode with an LAT scheme are [8]:

$$P_{f2} = Q\left(\frac{\varepsilon_0}{(1 + \beta \gamma_s)\sigma_a^2} - 1 \sqrt{f_s \cdot T_s}\right),$$

(5)

$$P_{d2} = Q\left(\frac{\varepsilon_0}{(1 + \beta \gamma_s + \gamma)\sigma_a^2} - 1 \sqrt{f_s \cdot T_s}\right).$$

(6)

where $\gamma_s$ is the secondary’s self-interference SNR.

In the FDTR mode, detection method is different from FDTS, and is based on NACK and Undecode events when a collision with primary happens. Since we assume that transmission errors in NACKs are negligible, the probability of not receiving a NACK at node $i$ (i.e. misdetection) despite the presence of a strong primary signal, is the probability that both channels from primary to SU1 and SU2 be in deep fade and the reverse channels (i.e. from SU1 and SU2 to PR) are not in deep fade. This probability is very small and may be neglected. However, when the PU signal is weak, it may not cause packet or frame errors in SUs’ transmissions (i.e. no NACKs would be declared), and the cognitive system may not be informed of the return of a PU signal. Hence the SUs will continue their transmissions despite collision with primary signal. To tackle this problem, the second detection threshold ($\varepsilon_1$) was introduced. For the presence of PU to be detectable by the SUs, the energy level of PU signal must be large enough to cause a considerable average packet error rate in the secondary’s transmission. This average packet error rate varies with the desired level of primary’s protection. To guarantee minimum collision with the primary, the system is allowed to operate in the FDTR mode, only if the return of the PU signal would cause enough packet error rate (NACK or Undecode events) in the secondary’s transmission. Then the system will stop full duplex transmission and switch back into the CS mode. The target value for secondary’s average packet error rate is a design factor based on the required primary’s protection and secondary’s throughput.

In order to set $\varepsilon_1$, first we need to relate the target average PER in secondary’s transmission to the primary’s signal level.

Here we use the approximate value for instantaneous packet error rate given in [20] which is widely used in the literature:

$$PER(\gamma) = \begin{cases} 1, & 0 < \gamma < \gamma_t, \\ a \exp(-g \gamma), & \gamma \geq \gamma_t, \end{cases}$$

(7)

where $\gamma$ is the instantaneous signal-to-noise ratio and $a, g, \gamma_t$ are mode dependent parameters found by least-squares fitting to the exact packet error rate. In a Rayleigh fading channel, $\gamma$ follows an Exponential distribution with the following probability density function:

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right),$$

(8)

where $\bar{\gamma}$ is the average received signal-to-noise ratio. For an SU with imperfect SIS where the instantaneous SNR is set to $\gamma_{S1}/(1 + \gamma_p + \beta \gamma_{S2})$, in which $\gamma_{S1}$ is the SNR at SU2 due to
SU1, $\gamma_p$ is the SNR at SU2 due to PU’s transmission, and $\beta \gamma_{S_2}$ is the residual self-interference of SU2 after partial cancellation, the average value of (7) is:

$$ \overline{PER}_{PS} = \mathbb{E}[PER(\gamma)] = \int_0^\infty \frac{\gamma S_2}{1 + \gamma P + \beta \gamma_{S_2}} f_{\gamma_S}(\gamma_S) d\gamma_S \exp \left( -\frac{(1 + \beta \gamma_{S_2}) \gamma_P}{\gamma_S} \right) \tag{9} $$

$\overline{PER}_{PS}$ is the target packet error rate in secondary transmission when both PU and SU signals are present. From (9) we can calculate the value of $\overline{\gamma}_P$ for a target $\overline{PER}_{PS}$. Then the value of $\epsilon_1$ would be:

$$ \epsilon_1 = \overline{\gamma}_P \sigma_u^2. \tag{10} $$

On the other hand, false alarm happens when an error in a frame triggers a NACK due to reasons other than collision with the primary. Within the FDTR mode, false alarm probability ($P_{fa}$) is the same as the average packet error rate. We assume that both SUs have the same average packet error rate. The average packet error rate of the secondary users when PU signal is not present ($\overline{PER}_{S}$) is:

$$ P_{fa} = \overline{PER}_{S} = 1 - \frac{\overline{\gamma}_P S_1}{1 + \beta \overline{\gamma}_{S_2}} \exp \left( -\frac{(1 + \beta \overline{\gamma}_{S_2}) \gamma_P}{\overline{\gamma}_S} \right). \tag{11} $$

### B. Distributions of Collision Probability and Collision Duration

As depicted in Fig. 7, collision with primary signal may occur at the beginning or the end of a secondary frame. Pre-collision happens when the network is in the CS mode and misdetection occurs, and post-collision occurs within the FDTS or FDTR modes. Therefore, probability of collision, $p_{col}$ in our proposed scheme would be:

$$ p_{col} = P(\mathcal{H}_0) \left[ a P_{o_1} \left( 1 - P_{f_2} \frac{T_o}{T} \right) + (1 - a) P_{o_2} \left( 1 - P_{f_3} \frac{T_o}{T} \right) \right] + P(\mathcal{H}_1)(1 - P_{di}), \tag{12} $$

where $P_{f_1} = 1 - e^{-T/2}$ and $P_{f_2} = 1 - e^{-T/2}$ are respectively the probabilities that primary signal reappears within a sensing slot of duration $T_o$ or within a SU frame of duration $T$. $\alpha$ is the probability that the system is operating in the FDTS mode (i.e. $P(\varepsilon \leq \epsilon_1)$, in which $\varepsilon$ is the measured energy of PU signal), and $1 - \alpha$ is the probability of operation in the FDTR mode ($P(\varepsilon \geq \epsilon_1)$). If we assume that $\epsilon$ varies uniformly between $\epsilon_0$ and $\epsilon_{max}$ due to random channel conditions or SUs’ movements, then $\alpha$ can be derived from:

$$ \alpha = \frac{\epsilon_1 - \epsilon_0}{\epsilon_{max} - \epsilon_0}. \tag{13} $$

$\epsilon_{max}$ is the maximum value for higher threshold which is equal to sum of PU’s transmission power and noise power. When $\epsilon_1 = \epsilon_0$ (a system with single threshold) $\alpha = 0$ and the system operates in FDTR mode only. When $\epsilon_1 = \epsilon_{max}$, then $\alpha = 1$ and the system operates in the FDTS mode only. For $\epsilon_0 < \epsilon_1 < \epsilon_{max}$, the system operates in both modes based on the detected primary’s signal strength.

In the traditional LBT scheme (HD or FD), probability of collision can be written as:

$$ p_{col}^LBT = P(\mathcal{H}_0)(1 - P_f)P_T + P(\mathcal{H}_1)(1 - P_{di}), \tag{14} $$

where $P_T = 1 - e^{-\lambda T/T_p}$ is the probability that primary signal reappears within a SU frame of duration $T - T_p$. $\mathcal{H}_0$ is the event that the PU is not active at the start of the frame, and its complement is $\mathcal{H}_1$. In our system where PU’s idle and busy times are exponential with parameters $\lambda$ and $\mu$ respectively, the respective a priori probabilities of PU activity at the start of each secondary frame are $P(\mathcal{H}_0) = \frac{\mu}{\lambda + \mu}$ and $P(\mathcal{H}_1) = \frac{\lambda}{\lambda + \mu}$.

Within the FDTS mode, the average collision duration would be $T_o$ seconds. When a collision happens in the FDTR mode, it would last until the first NACK signal is declared from any of the SUs. Having assumed that the NACK signals are free from errors, in asynchronous transmission mode when there is a lag of $T/2$ seconds between the SUs’ transmissions, the packet error rate in secondary users would be as follows:

$$ p_{col} = \begin{cases} \frac{\lambda e^{-\lambda T/T_p}}{1 - e^{-\lambda}} & 0 < \tau < \frac{T}{2} \\ 0 & \tau > \frac{T}{2} \end{cases}. \tag{15} $$

Collision duration may be longer than $T/2$ if there is no NACK at the end of a SU transmission frame. This may occur when the channel between the PU transmitter to only one of SU receivers has been in deep fade for at least the collision duration. If channels from the PU to both SU receivers had been in deep fade since PU has reappeared and collided with SUs signals, then the collision duration would be even longer than $T$ seconds. But such probability is low and may be ignored. A Rayleigh flat fading channel, the probability of outage $P_{out}(\tau_m)$ (i.e. when $SINR$ goes below a required threshold $R$ and stays there for at least $\tau_m$ seconds), is a function of the maximum Doppler frequency, threshold $R$ and the average fade duration. This can be found from equations (13-15) in [21] which are not repeated here for brevity. Hence, collisions with durations more than $T/2$ will happen when we have an outage at the same time on one of the channels. Then probability density function of the collision duration would be:

$$ p_T(\tau) = \begin{cases} \frac{\lambda e^{-\lambda T/T_p}}{1 - e^{-\lambda}} & 0 < \tau < \frac{T}{2} \\ \frac{\lambda e^{-\lambda T/T_p}}{1 - e^{-\lambda}} P_{out}(\tau) & \frac{T}{2} < \tau < T \\ 0 & \tau > T \end{cases}. \tag{16} $$

### C. Throughput

When PU is active, our proposed system operates in the CS mode and none of the SUs would attempt transmission. However, in case of misdetection, they may start transmitting...
while the PU is on. Since the probability of misdetection in the CS mode is very low, we neglect such rare possibility. On the other hand, when PU is idle and the system is in the FDTR mode with imperfect SIS capability, each secondary unit transmits in a full-duplex manner with nominal throughput $R_0 = \log(1 + SNR)_{\frac{T}{\tau}}$ during collision-free intervals, and with degraded throughput $R_1 = \log(1 + \frac{SNR}{1+SNR_0+p})$ during collision interval. $SNR_0$ is the signal to noise ratio due to one SU, and $SNR_0$ is that due to the primary signal at the SU’s receiver. In the FDTS mode, only one SU transmits with the rate $R_0$ during non-collision times and $R_1$ when collision happens. If any false alarm is announced in between, throughput will be zero during the subsequent sensing interval, $T_s$. Referring to Fig. 7, we denote the transmission time with no collision within a frame by $\bar{\tau}$, and collision duration by $\tau$. Then the average throughput of a frame in the FDTS and FDTR modes would be:

$$\bar{R}_{FDTS} = \frac{\bar{\tau}}{T} R_0 + R_1 + (1 - p_1) R_0$$  \hspace{1cm} (17)

$$\bar{R}_{FDTR} = \frac{\bar{\tau}}{T} R_0 + R_1 + (1 - p_1) R_0$$  \hspace{1cm} (18)

where $\bar{\tau}$ and $\bar{\tau}$ are, respectively, the average values of non-collision and collision durations in asynchronous FD operation, and are derived as follows:

$$\bar{\tau} = \mathbb{E}[\tau|\tau < \frac{T}{2}] = \int_0^{\frac{T}{2}} \tau p_\tau(\tau) d\tau = \int_0^{\frac{T}{2}} \frac{\lambda e^{-\lambda T} \tau}{1 - e^{-\lambda T}} d\tau = \frac{\lambda T + e^{-\lambda T} - 1}{2\lambda(1 - e^{-\lambda T})}$$  \hspace{1cm} (19)

$$\bar{\theta} = \mathbb{E}[\theta|\theta < T/2] = T - \bar{\tau} = \frac{1 + \lambda T - (2\lambda T + 1)e^{-\lambda T}}{2\lambda(1 - e^{-\lambda T})}$$  \hspace{1cm} (20)

The terms inside the first parenthesis in (17) and (18) correspond to the case when the frame encounters collision in the FDTS or FDTR modes, and the last terms to the frames with no collision. In order to calculate the average throughput, we need to consider the effect of false alarms. As we have three modes of operation and three types of false alarms corresponding to each mode, we need to differentiate between them.

False alarm effects on the first frame following the CS mode differently from other frames in this mode and in the other modes. When the primary ceases transmitting, the CR network would detect an opportunity with probability of $1 - P_{f1}$ and will start transmission for at least half of the frame duration, or will miss the opportunity for $T_s$ seconds (sensing period) with probability of $P_{f1}$. Within the FDTS mode, should a false alarm occur (with probability of $P_{f1}$), the system will go into the CS mode for at least one sensing slot (i.e. $T_s$ seconds), otherwise will continue transmission for $T_s$ seconds. In the FDTR mode if a false alarm occurs (with probability of $P_{f1}$), the system will go into the CS mode for at least one sensing slot (i.e. $T_s$ seconds), otherwise will continue transmission for $T/2$ seconds, by the end of which another ACK/NACK signal is issued. Given the above breakdown, we derive the average throughput for a bidirectional full duplex CR system, based upon our proposed scheme, as follows:

$$R_{avg} = P(\mathcal{H}_0) \left\{ \frac{\bar{\tau}}{T} \left[ 1 - (P_{f1} + P_{f2} + \cdots + P_{fT}) \right] \right\}$$  \hspace{1cm} (21)

in which $\frac{\bar{\tau}}{T}$ corresponds to the probability that the considered frame is the first frame following the CS mode, and $(1 - \frac{\bar{\tau}}{T})$ accounts for the frames in the FDTR or FDTS modes. (21) may be simplified more as follows:

$$R_{avg} = P(\mathcal{H}_0) \left\{ \alpha \bar{R}_{FDTS} \left[ 1 - \frac{T}{\tau} e^{-\lambda T} \right] \right\}$$  \hspace{1cm} (22)

Having inserted the respective values for $\bar{\tau}$ and $\bar{\theta}$ from (19) and (20) and for the case $\alpha = 0$ (i.e. when the system operates only in the CS and FDTR modes), the average throughput of a CR network with the proposed FD scheme, respectively in asynchronous and synchronous transmission modes, are derived as:

$$R_{sync} = 2 P(\mathcal{H}_0) \left\{ \frac{2T}{\lambda} P_{f1} \right\}$$  \hspace{1cm} (23)

For comparison purposes we have derived the throughput of a FD system using the LBT technique with similar duration of sensing slots $T_s$ and transmission frames $T$, as follows:

$$R_{lat} = 2 P(\mathcal{H}_0) \left\{ R_0 + (1 - P_{f1}) \right\}$$  \hspace{1cm} (25)

IV. SIMULATION RESULTS

For simulation purposes we have considered a CR network with a pair of SUs and a pair of PUs. The simulation parameters are shown in Table I. We have used coded BPSK modulation and the parameters presented in [20, Tab.1] for calculating packet error rate of SUs’ communication.

Fig. 8 depicts the probability of collision of a SU frame with different durations for our proposed scheme in

<table>
<thead>
<tr>
<th>TABLE II. SIMULATION SCENARIO PARAMETERS</th>
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<tr>
<td>PU ON State Duration ($\mu s^{-1}$)</td>
</tr>
<tr>
<td>PU OFF State Duration ($\lambda s^{-1}$)</td>
</tr>
<tr>
<td>Sampling rate ($f_s$)</td>
</tr>
<tr>
<td>Sensing slot duration ($T_s$)</td>
</tr>
<tr>
<td>Packet size</td>
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<td>Frame size</td>
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<tr>
<td>Primary SNR (at SUs)</td>
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<td>Secondary SNR (at SUs)</td>
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synchronous and asynchronous modes, and for the conventional LBT scheme in a full duplex CR scenario. As it is seen, when the frame length increases, it is more likely that primary signal reappears within a SU frame interval, and that collision occurs. This collision probability is the same for various false alarm probabilities \( P_f^2, P_f^3 \) in the FDT and FDTR modes, since the effect of \( P_f^2 \) and \( P_f^3 \) are reduced by \( T_s/T \) factor according to (12), and it is lower in asynchronous mode which was expected. But in the LBT scheme, false alarm probability \( P_f^1 \) has a considerable effect on the collision probability as seen in (14). In the conventional LBT scheme the probability of collision decreases as false alarm probability increases. However, this is not an advantage, since higher false alarm probability means more lost transmission opportunities and remarkable network throughput degradation, which is not desirable. Moreover, the adaptive ability of our scheme in switching between the FDTS and FDTR modes results in lower probability of collision as shown in Fig. 9. As we see in synchronous mode when \( \alpha = 0.3 \) (which means the system will operate in the FDTS mode in 30 percent of the times), the probability of collision is lower than in a LBT system with the same false alarm probability. Asynchronous transmission will result in even lower probability of collision without degradation in throughput of the system which is not shown here for brevity.

In Fig. 10, we have shown the average cumulative collision duration in a single SU transmission frame. As verified in Fig. 8, the false alarm probability has almost no effect on this metric in our proposed scheme for both synchronous and asynchronous modes, in contrast to that in the LBT scheme. When \( \alpha = 0 \) we experience lower average collision probability when we check the presence of PU before every SU transmission in the conventional LBT schemes. However, as can be seen from this figure, the average cumulative collision duration, which is more important in meeting PUs’
protection criteria, in our proposed scheme is less than that in
the LBT method. This metric improves even further for \( \alpha \neq 0 \).

Another important performance metric in a CR network is its throughput. Fig. 11 shows a comparison between the achievable SUs’ throughput in the asynchronous and synchronous modes of operation at different values of frame duration. We observe that the simulated average throughput is very close to the theoretical one derived in (22). We also observe that this average in the asynchronous mode is slightly more than in synchronous transmission which is due to the shorter collision duration in the asynchronous mode. For shorter frame lengths, the difference in collision duration between two modes is not considerable which results in less difference in throughput. Moreover, we observe the effect of SIS factor \( \beta \) on throughput; poorer SIS capability (higher \( \beta \)) results in throughput degradation.

In Fig. 12, we see the variation of the SU network throughput against frame time duration for different values of \( \alpha \). When the system has lesser chance of operation in the FDTR mode (i.e. for higher values of \( \alpha \)), the secondary network’s throughput is degraded. It would be expected since communication in the FDTS mode is in half duplex mode, and any value of \( \alpha \) other than zero, will force the system to operate in the FDTS mode with half the throughput compared to the FDTR mode.

Finally Fig. 13 shows the SU network throughput in our proposed scheme and the LBT scheme, versus SIS factor \( \beta \) for the case where \( \alpha = 0 \). Here we see a close match between the analytical and simulation results. As it is seen, our method outperforms the fixed sensing method for any value of false alarm probability. As self-suppression capability of SUs decreases, the average throughput of the CR network monotonically decreases as well. In the LBT schemes, SIS factor, \( \beta \), only influences the SU throughput through its effect on \( R_2 \) and \( R_1 \) according to (22), but in our protocol it has an additional effect on the false alarm probability, \( P_f \), according to (11). However, as shown in equations (23) and (24), the effect of \( P_f \) on the SU throughput in our protocol is negligible, which is in line with the behavior of SU throughput in our scheme and in the conventional LBT schemes according to Fig. 13.

V. CONCLUSIONS

In this paper we studied the full duplex paradigm in an opportunistic access cognitive radio network with imperfect self-interference suppression, and proposed a novel scheme incorporating cooperative sensing and asynchronous full duplex transmission. The system works in three modes: CS mode when PUs are active, and FDTR and FDTS modes when PUs are not active. As a result of cooperative sensing in the CS mode, the probability of detection is high enough to guarantee no collision with the primary signal when the PU is in ON state. In the FDTR mode, we applied asynchronous transmission for SUs with equal frame durations, in order to decrease the probability of collision and the average collision duration. We derived analytical closed forms for the probability of collision, the probability density function of collision duration, and the average throughput for the proposed scheme. Numerical results indicated that the proposed scheme outperforms both the conventional LBT and the synchronous transmission scheme in the FDTR mode, in terms of average collision duration and average secondary throughput.

ACKNOWLEDGMENT

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APPENDIX

Proof of Proposition: As it is shown in Figure A1 for a time difference of \( L \) seconds, the duration of collision in a frame would be:

\[
\tau = \begin{cases} 
L - \theta & \text{if } 0 < \theta < L, \\
T - \theta & \text{if } L < \theta < T 
\end{cases}
\]

Fig. A1. Duration of collision

in which random variable \( \theta \) with pdf \( p(\theta) = \frac{\lambda e^{-\lambda \theta}}{1 - e^{-\lambda T}} \) indicates the end time of primary’s OFF period. For \( 0 < \theta < L \) the maximum duration will occur at \( \theta = 0 \) and \( \tau_1 = L \). For \( L < \theta < T \) it occurs at \( \theta = L \) and \( \tau_2 = T - L \). The minimum would happen when \( \tau_1 = \tau_2 \), which will entail \( L_{\text{min}} = T/2 \).
The average collision duration within a single transmission frame would be:

\[
\bar{\tau} = \frac{P(0 < \theta < L)(L - \theta) + P(L < \theta < T)(T - \theta)}{\int_0^L p(\theta) (L - \theta) d\theta + \int_L^T p(\theta) (T - \theta) d\theta} = \frac{L \int_0^L p(\theta) d\theta + T \int_L^T p(\theta) d\theta - \int_0^T p(\theta) d\theta}{\int_0^L \left(\frac{1 - e^{-\lambda L}}{1 - e^{-\lambda L}} + \frac{e^{-\lambda L} - e^{-\lambda T}}{1 - e^{-\lambda T}}\right) + \left(\frac{\lambda T + 1}{\lambda (1 - e^{-\lambda T})} - 1\right)}
\]

In order to find the optimum value for \( L \) that minimizes the above average collision duration, we obtain its derivative with regard to \( L \) and set it to zero:

\[
1 + (\lambda L - 1)e^{-\lambda L} - \lambda T e^{-\lambda L} = 0
\]

\[
(\lambda L - 1 - \lambda T)e^{-\lambda L} = -1
\]

\[
(-\lambda L + 1 + \lambda T)e^{-(\lambda L + 1 + \lambda T)} = e^{1+\lambda T}
\]

\[
-\lambda L + 1 + \lambda T = W(e^{1+\lambda T}) - \lambda L = \frac{1 + \lambda T - W(e^{1+\lambda T})}{\lambda}
\]

where \( W(.) \) is the Lambert W function.

REFERENCES


