Accounting for Ambiguity and Trust in Partial Outsourcing: A Behavioral Real Options Perspective

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Concerned with the hidden costs of outsourcing, this paper examines the role of ambiguity and trust in partial outsourcing decisions from the perspective of real options theory. We study pricing and quantity dynamics between an ambiguity averse vendor and a less (more) trusting client in a leader-follower framework with fixed timing. We find that the client’s partial outsourcing quantity increases with the vendor’s ambiguity if outsourcing is meant for cost-saving purposes. The effect of trust on outsourcing quantity, meanwhile, is jointly moderated by the vendor’s ambiguity and quality of shared information forecasts when cost advantages are exaggerated. In terms of pricing effects, the vendor increases (decreases) her threshold with increasing ambiguity for long-term (short-term) contracts. These insights hold under the multiple-priors and worst-case ambiguity specification. When Choquet ambiguity and rank-dependent utility are considered, more complex and subtle dynamics are obtained. Ambiguity has additional non-linear effects on outsourcing quantity due to heterogeneity in ambiguity preferences (seeking vs. aversion) and probability weighting. The vendor’s price not only increases (decreases) with increasing ambiguity-seeking for long-term (short-term) contracts, but also with ambiguity aversion when specific risk-return ratio conditions are met. Trust effects are qualitatively similar under both ambiguity specifications.

Keywords: partial outsourcing; cost uncertainty; real options; trust; multiple-priors ambiguity; Choquet ambiguity

Introduction

Partial outsourcing is prevalent in many industries. Most manufacturers outsource their operations to some degree subject to changes in business and economic conditions. Samsung, for example, is known to manufacture its products in-house but also outsources some of its production processes whenever necessary. Toyota outsources 70% of the components of its vehicles and keeps 30% in-house to improve innovation (Xiao and Gaimon, 2013). According to Capgemini (2014), 72% of 3PL users have increased their reliance on outsourced logistics services either as a whole or in terms of individual activities, while several shippers source most, if not all, of their logistics activities. E-commerce companies like Amazon have their own fleet of trucks but also use traditional carriers such as FedEx and UPS to speed up product delivery and control shipping expenses.

While the above suggests that partial outsourcing has become second nature in manufacturing and services, stories about outsourcing failure are not uncommon (Barthelemy, 2003; Cabral et al., 2014).\footnote{Reports in the popular press (e.g., Forbes, IndustryWeek, Businessweek, CIO magazine, Computerworld) indicate that major companies such as IBM, JP Morgan Chase, EDS, Royal Bank of Scotland, Boeing, Accenture and Virgin Airlines have all experienced some kind of outsourcing failure in recent years.}

Reasons behind
such failures have been attributed to a mismatch in expectations between clients and vendors, unexpected and hidden operating costs, poor governance systems and cultural differences (Forbes, 2013; Larsen et al., 2013; Cabral et al., 2014). Several surveys of executives (Barthelemy, 2003; Benlian and Hess, 2011) especially highlight how most outsourcing programmes fail to meet their cost-saving targets because of unexpected and consistently increasing operating costs, lack of information clarity about outsourcing implementation, and fragile risk mitigation policies. This double-sided form of cost uncertainty together with information incompleteness and suboptimal behavior - as other hidden costs of outsourcing - creates ambiguity in the outsourcing process. Ambiguity also arises from external factors because of the increasingly complex and turbulent business environments firms (i.e., vendors and clients) find themselves operating in (Sargent and McGrath, 2011). Wages in several outsourcing hubs have more than doubled since 2008. Industrial and agricultural raw material prices have become more volatile (Boute and Van Mieghem, 2015). Exchange rates and shipping costs across the globe are also shifting frequently because of numerous structural economic changes. This renders the task of accurately predicting future costs and effectively managing outsourcing operations extremely difficult and uncertain for vendors. This, in turn, exacerbates the negative effects of cost-related uncertainty on outsourcing performance and increases the hidden costs of outsourcing. Another important source of uncertainty relates to the quality of outsourcing services, the degree of trust between clients and vendors, and the reliability of outsourcing providers (Zuñiga and Martinez, 2016; Drauz, 2016).

In such an ambiguous state of affairs and with the increasing rate of failure surrounding outsourcing, determining the right price and quantity of (partial) outsourcing is becoming a critical and especially challenging task for vendors (contract-receiving firms) and clients (contract-granting firms). In this paper, we examine the issue of the hidden costs of outsourcing, and its pricing and quantity implications, from an ambiguity perspective using real options theory. We are interested in the cost-saving and volume-based learning effects of partial outsourcing on pricing and quantity dynamics in a dyad consisting of an ambiguity averse vendor and a less (more) trusting client. Because of her perceived specialization, the vendor (she), while uncertain about her own future operating costs, makes positive forecasts about the cost-saving benefits associated with outsourcing. To clinch a bigger contract, she exaggerates the client’s in-house variable costs and communicates this exaggerated information to the client. The client (he), faced with incomplete information, becomes doubtful and less confident about the forecasts and decides whether and to which extent he should trust the vendor. As such, the client’s degree of trust towards the vendor’s forecasts and the vendor’s uncertainty or degree of ambiguity about the costs of outsourcing are fundamental to the outsourcing arrangement.
This paper proposes a behavioral real option model with trust for quantity and price under ambiguity that enables us to better understand some of the hidden costs of outsourcing, their consequences and the nature of outsourcing relationships in practice. Specifically, our ambiguity-based modeling provides insights into how unexpected costs, expectations’ mismatch and suboptimal/subjective behavior affect partial outsourcing decisions and their outcomes. The contribution of the paper is twofold. First, we address the problem of the hidden costs of outsourcing by unveiling the joint effects of ambiguity and trust on price and quantity dynamics in outsourcing decision making. Second, we compare and contrast these effects under the multiple-priors and Choquet ambiguity specifications using real options theory, thus introducing notions of cognition and subjective behavior to the real options literature concerned with outsourcing (e.g., Antelo and Bru, 2010, Li and Wang, 2010).

As a real option, outsourcing has the potential to lower costs, increase flexibility and enhance firm value (Choi et al., 2017). However, this might be at the expense of innovation, service quality and tacit know-how (Kenyon et al., 2016). In Xiao and Gainmon (2013), for example, future value is defined as a power learning function of an in-house production quantity that captures the incremental benefits of keeping manufacturing in-house. At the same time, the client can also learn from the vendor’s tacit knowledge through outsourcing (Gupta and Polonsky, 2014; Aubert et al., 2015). This learning effect is empirically highlighted by Kroes and Ghosh (2010) but has not received enough attention in normative outsourcing decision models. The trade-offs between the future value of in-house operations and that of outsourcing for the client - and their influence on outsourcing pricing and quantity decisions - are also studied in our paper using our ambiguity-based real options approach with trust. This is the first research to examine the partial outsourcing real option problem and the issue of the hidden costs of outsourcing from the perspective of behavioral theory (see e.g., Agliardi et al., 2016; Leiblein et al., 2017 and their behavioral valuation models) while considering ambiguity and trust jointly. Our behavioral real options model of partial outsourcing captures a number of features neglected in the existing literature: vendor’s ambiguity, client (dis)trust, and volume-based learning from in-house production and outsourced operations. Allowing for ambiguity in such a setting is important because it enables us to shed light on the role of subjective/suboptimal behavior in outsourcing relationships, account for unexpected cost uncertainty and lack of information clarity in outsourcing provision, and understand some of the antecedents of outsourcing failure.

In our fixed timing setting (see e.g., Agliardi and Koussis, 2011), the client acts as a leader in the sense that he controls the outsourcing quantity and timing while the vendor is a follower who sets her pricing conditions. We model ambiguity using the multiple-priors ambiguity specification or so-called “worst-case”
ambiguity aversion heuristic², addressing the following practical questions: What is the client's outsourcing quantity with and without trust? How to determine the outsourcing price and quantity of outsourcing under ambiguity when accounting for cost-savings and future learning benefits? How do trust and ambiguity (preferences) affect partial outsourcing outcomes?

We find that higher ambiguity from the vendor increases the client’s outsourcing quantity when the cost-saving index is positive but does not necessarily increase the vendor’s lowest offer price. The vendor’s price increases (decreases) with ambiguity for long-term (short-term) contracts. Trust affects the partial outsourcing quantity in a non-monotonic way as a result of joint moderating effects from ambiguity and information sharing quality. Most of these dynamics are altered under Choquet ambiguity and rank-dependent utility when ambiguity preferences (seeking vs. aversion) and probabilistic sophistication come into play. The effects of ambiguity on outsourcing quantity are exacerbated by heterogeneity in ambiguity preferences and the vendor’s price is found to not only increase (decrease) with ambiguity-seeking for long-term (short-term) contracts, but also with ambiguity aversion when specific risk-return ratio dynamics are met. Trust effects are maintained under rank-dependent utility. These findings are able to explain over-commitment and short-termism biases in outsourcing, provide guidance on how clients and vendors behave when faced with incomplete and asymmetric information, and point to the need to embed flexibility/performance clauses in outsourcing contracts and sequence commitment into contingent stages as ways of dealing with ambiguity and mitigating some of the hidden costs of outsourcing.

The paper is organized as follows. The next section reviews some relevant literature concerned with real options and outsourcing. In Section 2, we investigate the vendor’s outsourcing pricing strategy and the client’s outsourcing quantity decisions. The effects of ambiguity and trust on outsourcing outcomes are discussed in Section 3. The final section concludes with a summary of findings and research implications. A list of notations and all proofs are covered in the Appendix.

1. Related literature

There is substantial research on how real options affect outsourcing decisions in the presence of irreversibility and uncertainty. Real options theory has been applied to the analysis of clients’ outsourcing strategies in a number of economic settings and contractual frameworks (e.g., Nembhard et al., 2005; Alvarez and Stenbacka, 2007; Liu and Nagurney, 2011; Benaroch et al., 2012). These studies have provided a deeper understanding of clients’ managerial flexibility in outsourcing when faced with international risk and demand

² For comparison, Table 2 revisits our modeling from the perspectives of 1) Choquet ambiguity and 2) risk without ambiguity.
and cost uncertainty. Different from this stream of literature, Jiang et al. (2008), Jiang et al. (2010), Moon et al. (2011) and Shi and Feng (2016) have recently emphasized the role of vendors in setting outsourcing contracts and described how real options valuation for outsourcing can diverge if viewed from the vendor’s perspective. Little attention has been paid, however, to how subjective behavior and behavioral uncertainty affect option-based outsourcing outcomes. The decision variables in the above and other related articles are summarized and compared in Table 1. Most of the existing literature quantifies outsourcing outcomes based on exogenously determined outsourcing prices or quantities. Also, all the aforementioned real options studies share a common assumption that the distribution of uncertain factors is perfectly known (risk) to the client or vendor. The impact of ambiguity, as uncertainty beyond risk, on vendors’ and clients’ outsourcing decisions remains unclear and under-researched. Accounting for ambiguity in outsourcing is empirically relevant because of the hidden costs associated with outsourcing, lack of information clarity about vendors’ outsourcing provision and operating costs, vagueness in contractual relationships, and the high economic uncertainty in business environments. Recent qualitative empirical research also calls attention to the role of uncertainty, cognition and behavioral biases in outsourcing decisions (Musteen, 2016).

In this paper, we rely on the multiple-priors ambiguity specification, considering $\kappa$-ignorance with the worst-case decision rule, to elucidate how ambiguity affects outsourcing price and quantity dynamics. The ambiguity studied herein concerns the costs associated with outsourcing and relates to hidden and unexpected costs. For comparison, we also study the effect of Choquet ambiguity or rank-dependent utility (see e.g., Agliardi and Sereno, 2011; Kast et al., 2014; Driouchi et al., 2015), proxied by $c$-ignorance, on outsourcing outcomes in Section 3. The multiple-priors specification considers worst-case decision-making and appraisal whereas the Choquet framework accounts for probabilistic sophistication and probability weighting under ambiguity aversion and seeking. While studying multiple-priors ambiguity should be more suitable for situations where vendors feel less competent and less confident about performing new or untested/unknown outsourcing tasks (i.e., akin to Ellsberg “unknown” urns), Choquet-based analysis should fit cases of more confident vendors with more relevant outsourcing expertise and experience.

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3 Our paper is also related to recent real options research on revenue sharing (e.g., Shibata and Nishihara, 2011; Banerjee et al., 2014).

4 Several papers have recently studied real options in the context of ambiguity averse decision makers, including Nishimura and Ozaki (2007), Trojanowska and Kort (2010), Asano (2010), Miao and Wang (2011) and Gao and Driouchi (2013).

5 See detailed discussions in Agliardi and Sereno (2011), Kast and Lapied (2010) and Kast et al. (2014). Compared with multiple-priors ambiguity (e.g., Chen and Epstein, 2002; Nishimura and Ozaki, 2007), Choquet utility allows for different attitudes towards ambiguity, probability weighting and meets the dynamic consistency requirement of real options analysis.
Table 1. Real options in outsourcing

<table>
<thead>
<tr>
<th>Citation</th>
<th>Uncertainty type</th>
<th>Client decision variables</th>
<th>Vendor decision variables</th>
<th>Considering trust or volume-based learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nembhard et al. (2003)</td>
<td>Risk</td>
<td>Outsourcing timing</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>Nembhard et al. (2005)</td>
<td>Risk</td>
<td>Option implementation</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>Alvarez and Stenbacka (2007)</td>
<td>Risk</td>
<td>Outsourcing proportion</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>Jiang et al. (2008)</td>
<td>Risk</td>
<td>-</td>
<td>Outsourcing timing</td>
<td>No</td>
</tr>
<tr>
<td>Yao et al. (2010)</td>
<td>Risk</td>
<td>Outsourcing price and timing</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>Jiang et al. (2010)</td>
<td>Risk</td>
<td>Outsourcing price and timing</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>Antelo and Bru (2010)</td>
<td>Risk</td>
<td>Restructuring or outsourcing</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>Li and Wang (2010)</td>
<td>Risk</td>
<td>Production quantities</td>
<td>Capacity</td>
<td>No</td>
</tr>
<tr>
<td>Moon et al. (2011)</td>
<td>Risk</td>
<td>-</td>
<td>Outsourcing timing</td>
<td>No</td>
</tr>
<tr>
<td>Benaroch et al. (2012)</td>
<td>Risk</td>
<td>The probability and timing of switching</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>Moretto and Rossini (2012)</td>
<td>Risk</td>
<td>Outsourcing timing, proportion and capacity</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>Liu and Nagurney (2013)</td>
<td>Risk</td>
<td>Sales, outsourcing and fast-response production quantities</td>
<td>-</td>
<td>No</td>
</tr>
<tr>
<td>Shi and Feng (2016)</td>
<td>Risk</td>
<td>-</td>
<td>Outsourcing timing</td>
<td>No</td>
</tr>
<tr>
<td>Shi (2016)</td>
<td>Risk</td>
<td>-</td>
<td>Entry and exit timing</td>
<td>No</td>
</tr>
<tr>
<td>This paper</td>
<td>Risk and ambiguity</td>
<td>Outsourcing quantity</td>
<td>Outsourcing price</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Besides the significance of ambiguity in buyer-seller relationships (see e.g., Hazen et al., 2011; Gao et al., 2018), trust is also considered a key factor in outsourcing but it is rarely studied in extant real options research. As explained by McEvily (2011), it is more realistic and appropriate to combine uncertainty and trust when explaining decision-making behavior than focus on only one or the other. Prior literature highlights the importance of manufacturers’ demand forecast information sharing and its influence on suppliers’ capacity decisions. Özer et al. (2011) pioneer the analytical trust-embedded model for capacity investment decisions and demonstrate that a continuum exists between absolute trust and complete distrust. Özer et al. (2014) further confirm the existence of this continuum between supply chain members from China and those of the United States in an experimental setting. Ebrahim-Khanjari et al. (2012) use trust to evaluate a retailer’s optimal order quantity when a sales-person shares information forecasts on behalf of a manufacturer. They find that trust
alters standard optimality conditions. Similarly, Pezeshki et al. (2013) use trust to filter retailers’ forecasted demand in divergent supply chains and establish a significant link between trust and financial performance. These studies focus on forecast information sharing from a demand perspective. We address the issue of trust in cost information sharing from the client’s viewpoint and consider ambiguity in the vendor’s variable costs (i.e., accounting for unexpected and hidden costs). We define trust as the client’s willingness to adopt the vendor’s forecast report when he evaluates his in-house variable costs. Our behavioral real options framework determines the vendor’s lowest offer price for a given outsourcing timing and the client’s corresponding outsourcing quantity, and offers several new insights into the effects of ambiguity and trust on partial outsourcing decisions.

2. Partial outsourcing under ambiguity and trust
2.1. Model formulation and problem description
Consider a two-tier supply chain with a vendor and client. The client’s total demand is denoted by $Q$. It consists of outsourcing quantity $q$ and in-house operation quantity $Q - q$. The vendor’s and client’s fixed costs are represented by $I$ and $M$, respectively. The contract duration is $D$. Let \((\Omega, \mathcal{F}_T, P)\) denote a probability space and \((B_t)_{0 \leq t \leq T}\) represent a standard geometric Brownian motion with respect to $P$. The benchmark dynamics of the vendor’s variable cost $X(t)$ follow $dX(t) = \mu X(t) dt + \sigma X(t) dB_t$, where $\mu$ and $\sigma$ are the growth rate and volatility, respectively. Because of uncertainty in cost expectations, this lognormal diffusion process gets distorted when accounting for the vendor’s ambiguity and cognition in our outsourcing setting (see Assumption 1). Other notations are summarized in Table A1 in Appendix A. The following assumptions apply.

**Assumption 1.** Ambiguity in the vendor’s variable cost is characterized by the set of probability measures $\mathcal{P} = \{Q^\theta \mid \theta \in \Theta\}$, where $(\theta)_{0 \leq t \leq T} \in \Theta$ satisfies Novikov’s condition, $Q^\theta$ is equivalent to $P$, $\Theta$ is a set of density generators, $\Theta = [-\kappa, \kappa]$, $\kappa$ is a constant used to limit the scope of $\theta$, $\kappa \geq 0$. This form of ambiguity is known as $\kappa$-ignorance in the multiple-priors ambiguity framework (see e.g., Chen and Epstein, 2002). It implies that the vendor is uncertain about the drift of the Brownian motion driving her costs function $X(t)$ and that there are unexpected outsourcing costs. $\kappa$-ignorance further reflects the degree of “contamination” of the vendor’s confidence (see e.g., Nishimura and Ozaki, 2007) about her future variable costs and her subjective/behavioral deviations from the benchmark probability. We use the following multiple-priors-based Brownian motion formulation to capture ambiguity in the vendor’s variable costs (see also Nishimura and Ozaki, 2007; Cheng and Riedel, 2013):
\[ dX(t) = (\mu - \sigma\theta_t)X(t)dt + \sigma X(t)dB^\theta_t \]  
(1)

where \( dB^\theta_t = dB_t + \theta_t dt \) for \( t \in [0, T] \) based on Girsanov’s theorem, \( dB^\theta_t \) is a Brownian motion with respect to \( Q^\theta \), the drift or growth rate is subjectively evaluated as \( \mu - \sigma\theta_t \) because of the vendor’s ambiguity. \( \theta \) is restricted to \( \Theta = [-\kappa, \kappa] \). A greater \( \kappa \) means the vendor is less confident and more ambiguity averse about the growth rate and considers a range of possible drifts under uncertainty (Cheng and Riedel, 2013). Eq. (1) hence aims to also reflect the unexpected or additional costs to be incurred by the ambiguity prone vendor. Behavioral adjustment factor \( \sigma\theta_t \) accounts for her ambiguity aversion/pessimism and subjective deviations from the benchmark cost specification. This subjective factor can help prepare for worst-case eventualities and aid in avoiding the winner’s curse in outsourcing (see e.g., Kern et al., 2002; Jiang et al., 2010).

The worst case of the vendor’s cost \( X(t) \) over \( [t, t + D] \) can be obtained using the maxmin expected utility as follows (Nishimura and Ozaki, 2007; Asano, 2010; Trojanowska and Kort, 2010):

\[ W(t) = \sup_{\theta \in [-\kappa, \kappa]} E_t^Q\left[ \int_t^{t+D} X(\tau)e^{-r(\tau-t)}d\tau \left| \mathcal{F}_t \right. \right] = \lambda X(t) \]  
(2)

where \( r \) is the discount rate, the ambiguity multiplier \( \lambda = \frac{1-e^{-hD}}{h} \), \( h = r - \mu - \kappa\sigma \ll 1 \).

From Eq. (2), Eq. (1) can be rewritten as:

\[ dW(t) = (\mu - \sigma\theta_t)W(t)dt + \sigma W(t)dB^\theta_t \]  
(3)

Sharing forecast information is a common practice in global supply chains. Clients need accurate cost forecasts to plan their future operations (see also Yan and Ghose, 2010), including those potentially related to outsourcing. Outsourcing vendors can also share their cost forecasts with clients. For example, the vendor could deliver her forecast about the client’s variable costs. However, it is also well known that vendors usually inflate clients’ in-house operation costs to highlight their own cost advantages. Consequently, trust plays a key role in deciding the client’s order policy and position vis-à-vis the vendor. It is observed that there is a continuum between complete trust and no trust (Özer et al., 2011). This is reflected in Assumption 2.

**Assumption 2.** The client communicates with the vendor via cheap talk. He is faced with two forecast values for his in-house costs: his own judgment \( W_{IH}(t) = A + \rho_{IH}W(t) \) and the vendor’s report \( W_V(t) = \rho_VW(t) \) (see e.g., Yao et al., 2009). Here \( A \) denotes the fixed component of \( W_{IH}(t) \) that does not need to be forecasted. \( \rho_{IH}W(t) \) represents the dynamic part of \( W_{IH}(t) \) that could also be associated with the state of the economy (see e.g., Pindyck, 1993). \( \rho_{IH} \) (\( \rho_V \)) denotes the ratio of the client’s forecast (vendor’s report) to the vendor’s variable cost. If \( \rho_V \) is greater (smaller) than 1, the vendor claims (communicates) that she has a variable cost advantage (disadvantage) over the client. The client’s trust-embedded forecast about his in-house variable cost \( W_{IH}^\tau(t) \) is as follows:
\[ W_{IH}^T(t) = \delta W_V(t) + (1 - \delta)W_{IH}(t) \]  

(4)

where \( \delta \) is the trust factor, \( \delta \in [0,1] \). If \( \delta = 1 \) (\( \delta = 0 \)), the client completely trusts (distrusts) the vendor’s forecast report.

Rearranging Eq. (4), we get:

\[ W_{IH}(t) = \delta \rho_V W(t) + (1 - \delta)(A + \rho_{IH} W(t)) = (1 - \delta)A + \rho(\delta)W(t) \]  

(5)

where the weighted ratio \( \rho(\delta) = \delta \rho_V + (1 - \delta) \rho_{IH} \).

Besides cost-saving considerations, the client has an opportunity to learn from the vendor and acquire new knowledge through outsourcing. Meanwhile, the client can also transfer his in-house production expertise for innovation. To capture such effects, we make the following assumption.

**Assumption 3.** The client’s volume-learning effects derived from outsourcing and in-house operations are:

\[
\int_t^{t+D} f_{OH} q^\xi e^{-r(t-\tau)} d\tau = \varphi f_{OH} q^\xi \quad \text{and} \quad \int_t^{t+D} f_{IH}(Q - q)^\gamma e^{-r(t-\tau)} d\tau = \varphi f_{IH}(Q - q)^\gamma,
\]

respectively. Here \( \varphi \) is the present value multiplier \( \varphi = \frac{1-e^{-rD}}{r} \). \( \xi \) and \( \gamma \) denote decreasing returns from learning benefits. \( \xi, \gamma \in (0,1) \). \( f_{OH} \) and \( f_{IH} \) are scaling factors for volume-learning from outsourcing and in-house operations, \( f_{OH} \geq 0 \), \( f_{IH} \geq 0 \).

The power functions in Assumption 3 take similar forms as the future value function in Xiao and Gainmon (2013) but differ in considering contract duration and the volume-based learning benefits of outsourcing. The client’s volume-based learning effects due to in-house production and outsourcing reflect how volume-based knowledge can improve products and services, and create value through in-house production or partial outsourcing (Aubert et al., 2015).

### 2.2. The vendor’s pricing and the client’s outsourcing quantity decisions

The client aims to start outsourcing immediately and announces the outsourcing timing \( t_0 \). From this, the vendor determines the outsourcing price and provides her forecast report. The client filters the vendor’s report and optimizes his outsourcing quantity \( q \). Given the outsourcing timing \( t_0 \), the client gives up the pricing rights to the vendor and only determines his outsourcing quantity.

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\(^6\) A recent stream of research concerned with demand forecast sharing (e.g., Ebrahim-Khanjari et al., 2012; Fu et al., 2017) updates the trust factor using information mismatch criteria. Following these studies, let \( \Delta \) denote the relative distance between the vendor’s report \( \rho_V W(t) \) and the client’s benchmark report \( \hat{\rho}_V W(t) \), where \( \Delta = (\rho_V - \hat{\rho}_V) / \hat{\rho}_V \). \( \hat{\rho}_V \) proxies for the client’s standard assessment of \( \rho_V \). \( \delta = \delta(a,b,\Delta) \) is the trust factor function with updating, \( a \) denotes the trust factor without updating based on information sharing levels, reputation, prior experience, network position etc. (Chang et al., 2014; Schoenherr et al., 2015), and \( b \) indicates the client’s sensitivity to \( \Delta \). As ambiguity \( \kappa \) does not appear in \( \delta = \delta(a,b,\Delta) \), using this trust function does not alter our main conclusions. Therefore, and for ease of exposition, we follow Özer et al. (2011) and do not account for the determinants of trust in our fixed timing setting.
The vendor uses real option rules to evaluate whether to accept the outsourcing contract or not. She maximizes her utility as follows:

$$F(W(t)) = \max_{t' \geq t} \left\{ \min_{\tau' \leq \tau} \left[ \int_{t'}^{t'+\delta} (pq - ql - X(\tau)q)e^{-r(\tau-t')}d\tau \right] \right\}$$

(6)

The vendor chooses between committing to the outsourcing contract at time $t$ or postponing exercise. This can be expressed as:

$$F(W(t)) = \max \left\{ p\varphi q - l\varphi q - W(t)q, \min_{\theta \in [-\lambda, \lambda]} E^{\theta} \left[ df | F_{t} \right] + F - rFdt \right\}$$

(7)

After solving a standard optimal stopping problem (e.g., Dixit and Pindyck, 1994), we estimate the vendor’s timing threshold and her put option value under ambiguity in finite-time as follows (see the proof in Appendix B.1):

$$F(W(t)) = \begin{cases} (p\varphi - \varphi l - \lambda X(t))q & W(t) \leq W^* \\ (p\varphi q - \varphi lq - W^* q) \left( \frac{W(t)}{W^*} \right)^\beta & W(t) > W^* \end{cases}$$

(8)

where $W^*$ is the vendor’s variable cost trigger $W^* = \frac{\beta}{\beta - 1} (p\varphi - \varphi l)$ and the ambiguity-adjusted option value parameter $\beta = \frac{1}{2} - \frac{\mu + \kappa \sigma}{\sigma^2} - \chi < 0$, $\chi = \sqrt{\left( \frac{1}{2} - \frac{\mu + \kappa \sigma}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}}$.

Under the real options lens, if $W(t_0)$ is smaller than $W^*$ it is optimal to start the outsourcing arrangement, otherwise the vendor should wait. Since the contract must be exercised at $t_0$, the vendor needs to be compensated for the opportunity cost of waiting and potential loss in option value. Rearranging $W(t_0) \leq W^*$, the vendor’s price satisfies the following condition:

$$p \geq p(t_0) = \frac{\beta - 1}{\beta} \frac{\lambda X(t_0)}{\varphi} + I$$

(9)

where $p(t_0)$ denotes the vendor’s maxmin-based lowest offer price or threshold for the fixed outsourcing timing $t_0$.

Eq. (9) ensures that it is attractive for the vendor to accept the outsourcing contract at time $t_0$. The vendor’s price resulting from real options effects $p(t_0)$ is higher than its counterpart under the standard NPV approach $p_{NPV} = \frac{\lambda X(t_0)}{\varphi} + I$ when option value is neglected. If the ambiguity averse vendor is not ready to undertake the outsourcing contract at time $t_0$, $p(t_0)$ may compensate for the client’s hurriedness.

The client’s utility comprises his revenues, costs and volume-based learning effects. Let $P_c$ denote the product’s selling price. $P_c > M$ and $P_c > I$ for the problem to be economically meaningful. The term $P_c \varphi Q$ is the client’s total revenues. His in-house costs over $D$ is $\left( \varphi M + W_{IH}^T(t) \right)(Q - q)$, where $\varphi M$ and $W_{IH}^T(t)$
are the expected values of the client’s fixed costs and variable costs from in-house operations.

Following Alvarez and Stenbacka (2007) and Moretto and Rossini (2012), there is an organizational redesign cost for the client $K \frac{q^2}{Q}$. $K > 0$ is a scaling factor indicating the degree of organizational redesign. The client’s problem is to choose the outsourcing quantity with the highest utility:

$$\max_q U(q) = P_c \phi Q - (\phi M + W_{IH}^T(t))(Q - q) - p(t_0)\phi q - \frac{K}{2} \left(\frac{q}{Q}\right)^2 + \phi f_{IH}(Q - q)^\gamma + \phi f_{OH}q^\xi$$

s.t. $0 < q \leq Q$  \quad (10)

Condition (11) means the client’s outsourcing quantity $q$ should be greater than zero and smaller than or equal to his total demand. The optimization problem for the client is a non-linear function of outsourcing quantity $q$. We assume $U(q^*) \geq 0$ to ensure the client can implement outsourcing. Let $\tilde{K} = \frac{q^2}{(Q - \omega)} \{\xi \phi f_{OH}(Q - \omega)^{\xi - 1} - \gamma \phi f_{IH}(Q - \omega)^{\gamma - 1} + \phi M - \phi p(t_0)\}$, where $\omega$ denotes a sufficiently small positive constant. We use the Karush-Kuhn-Tucker-conditions to obtain the client’s optimal outsourcing quantity as shown in Theorem 1 (see the proof in Appendix B.2).

**Theorem 1**

Suppose the organizational cost $K > \tilde{K}$ and $U(q^*) \geq 0$. If $W_{IH}(t_0) < W_{IH}^T(t_0) < \overline{W}_{IH}(t_0)$, the optimal partial outsourcing quantity $q^*$ is a solution to:

$$\phi M + W_{IH}^T(t_0) - \frac{\beta - 1}{\beta} \lambda X(t_0) - \phi I - \frac{K}{Q} \left(\frac{q}{Q}\right)^\gamma + \xi \phi f_{OH}q^{\xi - 1} = 0$$

(12)

$q^*$ is an interior solution that maximizes the client’s objective value in our setting. When $W_{IH}^T(t_0) > \overline{W}_{IH}(t_0)$, the client goes for full outsourcing. Here $W_{IH}(t_0) = \phi p(t_0) - \phi M + \frac{K\omega}{Q^2} + \gamma \phi f_{IH}(Q - \omega)^{\gamma - 1} - \xi \phi f_{OH}\omega^{\xi - 1}$ and $\overline{W}_{IH}(t_0) = \phi p(t_0) - \phi M + \frac{K}{Q^2} (Q - \omega) + \gamma \phi f_{IH}\omega^{\gamma - 1} - \xi \phi f_{OH}(Q - \omega)^{\xi - 1}$. $W_{IH}(t_0)$ and $\overline{W}_{IH}(t_0)$ are lower and upper limits of the client’s in-house variable costs.

Theorem 1 indicates that depending on the relationships among $W_{IH}^T(t_0)$, its lower limit $W_{IH}(t_0)$ and upper limit $\overline{W}_{IH}(t_0)$, there are two outsourcing strategies. Note $W_{IH}^T(t_0)$ also incorporates the trust factor $\delta$. If the client believes his variable costs of in-house operation is moderate such that $W_{IH}(t_0) < W_{IH}^T(t_0) < \overline{W}_{IH}(t_0)$, he will opt for partial outsourcing. The condition $W_{IH}^T(t_0) > \overline{W}_{IH}(t_0)$ ensures that the client can save costs through outsourcing. In the presence of high variable costs $W_{IH}^T(t_0) \geq \overline{W}_{IH}(t_0)$, it is optimal for the client to fully outsource. Moreover, the client will keep all activities in-house for low variable costs ($W_{IH}^T(t_0) \leq W_{IH}(t_0)$). The assumption of $K > \tilde{K}$ is equivalent to $\overline{W}_{IH}(t_0) > 0$. Theorem 1 also implies that
the lower and upper limits of the client’s in-house variable costs are sensitive to the vendor’s minimum price \( p(t_0) \) which accounts for ambiguity. If the client focuses on cost reduction only and overlooks volume-based learning effects, we obtain Corollary 1.

**Corollary 1**

Let \( f_{IH} = f_{OH} = 0 \) and \( K > \varphi MQ - p(t_0)\varphi Q \). Assuming \( U(q^*_c) \geq 0 \), when \( W_{IH'C}(t_0) < W_{IH}(t_0) < \bar{W}_{IH'C}(t_0) \), the client’s optimal partial outsourcing quantity aimed at saving costs is:

\[
q^*_c = \left( \varphi M + W_{IH}(t_0) - \varphi p(t_0) \right) \frac{Q^2}{K}.
\]

When \( W_{IH}(t_0) \geq \bar{W}_{IH'C}(t_0) \), the client chooses full outsourcing. Here \( \bar{W}_{IH'C}(t_0) = \varphi p(t_0) + \frac{K}{Q} - \varphi M, \)

\( W_{IH}(t_0) = \varphi p(t_0) - \varphi M. \)

Corollary 1 suggests that the client’s partial outsourcing quantity aimed at saving costs increases with the client’s trust-embedded forecast \( W_{IH}(t_0) \) and his fixed cost of in-house operations \( M \). An increase in the vendor’s minimum price \( p(t_0) \) decreases the client’s partial outsourcing quantity. This accords with the intuition: if the vendor asks for a greater price, the client places a smaller order.

### 3. The effects of ambiguity and trust on partial outsourcing outcomes

This section describes how partial outsourcing quantity and pricing decisions are affected by the vendor’s ambiguity (\( \kappa \)-ignorance in the first instance) and the client’s trust factor \( \delta \) (see Propositions 1-3). We suppose vendor A is located in an outsourcing hub, while Client B is a multinational enterprise with headquarters in Europe. Client B has his own subsidiary in A’s market, purchases products from vendor A and sells them to European and North American customers. For illustration (Figures 1-3), the vendor and client aim to sign a fixed price contract with contract duration \( D = 10 \) months (e.g., case of household cleaning products).\(^7\)

Initial parameters values for the vendor are as follows: the discount rate \( r = 0.06 \), her fixed cost \( I = 1 \) $/unit, variable cost \( X(0) = 10 \) $/unit, the objective drift rate \( \mu = 0.02 \), the volatility \( \sigma = 0.05 \). We set the initial/base level of ambiguity to \( \kappa = 0.2 \), hence the drift rate varies from 0.01 to 0.03. The other parameter values are as follows: total demand \( Q = 100,000 \) units per month, the organizational redesign cost \( K =

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\(^7\) Household cleaning products are subject to high raw material price volatility (i.e., primary and intermediate petrochemicals). As monthly data is publically available and suppliers and clients can adjust supply prices and order quantities on a regular basis, a monthly frequency can be used to model related variable costs.
1500 000 dollars, the client’s fixed cost $M = 1.2$ $$/\text{unit}$, parameters for volume-based learning effects $\gamma = \xi = 0.5, f_{IH} = f_{OH} = 8.5$. Let $\pi$ denote the variable cost-saving index, where $\pi = \rho(\delta) - \frac{\beta - 1}{\beta}$.

**Proposition 1. The effect of vendor’s ambiguity on the client’s outsourcing quantity**

When the variable cost-saving index $\pi \geq 0$, the client’s partial outsourcing quantity $q^*$ increases with ambiguity $\kappa$.

If $\pi < 0$, the relationship between $q^*$ and $\kappa$ is equivocal as follows: $\frac{dq^*}{d\kappa} \equiv 0 \Leftrightarrow \pi \equiv \frac{\lambda}{\sigma \beta_X \epsilon_\lambda}$, where

\[ \epsilon_\lambda = \sigma \frac{1 - e^{-hD(1+hD)}}{h^2}. \]

Proposition 1 implies the client’s partial outsourcing quantity is increasing with the vendor’s ambiguity if the variable cost-saving index $\pi$ is greater than zero (see the proof in Appendix B3). The positive cost-saving index suggests that the client can reduce costs through outsourcing and the vendor has a variable cost advantage. This advantage appears to be more significant, or is inflated, with higher ambiguity or pessimism from the vendor. Figure 1 illustrates such dynamics for different trust levels. Note $\pi$ incorporates trust factor $\delta$, the ratios $\rho_V$ and $\rho_{IH}$ and the vendor’s judgment (i.e., ambiguity $\kappa$, growth rate $\mu$ and volatility $\sigma$). This means that if made aware of the vendor’s ambiguity, the client needs to re-evaluate the quality of the shared information forecasts and revisit whether he should really increase his outsourcing quantity or not. Not doing so can result in over-commitment to outsourcing when the vendor’s ambiguity is high. When outsourcing is aimed at saving costs, the vendor’s ambiguity can be a source of hidden costs for the client.

Recall that $\frac{dq^*}{d\kappa} = \frac{\pi \epsilon_\lambda X(t_0)}{B} + \frac{\lambda X(t_0)}{\sigma \beta_X \epsilon_\lambda}$, $B > 0$, $\epsilon_\lambda > 0$ and $\beta < 0$ in Appendix B.3. The first term in the bracket is the *present value effect* and the second term is the *option value effect* (Trojanowska and Kort, 2010). If $\pi$ is negative, the first term decreases with $\kappa$ while the second term increases with $\kappa$. This creates an equivocal and non-linear relationship between ambiguity and quantity when outsourcing is not driven by cost-saving considerations.

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8 To minimize the risk of operational failure from the vendor and not over-commit to outsourcing, the client can place the order in stages and in a contingent manner or simply switch to a less ambiguity prone vendor.
Proposition 2. The effect of trust on the client’s outsourcing quantity

When $A > 0$, the partial outsourcing quantity $q^*$ increases with the trust factor $\delta$ if $\kappa \geq \kappa_0$ and $\rho_V > \rho_{IH}$, and decreases with $\delta$ if (i) $\rho_V \leq \rho_{IH}$; (ii) $\kappa < \kappa_0$ and $\rho_V > \rho_{IH}$. That is:

$$
\frac{dq^*}{d\delta} \geq 0 \quad \text{if} \quad \kappa \geq \kappa_0, \quad \rho_V > \rho_{IH}$$

$$
\frac{dq^*}{d\delta} < 0 \quad \text{if} \quad \kappa < \kappa_0, \quad \rho_V > \rho_{IH} \quad \text{or} \quad \rho_V \leq \rho_{IH},$$

where $\kappa_0$ is the solution to:

$$A \frac{1-e^{-(r-\mu-\kappa\sigma)D}}{r-\mu-\kappa\sigma} = 0.$$

When $A = 0$, the partial outsourcing quantity $q^*$ increases (independently of $\kappa$-ignorance) with the trust factor $\delta$ if $\rho_V \geq \rho_{IH}$ and decreases with $\delta$ if $\rho_V < \rho_{IH}$, that is:

$$
\frac{dq^*}{d\delta} \geq 0 \quad \text{if} \quad \rho_V \geq \rho_{IH},$$

$$
\frac{dq^*}{d\delta} < 0 \quad \text{if} \quad \rho_V < \rho_{IH}.$$

When the fixed component $A$ is greater than zero, the client adjusts his outsourcing quantity $q^*$ non-monotonically based on the vendor’s ambiguity and the relationship between the two forecast ratios (see the proof in Appendix B3). If the vendor’s report $\rho_V W(t_0)$ is greater than the client’s forecast $\rho_{IH} W(t_0)$ and $A > 0$, the vendor is more likely to inflate her variable cost advantage. In this situation, the vendor’s ambiguity will moderate the relationship between trust and outsourcing quantity. More specifically, a more trustful client would opt for a smaller (an even greater) outsourcing quantity when the vendor’s ambiguity is low $\kappa < \kappa_0$ (high $\kappa \geq \kappa_0$) (see Figure 2a). Ambiguity effects matter more when the ambiguous vendor communicates a variable cost advantage. If $\rho_V$ is smaller than $\rho_{IH}$, the vendor is less likely to have a variable cost advantage. In this case, the client trusts the vendor and places a lower order (see Figure 2b).
\[ \rho_{IH} = 1.1 \] in Fig. 2. \( \rho_v = 1.123 > \rho_{IH} \) and \( \rho_v = 1 < \rho_{IH} \) in Figs. 2a and 2b, respectively, where \( A = 2 \). In Fig. 2c, \( A = 0 \), \( \rho_v = 1.05 \) or 1.18.

When \( A = 0 \), more trust can either increase or decrease the client’s outsourcing quantity as shown in Figure 2c. Its influence depends on the difference between \( \rho_v \) and \( \rho_{IH} \). Higher trust levels decrease the client’s outsourcing quantity when the vendor is less likely to communicate a variable cost advantage. Ambiguity does not moderate the relationship between trust and outsourcing quantity if \( A = 0 \) (e.g., the client does not have raw material in stock). Proposition 2 is unchanged with or without volume-based learning effects. The vendor’s and client’s utilities are also non-monotonic with respect to trust. This is caused by the non-linear association between trust and the client’s “optimal” partial outsourcing quantity.

Contract duration is also an important dimension of outsourcing. Prior literature such as Jiang et al. (2008), Shi and Feng (2016) and Shi (2016) discusses how the time horizon for outsourcing affects the vendor’s acceptance decision and commitment level under risk. In particular, Shi and Feng (2016) find there is a contract trigger that induces a non-monotonic relationship between the supplier’s timing threshold and contract duration. We add to these prior studies by considering the effect of ambiguity on the vendor’s pricing
for long-term and short-term contracts in Proposition 3.

**Proposition 3. The effect of ambiguity on the vendor’s pricing**

The vendor’s lowest offer price $p(t_0)$ increases with ambiguity $\kappa$ if $D \geq D_v$ and decreases with $\kappa$ if $D < D_v$, that is: $\frac{\partial p(t_0)}{\partial \kappa} \geq 0$ if $D \geq D_v$ and $\frac{\partial p(t_0)}{\partial \kappa} < 0$ if $D < D_v$, where the contract duration threshold $D_v$ is a solution to:

$$1 - e^{-hD(1+hD)} + \frac{h^2}{(\beta-1)\sigma^2\chi} = 0, \quad h = r - \mu - \kappa \sigma, \quad \chi = \sqrt{\frac{1}{2} - \frac{\mu + \kappa \sigma}{\sigma^2}} + \frac{2r}{\sigma^2}.$$

Proposition 3 indicates that an increase in ambiguity results in a higher vendor’s minimum price $p(t_0)$ when $D \geq D_v$ (long-term contract) and a lower vendor’s price when $D < D_v$ (short-term contract) (see the proof in Appendix B3). This is in line with evidence of vendors sometimes preferring short-term outsourcing contracts over long-term ones (i.e., short-termism bias) when faced with high cost uncertainty. For long-term contracts, the vendor demands a higher price because of her ambiguity aversion. This results from the vendor’s short-termist predisposition and the perception or bias that some of her idiosyncratic uncertainty will resolve itself faster in short-term arrangements. As ambiguity increases, $p(\mu=0)$ will first decrease then increase as depicted in Figure 3a. This is because $D_v(\mu=0)$ is first greater than $D$ and then smaller than $D$ (see Figure 3b where $D = 10$ months). The other lines in Figure 3a are increasing in ambiguity since $D$ is greater than the contract duration thresholds. The relationship between ambiguity and the price trigger is, therefore, conditional on contract duration. This adds to extant real options research on the equivocal relationship between ambiguity and the investment trigger under $\kappa$-ignorance in finite-time (see e.g., Nishimura and Ozaki 2007; Trojanowska and Kort, 2010). We show that the existence of a positive association is conditional on contract duration. In terms of practical implications, the client will be charged a higher price with higher ambiguity from the vendor if he favors long-term contracts. This once again points to the value of sequencing outsourcing commitment into contingent stages as a way of mitigating the hidden costs associated with the vendor’s pricing strategy.
Having examined how ambiguity and trust affect outsourcing quantity and price under the multiple-priors specification, we revisit our propositions under Choquet ambiguity and rank-dependent probability weighting. This is justified by the fact that the maxmin multiple-priors or worst-case ambiguity ignores differences in ambiguity preferences (e.g., ambiguity-seeking) and sensitivity to likelihood (see Abdellaoui et al., 2011; Baillon et al., 2017). Also, the multiple-priors ambiguity specification should correspond to cases where vendors feel less competent and less confident/knowledgeable about performing new or untested outsourcing tasks (e.g., advanced IT outsourcing or outsourcing of complex corporate functions), while the Choquet specification should match cases with more confident, experienced and specialized vendors (e.g., outsourcing of financial services plans and outsourcing in knowledge-intensive industries). We use the notion of c-ignorance to differentiate between ambiguity-seeking and aversion and study the influence of Choquet ambiguity on outsourcing pricing and quantity. Table 2 reports our findings (the risk case is also shown for comparison). In the Choquet setting, the vendor’s variable cost evolves according to the Choquet–Brownian motion developed by Kast and Lapied (2010) and Kast et al. (2014):

$$dX(t) = (\mu + m\sigma)X(t)dt + s\sigma X(t)dB_t$$

(13)

where the (range of) drift and volatility terms are subjectively evaluated as $\mu + m\sigma$ and $s\sigma$. $m=2c-1$, $s^2 = 4c(1-c)$, $0 < c < 1$, $\mu > 0$, $\sigma > 0$. Ambiguity affects both the drift and volatility of the Brownian motion this time. $c$ is a non-additive capacity that captures decision makers’ ambiguity preferences through the weights assigned to probabilities of up and down movements. When $c > 0.5$ ($c < 0.5$), the vendor is
ambiguity-seeking (averse) and, in our setting, is bullish (bearish) about the cost-saving benefits of outsourcing.\textsuperscript{9}

We find that when rank-dependent probability weighting - aka Choquet ambiguity - is considered, the non-linear effects of ambiguity on outsourcing quantity are exacerbated by heterogeneity in ambiguity preferences. The client’s outsourcing quantity tends to increase (decrease) with increasing ambiguity-seeking (aversion: $c < 0.5$) from the vendor if the cost-saving index is positive (negative and the risk-return ratio $z(c) > 0$). Also, the vendor’s price not only increases (decreases) with increasing ambiguity-seeking $c > 0.5$ for long-term (short-term) contracts, but also with aversion to ambiguity when the risk-return relation $z(c)$ is positive. This highlights the importance of probability weighting and separating ambiguity preferences from ambiguity in decision-making models. Trust dynamics are generally maintained under Choquet ambiguity.

Table 2 summarizes the effects of risk, $\kappa$-ignorance, Choquet ambiguity and trust on outsourcing outcomes for comparison (see proofs in Appendix C). Compared to the unequivocal relationship between risk and outsourcing quantity documented in the normative real options literature, both $\kappa$-ignorance and Choquet ambiguity alter the outsourcing quantity non-linearly and conditionally. The same holds for the associations between price and risk vs. price and ambiguity (due to the role of contract duration in the ambiguous settings). Trust effects interact with ambiguity effects only when the vendor communicates a variable cost advantage or inflates her forecasts.

\textsuperscript{9} $c = 0.5$ reduces to the classical geometric Brownian motion under risk or uncertainty-neutrality.
Table 2. Comparing risk versus ambiguity specifications

<table>
<thead>
<tr>
<th>with risk and trust</th>
<th>with multiple-priors ambiguity and trust</th>
<th>with Choquet ambiguity and trust</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{dq^*_R}{d\sigma} &lt; 0$</td>
<td>If $\pi \geq 0$, $\frac{dq^<em>_R}{d\kappa} &gt; 0$; $\frac{dq^</em>_R}{d\sigma} &lt; 0$ or if $\pi &lt; 0$. $\pi = \rho(\Delta) - \frac{\beta}{\alpha}$.</td>
<td>$\frac{dq^<em><em>CA}{dc} &gt; 0$ if $\pi</em>{CA}(\delta) &gt; 0, c \geq 0.5$ $\frac{dq^</em><em>CA}{dc} &lt; 0$ if $\pi</em>{CA}(\delta) \leq 0, c &lt; 0.5$ and $z(c) &gt; 0$. In the other cases, $\frac{dq^*<em>CA}{dc} \geq 0 \Leftrightarrow \pi</em>{CA}(\delta) \geq \frac{\kappa_a}{\alpha}$.</td>
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</table>

(1) For $A > 0$, $\frac{dq^*_\kappa}{d\kappa} \geq 0$ if $\mu \geq \mu_0$, $R_{\mu} > R_{\mu}$ $\frac{dq^*_\kappa}{d\sigma} < 0$ or if $\rho_{\mu} \leq R_{\mu}$. Where $\mu_0$ is the solution of $1-e^{-(r-\mu)\sigma} - R = 0$ (2) If $A = 0$, $\frac{dq^*_\kappa}{d\kappa} \geq 0$ if $\rho_{\mu} \geq R_{\mu}$ $\frac{dq^*_\kappa}{d\sigma} < 0$ if $\rho_{\mu} < R_{\mu}$.

$\frac{\partial P_R(t_0)}{\partial \sigma} > 0$ If $D \geq D_v$, $\frac{\partial P_R(t_0)}{\partial \kappa} \geq 0$. Otherwise, $\frac{\partial P_R(t_0)}{\partial \kappa} < 0$, $\Delta$ is a solution to: $1-e^{-\alpha(1+D_b)} + \frac{h^2}{(\beta-1)\alpha^2} = 0$ $\frac{\partial P_C(t_0)}{\partial c} > 0$ if $c < 0.5$ and $z(c) > 0$ $\frac{\partial P_C(t_0)}{\partial c} < 0$ if $c \geq 0.5$ and $D > D_C$.

Conclusions

In this paper, we propose a behavioral real options model for partial outsourcing that integrates ambiguity in costs, trust in cost information sharing and volume-based learning to explain some of the hidden costs of outsourcing. The effects of maxmin ambiguity and trust on partial outsourcing outcomes are studied. First, we find that higher $\kappa$-ignorance from the vendor increases the client’s outsourcing quantity when the cost-saving index is positive but does not necessarily increase the vendor’s lowest offer price. The vendor’s lowest offer price increases (decreases) with $\kappa$-ignorance for long-term (short-term) contracts only. Second, we show that trust affects partial outsourcing quantities in non-monotonic ways. Higher trust can encourage (reduce) outsourcing if the vendor’s forecast is higher (lower) than the client’s. This relationship is, however, likely to
be moderated by $\kappa$-ignorance when the vendor exaggerates her cost advantages. Finally, compared with the effects of $\kappa$-ignorance, the relationships between Choquet ambiguity and the corresponding outsourcing outcomes are more complex and subtle because ambiguity preferences (seeking vs. aversion) and probabilistic sophistication come into play. In the absence of ambiguity, the client’s outsourcing quantity unequivocally decreases with risk while the vendor’s lowest price increases with volatility. Our behavioral real options model shows that ambiguity, trust and ambiguity preferences affect outsourcing outcomes in many different ways and that volatility is only part of the uncertainty story. The omission of behavioral factors from normative or rational option valuation models are part of the reason why real options theory predictions for outsourcing do not fully hold in practice (see e.g., Musteen, 2016; Choi et al., 2017). Our modeling insights also help to clarify recurrent biases in outsourcing (e.g., over-commitment and short-termism) and contribute to understanding some of the cost determinants of outsourcing failure. In terms of practical implications, our findings provide guidance on how clients and vendors behave when faced with incomplete and asymmetric information, and point to the need to embed flexibility/performance and exit clauses in outsourcing contracts and sequence commitment into contingent stages as ways of dealing with ambiguity and mitigating some of the hidden costs of outsourcing.

Compared with extant studies under risk, the real options model we propose should be more reflective of reality because it considers the behavior and potential biases of vendors and clients - through ambiguity and trust - in outsourcing relationships and explains some of the hidden and unexpected costs of outsourcing. Our work can be extended in several directions. The client’s selling price and total demand are assumed to be constant. Such factors can be uncertain and affected by product and service quality. Our framework is restricted to interactions between a vendor and client. Theoretical extensions could account for multiple vendors and clients and incorporate competition or learning-by-doing in the various dynamics. The role of cultural attributes can also be examined given the cultural differences between clients and vendors in international outsourcing. Finally, it would be interesting to empirically validate our research predictions using a global outsourcing dataset of clients and vendors.
Appendix A.

<table>
<thead>
<tr>
<th>Table A1. Key notation</th>
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<tbody>
<tr>
<td>$Q$</td>
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<tr>
<td>$D$</td>
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<tr>
<td>$I, M$</td>
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<tr>
<td>$P_c$</td>
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<tr>
<td>$X(t)$</td>
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<td>$W(t)$</td>
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<tr>
<td>$\kappa$</td>
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<tr>
<td>$\delta$</td>
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<tr>
<td>$\rho_{hV}$, $\rho_V$</td>
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<tr>
<td>$A$</td>
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<td>$\beta$</td>
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<td>$\pi(\delta)$</td>
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<td>$r$</td>
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<tr>
<td>$\varphi$</td>
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<tr>
<td>$f_{hiH}$, $f_{oh}$</td>
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<tr>
<td>$\gamma, \xi$</td>
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Appendix B.

Appendix B.1. The vendor’s option value

Let $Y(t) = W(t)q$. According to Eq. (2), the vendor’s total variable cost $Y(t)$ follows: $dY(t) = (\mu - \sigma \theta t)Y(t)dt + \sigma Y(t)dB_t^\theta$. In the continuation region, $\min_{\theta \in [-\kappa, \kappa]} E^{Q^\theta}[dF|\mathcal{F}_t] = rF(Y(t))dt$. Suppose that $F$ is twice differentiable in the continuation region and $F' = \partial F/\partial Y(t) < 0$. The expectation of $dF$ can be written as (Nishimura and Ozaki, 2007; Trojanowska and Kort, 2010):

$$\min_{\theta \in [-\kappa, \kappa]} E^{Q^\theta}[dF|\mathcal{F}_t] = \frac{1}{2} \sigma^2 (Y(t))^2 F'' dt + (\mu + \kappa \sigma) Y(t) F' dt,$$

where $F'' = \partial^2 F/\partial (Y(t))^2$.

From the Bellman equation, we get $\frac{1}{2} \sigma^2 (Y(t))^2 F'' + (\mu + \kappa \sigma) Y(t) F' - rF(Y(t)) = 0$. The vendor’s
option value $F(Y(t))$ should satisfy the following value matching, smooth-pasting and barrier conditions: $F(Y^*)=p\varphi q-l\varphi q-Y^*, \ T'(Y^*)=-1, \ F(\infty)=0.$ Thus, we obtain the Eq. (8).

**Appendix B.2. Proof of Theorem 1**

Since $\xi < 1$ and $\gamma < 1$, $\frac{\partial^2 u}{\partial q^2} < 0.$ The client’s objective function is strictly concave in $q$. Let $L=P_c\varphi q - (\varphi M + W^T_{ih}(t_0))(Q-q) - p(t_0)\varphi q - \frac{K}{2}(\varphi q)^2 + \varphi f_{ih}(Q-q)^\gamma + \varphi f_{oh}q^\xi + y_1q + y_2(Q-q)$ denote the Lagrangian of the client’s problem. The complementary slackness conditions are (Chiang and Wainwright, 2005):

$$\frac{\partial L}{\partial q} \leq 0, \ q \geq 0; \ \frac{\partial L}{\partial y_1} \geq 0, \ y_1 \geq 0; \ \frac{\partial L}{\partial y_2} \geq 0, \ y_2 \geq 0; \ \frac{\partial L}{\partial y_2} = 0.$$

The first order derivative of $L$ with respect to $q$ is:

$$l(q)=\frac{\partial L}{\partial q} = \varphi M + W^T_{ih}(t_0) - \frac{1}{\beta} - \lambda X(t_0) - \varphi I - \frac{K}{Q} - \gamma \varphi f_{ih}(Q-q)^{\gamma-1} + \xi \varphi f_{oh}q^{\xi-1} + y_1 - y_2$$

If $y_1 = 0$ and $y_2 = 0$, $q^*$ is the solution of $l(q) = 0$. $\xi, \gamma \in (0, 1)$. $l(q)$ goes to positive (negative) infinity if $q$ approaches 0 ($Q$). Let $\sigma$ denote a sufficiently small positive constant, where $\sigma < Q - \sigma$.

From $\frac{\partial^2 l}{\partial q^2} < 0$, we know that $\frac{\partial l}{\partial q} = 0$ exists between $q \in (\sigma, Q - \sigma)$ if $\frac{\partial l}{\partial q} |_{q=\sigma} > 0$ and $\frac{\partial l}{\partial q} |_{q=Q-\sigma} < 0$.

This implies the interior solution exists if: $W^T_{ih}(t_0) > W_{ih}(t_0) = \varphi(p(t_0) - M) + \frac{\kappa}{Q^2} + \gamma \varphi f_{ih}(Q-\sigma)^{\gamma-1} - \xi \varphi f_{oh}(Q-\sigma)^{\xi-1}$ and $W^T_{ih}(t_0) < \bar{W}_{ih}(t_0) = \varphi(p(t_0) - M) + \frac{\kappa}{Q^2} (Q-\sigma) + \gamma \varphi f_{ih}(Q-\sigma)^{\gamma-1} - \xi \varphi f_{oh}(Q-\sigma)^{\xi-1}$. As $\xi, \gamma \in (0, 1)$ and $\bar{W}_{ih}(t_0) - W_{ih}(t_0) = \frac{\kappa}{Q^2} (Q-2\sigma) + \gamma \varphi f_{ih}(Q-\sigma)^{\gamma-1} - (Q-\sigma)(\gamma-1) + \xi \varphi f_{oh}(Q-\sigma)^{\xi-1} - \xi \varphi f_{oh}(Q-\sigma)^{\xi-1} > 0$. To ensure $\bar{W}_{ih}(t_0) > 0$, we require $K > \bar{K} = \frac{\sigma^2}{(Q-\sigma)} \left[ \xi \varphi f_{oh}(Q-\sigma)^{\xi-1} - \gamma \varphi f_{ih}(Q-\sigma)^{\gamma-1} + \varphi M - \varphi p(t_0) \right]$. If $y_1 > 0$ and $y_2 = 0$, $q = 0$ and $l(q) \leq 0$. This means $W^T_{ih}(t_0) \leq W_{ih}(t_0)$. If $y_1 = 0$ and $y_2 > 0$, we obtain $q = Q$ and $W^T_{ih}(t_0) \leq W_{ih}(t_0)$. If $y_1 > 0$ and $y_2 > 0$, the optimal solution does not exist. This leads to Theorem 1.

**Appendix B.3. Proofs of Propositions 1-3**

**Proposition 1**

The first order derivative of partial outsourcing quantity $q^*$ with respect to $k$ follows: $\frac{dq^*}{dk} = \xi \beta \left( \pi - \frac{\lambda}{\sigma \beta \chi \varepsilon} \right) X(t_0)$, where $\varepsilon = \frac{\partial \lambda}{\partial k} = \sigma \frac{1 - e^{-\beta D(1+H)}}{h^2} > 0$, $\beta < 0$, $\pi = p(\delta) - \frac{\beta - 1}{\beta}$, $B = \frac{\kappa}{Q^2} + \gamma (1 - \gamma) \varphi f_{ih}(Q-$
\( q^{\gamma-2} + \xi (1 - \xi) \varphi f_{\text{on}}q^{\xi-2} > 0, \; \gamma < 1, \; \xi < 1. \) Then, \( \frac{dq^*}{d\kappa} > 0 \) if \( \pi \geq 0. \) If \( \pi < 0, \) \( \frac{dq^*}{d\kappa} \geq 0 \iff \pi \geq \frac{\lambda}{\sigma \beta \chi \kappa}. \)

**Proposition 2**

From Theorem 1, \( \frac{dq^*}{d\delta} = \frac{1}{B} \left[ (\rho_V - \rho_{IH}) \lambda X(t_0) - A \right]. \) Recall \( B > 0. \) If \( A = 0, \) the sign of \( \frac{dq^*}{d\delta} \) depends on \( \rho_V - \rho_{IH}. \) When \( A > 0 \) and \( \rho_V \leq \rho_{IH}, \) \( \frac{dq^*}{d\delta} < 0. \) When \( A > 0 \) and \( \rho_V > \rho_{IH}, \) we know that \( \frac{dq^*}{d\delta} \geq 0 \) if \( \lambda \geq R \) and \( \frac{dq^*}{d\delta} < 0 \) if \( \lambda < R, \) where \( R = \frac{A}{(\rho_V - \rho_{IH})X(t_0)}. \) Let \( \kappa_0 \) denote the solution to \( R - \lambda = 0. \) As \( \frac{\partial \lambda}{\partial \kappa} > 0, \) we know that \( \lambda \geq R \) is equivalent to \( \kappa \geq \kappa_0 \) and \( \lambda < R \) means \( \kappa < \kappa_0. \) Thus we obtain proposition 2.

**Proposition 3**

We compute the derivative of the vendor’s lowest offer price \( p(t_0) \) with respect to \( \kappa \) as follows:

\[
\frac{\partial p(t_0)}{\partial \kappa} = \frac{\lambda X(t_0)(1-\beta)\sigma}{-\beta \phi h^2} \left( \hat{L} + \frac{h^2}{(\beta - 1)\sigma^2 \chi} \right),
\]

where \( \hat{L} = \frac{1 - e^{-hD}}{(1+hD)} \). As \( \hat{L} \) is a function of contract duration \( D, \) we have \( \frac{\partial \hat{L}}{\partial D} = e^{-hD} \frac{e^{-hD} - (1-hD)}{(1-hD)^2} > 0. \) Recall \( \lambda > 0 \) and \( \beta < 0. \) When \( \hat{L} \geq \frac{h^2}{(\beta - 1)\sigma^2 \chi}, \) we have \( \frac{\partial p(t_0)}{\partial \kappa} \geq 0. \) If \( \hat{L} < \frac{h^2}{(1-\beta)\sigma^2 \chi}, \) we get \( \frac{\partial p(t_0)}{\partial \kappa} < 0. \) Let \( D_V \) denote the solution of \( \hat{L} + \frac{h^2}{(\beta - 1)\sigma^2 \chi} = 0. \) We get \( \frac{\partial p(t_0)}{\partial \kappa} \geq 0 \) if \( D \geq D_V \) and \( \frac{\partial p(t_0)}{\partial \kappa} < 0 \) if \( D < D_V. \)

**Appendix C. The effects of Choquet ambiguity and trust on partial outsourcing**

When \( \kappa = 0, \) we use the subscript \( R \) to denote results under risk (no ambiguity): \( \beta_R = \frac{1}{2} - \frac{\mu}{\sigma^2} \) and \( \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0. \) Note \( \frac{\partial \beta_R}{\partial \sigma} > 0. \) (Moon et al., 2010). \( \frac{\partial p_R(t_0)}{\partial \sigma} > 0 \) and \( \frac{dq_R^*}{d\sigma} = -\frac{\lambda R X(t_0)}{(\beta R)^2 B} \frac{\partial \beta_R}{\partial \sigma} < 0. \) For \( A > 0 \) and \( \rho_V > \rho_{IH}, \) \( \frac{dq_R^*}{d\delta} \geq 0 \) if \( \mu \geq \mu_0 \) and \( \frac{dq_R^*}{d\delta} < 0 \) if \( \mu < \mu_0. \) If \( \rho_V \leq \rho_{IH}, \) \( \frac{dq_R^*}{d\delta} < 0, \) where \( \mu_0 \) is the solution of \( \frac{1 - e^{-(r-\mu)D}}{r-\mu} = R = 0. \) If \( A = 0, \) the sign of \( \frac{dq^*_R}{d\delta} \) is the same as the sign of \( \rho_V - \rho_{IH}. \) Thus, we obtain the first column in Table 2.

Using a similar logic, the vendor’s lowest offer price under Choquet ambiguity is \( p_{CA}(t_0) = \frac{\sigma - 1}{\alpha} \eta X(t_0) \ast I, \) where the subscript \( CA \) denotes Choquet ambiguity, \( \alpha = \chi_{CA} = \sqrt{\chi_{CA}^2 + \frac{2r}{s^2 \sigma^2}} < 0, \chi_{CA} = \frac{1}{2} - \frac{\bar{\mu}}{s^2 \sigma^2}, \bar{\mu} = \mu + m \sigma, \)
\[ \eta = (1 - e^{-(r-\mu)D})/(r-\mu). \]

Suppose \( K > \tilde{K}_{CA} \) and \( U(q_{CA}^*) \geq 0 \). If \( W_{IH}(t_0) < W_{CA}^T(t_0) < \tilde{W}_{IH}(t_0) \), the optimal partial outsourcing quantity under Choquet ambiguity \( q_{CA}^* \) is a solution to:

\[ \varphi M + W_{CA}^T(t_0) - \frac{\alpha-1}{\alpha} \eta X(t_0) - \varphi I - \frac{K^2}{Q(Q')} q_{\xi}^* - \varphi q_{\tilde{f}_{OH}}(Q-q)^{\xi-1} + \varphi q_{\tilde{f}_{OH}} q_{\xi}^* = 0, \]

where \( W_{CA}^T(t_0) = (1 - \delta)A + \rho(\delta)\eta X(t_0) \), \( \tilde{K}_{CA} = \frac{q^2}{(Q-\sigma)} [\xi q_{f_{OH}}(Q-\sigma)^{\xi-1} + \varphi q_{f_{OH}}(Q-\sigma)^{\xi-1} - \varphi q_{f_{OH}}(Q-\sigma)^{\xi-1} - \varphi q_{f_{OH}} q_{\xi}^* - \varphi M - \varphi p_{CA}(t_0)] \), \( W_{CA}(t_0) = \varphi (p_{CA}(t_0) - M) + \frac{\kappa_0}{Q} + \gamma q_{f_{OH}}(Q-\sigma)^{\xi-1} - \xi q_{f_{OH}} q_{\xi}^* \).

Taking the derivative of \( q_{CA}^* \) with respect to \( c \), we get:

\[ \frac{dq_{CA}^*}{dc} = \frac{\varepsilon}{B} \left( \frac{\pi_{CA}(\delta) - \frac{\eta\varepsilon}{\alpha^2}}{\eta} \right) X(t_0), \]

where \( \pi_{CA}(\delta) = \rho(\delta) - \frac{\alpha-1}{\alpha}, \quad \eta = \frac{\partial \eta}{\partial c} = 2e^{-(r-\mu)D} \sigma e^{(r-\mu)D-(r-\mu)D-1} (r-\mu)^2 > 0, \quad \varepsilon = \frac{\partial \varepsilon}{\partial c} = \frac{-az(c) + r(1-2c)}{4\sigma^2 c^2(1-c)^2} \left( \frac{\eta}{\eta^2} \right) B \]

defined in Appendix B.3, \( z(c) = \mu(1 - 2c) - \sigma[2(c - 0.5)^2 + 0.5] \). Then if \( c \geq 0.5, z(c) < 0 \) and \( \frac{\partial \alpha}{\partial c} < 0 \). In this case, \( \frac{dq_{CA}^*}{dc} > 0 \) if \( \pi_{CA}(\delta) > 0 \). If \( \pi_{CA}(\delta) \leq 0, c < 0.5 \) and \( z(c) > 0 \), \( \frac{dq_{CA}^*}{dc} < 0 \). Otherwise, \( \frac{dq_{CA}^*}{dc} \approx 0 \) if \( \pi_{CA}(\delta) \approx 0 \) if \( \frac{\eta\varepsilon}{\alpha^2} \).

The effect of trust on \( q_{CA}^* \) is determined using Appendix B.3. The derivative of \( p_{CA}(t_0) \) with respect to \( c \) is:

\[ \frac{dp_{CA}(t_0)}{dc} = \frac{\lambda}{Q} \left( (\alpha^2 - \alpha) \frac{\varepsilon}{\eta} + \varepsilon_{\alpha} \right). \]

If \( c < 0.5 \) and \( z(c) \), \( \varepsilon_{\alpha} > 0 \) and \( \frac{dp_{CA}(t_0)}{dc} > 0 \). If \( c < 0.5 \) and \( z(c) \leq 0 \), \( \frac{dp_{CA}(t_0)}{dc} < 0 \) if \( \varepsilon_{\alpha} \approx (\alpha - \alpha^2) \frac{\varepsilon}{\eta} \). Let \( \Lambda = \frac{\varepsilon}{\eta} = 2\sigma \frac{1-(r-\mu)D - e^{-(r-\mu)D}}{(r-\mu)(1-e^{-(r-\mu)D})} \). As \( (r-\mu)D \ll 1 \), \( \frac{\partial \Lambda}{\partial D} > 0 \). Let \( D_{CA} \) denote the solution to: \( (\alpha^2 - \alpha) \frac{\varepsilon}{\eta} + \varepsilon_{\alpha} = 0 \). If \( c \geq 0.5 \) and \( D > D_{CA} \), \( \frac{dp_{CA}(t_0)}{dc} > 0 \).

If \( c \leq 0.5 \) and \( D < D_{CA} \), \( \frac{dp_{CA}(t_0)}{dc} < 0 \). This leads to the third column in Table 2.

References


