Three Dimensional Force Estimation for Steerable Catheters through Bi-point Tracking

Junghwan Back¹, Lukas Lindenroth¹, Kawal Rhode² and Hongbin Liu¹*

¹Department of Informatics, King’s College London, London, UK

²Biomedical Engineering, King’s College London, London, UK

E-mail for the corresponding author: Hongbin.Liu@kcl.ac.uk
Three Dimensional Force Estimation for Steerable Cardiac Catheters through Bi-point Tracking

Contact forces play a significant role in the success of cardiac ablation. However, it is still challenging to estimate the applied contact force during the intervention when a catheter is under large bending or experiences multiple contact points along its body. A multi-element kinetostatic model of a tendon-driven catheter is proposed for real-time intrinsic force sensing. The model is able to accurately predict the steerable section shape of the catheter for given tendon tensions as well as the contact force at any known location on the steerable section. An algorithm is proposed which estimates the contact force on the steerable section using the model-based shape prediction in combination with end-position tracking of the steerable section. In this paper, undefined parameters and contact states of the force and shape estimation are defined and investigated. The shape prediction is validated in 3D space. The contact force estimation is validated with different catheter shapes, contraint catheter and buckling. It can be seen that end-position of the steerable section can be predicted with an accuracy of about 2.3mm. In the validations, the 3-dimensional contact forces can be estimated accurately with an error of about 0.018N and 1.6ms computation time. Furthermore, the contact force estimation algorithm are able to incorporate external physical constraints along the catheter, which is validated in an experimental setup.

Keywords: Intrinsic force sensing, catheter contact force, continuum kinematics, catheter robot and catheter ablation

Subject classification codes: Medical robotics

1. Introduction

A conduction system in the healthy human heart uses the sinus and atrioventricular nodes to regulate the heart rhythm as a natural pacemaker. Chaotic electrical impulses in the conduction system cause an abnormal heart rhythm, known as heart arrhythmia. To treat arrhythmia, steerable catheters are navigated to a target area in the heart to block the arrhythmic electrical impulses and the additional electrical pathways by tissue ablation using RF(radio frequency) energy.
Recent catheter ablation studies suggest that applying sufficient force over a desired time during the radio frequency ablation is highly correlated to the procedure outcomes, such as the size and depth of ablation lesion as well as the overall success rate and risk of complications [1-2]. The contact force applied must be between 0.2N and 0.3N to achieve a desirable quality of the lesion. The application time, however, must be strictly controlled to ensure a positive outcome [1]. Also, the inherent flexibility in the catheter causes that the patient is safe from the contact force by collision in the heart. The contact force during delivering RF energy is only interested. The contact force is generated by bending of the steerable section of the catheter.

The importance of providing contact force feedback during ablation has motivated the development of miniature force sensors for catheters. The Thermocool® SMARTTOUCHTM catheter from Biosense Webster and the Sensei® catheter system, for instance, provide both magnitude and direction of contact forces in their respective
3D visualized environments Carto™ and NIOBE®. Furthermore, optical catheter tip force sensors have been presented in [3-5]. Especially, in [6], MR-compatible force and temperature sensor is proposed for cardiac catheter. Also, another multi-parameter sensor based on MEMS have been also proposed in [7] for pressure and flow measurements on a catheter tip.

However, creating miniature tip force sensors can be technically challenging, particularly considering that catheters are disposable instruments. Low-cost sensor designs are therefore of paramount importance. Instead of embedding physical sensors, the inherent flexibility of the catheter can be utilized to achieve intrinsic force sensing. The flexible deformation of the catheter allows the usage of the catheter body as a sensor to estimate contact forces. Various efforts have been made in line with this concept. Intrinsic force sensing for a thin continuum robot using a probabilistic approach has been proposed in [8]. Another intrinsic force sensing approach for inextensible continuum-like robots has been simulated in [9] based on quasi-static modelling of the robot assuming multiple constant curvature segments and has been implemented in control of a continuum robot for compliant interaction using intrinsic force feedback in [10-11]. Khoshnam et al [12] modelled a catheter as a continuum constant-curvature segment with additional rigid links. Our recent work also shows that contact forces at the catheter can be accurately estimated using a discretized Cosserat rod model with a given catheter shape [13]. Kinetostatic modelling of a catheter is the key factor to determine the accuracy of force estimation. Existing analytical kinetostatic models for contact force estimation are mostly based on the assumption of constant curvature [9, 14-15]. Previous research [16] and experimental observation have shown, however, that the constant curvature assumption is invalid with the existence of internal friction and contact forces. The empirical rigid-link model proposed in [12] can cope with variable curvature, but the model parameters
Another variable curvature continuum kinetostatic model, which describes the catheter in a multi-section fashion, has been derived for real-time control in [17]; contact force and local torque were not considered, though. In [18], a kinetostatic modelling framework has been proposed to derive the generalized space Jacobians of a multi-backbone continuum robot, which requires additional optimization. The Cosserat Rod model [19] and co-rotational beam model [20] have been employed for continuum robot kinematics with variable curvature, but the prediction of shape deflection from applied forces requires additional optimization. Thus, these models are computationally expensive and not suitable for real-time applications. Moreover, existing models for catheters assume that the contact occurs at the distal tip only. However, since the catheter is operated within confined spaces in the heart, there are often multiple contacts along the catheter.

In consideration of the limitations of the state-of-the-art work presented above, we proposed the multi-element kinetostatic model for a novel intrinsic force sensing method using bi-point position tracking and tension measurements. The model predicts the 3-dimensional shape configurations of the steerable section in the catheter with a given tension on the tendons and the applied contact forces at arbitrary locations in real-time. This model can be used for steering the catheter. In this study, however, the model mainly contributes to the bi-point contact force estimation algorithm. The “bi-point” referred to in this work are the catheter tip position and the stiff shaft end position as shown in Fig. 1-(a). When the steerable section is in contact to generate a desired contact force, the catheter tip position is constrained by the contact. The contact force is estimated using the position difference between the position sensing feedback and the predicted tip position from the model calculation based on given tension values. The catheter tip position relative to a 3D model of the patient’s heart is continuously available during the
ablation procedure using electromagnetic (EM) tracking systems such as Carto® and ENSite™ Velocity™ Cardiac mapping systems as shown in Fig. 1-(b). The use of these systems with the bi-point force estimation proposed here is more reliable than image-based force estimation methods. Therefore, the contributions of this work can be summarized as: 1) real-time 3-dimensional catheter contact force estimation from given catheter tip position measurement, contact location and tension load, 2) catheter tip contact force estimation without entire shape feedback, which is more suitable to catheter mapping systems already available during the cardiac ablation procedure and 3) real-time shape estimation of the 3D steerable section from measured tension loads and applied contact information. The preliminary results of this study are presented in [21]. However, many parameters and contact states of the force and shape estimation were not defined or investigated but are nevertheless required to justify the efficacy of the method. In this paper, we extend the previous work of [21] by investigating the following aspects: 1) the effect of gradual increase of the number of elements in the discreet model on error convergence; 2) a 3D shape estimation evaluation; 3) further evaluation of the force estimation algorithm for different bending configurations of the catheter; 4) the performance of the contact force estimation algorithm for the scenarios in which an arbitrary, known location on the catheter is constrained; and 5) the limitations of the force estimation. These investigations further support the usefulness and more rigorously define the capabilities of using the difference between expected and actual tip position to estimate contact force.

2. MULTI-ELEMENT KINETOSTATIC MODEL FOR CATHETER
Most commonly used medical catheters consist of a stiff shaft (unactuated) and a steerable section (actuated). The steerable section have over 90 degree bending slope at the tip by applying maximum tension. These facts lead to the assumption that the kinetostatic
behaviour of the catheters can be described by employing a large deflection model for cantilever beams. The previously mentioned efforts in deriving analytical kinetostatic models for catheters which consider external contact and internal friction is still considered a major challenge in the field. In the context of previous research, we develop a multi-element kinetostatic model based on the Bernoulli-Euler hypothesis to describe large deflection of the steerable section considering external contact, compression and friction in the tendon channels [21]. The multi-element kinetostatic model for the catheter deflection in 3D is solved by analytical equations and used to estimate the contact forces applied to the steerable section.

2.1 Tension-based multi-elements model
A multi-element kinetostatic model for the catheter has been derived by evaluating the physical behavior of the helical segment in the steerable section as proposed in [22]. The helix structure made it possible to achieve flexibility and variable stiffness by compression as shown in Fig. 2.

![The catheter and individual helical segment](image)

Figure.2 The catheter and individual helical segment

Therefore, friction inside the tendon channels, deformation of a single element, and variable Young’s modulus due to compression in the steerable section were considered in this model. There are four tendons in the steerable section. These tendons are arranged
along the four cardinal directions, resulting in a 3-dimensional workspace of the steerable section. Each coupled pair of tendons controls the direction of the steerable section bending about one axis. Thus, the tension forces on x and y-axis can be represented as follows:

\[ F_{tx} = T_1 - T_2, \quad F_{ty} = T_3 - T_4 \]  \hspace{1cm} (1)

where, \( T_{1,4} \) corresponds to tension applied to tendon 1 to 4, as shown in Fig. 3.

Figure.3 Diagram of the proposed multi-elements model
The multi-element model assumes that the bending of a single element lies in a 2-dimensional bending plane as depicted in Fig 3. The bending moment by the applied tensions in the bending plane at any element in the steerable section is calculated using the magnitude of the tensions as Eq. (2).

\[ M_b^{(i)} = c \sqrt{F_{tx}^2 + F_{ty}^2} \]  \hspace{1cm} (2)
where \( c(=0.75\text{mm}) \) is a distance between centre of a cross-sectional view of the steerable section and a tendon (see Figure 3), and \( i \) is the element number. The equation for a bending angle and moment \( (\theta = ML/EI) \) is discretized by the element length \((ds)\) as shown in Eq.(3).

\[
d\theta^{(i)}_{b} = \frac{M^{(i)}_{b}}{EI} ds
\]

Based on Eq.(3), we assume that the assembly of the rotated, small element length \((ds)\) can represent a geometrical curve which corresponds to the steerable section shape as shown in Fig 4.

![Figure 4](image.png)

Figure 4 Assembly of the bending angle at each element to describe curved shape of the steerable section.

Thus, a position \( P^{(i)}_{b} = [x_{b} \quad z_{b}] \) of an element in the bending plane is defined using a rotation about the bending angle \( \beta^{(i)}_{t} \) by a tension which is defined by the previous bending angle at \( \beta^{(i-1)}_{t} \) with Eq. 3 as:

\[
\beta^{(i)}_{t} = \beta^{(i-1)}_{t} + d\theta^{(i)}_{b}
\]

Position \( P^{(i)}_{b} \) is updated as:

\[
P^{(i)}_{b} = [\sin \beta^{(i)}_{t} \quad ds \quad \cos \beta^{(i)}_{t} \quad ds] + P^{(i-1)}_{b}
\]
The rotation of $P_b^{(i)}$ around the z-axis angle $\alpha_t$ by the tension represents the 3-dimentional element shape (see figure. 3), thus the 3-dimensional position ($P^{(i)} = [x_i \ y_i \ z_i]$) at an element is

$$P^{(i)} = \begin{bmatrix} \sin \alpha_t P_b^{(i)}_{b,x} & \cos \alpha_t P_b^{(i)}_{b,x} & P_b^{(i)}_{b,z} \end{bmatrix} + P^{(i-1)}$$  

(6)

The angle $\alpha_t$ is calculated using the decomposition of tendon force as Eq. (7)

$$\alpha_t = \tan^{-1} \left( \frac{F_{tx}}{F_{ty}} \right)$$  

(7)

If $\beta_t^{(i)}$ is purely linear along the steerable section, $d\theta_b^{(i)}$ is of equal value at all elements.

The calculation result in Eq. (6) is similar to the constant curvature deflection model. However, the bending angles at each individual element have different values due to friction in the tendon channels. Moreover, compression and variable Young’s modulus due to the helical structure in the used catheter have to be considered, which is derived below.

2.2 Tendon friction

A kinetic friction force ($F_k$) is the primary cause of friction during catheter steering. $F_k$ results from the normal force $F_n$ on the surface of the tendon channel [16].

Figure.5. The kinetic friction description in the tendon channel
To calculate $F_n$, we consider the body force $F_b$ as shown in Fig. 5. Elasticity of the steerable section generates a body force $F_b$ when the steerable section is deflected. Thus we assume that $F_b$ acts as normal force $F_n$ between the tendon and the surface ($F_b = F_n$).

The internal body moment $M_b^{(i)}$ is defined by $F_b L^{(i)}$, where $L^{(i)}$ is the length between the first element ($i=1$) and any point along the steerable section. Therefore, $F_b$ can be derived using the substitution of $F_b L^{(i)}$ into Eq. (3), and $F_k$ can be calculated as Eq.(8).

$$F_k^{(i)} = \frac{dg^{(i)}EI}{L^{(i)}} \frac{1}{ds} \mu^k + F_k^{(i-1)}$$  \hfill (8)

where the kinetic coefficient $\mu^k$ is considered as friction between two different materials.

The kinetic friction at an element ($i$) involves the kinetic friction at the previous element ($i - 1$). Now, the bending moment $M_b^{(i)}$ in Eq. (2) is re-updated using both kinetic friction forces.

$$M_b^{(i)} = r \sqrt{F_{tx}^2 + F_{ty}^2 - F_k^{(i)}}$$ \hfill (9)

### 2.3 Helix compression

When tension is applied, all of the discretized elements are simultaneously bent and compressed. The direction of the compression follows the neutral axis the steerable section. Thus, Eq. (5) involves compression as shown in Eq. (10), $P_b^{(i)}$ can be rewritten considering compression as

$$P_b^{(i)} = [\sin \beta_t^{(i)} (ds - \frac{F_t A}{ks}), \cos \beta_t^{(i)} (ds - \frac{F_t A}{ks})] + P_b^{(i-1)}$$ \hfill (10)

Where $A$ is the area on which the force acts, and $F_t$ the tension including the friction forces. The stiffness of the helix is altered by the compression of the helix structure, for
which E has been obtained experimentally. Based on experimental observation, the 
Young’s modulus varies with the applied tension as shown in Eq. (11).

\[ E^{(i)} = E_0 e^{\frac{F_t A}{E^{(i-1)}} ds} \] 

(11)

Where \( E_0 \) is the initial Young’s modulus. The difference of the bending angle between 
two elements, \( d\theta_p^{(i)} \) is re-updated using Eq. (11). The steerable section shape prediction 
by given tension is presented in Algorithm 1, and \( P_t \) indicates the predicted steerable 
section shape by given tension.

2.4 Shape prediction using Contact force

Steering the catheter in a confined space such as the heart chamber often results in contact 
forces along the steerable section. The steerable section shape deforms based on the 
applied contact forces, which is considered in the modelling. In this model, the shapes 
generated by tension and contact force are separated; the contact force is applied to the 
already estimated catheter shape, derived from the tension forces.

A contact force on the steerable section is assumed as a point load at any point on the 
steerable section. The contact force \( (F_c = [F_x, F_y, F_z]) \) acts on the catheter between the first 
element \((i=1)\) and the contact location \((P_c)\). Deformation \( P_e^{(i)} \) by \( F_c \) is calculated as Eq. 
(12).

\[ P_e^{(i)} = diag \left[ \frac{ds}{GA}, \frac{ds}{GA}, \frac{A}{E ds} \right] F_c \]

(12)

where \( G \) is shear modulus. The Young’s modulus (E) in Eq.(11) is recalculated using the 
z-axis contact force \( F_{c,z} \) as:

\[ E^{(i)} = E_0 e^{(F_t + F_{c,z}) \frac{A}{E^{(i-1)}} ds} \]

(13)
The bending moment at each element is recalculated by a cross product of the distance between an element position and the contact location \((P_c - P^{(i)})\) and contact force \(F_c\) as 
\[M_e^{(i)} = (P_c - P^{(i)}) \times F_c,\] as shown in Fig.6. Furthermore, we assume that each element has an individual bending plane. Therefore, the bending moment \((M_{eb}^{(i)})\) in the bending plane of each element is calculated as

\[M_{eb}^{(i)} = \frac{F_z}{|F_z|} \sqrt{M_{e,x}^{(i)} + M_{e,y}^{(i)}} \quad (14)\]

The z-axis contact force \((F_z)\) is used to define the direction of \(M_e\). The bending angle \((\beta_e^{(i)})\) caused by the contact force at each element is calculated using the Young’s moduli from Eq.(4-5). The angle \((\alpha_e^{(i)})\) from the x-axis to the bending plane caused by the contact force is calculated using Eq. (11) with \(F_x\) and \(F_y\) instead of the tensions \(F_{tx}\) and \(F_{ty}\). The existence of z-axis torsion moment \((M_{e,z}^{(i)})\) at each element contributes to torsion angle \((\alpha_{tor}^{(i)})\). The torsion angle is calculated independently using Eq.(3) with torsional stiffness.
GJ instead of EI, where J is the torsional constant. Thus, the angles $\rho_e^{(i)}$, $\alpha_e^{(i)}$ and $\alpha_{tor}^{(i)}$, which result from the contact force are summed with the $\beta_t^{(i)}$ and $\alpha_t^{(i)}$ as Eq. (15-16):

$$\alpha^{(i)} = \alpha_t^{(i)} + \alpha_e^{(i)} + \alpha_{tor}^{(i)}$$ (15)

$$\beta^{(i)} = \beta_t^{(i)} + \beta_e^{(i)}$$ (16)

The resulting angles and deformations are applied to Eq.(10) to reshape the steerable section in the bending plane.

$$P_b^{(i)} = \left[ \sin \beta^{(i)} \left( ds - (F_t + F_c) \frac{A}{E_d} \right), \cos \beta^{(i)} \left( ds - (F_t + F_c) \frac{A}{E_d} \right) \right] + P_b^{(i-1)}$$ (17)

And represented in 3D

$$P^{(i)} = \left[ \sin \alpha^{(i)} P_{b,x}^{(i)} \right] \left[ \cos \alpha^{(i)} P_{b,z}^{(i)} \right] + P_e^{(i)} + P^{(i-1)}$$ (18)

As depicted in Fig 6, the steerable section shape is therefore determined by a combination of tension force and contact force, and order of the steerable section shape prediction by given contact force is presented in Algorithm 1. Note that, in the Algorithm 1, $P$ indicates the predicted steerable section shape by given contact force.

This kinetostatic model computes the steerable section shape by accounting for tendon friction and the deformations due to the pure bending of each small element. Most tendon driven commercial catheters have the tendon channels similar to the catheter in this paper. Thus, the friction calculation can be applied to the other tendon driven catheters. Amounts of torsion, compression and shear deformations of the other catheter can be calculated by providing the material properties.
2.5 Contact Force Estimation

The catheter tip position variation by the contact force can be used to determine the contact force ($F_c$) employing catheter tip position sensing instead of entire shape sensing. Key principle of the contact force estimation based on position sensing is that the estimated catheter tip position $P_t$ from tension and the measured catheter tip position $P_m$ have a difference $P_d$. This force estimation assumes that $P_d$ is proportional to the magnitude of $F_c$. The first step in the calculation is the determination of the steerable section shape based on the applied tensions $P_t$. The unit direction of the contact force ($\hat{F}_c$) is equal to the direction of the position difference ($\hat{P}_d$) as Eq. (19) as shown in Fig. 6.

$$\hat{F}_c = \hat{P}_d$$ (19)

Applying $F_c$ ($\|F_c\| = 1$N) to the contact location in the configured steerable section shape causes a larger displacement in the catheter tip position. Thus, $\hat{F}_c$ is downscaled by scale factor $u(< 0.01)$ and applied to calculate $M_{eb}^{(i)}$. The new catheter position $P^{(i)}$ caused by $\hat{P}_d u$ is inserted into Eq.(18) and another position difference ($\Delta P_f = P^{(i)} - P_t$) is calculated. Finally, based on the linear assumption, $F_c$ is estimated using the measured position $P_m$ and the catheter tip position difference $P_f$ as Eq. (20).

$$F_c = \hat{P}_d u \frac{P_m}{\Delta P_f}$$ (20)

Therefore, algorithm for estimating the contact force and the steerable section shape can represented as below.
Algorithm 1: Shape prediction and contact force estimation

1. **Input:** $F_{tx}$, $F_{ty}$, $P_m$
2. **Output:** $F_c$.
3. **Shape prediction by tension** $P_t^{(i)} \leftarrow F_{tx}$ and $F_{ty}$
   4. for $i = 1$: elements number
      5. $M_b^{(i)} \rightarrow d\theta_b^{(i)} \rightarrow F_k^{(i)}$
      6. Update $M_b^{(i)}$ and $E$
      7. $P_t^{(i)} = [\sin \alpha_t P_{b,x}^{(i)} \cos \alpha_t P_{b,x}^{(i)} P_{b,z}^{(i)}] + P^{(i-1)}$
   end

4. **Shape prediction by contact force**, $P^{(i)} \leftarrow F_c$
   5. for $i = 1$: elements number
      6. Calculate $P_e^{(i)} \leftarrow F_c$
      7. Update $E$ and $M_{eb}^{(i)} \leftarrow F_c$
      8. $P^{(i)} = [\sin \alpha^{(i)} P_{b,x}^{(i)} \cos \alpha^{(i)} P_{b,x}^{(i)} P_{b,z}^{(i)}] + P_e^{(i)} + P^{(i-1)}$
   end

5. **Contact force estimation**
   6. Receive tip position feedback, $P_m$
   7. Calculate $P_t^{(i)} \leftarrow F_{tx}$ and $F_{ty}$
   8. $\hat{P}_d = P_t^{(i)} - P_m$
   9. $P^{(i)} \leftarrow \hat{P}_d u$
   10. Position difference, $P_f = P^{(i)} - P_t$
   11. Estimate contact force, $F_c = \hat{P}_d u \frac{P_m}{P_f}$

3. **MODEL VALIDATION**

In practice, the catheter tip position and its shape can be obtained through fluoroscopy imaging or 3D electroanatomic mapping systems, such as the Carto™ [19-20]. Most existing ablation catheters, however, can only provide confirmation of the catheter tip in contact. In this work, the contact force on the steerable section in the catheter is estimated according to the position difference between the model prediction and the position sensing. Position accuracy in the model is therefore of paramount importance in estimating the contact force accurately. Thus, the multi-element kinetostatic model is validated to determine the error between prediction and physical system. In the following validation, the contact force estimation is examined with different shape configurations on two different bending planes as shown in Fig. 13. Moreover, the contact force with an
additional position constraint along the steerable section body as well as loading resulting in buckling conditions along the longitudinal axis of the catheter are tested.

3. EXPERIMENT

3.1 Contact force estimation validation setup

A catheter for this validation has been prototyped based on the catheter design using helical segments as shown in Fig. 2. The steerable section’s helical segment is 1.5 mm in radius with a 10.2 mm length including 4.5mm length of the helix structure and an internal channel of 1.2 mm for the insertion of optional elements. Each tendon guiding channel is 0.25mm in diameter. The steerable section length (L) is 102mm with 10 helical segments. The other physical variables in the kinetostatic multi-element model are measured experimentally. A single helical segment is placed on a the 6-axis ATI nano17 force/torque sensor (Calibration SI-25-0.25, resolution: 1/160N for $F_x, F_y, F_z$, 1/32 Nmm for $M_x, M_y, M_z$, range: $F_x, F_y = \pm 25N$, $F_z = \pm 35N$, $M_x, M_y, M_z = \pm 250Nmm$) and moved with a linear guide as shown in Fig 7. Young’s modulus is calculated using measured z-axis force and displacement from the linear guide. The Young’s modulus of the segment was measured three times and compared to the estimated Young’s modulus using Eq.11 with constant ds (=9mm) for two helix structures. The error to the measured E was about $5.2\pm 1.2\%$ ($0.43\pm 0.09 \times 10^6 N/mm^2$).

In addition, shear modulus (G) is measured by applying lateral force as shown in Table-1.
Figure 7 The Young’s modulus experiment setup using the 6-axis force sensor and the linear guide (left) and the results (right), the predicted Young’s modulus using Eq. 11 agrees well the measurement.

The second moment of area ($I$) and torsion constant ($J$) are calculated based on the geometry of the structure. It is difficult to measure the kinetic coefficient $\mu^k$ between the tendon channel surface and the tendon. Thus, the kinetic coefficient $\mu^k$ is chosen by manually tuning the value to best fit the shape estimation data during the shape estimation experiment.

Table 1 shows the defined values of the variables.

<table>
<thead>
<tr>
<th>Physical property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial young’s modulus ($E_0$)</td>
<td>$5.1 \times 10^6 N/m^2$</td>
</tr>
<tr>
<td>Shear modulus ($G$)</td>
<td>$2 \times 10^6 N/m^2$</td>
</tr>
<tr>
<td>Second moment of area ($I$)</td>
<td>$5.153 \times 10^{-13} m^4$</td>
</tr>
<tr>
<td>Torsion constant ($J$)</td>
<td>$1.0306 \times 10^{-12} m^4$</td>
</tr>
<tr>
<td>The steerable section length ($L$)</td>
<td>102mm</td>
</tr>
</tbody>
</table>

The catheter is integrated with the linearly-actuated tendon actuation platform based on [23] to provide tension feedback control as shown in Fig. 8-(a). It introduces advantages such as backlash prevention, less slack and tangled tendon due to the linear actuation [23]. Furthermore, a near frictionless ball bearing is placed in the location where the tendon contacts a surface in the platform. The maximum friction between the bearing and the tendon is measured using a two arm scale mechanism, with the friction force being
0.037N at a 5N tension. The friction on the bearing is less than 1% of the applied tension and therefore ignored in this study.

The catheter is guided into the heart from the femoral vein through the aorta to the right atrium. When the catheter ablates in the right atrium, the stiff shaft is mostly straight with small deflection, which results in low internal friction between the tendons and stiff shaft. In this validation, the stiff shaft, which is comprised of a 60 cm flexible tube, is clamped between the steerable section and the platform to form a straight line (see figure 8). The measured tendon friction in the straightened shaft is 0.02N under a 5N tension. Therefore, the friction caused by deflection of the stiff shaft is ignored. If the catheter is inserted into the ventricles, the stiff shaft may have a considerable curve. Then, the friction from the bending of the stiff shaft must be considered.

Figure 8 (a) Linearly-driven tendon actuation platform, (b) Force estimation experiment setup with 6-axis force sensor (electromagnetic (EM) tracker 1 and 2 are the bi-point tracking)

### 3.2 The steerable section shape prediction by tension

The shape prediction accuracy could potentially change with number of elements in calculation. To determine the error convergence of the tip position accuracy with respect to the number of elements, the steerable section position is fixed with a constant 0.8N tension and the number of elements is increased from 20 to 110.
Each segment in the steerable section includes a helix part (4.5mm in length) and a rigid part which serves as a linkage to the neighbouring elements. Length of the rigid part between the two helical segments is 5.7mm ($ds$) and is always considered as a single element in the model. The total number of rigid elements in the model is 10, including both ends of the steerable section. In order to increase the number of elements in the model, the number of elements across the helix portion of each segment is increased. The length of each sub-segment depends on the total number of elements for the entire steerable section. For example, for a steerable section with 10 segments, there would be 10 rigid parts and 10 helical parts. A 50 element model would then consist of the 10 elements from the 10 rigid parts and 40 elements from 10 helix structures. Thus, the length of element in a single helix structure would be $1.125mm (=4.5/4)$ as described in Fig.9.

![Figure 9](image)

(a) Figure 9 Description of the element division: (a) the total element number is 50 and (b) the total element number is 70

The results are presented in form of the prediction error between the estimated and measured positions against the element number as shown in Fig. 10-(a). The prediction error decreased by about 0.5mm to 2 mm when the number of element is 50, and then remained constant. Moreover, the computation frequency changed from 1.7 kHz to 1.53 kHz for 20 elements to 110 elements. Therefore, 50 elements provide a good tip position accuracy with an efficient computational speed.
Figure 10 (a) Prediction error of catheter tip position with respect to the number of elements in the model (b) Prediction error of the catheter tip position depending on applied tension.

Figure 11 Comparison between model prediction and measured the steerable section shape

To further validate the model accuracy under different levels of tension, the x-axis tension is increased from 0 N to 1 N at 0.1 N intervals. The steerable section shape was measured using a RGB camera as shown in Fig. 11. The RGB camera was placed to observe the entire shape on x-z plane in the experiment coordinate and the shape was extracted by removing background. The shape extraction was calibrated, it showed $0.1 \pm 0.07$mm position error. The prediction error varied from 1.8mm (0N) to 2.7mm (0.4N) with an average position error of 2.3mm, as shown in Fig. 11-(b), with a small standard deviation of the prediction error of about 0.3mm.
Figure 12 (a) The estimated and measured (by EM tracker) 3D position of the steerable section tip position are presented (b) The position magnitude errors: The distance from origin to each point is amount of absolute position error. The bending plane angle ($\alpha$) is calculated as in Fig 12-(a)

The shape estimation was validated in 3D space. The tensions were increased in 0.1N intervals up to 1.0N. The steerable section was deflected in 8 different directions as depicted by red arrows in Fig 12-(a). The absolute position errors are presented in Fig 12-(b). The maximum, minimum and mean absolute errors are 4.14mm, 2.92mm and 1.87mm ±0.67 respectively. Deflections in bending planes that resulted from two neighbouring tendon motion (deflection directions 2, 4, 6 and 8) showed a larger mean absolute position error of about 3.3mm on average compared 2.4mm on average in directions 1, 3, 5 and 7 where bending is resulted from a pair of antagonistic tendons. A reason for this error could be due to the additional friction generated when two tendons both apply compressive pressure to the tendon channels, where the model assumes friction is generated by a single tendon pulling. Another reason of the error is uncompensated gravity. These directions also showed a larger bending plane angle ($\alpha$) error of about 2.4±0.4° following direction of the gravity.

In the cardiac ablation procedure, however, radio frequency energy delivery during ablation is spread over a region and not confined to individual points. Therefore, a certain
error margin (~ 5 mm) is acceptable during the cardiac ablation procedure [24]. Thus, the shape estimation using the multi-element model is applicable under the required accuracy.

3.3 Contact force estimation

Both ends of the catheter tip positions are provided using a magnetic position tracker (NDI Aurora tracker) as shown in Fig. 8-(b). The catheter is steered using the linearly-actuated catheter steering platform to generate contact forces on the 6-axis force sensor. An additional aurora tracker is attached to the 6-axis ATI nano17 force/torque sensor to measure its orientation with respect to the catheter tip, so that the 3-dimensional contact force on the 6-axis force sensor are re-oriented and used as benchmark force to be compared with the estimated contact force.

The contact force estimation method could have different accuracies for different bending planes and the steerable section shape configurations. To evaluate this, two different bending planes are tested with seven different initial the steerable section deflections, which are presented in Fig. 13-(a) and (b). The 6-axis contact force sensor is placed at the catheter tip position under the given tip angle and the tension is repeatedly increased from 0N to 3.5N.
Figure 13 (a) The steerable section shape configurations on the plane-1, (b) The steerable section configurations on the plane-2, (c) The contact force estimation results on the plane-1 with 130° the catheter tip orientation, (d) The contact force estimation results on the plane-2 with 60° catheter tip orientation

Plane-1 lies in the x-z-plane and the steerable section is brought into contact with the 6-axis force torque sensor under different tip deflections (45°, 65°, 90° and 130°) as shown in Fig. 13-(a). $F_{tx}$ is the major tension value and repeatedly increased from 0N to 3.5N. Plane-2 is rotated around the z-axis by 45° as shown in Fig 13-(b), and similarly, the magnitudes of $F_{tx}$ and $F_{ty}$ are repeatedly increased between 0N and 3.5N. Three different orientations are tested with 45°, 60° and 80°, which is shown in Fig. 13-(b). During the evaluations in both planes, the catheter tip orientations are varied while the initial contact angle between the tip and the 6-axis force torque sensor is constant.
A notable result is that the 3-dimensional contact force is distributed with respect to the bending plane orientation. In plane-1, the contact force is mostly distributed over $F_x$ and $F_z$, as shown in figure Fig. 13-(c). In plane-2, the contact force is evenly distributed over $F_x$, $F_y$ and $F_z$ as shown in Fig. 13-(d). The contact force results show that the contact force estimation approach works reliably for 3-dimensional cases with different tip orientations. The comparison between estimation and measurement show accuracies of 83%, 92% and 80% for x, y and z-axis forces respectively.

A notable error in the results can be seen in the early decrease in the contact force during the application. This could be caused by the catheter tip slipping over the force sensor while the contact force is being applied. Friction and compression still remain after the tension release and the tip stays in contact with the silicone rubber after the tension application, which causes the force torque sensor to remain constant even though the contact force reaches zero, as shown in Fig. 14-(c) between 35s and 40s. The falling edge in the contact force estimation is therefore neglected in the comparison of the results.

Figure 14 Contact force estimation accuracies for different bending configuration of the catheter tip orientation
The three dimensional contact forces estimation showed accurate results in comparison with the benchmark contact force as shown in Fig. 14; mean of the accuracies are 84.23±5.5% and 84.5±5.73% for planes 1 and 2 respectively. The average accuracy of all cases is 84.37±5.62%. Mean average error with standard deviation of the errors from all cases is 0.018±0.015N. The MAE from both planes comprises of a similar error range. Hence, the accuracy of the contact force estimation does not vary with the catheter tip orientation or the orientation of the bending plane, but the estimated contact force has a 15% error with respect to the 6-axis force sensor. During the catheter ablation procedure the desired contact force range (0.2-0.3N) can be achieved with a force estimation error 15±5% (0.03±0.01N). Therefore, the force estimation error is acceptable.

The error in the force estimation can be attributed to the position error in the kinetostatic model. This is caused by 1) gravity is not considered 2) the Yong’s modulus estimation error and 3) only single tendon friction in the tendon channel was considered in the kinetostatic model during the application of contact force using two tendons. In future work, the kinetostatic model should consider these sources of error to improve the accuracy.

The computational time of force after receiving sensors measurements is 1.3ms and 750Hz. The overall update rate depends on the tension sensor update rate and the EM position tracking frequency. In this experiment, the tension sensor update rate was 160Hz and the NDI aurora position tracking (EM tracker) system showed 60Hz update rate. Therefore, the force estimation update rate overall in this work is 60Hz.
3.4 Contact force estimation with constraints

During the cardiac ablation procedure, described above, the steerable section can be physically constrained by its environment. For instance, the catheter may need to be steered through the septum wall to reach the left atrium from the right atrium as shown in Fig. 1-(a). The septum wall then constraints the movement of the catheter at the intersecting location. The discretization in the force estimation model allows the algorithm to be easily adapted for constraining a number of segments in the physical catheter.

To examine such cases, the third and sixth segments in the steerable section are constrained by a 3d-printed fixture as shown in Fig. 15-(a) and (b). As defined above, the accuracy of the force estimation does not change significantly with the orientation of the bending plane. Fig.15-(a) and (b) show the results; the magnitudes of the contact forces are estimated with 85.4% and 88.9% accuracy compared to the 6-axis force sensor for 66 mm and 42 mm respectively.

Figure 15 Contact force estimation results; (a) 66mm length with constraint on the sixth segment and (b) 42mm length with constraint on the third segment
Given the proposed method, as long as the catheter tip can be tracked fast enough, the force estimation should work for dynamic cardiac motion (normally <1.6 Hz, [25]) as well, since the computation speed is more than 700Hz and the catheter itself has neglectable weight.

Moreover, as mentioned in the introduction with Fig 1-(b), the existing electroanatomic mapping systems such as Carto® provide a position of the multiple electrodes along the catheter in the 3D cardiac anatomy using multiple electromagnetic (EM) tracking as in this work. During the ablation procedure, deep sedation or general anaesthesia is generally used to suppress beating heart motion [26-27]. Heart beat motion compensation techniques are also available in [28]. Therefore, heart beat motion compensation, the electroanatomic mapping system and deep sedation together all ways of mitigating the effects of cardiac motion. As a result, this method described in this paper is applicable to force estimation under external constraints with the cardiac motion.

3.5 Buckling under tip constraints – Method limitations

The catheter steering within a constrained environment induces collisions. For each catheter bending configuration listed in Fig 13, we carried out tests by pushing the catheter tip using the force sensor, while fixing the tension on the tendons to simulate the collision. It was found that the collision force generated by collision is trivial (< 0.02 N),

Figure 16 The catheter shape change during the buckling experiment and the contact force result
this value is close to the error margin of the force estimation algorithm. Thus, the force estimation algorithm has not been validated for these cases.

One special case is when axial loading is applied to the catheter when it is in a straight configuration, upon which buckling occurs. Buckling by compressive stress is not considered in the model; the steerable section shape under buckling will therefore be estimated with a considerable error. An experiment to investigate the buckling behaviour is conducted with the steerable section being translated against a force torque sensor with a 20mm translation, as shown in Fig. 16. In this case, the estimation accuracy is only 27% with respect to the measured benchmark forces. The estimated force is 0.18 N lower than the benchmark. This study shows that when buckling occurs, a considerable contact force error can be generated. The considerable error can be from an additional deflection in the shaft and that the model does not include account for the behaviour of the steerable section buckling.

Buckling could potentially occur when the catheter tip is mostly straight and encountered with an axial force. It may happen during the catheter insertion stage, in which case, the inherent softness of the catheter is used to avoid tissue perforation and the force sensing is not required. As the interaction force during buckling is uncontrollable, clinicians avoid to carry out tissue ablation when the catheter has a “prone-buckling” configuration.

4. DISCUSSION

While the algorithm shows good results for most of the evaluated scenarios, limitations have been identified which are impact force by collision, shaft deflection, multiple external forces by steering the catheter in the confined space and buckling occurs.

The purpose of the contact force estimation aims for applying adequate force to deliver the right amount of RF energy to avoid thermal complications [1]. In practice, collision
forces are managed to a minimum amount by clinician via careful steering the catheter and the inherent flexibility of the catheter during insertion.

If the stiff shaft deflects significantly, the friction in the stiff shaft must be compensated additionally. As defined during the 3D shape estimation, gravity and additional friction in tendon channel must be considered in the multi-element kinetostatic model. The bi-point contact force estimation is limited in single contact scenario. To estimate multiple external forces simultaneously on the steerable section requires complete knowledge of the catheter 3D shape. Applying multiple $F_c$ in Eq.19 to the selected elements which correspond to the physical contact locations estimates the catheter shape, and calculates the position differences between the model and the measured position as the tip force estimation. Then, multiple contact forces can be estimated using Eq. 20 using the position differences between the model and the measured position. In practice, catheter shape could be achieved using fluoroscopic image processing [29-30] or existing shape sensing technologies [31-32]. To estimate the contact force when buckling occurs, additional tracking points on the catheter are required to reconfigure the buckled shape, and the external force applied to the catheter may be inferred inversely from the shape mechanical modelling. This will be investigated in our future work and beyond the scope of this paper.

5. CONCLUSION
A multi-element model for a tendon driven catheter is introduced to accurately predict 3-dimensional shape of the catheter considering internal friction, tendon tensions and the contact force. A bi-point contact force estimation algorithm is derived and validated with various experiment scenarios.

The undefined relationship between position accuracy and number of elements in the model in [23] was examined. The result in the examination is that the error remains
constant at 2mm with more than 50 elements. During the experiments with different bending planes and with different the steerable section configuration, the contact force was estimated with 15% error compared to the benchmark force and the accuracy of the estimation did not change significantly with the steerable section shape configuration or the orientation of the bending plane. Furthermore, the contact force was estimated with a similar error range (88% and 84%) when external constraints were introduced. About 15% contact force error (∼0.03N) is acceptable based on the fact that cardiologists only require the applied force 0.2 – 0.3 N for a given time period, which has been shown to be achievable with the given setup.

6. REFERENCES


Elizaveth MC Ashley, Anaesthesia for electrophysiology procedures in the cardiac catheter laboratory, Continuing Education in Anaesthesia Critical Care & Pain, 12(5): 230-236, 2012


