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Modelling and mitigating flux jumps in bulk high-temperature superconductors during quasi-static, high-field magnetisation

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Abstract

\textit{(RE)}-Ba\textsubscript{2}Cu\textsubscript{3}O\textsubscript{7−δ} bulk superconductors acting as permanent trapped field magnet analogues have been shown to trap fields in excess of 17 T. However, the occurrence of thermomagnetic instabilities during their magnetisation process can limit their performance. In 2019, Naito et al trapped 15.1 T in a (Y)-BaCuO bulk pair under applied fields of up to 22 T (2019 J. Appl. Phys. 126 243901), whilst also documenting the mechanical failure of the bulks due to a flux jump. Here, we accurately numerically model this experiment and the observed flux jump, providing insight into the experimental results, such as the role of heating, the presence of thermal gradients, and a possible reason for the location of the bulk-pair fracturing. Extending the study with an investigation into the role of enhanced cooling of the bulk-pair, the influence of cooling power on the calculated trapped field and flux jump is explored. Finally, we propose and numerically investigate a new composite bulk configuration, consisting of stacks of (Y)-BaCuO bulk and interspaced metallic discs, to enhance the thermal stability of the bulk pair, to ultimately improve the trapped field performance.

Keywords: bulk high-temperature superconductors, flux jumps, field-cooled magnetisation, trapped field, numerical modelling, finite element method, composite structures

(Some figures may appear in colour only in the online journal)

1. Introduction

1.1. Background

Bulk superconductors comprising single grain (RE)-Ba\textsubscript{2}Cu\textsubscript{3}O\textsubscript{7−δ} materials (where RE is the rare earth element, RE123 the superconducting phase, and is herein referred to as (RE)-BaCuO) are capable of acting as permanent trapped field magnet analogues, and show great promise for a number of engineering applications, including rotating machines \cite{1, 2}, NMR and MRI \cite{3, 4}, and field-enhancing techniques such as magnetic lensing \cite{5, 6}.

However, effectively trapping a useful, high magnetic field within (RE)-BaCuO bulk superconductors, using a suitable magnetisation technique, can present a number of challenges \cite{7, 8}. First, significant electromagnetic stresses associated with the Lorentz force are exerted on the sample during magnetisation, which deteriorates the superconducting properties and field-trapping performance (i.e. the critical-current
density, \( J_c \), is dependent on magnetic field, temperature and mechanical strain [9, 10], and can cause mechanical failure due to the brittle ceramic nature of these materials. Second, the motion of flux through the sample during magnetisation—even under slow, quasi-static conditions—can induce significant heat, which may generate thermomagnetic instabilities such as flux jumps, leading to quenching (and often catastrophic mechanical failure) of the sample [11–13]. It has been shown in the literature that not only can thermal stress generated during a flux jump lead to the mechanical failure of a bulk [14], but also the induced electromagnetic stresses can cause mechanical failure in unreinforced bulks at 7 to 9 T [15–17].

The typical fabrication methods used to create bulk (RE)-BaCuO (such as TSMG [18], QMG [19] and TSIG [20]), create inhomogeneous samples containing cracks, voids, pores, and nonsuperconducting inclusions, which naturally causes a variation in trapped field performance between similarly produced samples [21, 22]. After fabricating a bulk, processing it for high-trapped field performance often involves the use of mechanical reinforcement, such as from carbon-fibre wrapping and resin-impregnation [23], or ‘shrink-fit’ rings.

Using the latter reinforcement, a trapped field of 17.6 T was achieved between a (Gd)-BaCuO-Ag bulk-pair, showing the significant potential of these superconducting bulks when suitably reinforced [24]. Naito et al also achieved high-trapped fields within a (Y)-BaCuO bulk-pair (fabricated by Nippon Steel [25]), by using the FCM (field-cooled magnetisation) method, using compressive stainless-steel ‘shrink-fit’ ring mechanical reinforcement and full sample encapsulation [26]. However, despite this seemingly robust mechanical reinforcement technique, the experiment was blighted by the presence of a flux jump causing the catastrophic mechanical failure of the bulk pair.

### 1.2. Motivation

Naito et al [26] reported two unexplained problems with the observed flux jump. First, the electromagnetic hoop stress, and thus the required compressive force from the stainless-steel rings, were calculated to be sufficient for a trapped field of 22 T [27]. Second, analytical calculations suggested the observed temperature increase was not sufficient to induce the resulting cracks from the sudden increase in thermal stress [26]. There are possible explanations for these observations, including the inhomogeneous sample microstructure discussed above, however, the investigative power of numerical methods can also provide further insight.

Here, using numerical modelling, we accurately describe the experimental setup and time-dependent measured data from the experiment by Naito et al [26]. The experimentally observed flux jump was modelled with high accuracy, through careful control of the flow of thermal energy within the bulk pair. Utilising the flexibility of numerical methods, we analyse the evolution of the flux jump, and suggest reasons for the observed cracking. With the accurate replication of the experiment by Naito et al we extend the numerical models to make predictions of the trapped field performance of bulk pairs subject to different cooling powers, and investigate the trapped field performance of various composite sample configurations, consisting of stacks of (Y)-BaCuO and copper layers.

### 2. Modelling framework and assumptions

Using the finite-element method and the commercial software package COMSOL\textsuperscript® Multiphysics, the experimental setup and procedure described by Naito et al [26], was replicated numerically. Here, a 2D-axisymmetric thermomagnetic coupled-physics model based on the \( H \)-formulation [28] is developed to describe the (Y)-BaCuO bulk-pair, and the stainless-steel rings and plates (referred to as the bulk-pair assembly), which are shown schematically in figure 1.

The magnetic flux density and temperature were calculated in four locations (i.e. ‘Hall’ and ‘temperature probes’, see figure 1). The Hall probes’ (or H.P. 1 and 2) were placed between the bulk-pair along the line \( z = 0 \), at \( r = 0 \), 2 mm respectively. Meanwhile, the ‘temperature probes’ (T.P. 1 and 2) were placed along the line \( r = 0 \), at \( z = 1.6, -13 \) mm, respectively. These positions are consistent with those used by Naito et al [26] experimentally, with the exception of T.P. 1 which is additional to the modelling, and inserted just inside the upper bulk.

#### 2.1. Electromagnetic considerations

The governing equations for the 2D-axisymmetric \( H \)-formulation are given by Ampère’s and Faraday’s laws, shown in equations (1) and (2), respectively:

\[
\nabla \times \mathbf{H} = \mathbf{J} \quad (1)
\]

\[
\nabla \times \mathbf{E} = -\frac{\partial(\mu \mathbf{H})}{\partial t} \quad (2)
\]

where \( \mathbf{H} = [H_r, H_z] \), represents the magnetic field strength components, \( \mathbf{J} = [J_\phi] \) represents the current density, \( \mathbf{E} = [E_\phi] \) represents the electric field, and \( \mu = \mu_r \mu_0 \), the relative and vacuum permeabilities, respectively.

The electromagnetic behaviour of the bulk superconductor is described by the critical current density distribution, \( J_c(B, T) \), and was modelled using equation (3) [29, 30]:

\[
J_c(B, T) = \frac{J_{c0} \left(1 - \frac{E}{T_c} \right)^2}{\left(1 + \frac{B}{B_o(T)} \right)^2}^{1.5} \quad (3)
\]

where the assumed values of \( J_{c0} = 1 \times 10^{10} \text{ A m}^{-2} \), and \( B_o(0 \text{ K}) = 16 \) T (such that \( B_c = 16 \) T, see table 1) were found to describe the experimental data well. The nonlinear resistivity of the bulk meanwhile was modelled using the modified \( E-J \) power law [31], with a temperature-dependent
Figure 1. Geometry and basic physical assumptions used in the model, illustrating the size of the (Y)-BaCuO bulk-pair, SUS316 stainless-steel rings and plates, and their positioning. Thermal contacts are assumed on all boundaries of the bulks, whilst other boundary conditions enforce axial symmetry, applied magnetic field, and heat flux.

Table 1. Table of thermomagnetic-coupled relationships employed within the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_c$</td>
<td>(Y)-BaCuO critical temperature</td>
<td>92 K</td>
</tr>
<tr>
<td>$J_c(B,T)$</td>
<td>Critical current density</td>
<td>$1 \times 10^{10}$ A m$^{-2}$</td>
</tr>
<tr>
<td>$J_{co}$</td>
<td>$J_c(B = 0 \text{ T})$ (equation 3)</td>
<td>$B_c \left( 1 - \left( \frac{T}{T_c} \right)^2 \right)^2$ T</td>
</tr>
<tr>
<td>$B_a(T)$</td>
<td>Material-dependent equation (equation 3)</td>
<td>$B_c \left( 1 - \left( \frac{T}{T_c} \right)^2 \right)^2$ T</td>
</tr>
<tr>
<td>$B_c$</td>
<td>Material-dependent constant (equation 3)</td>
<td>16 T</td>
</tr>
<tr>
<td>$t_c$</td>
<td>Time to cool bulk and ring</td>
<td>500 to 1500 s</td>
</tr>
<tr>
<td>$t_{fc} - t_c$</td>
<td>Time to remove applied field</td>
<td>120 to 220 min</td>
</tr>
<tr>
<td>$T_{amb}$</td>
<td>Ambient temperature</td>
<td>23 to 77 K</td>
</tr>
<tr>
<td>$B_{app}$</td>
<td>Applied magnetic flux density</td>
<td>15 to 22 T</td>
</tr>
<tr>
<td>$\rho_{nm}$</td>
<td>(Y)-BaCuO normal-state resistivity</td>
<td>$3.5 \times 10^{-6}$ Ω m</td>
</tr>
<tr>
<td>$E_o$</td>
<td>Characteristic voltage ($E-J$ power law)</td>
<td>$1 \times 10^{-4}$ V m$^{-1}$</td>
</tr>
<tr>
<td>$n(T)$</td>
<td>n-value ($E-J$ power law)</td>
<td>$30 \left( 1 - \left( \frac{T}{T_c} \right)^2 \right)^2 + 1.5$</td>
</tr>
</tbody>
</table>

The magnetic field and temperature constraints were applied following a field-cooled magnetisation procedure, shown in figure 2. The peak applied magnetic flux density, $B_{app}$, was applied as an initial condition and held constant until time $t_c$, during which the bulk was cooled to the chosen ambient temperature $T_{amb}$. The applied field ($B_a$, i.e. the value
of the field for \( t > t_c \) was then gradually decreased at a rate consistent with the experiment in [26], applied as a boundary condition from the outer periphery of the ‘air’ domain shown in figure 1. Additionally, axial symmetry was assumed along the line \( r = 0 \). All these described conditions and constraints were, and can be, readily implemented with the ‘MFH’ (magnetic field formulation) interface in the AC/DC module in COMSOL®.

### 2.2. Thermal considerations

As shown in figure 2, the initial temperature was set to 92 K, with the bulk-pair assembly permitted to cool via the ring outer radius (see figure 1). The value of \( t_c \) was dynamically adjusted to ensure the bulk cooled to the chosen ambient temperature prior to the removal of the field. The rate of flow of heat from the outer ring boundary is defined with Newton’s law of cooling, equation (4), where \( q_o \) is the rate of flow of heat energy per unit area, and \( A = 4 \text{ W m}^{-2} \text{ K}^{-1} \) is the heat transfer coefficient. Thermal contact boundary conditions, meanwhile, were assumed for all internal boundaries (between the plates, bulks and rings), with the contact conductances set between \( A_o = 4 \) to \( 12 \text{ W m}^{-2} \text{ K}^{-1} \) [36], i.e. higher at the bulk radius due to the highly compressed ‘shrink-fit’ ring, as these values were found to model the experimental data well.

\[
q_o = A(T - T_{\text{amb}}). \tag{4}
\]

The thermal transient equation, meanwhile, governs the flow of heat throughout the media modelled, and is given by equation (5):

\[
\rho \cdot C \frac{dT}{dt} - \nabla \cdot (\kappa \nabla T) = Q_s \tag{5}
\]

where \( \rho \) represents the material density, \( C \) the specific heat capacity, \( \kappa \) the thermal conductivity, and \( Q_s \) the total heating sources (see table 2). It should be noted the thermal conductivity in the \( ab \)-plane (\( \kappa_{ab} \)) was related to the \( c \)-axis thermal conductivity (\( \kappa_c \)) by the relation \( \kappa_c = \kappa_{ab}/3 \). The heating of the bulk by the removal of the field was calculated using equation (6), which further couples the electromagnetic and thermal components of the model (along with equation (3)):

\[
Q = E_{\varphi} \cdot J_{\varphi}. \tag{6}
\]

These conditions and constraints were, and can be, readily implemented with the ‘HT’ (heat transfer) module in COMSOL®. Finally, table 1 lists the various relationships assumed throughout the model, whilst table 2 lists the material thermal properties assumed.

### 2.3. Numerical procedure

Naito et al [26] utilised the FCM procedure experimentally, applying a magnetic flux density of \( B_{\text{app}} = 18 \text{ T} \) at temperature \( T_{\text{amb}} = 28 \text{ K} \), and additionally \( B_{\text{app}} = 15 \text{ T} \) at two temperatures of \( T_{\text{amb}} = 30 \) and 40 K, to achieve trapped fields in excess of 15 T between the bulk pair. During a final FCM procedure, with an ambient temperature of \( T_{\text{amb}} = 23 \text{ K} \) and \( B_{\text{app}} = 22 \text{ T} \), a flux jump occurred at approximately \( B_{\text{app}} = 6.9 \text{ T} \) causing the observed cracks in each bulk. Before and after the flux jump, two more FCM procedures were performed at \( T_{\text{amb}} = 77 \text{ K} \) in order to measure the surface distribution of magnetic flux density, and hence infer the sample homogeneity.

All of these described experimental procedures are numerically replicated and accurately represented herein, indicating that the behaviour of the bulk, and indeed the observed flux jump, have been accurately captured numerically. The
The remainder of this manuscript is subdivided into three main sections;

(a) The accurate replication of the experiment by Naito et al [26], over the full range of applied temperatures and magnetic fields, including the accurate modelling of the observed flux jump.

(b) Utilising the developed model from (a), an analysis of the evolution of the observed flux jump is presented with magnetic flux density and temperature distributions along the bulk cross-section.

(c) An extension study on how the effective cooling of the bulk-pair effects the calculated flux jumps, including an analysis of a newly proposed composite-bulk exploiting thin layers of copper with high thermal conductivity, and stacks of (Y)-BaCuO bulks.

### 3. Accurate modelling of the FCM experiments

#### 3.1. Bulk surface trapped field

Measuring the surface magnetic flux density of a bulk using FCM is an established method for examining the distribution and homogeneity of \( J_c \), and also inferring the interior condition for the presence of fractures or cracks [41]. Using this procedure, Naito et al [26] examined the bulks before and after the observed flux jump, showing the samples trapped similar fields with symmetric distributions on both sides of the bulk, implying a (near) spatially-invariant \( J_c \) distribution.

Repeating this procedure numerically with \( B_{app} = 1 \) T at \( T_{amb} = 77 \) K, figure 3(a) illustrates the calculated distribution of magnetic flux density across the top surface of the bulk. Meanwhile, figure 3(b) compares the numerically calculated and experimentally measured distribution of magnetic flux density by Naito et al [26], before the observed flux jump, showing excellent agreement. Figures 3(a) and (b) justify well the choice of a 2D-axisymmetric model given the \( J_c \) distribution is homogeneously distributed and appears to have a very small spatial-dependence.

Figure 8 in [26] shows the same procedure completed on the bulks after the observed flux jump, indicating that both bulks were fractured along the radius, close to the ‘A-plate’ surfaces (i.e. the \( ab \)-plane surfaces closest to \( z = 0 \) in figure 1). The suspected location of the cracks become important later in section 4, when the evolution of the flux jump is analysed.

#### 3.2. High-field FCM models

Next, the FCM procedures at \( T_{amb} = 28 \) K, 30 K, and 40 K are numerically replicated. Figure 4(a) shows the numerically calculated trapped magnetic flux density (i.e. \( B_{trap} \)) against bulk radius for \( B_{app} = 18 \) T (\( T_{amb} = 28 \) K) and \( B_{app} = 15 \) T (\( T_{amb} = 30 \) K, 40 K). The observed agreement between the experimentally-measured and numerically-calculated trapped field is excellent, with a minor over-prediction of the trapped field by the models towards the periphery of the bulk. This is likely due to the critical current density of the real bulk decreasing slightly towards the outer radius (as is expected in a typical (RE)-BaCuO bulk [22]). Additionally, the time-dependence of the temperature of the bulks during magnetisation were calculated for the same three FCM experiments described above, at the positions of T.P. 1 and T.P. 2 (see figure 1), shown in figure 4(a).

The position of T.P. 1 is inside the upper bulk, meanwhile T.P. 2 is located close to the outer surface of the lower ‘B-plate’. The calculated discrepancy in temperature becomes increasingly apparent later, and is due to the modelled preferential flow of thermal energy through the ring, and other thermal assumptions such as the contact conductances and thermal conductivities. A noticeable feature of figure 4(b) is that no flux jumps were calculated at these temperatures, applied fields, and ramp-rates (\( B_d \)) (i.e. flux jumps did not appear in these numerical calculations at any point).

Next, the FCM experiment with \( B_{app} = 22 \) T at \( T_{amb} = 23 \) K was numerically replicated (referred to as the ‘22 T–23 K’ model), with the accurate modelling of the experimentally observed flux jump, shown in figures 5(a) and (b). Numerically, a continuous \( B_d \) of \( \approx -0.1 \) T min\(^{-1} \) was assumed, with the same thermal and electromagnetic assumptions used in the previous FCM models. Figure 5(a) compares the numerically calculated magnetic flux density at locations H.P. 1 and H.P. 2 (solid-line plots), to the experimentally measured magnetic flux density (scatter plots), whilst the dashed grey line shows the ambient magnetic flux density (i.e. \( B_d \)). The numerical data agree well, showing slightly higher flux creep, prior to the flux jump occurring at \( B_d = 6.9 \) T, where the line-plots intersect with the dashed line. Naito et al [26] indicated that the Hall
Figure 3. (a) and (b) show the surface magnetic flux density, at 2.5 mm above the bulk surface, after FCM at 77 K and a 15 min wait period. The distribution is symmetric around the $z$-axis as expected due to the 2D-axisymmetric method. Experimental data from 3(b) corresponds to figure 3(a) in [26]. Reprinted from [26], with the permission of AIP Publishing.

Figure 4. In (a), the numerically-calculated (along $z=0$) trapped magnetic flux density is compared to the experimentally-obtained data (figure 4 in [26]), and in (b), the numerically calculated temperatures are given during the models of FCM at $T_{amb}=28$ K, 30 K, 40 K, with $B_{app}=15$ T and 18 T. The annotated values of $t_{fc}$ given are in minutes. Reprinted from [26], with the permission of AIP Publishing.

probes malfunctioned during the flux jump as they maintained an erroneous measurement of 5 T, even after heating to 100 K (i.e. above $T_c$). Additionally, the bulks cracked due to the flux jump, which is not accounted for in the models (and therefore why, numerically, the bulks continue to trap $\approx 6.9$ T after the flux jump).

Finally, figure 5(b) compares the experimentally-observed temperature of the bulk (measured from the lower ‘B-plate’) with the numerically-calculated temperature at locations T.P. 1 and T.P. 2 (where T.P. 2 is in a similar location to that measured experimentally). The discrepancies in measured temperature are evident, with a temperature of $\approx 80$ K measured at T.P. 1, $\approx 40$ K measured at T.P. 2, whilst the experimentally-measured temperature reaches $\approx 60$ K. This appears to support the claim by Naito et al that the measured temperature was likely not accurate, as the modelled preferential flow of thermal energy (towards the ring) and assumed thermal boundary conditions create thermal gradients across the bulk.

The high accuracy of the location of the flux jump was achieved with careful moderation of the values $A_o$ and $A$. 

Once the accuracy was achieved in the ‘22 T–23 K’ model, the values were kept constant for all other models shown (i.e. the values of $A_0$ and $A$ did not change with applied field or ambient temperature). Additionally, the value of $J_c$ was not altered to achieve the accuracy of the flux jump. This indicates that the balance of thermal energy throughout the bulk-pair assembly may be responsible for the observed thermomagnetic instabilities.

### 4. Evolution of thermomagnetic instabilities

#### 4.1. Distributions of temperature and magnetic field

Numerically, the distributions in temperature and magnetic flux density across the $r$-$z$ cross-section of the bulk pair can be analysed both before and after the flux jump, facilitating analysis of the thermomagnetic instability. During the ‘22 T–23 K’ model, the following time-frames around the flux jump are defined: $t_1 = -40$ s, $t_2 = -10$ s, $t_3 = -1.5$ s, $t_4 = -0.05$ s and $t_5 = +0.05$ s, $t_6 = +0.5$ s, $t_7 = +3$ s, $t_8 = +10$ s, where negative values of time are before the flux jump and positive values are after. Using these defined timeframes, figure 6 shows the evolution of the temperature distribution before the flux jump.

At timeframe $t_1$ in figure 6, the temperature is distributed near-uniformly, whilst at $t_2$, the hottest point of the distribution within the bulk (referred to as the ‘hot spot’), begins to form towards the line $z = 0$. At $t_3$, the temperature ‘hot spot’ is seen to migrate and concentrate further towards the point $z = 0$ and $r = 10$ mm (i.e. the outer radius) in both bulks. Finally, at $t_4$, the ‘hot spot’ has concentrated with approximately a 4 K difference in temperature at the points $z \approx \pm 2$ mm and $r = 10$ mm in each bulk. The modelled flux jump thus nucleates at these two points within the bulk pair, before propagating through the bulks. There is a minor calculated asymmetry in temperature between the two bulks, which causes the flux jump in the lower bulk to develop fractionally before the upper bulk, and the effect of which is made clear by figure 7, illustrating the bulk temperature distributions at $t_5–8$.

The distributions of temperature are more evidently asymmetric post-flux jump, showing a greater temperature in the upper bulk, by 2 K at $t_8$. At time $t_5$, the ‘hot-spot’ is distributed over a larger area in the upper bulk than in the lower bulk, which is also seen at $t_6$. At times $t_7$ and $t_8$, the bulk temperature is more uniformly distributed, with a temperature gradient between the two bulks. The bulks, and all thermal and electromagnetic conditions and constraints placed upon them, have been modelled symmetrically. Despite which, an asymmetric temperature distribution is calculated between the two bulks (see section 4.2).

Figure 8 illustrates the distribution in magnetic flux density across the bulk pair, at times $t_1$, $t_4$ and $t_8$. The distribution of magnetic flux density remains symmetric and near-constant in the times leading to the flux jump, as seen at $t_1$ and $t_4$. However, after the flux jump at $t_8$, the lower bulk is seen to retain a marginally greater magnetic flux density ($\approx 0.4$ T), indicating a greater movement of flux occurred through the upper bulk. The final subfigure in 8 shows the distribution of magnetic flux density with a scale normalised to those presented in $t_1$ and $t_4$, showing the resulting asymmetry is small compared to the overall change in flux density throughout the flux jump.
Figure 6. Distributions of temperature across the $r$–$z$ plane of the bulk pair, taken at times $t_{1-4}$ ($-40$ s, $-10$ s, $-1.5$ s, and $-0.05$ s, respectively), i.e. before the calculated flux jump, showing a concentration of temperature towards the line $z = 0$ and the outer radius in each bulk. Scales are normalised.

Figure 7. Distributions of temperature across the $r$–$z$ plane of the bulk pair, taken at times $t_{5-8}$ ($+0.05$ s, $+0.5$ s, $+3$ s, and $+10$ s, respectively), i.e. after the calculated flux jump, showing a large rise in temperature within the bulk and resulting asymmetric distributions (discussed in section 4.2). Scales are normalised.

Figure 8. Distributions of magnetic flux density across the $r$–$z$ plane of the bulk pair, taken at times $t_1$, $t_4$, and $t_8$ ($-40$ s, $-0.05$ s and $+10$ s, respectively) showing the change in magnetic flux density in each bulk both before and after the observed flux jumps. The final figure for $t_8$ is normalised to the same scale as $t_{1/4}$. 
Figure 9. The distribution of $J/J_c$ (i.e. the current density normalised to the critical current density) across the $r$–$z$ plane of the bulk-pair, taken at times $t_1$, $t_4$, and $t_8$ ($-40\,\text{s}$, $-0.05\,\text{s}$ and $+10\,\text{s}$, respectively) showing the evolution of this ratio in each bulk both before and after the observed flux jumps.

Finally, figure 9 shows the distribution of $J/J_c$ (i.e. the current density normalised to the critical current density) across the $r$–$z$ plane of the bulk-pair, and the evolution of this value before and after the flux jump. It can be seen at $t_1$ and $t_4$ that the large centrally trapped field has not changed since the beginning of the magnetisation procedure before the flux jump (i.e. $J/J_c$ is small). The peak value of $J/J_c$ is approximately 0.98 at $t_1$. Just before the flux jump, the ratio exceeds 1 and rapidly climbs throughout the flux jump (with the peak value of $J/J_c$ at $t_4$ approximately 1.14). Post-flux jump, the distributions show the same asymmetry observed in the earlier figures.

4.2. Discussion

Figures 6, 7 and 8 provide some insight into the experiment conducted by Naito et al. [26]. From figure 6, the ‘hot spot’ is seen to migrate towards the centre line of $z = 0$, and to the outer radius of both bulks. The outer radius of the bulk during FCM is where the value of $J_c(B, T)$ is highest (due to the lower local magnetic field in this region). Our results therefore reinforce the speculation of Naito et al. [26] regarding the location of the fracture, as the modelled results show the region of greatest temperature coincides with the observed location of the bulk-pair cracks (figure 8 of [26]). A possible reason for the observation of the fracture locations coinciding with the location of concentrating temperature is due to the known correlation of flux jumps with high-$J_c$ bulks (such as in those irradiated to form columnar pinning-centres [42, 43]). This is also congruent with the chosen orientation of the bulks by Naito et al. [26], placed with their ‘seed-surface’ facing the line $z = 0$, which is known to be the surface of greatest local $J_c$ values in typical (RE)-BaCuO bulks [22].

Additionally, the asymmetry observed in figures 6, 7 and 8 may be explained by inherent limitations of the finite-element method, and the highly nonlinear nature of the problem being solved. The presence of calculation errors can be reduced (such as through the use of high-density discretisation, or tighter error tolerances), but ultimately cannot be entirely removed as the numerical solution is a mathematical approximation of the solution to the underlying physical equations [44]. Due to the presence of small and accumulated errors, there is a fractionally asynchronous timing of the flux jump (in each bulk), which develops over the entire progression of the model. However, this is merely one possible answer, and there may be some underlying mechanism causing this effect that the authors are unaware of. The asymmetry may be a direct physical result of flux jumps developing in close proximity (Naito et al. [26] did observe that the crack in the lower bulk appeared to be larger than in the upper bulk). It should be noted that this is not a rigorous examination, and merely a point of curiosity presented for the interest of the scientific community.

To summarise, it was found that the flux jump can ‘initiate’ in either of the two bulks (and was found to with equal probability from 50 repetitions of the model), with the same result obtained regardless. That is, explicitly, the same magnetic field and temperature distributions are calculated at the same time, such that mirroring the distributions of figures 6, 7 and 8 along the line $z = 0$ accurately represents the situation of the flux jump initiating in the top bulk first. Further, it should be once again emphasised that whilst this may warrant further investigation into the effect, it should not distract from the excellent modelled agreement with the experimental results of the flux jump shown throughout.
5. Extension study

The following extension study examines the effect of enhanced cooling, through modifying the values $A$ and $A_o$ (which correspond to the heat transfer coefficients of the cryogenic system and the stainless-steel, respectively) by factors of between 0.4 and 2.2, and also through investigating a proposed composite structure of stacked (Y)-BaCuO bulks and high-conductivity metallic layers, which have been shown to reliably trap high-magnetic fields [45].

5.1. Influence of enhanced cooling

First, the values of $A$ and $A_o$ (representing how effectively the bulk-pair assembly cools from the cryogenic system, $A$, and internally at boundaries within the bulk-pair assembly, $A_o$) which were used in the ‘22 T–23 K’ model to accurately describe the experimentally observed flux jump are varied to investigate the effect of enhanced or reduced cooling to the bulk-pair assembly. In figure 10 the effect of modifying the values of $A$ and $A_o$ (written singularly as $A_t$, where values of $A_t$ represent the ratio from the initial values of $A$ and $A_o$) by the same factor of between 0.6 and 2.0, is shown against the calculated temperature of the bulk-pair assembly (at T.P. 1; see figure 1). Increasing $A_t$, monotonically increases both the time delay in the calculated flux jump, and the resulting increase in maximum temperature (dashed orange line). It can also be seen that there is no flux jump for the $A_t = 2$ dataset, as the bulk-pair is seen to be ‘critically cooled’, i.e. initially enhanced cooling increases the magnitude and maximum temperature of the resulting flux jump, up to a critical value of $A_t = 2$.

Figure 11 shows the effect of reducing the value of $A_t$ further, to $A_t = 0.4$, resulting in the calculation of two flux jumps whilst the applied field is removed during the FCM process. The rise in temperature from the flux jump is seemingly correlated to the magnitude of the motion of the magnetic flux density during the flux jump (approximately 12.5 T for the second flux jump). Meanwhile, the series where $A_t = 2.2$ is also seen to be ‘critically cooled’, as would be expected, where no flux jumps were calculated with a negligible rise in temperature during the model.

Finally, figure 12 illustrates both the final trapped magnetic flux density ($B_{trap}$) and the maximum temperature rise during the model, against the value of $A_t$. Interestingly, the resulting trapped field appears to near-linearly decrease with effective cooling (due to the delaying of the flux jump, and thus increase in magnitude). This is true up to a value of $A_t = 2$ where, as indicated in figure 12, the bulk pair becomes ‘critically cooled’ (indicated by the shaded region), and with no calculated flux jumps, a reversal of the relationship is observed (i.e. increasing values of $A_t$ reduce maximum temperature and increase trapped field). A useful conclusion from figures 10, 11 and 12, which may be applied by researchers experimentally, is that the cooling architecture must be improved to realise the results of the $A_t = 2$ and $A_t = 2.2$ series (i.e. a ‘critically cooled’ bulk). The presented model describes a perfectly homogeneous material where thermal flow is not impeded by microstructure or internal weak-links, enabling the entire bulk to be effectively cooled through cooling power applied only at the bulk-pair radius. Experimentally however, researchers may wish to devise methods of improving thermal flow internally within the bulk (such as impregnation of a high-thermal conductivity fluid, e.g. an epoxy dispersed with copper powder, or by milling holes in the bulk [46, 47], and subsequently filling them with a thermally conductive material, which has been found to improve heat dissipation and trapped field performance [48, 49]). It should be noted here that increasing the ramp-time ($t_{rb}$) should reduce the heating rate, and thus also delay the onset of the flux jump. However, this investigation focussed on improving the thermal
stability of the bulk, as experimental constraints may exist preventing easy ramp-rate modification (such as during experiments with very high-field magnets such as these, or during pulsed-field magnetisation).

5.2. Composite bulk of stacked (Y)-BaCuO and copper layers

The results of Huang et al [45] show that it is possible to reliably trap large magnetic fields with stacks of (Gd)-BaCuO-Ag bulks, interspaced by thin steel discs, and it was shown numerically this decreases the thermal and hoop stress applied to the bulks. Here, the influence of thermodynamic instabilities on composite, stacked (RE)-BaCuO bulks are analysed, and the resulting thermal behaviour is compared to the non-composite (or unmodified bulk-pair) case. The purpose of this investigation is to seek the number of copper layers at which the composite bulk-pair is stable against flux jumps, during a quasi-static, high-field magnetisation process. An improved cooling architecture, such as with layers of copper in a composite MgB$_2$ [50], or (RE)-BaCuO bulk [51], and with full-encapsulation of the sample [52], has been shown to improve trapped field performance (albeit under pulsed-field magnetisation, where heating is a prominent issue). This is also one of the merits of high-temperature superconducting coated-conductor tape stacks [53, 54], which have been proposed as an alternative to bulk superconductors, and can typically be magnetised with high fields at lower temperatures than bulks.

Figure 13 shows the geometry of the modelled composite bulks, using the parameter $L$, which denotes the number of (Y)-BaCuO bulk ‘layers’ within each bulk in the bulk-pair, interspaced by thin (0.5 mm) layers of high thermal conductivity copper (where $L - 1$ indicates the copper layer number). Unlike in the work of Huang et al [45], the volume of superconducting material is kept constant and thus the height of the bulk-pair assembly increases with the addition of layers...
of copper. Figure 13 shows the modelled composite bulks schematically, with different values of the parameter $L$. The added copper layers (black) can be seen interspaced between layers of (Y)-BaCuO bulk (orange), where the upper right hand side of the bulk only is shown, such that $z, r = 0$ are lines of symmetry.

Varying the value of $L$ between 2 and 5 (see figure 13), the influence this new composite geometry has on the calculation of the flux jump is investigated. The thermal properties and assumptions of the bulk-pair assembly were kept constant, including the heat flux applied to the SUS316 ring outer radius (i.e. using $A_c = 1$, as per section 3). The copper layers, however, were assumed to have a thermal conductivity (from an experimental source, see table 2) much larger than the SUS316 components at these temperatures. Thus, using the same assumptions as the ‘22 T–23 K’ model, presented below are the results of adding incrementally more layers of copper to the composite stack.

Figure 14(a) shows the modelled temperature of the composite-bulk stack, calculated at the location of T.P. 1, with increasing (Y)-BaCuO layer number, $L$. The value of $L = 1$ therefore refers to the unmodified bulk-pair configuration, with no added copper layers. It can be seen in figure 14(a) that increasing values of $L$ (and hence the number of copper layers) delays the onset of the flux jump, until the value of $L = 4$ and above, where flux jumps are no longer observed. This should be expected, due to the increased effective cooling, as shown earlier by increasing the factor of $A_c$. Contrasting the results of figure 10 however, despite the increased magnitude of the flux jump, the maximum temperature of the flux jump decreases with increasing values of $L$ (can be seen by the difference between the circle and square dotted scatter points). Figure 14(b) shows similarly to figure 12 the region of flux jump stability for $L \geq 4$, where a decrease in temperature is observed approaching the stability limit, despite the increase in flux jump magnitude. From figure 14, it is clear that the presence of thinner (Y)-BaCuO bulks, and interspersed copper layers, inhibits the maximum temperature rise of the bulk and further suppresses flux jumps past a critical number of copper layers.

Exploring this effect further, figure 15 illustrates the distribution of temperature at time $t_8$ (i.e. 10 seconds after the observed flux jump) in the composite bulk-pair with $L = 3$, and the unmodified bulk-pair case, $L = 1$. In the latter case, the change in magnetic flux density at the flux jump (calculated at H.P. 1) is $\approx 13$ T (see figure 5(a)), whilst for the composite bulk ($L = 3$), the change in magnetic flux density is $\approx 16$ T. However, in figure 15, the temperature is more uniformly distributed (with a range of $\approx 1$ K for the composite bulk), and at a lower temperature than the noncomposite bulk 10 s after the flux jump. By subdividing the bulk, it is seen from $L = 3$ in figure 15, that individual stacks have different but lower temperature distributions throughout the flux jump due to the presence of the metallic layers. This is likely why the maximum temperature was seen to decrease despite the increased change in magnetic flux density during the flux jump (opposite to that seen in figure 10).

It has been shown numerically in this section that the use of thermally-conductive layers makes the bulk more stable to thermomagnetic instabilities (which may also be combined with mechanical reinforcement against high electromagnetic stress). It would be prudent for researchers to investigate this thoroughly, as it is likely we can reduce the typical operating temperature of high-field bulk magnetisation ($\approx 30$ K), with a real possibility of surpassing existing trapped field records, towards 20 T and beyond.
Figure 15. Comparison of the temperature distribution at time $t_8$ (i.e. 10 seconds after the calculated flux jump) in the composite bulk with $L = 3$, to the noncomposite (single bulk, $L = 1$). The scale has been normalised for both figures.

6. Conclusions

Despite the promise of bulk (RE)-BaCuO materials to act as permanent trapped field magnet analogues, challenges remain in trapping a useful, high magnetic field within them (due to the presence of thermomagnetic instabilities, and the risk of mechanical failure under large electromagnetic stresses). Here, we attempt to address some of these issues, and accurately model the experimental investigation by Naito et al [26]. The full range of temperatures and applied fields were modelled, including the accurate numerical replication of the flux jump observed during the ‘22 T–23 K’ FCM experiment. Using the flexibility of numerical methods, the observed flux jump during the ‘22 T–23 K’ FCM experiment was closely analysed by examining the distributions of temperature and magnetic flux density across the bulk-pair assembly cross-section. With this analysis, a possible explanation for the location of the experimentally-observed fractures was provided, by examining the generation of heat prior to the flux jump. Next, the experimentally-validated model was used to explore the role of thermal energy in the bulk-pair assembly, showing that incrementally enhanced cooling delays the onset of the calculated flux jump, and thus increases the maximum temperature. It was shown that past a certain cooling power ($A_c$), the bulk-pair assembly could become ‘critically cooled’ and was stable to flux jumps. Finally, a composite bulk consisting of a stack of (Y)-BaCuO bulks and interspaced copper layers was shown to change the way thermal energy is distributed throughout the bulk during the flux jump, and provide enhanced cooling to the bulk assembly, preventing flux jumps when three or more layers of copper were added. The results provide experimental researchers with a useful method to improve the trapped field performance of bulk high-temperature superconductors when magnetised in quasi-static, high magnetic fields. The flexible numerical modelling framework presented here provides a cost-effective tool for predicting and analysing flux jumps, as well as mitigating them through advanced design and optimisation.

Data availability statement

The data that support the findings of this study are openly available at the following URL/DOI: www.doi.org/10.18742/21215801.

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