DOI:
10.1109/TCSII.2024.3378280

Document Version
Peer reviewed version

Fuzzy Observer-based Command Filtered Adaptive Control of Flexible Joint Robots with Time-varying Output Constraints

Junhao Su, Hak-Keung Lam, *Fellow, IEEE*, Jiapeng Liu, Jinpeng Yu , *Senior Member, IEEE*

***Abstract*—Flexible joint robots (FJR) systems are used in many aspects of actual production due to its high compliance, low energy consumption, human-computer interaction safety and other characteristics. A fuzzy observer-based command filtered adaptive control method is applied to make FJR systems with time-varying output constraints (TVOC) and model uncertainties operate safely in a complex environment in this paper. Chiefly, a fuzzy observer is developed to estimate the link's angle velocity and motor angle velocity of the FJR. Next, by combining time-varying barrier Lyapunov function (TVBLF) with fuzzy logic systems, the uncertainties of the FJR model are approximated without violating the TVOC. Besides, the command filtered method with error compensation signal resolves the issue of "explosion of complexity" and removes the impacts of filtering errors. The stability of the FJR system is verified by Lyapunov stability theory. Simulation shows that the devised approach can insure the TVOC, the validity of the observer and position tracking accuracy of the system.**

***Index Terms*—Fuzzy observer, Adaptive control, Flexible-joint robot, Time-varying output constraint, Command filtered.**

I. Introduction

FLEXIBLE-joint robots (FJR) are widely used in medical and health, aerospace, education industry, life services and so on, on account of its light weight, high compliance, low energy consumption, human-computer interaction safety and other characteristics [1]. At the same time, the FJR system is a high-order nonlinear system with strong coupling, which limits the control performance. Therefore, the control design problems for FJR is of essential importance, and becoming a research hot spot. There are a number of studies on the tracking control of FJR systems, such as singular perturbation method [2], sliding mode control [3]-[4], variable structure control [5], feedback linearization method [6], backstepping control [7]-[9] and so on.

The combination of adaptive backstepping control and fuzzy approximation theory is used in the tracking control of FJR extensively. However, the traditional backstepping method used in [7]-[9] has the problem of "explosion of complexity (EOC)", which increases the computation burden and leads to limitations in the application of the backstepping control of FJR. To solve EOC, the dynamic surface control (DSC) method [10], [15] has been presented, but the filtering errors in the DSC will affect the tracking performance. The command filtered control (CFC) method with the error compensation mechanism (ECM) has been presented in [11]-[13] to remove the impacts of filtering errors. Combined with adaptive control, a new CFC method has been expanded into nonlinear multi-agent systems by Zhao *et al*. [11]. Considering the tracking control of the FJR, CFC method has been introduced in [12]-[13]. However, the above results did not consider the complex and changeable work environment of FJR.

The working environment of FJR is usually complicated and volatile, which may cause safety risks to the robot itself, and even personal. As a result, the FJR system's TVOC must be ensured. The TVBLF has been used to fulfill the output constraints [14]. [15] designed an output-feedback-based predefined time full-state error constrained controller. The combine of a tangent barrier Lyapunov-Krasovskii function and DSC has been used in [19]. In addition, it is arduous to obtain the velocity terms of FJR. Observer technique [15]-[16] solves the problem and has been applied to FJRs [16]. Considering the factors of highly nonlinear and the velocity terms that is difficult to measure in FRJ system with complex work environment, the fuzzy observer-based CFC with TVOC in FJR system is worth discussing.

A fuzzy observer-based CFC for FJR with TVOC is studied. The proposed scheme has the following advantages.

(1) Compared with [12], [13], [17], the combine of TVBLF and Lyapunov stability theory are used for the n-link FJR system to make it suitable for complex working environments and ensure the boundness of all signals, which . In [12]-[13], the problem of output constraints has not been considered. In [17], the time-varying nature of the constraints has not been taken into account.

(2) Compared with [7]-[10], the CFC method with ECM is used to solve the problem of EOC caused by backstepping as well as remove the effect of filtering errors to obtain better tracking effect of the n-link FJR system.

(3) Fuzzy state observer technique is used to solve the problem that the velocity terms of n-link FJR are difficult to measure, which can improve the reliability.

This work was supported in part by the National Natural Science Foundation of China under Grant 61973179, in part by the Chang Jiang Scholars Program, in part by the Major Innovation Project of Shandong Province under Grant 2022CXGC020901, in part by the Taishan Scholar Special Project Fund under Grant TSTP20221120, and in part by the Qingdao Key Technology Research and Industrialization Demonstration Project under Grant 23-1-4-xxgg-6-gx. *(Corresponding author: Jinpeng Yu.)*

Junhao Su, Jiapeng Liu and Jinpeng Yu are with the School of Automation, Qingdao University, Qingdao 266071, China (e-mail: 13296360412@163.com; ljpqdu1990@qdu.edu.cn; yjp1109@hotmail.com;).

Hak-Keung Lam is with the Department of Engineering, King's College London, Strand, London, WC2R 2LS, U.K. (e-mail: hak-keung.lam@kcl.ac.uk).

II. Mathematical Model and Preliminaries

A. FJR system description

Considering the following dynamic model of n-link FJR:

$$\begin{cases} M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + Kq = Kq_m \\ J\ddot{q_m} + B\dot{q_m} + K(q_m - q) = u \\ y = q \end{cases} \tag{1}$$

where the definitions and values of q, $\dot{q}$, q_m, $\dot{q_m}$, $M(q) \in R^{n\times n}$, $C(q,\dot{q}) \in R^{n\times n}$, $G(q) \in R^n$, $F(\dot{q}) \in R^n$, $K \in R^{n\times n}, J \in R^{n\times n}$, $B \in R^{n\times n}$ and $u \in R^n$ can be found in [13]. Suppose that B contains uncertainty.

Let $\varsigma_1 = q = [\varsigma_{1,1}, \varsigma_{1,2}, ..., \varsigma_{1,n}]^T$, $\varsigma_2 = \dot{q} = [\varsigma_{2,1}, \varsigma_{2,2}, ..., \varsigma_{2,n}]^T$, $\varsigma_3 = q_m = [\varsigma_{3,1}, \varsigma_{3,2}, ..., \varsigma_{3,n}]^T$, $\varsigma_4 = \dot{q_m} = [\varsigma_{4,1}, \varsigma_{4,2}, ..., \varsigma_{4,n}]^T$, (1) is rewritten as

$$\begin{cases} \dot{\varsigma}_1 = \varsigma_2 \\ \dot{\varsigma}_2 = h_2\varsigma_3 + f_2 \\ \dot{\varsigma}_3 = \varsigma_4 \\ \dot{\varsigma}_4 = h_4 u + f_4 \\ y = \varsigma_1 \end{cases} \tag{2}$$

where $h_2 = KM^{-1}(\varsigma_1)$, $h_4 = J^{-1}$, $f_2 = M^{-1}(\varsigma_1)$ $(-C(\varsigma_1,\varsigma_2)\varsigma_2 - G(\varsigma_1) - K\varsigma_1 - F(\varsigma_2))$, $f_4 = J^{-1}(K(\varsigma_1 - \varsigma_3) - B\varsigma_4)$

Remark 1. The FJR system is a 4-order nonlinear system with uncertain terms, which brings some difficulties when controlling it, meanwhile, measuring the velocity terms of an operating FJR system, dealing with the uncertainties and the problem of TVOC without affecting the tracking effect brings challenges when designing the controller.

B. Preliminaries

Assumption 1. The uncertain term $F(\dot{q})$ is continuous and bounded.

Assumption 2. The desired trajectory $y_{d,i}$ and its first derivative $\dot{y}_{d,i}$ are continuous and bounded with $|y_{d,i}| \le Y_0 < k_{c,i}$, $|\dot{y}_{d,i}| \le Y_1$, where Y_0 and Y_1 are positive constants.

Assumption 3. [15] Based on Lipschitz conditions, for a functions $f_i(\Im)$ and constants $m_i > 0, i = 1, 2..., n$, the inequality holds $\left|f_i(\Im) - f_i(\hat{\Im})\right| \le m_i||\Im - \hat{\Im}||$, where $\hat{\Im}$ is the estimated value of $\Im$, and $||\cdot||$ denotes the 2-norm.

Lemma 1. [11] The command filter is defined as follows:

$$\begin{cases} \dot{\kappa}_{i,1} = \omega_n \kappa_{i,2} \\ \dot{\kappa}_{i,2} = -2\zeta\omega_n\kappa_{i,2} - \omega_n(\kappa_{i,1} - \alpha_i) \end{cases} \tag{3}$$

where α_i is the input of the filter and $\kappa_{i,1}(0) = \alpha_i(0), \kappa_{i,2}(0) = 0$, if $\varpi_1 > 0$, $\varpi_2 > 0$ satisfy $|\dot{\alpha}_i| \le \varpi_1$ and $|\ddot{\alpha}_i| \le \varpi_2$ for all $t \ge 0$, then for any $\varrho > 0$, existing $\omega_n > 0$ and $\zeta \in (0, 1]$, so that $|\kappa_i - \alpha_i| \le \varrho$, $|\dot{\kappa}_i|$ and $|\ddot{\kappa}_i|$ are bounded.

Lemma 2. [14] Defining a continuous function $f(\Phi)$ on the set Ω_Φ, for $\forall\varepsilon > 0$, it always has a fuzzy logic system $W^T S(\Phi)$ such that

$$f(\Phi) = W^T S(\Phi) + \sigma(\Phi) \tag{4}$$

where $\sigma(\Phi)$ is the approximation error and $|\sigma(\Phi)| \le \varepsilon$.

III. Design of the fuzzy observer

In order to obtain the link's angle velocity and motor angle velocity of the running FJR, the fuzzy observer is designed as follows:

$$\begin{cases} \dot{\hat{\varsigma}}_1 = \hat{\varsigma}_2 + \bar{d}_1(y - \hat{y}) \\ \dot{\hat{\varsigma}}_2 = h_2\varsigma_3 + \hat{f}(\hat{\varsigma}_2|\hat{\vartheta}_2) + \bar{d}_2(y - \hat{y}) \\ \dot{\hat{\varsigma}}_3 = \hat{\varsigma}_4 + \bar{d}_3(\varsigma_3 - \hat{\varsigma}_3) \\ \dot{\hat{\varsigma}}_4 = h_4 u + \hat{f}(\hat{\varsigma}_4|\hat{\vartheta}_4) + \bar{d}_4(\varsigma_3 - \hat{\varsigma}_3) \\ \hat{y} = \hat{\varsigma}_1 \end{cases} \tag{5}$$

where $\hat{\varsigma} = [\hat{\varsigma}_1, \hat{\varsigma}_2, \hat{\varsigma}_3, \hat{\varsigma}_4]^T$ is the estimate of ς, then, the observer is rewritten as

$$\begin{cases} \dot{\hat{\varsigma}} = A\hat{\varsigma} + Dy + \Gamma\varsigma_3 + Hu + \beta \\ \hat{y} = C\hat{\varsigma} \end{cases} \tag{6}$$

where $A = [-\bar{d}_1 \ -\bar{d}_2 \ 0 \ 0;\ 1 \ 0 \ 0 \ 0;\ 0 \ 0 \ -\bar{d}_3 \ -\bar{d}_4;\ 0 \ 0 \ 1 \ 0]$, $D = [\bar{d}_1, \bar{d}_2, 0, 0]^T, \Gamma = [0, h_2, \bar{d}_3, \bar{d}_4]^T, H = [0, 0, 0, h_4]^T, \beta = [0, \hat{f}(\hat{\varsigma}_2|\hat{\vartheta}_2), 0, \hat{f}(\hat{\varsigma}_4|\hat{\vartheta}_4)]^T$, and $C = [1, 0, 0, 0]$.

Define $\hat{f}_i(\hat{\bar{\varsigma}}_i|\vartheta_i^*) = \vartheta_i^{*T}\varphi_i(\hat{\bar{\varsigma}}_i)$, $\hat{f}_i(\hat{\bar{\varsigma}}_i|\hat{\vartheta}_i) = \hat{\vartheta}_i^T\varphi_i(\hat{\bar{\varsigma}}_i)$, where $i = 2, 4$, $\hat{\bar{\varsigma}}_i = [\hat{\varsigma}_1, \hat{\varsigma}_2, ..., \hat{\varsigma}_i]^T$, $\tilde{\vartheta}_i = \vartheta_i^* - \hat{\vartheta}_i$, ϑ_i^* is the optimal parameter, $\tilde{\sigma}_i = f_i(\hat{\bar{\varsigma}}_i) - \hat{f}_i(\hat{\bar{\varsigma}}_i|\hat{\vartheta}_i)$, where $|\tilde{\sigma}_i| \le \sigma_0$, and σ_0 is a positive constant. Define $e = [e_1, e_2, e_3, e_4]^T$ as observer error vector, $e_j = \varsigma_j - \hat{\varsigma}_j, j = 1, 2, 3, 4$. The error expression of the observer is as follows:

$$\dot{e} = Ae + \Xi + \tilde{\sigma} \tag{7}$$

where $\Xi = [0, \ f_2(\bar{\varsigma}_i) - f_2(\hat{\bar{\varsigma}}_2), \ 0, \ f_4(\bar{\varsigma}_i) - f_2(\hat{\bar{\varsigma}}_4)]^T$, $\tilde{\sigma} = [0, \ f_2(\hat{\bar{\varsigma}}_2) - \hat{f}_2(\hat{\bar{\varsigma}}_2|\hat{\vartheta}_2), \ 0, \ f_4(\hat{\bar{\varsigma}}_4) - \hat{f}_4(\hat{\bar{\varsigma}}_4|\hat{\vartheta}_4)]^T$.

Based on Assumption 3, $\left|f_i(\bar{\varsigma}_i) - f_i(\hat{\bar{\varsigma}}_i)\right| \le \bar{m}_i \left\|\bar{\varsigma}_i - \hat{\bar{\varsigma}}_i\right\| = \bar{m}_i \|e\|, (i = 2, 4)$. *And* $\tilde{\sigma} = [0, \tilde{\sigma}_2, 0, \tilde{\sigma}_4]^T, (\tilde{\sigma})^2 \le 2\sigma_0^2$. A is a standard Hurwitz matrix by choosing the right d_1, d_2, d_3 and d_4. Given a matrix $\check{Q}^T = \check{Q} > 0$, there exists $\check{P}^T = \check{P} > 0$ such that $A^T\check{P} + \check{P}A = -\check{Q}$.

Choosing the Lyapunov function $V_0 = e^{\mathrm{T}}\check{P}e$, and then

$$\dot{V}_0 = \dot{e}^T\check{P}e + e^{\mathrm{T}}\check{P}\dot{e} = -e^T\check{Q}e + 2e^T\check{P}(\Xi + \tilde{\sigma}) \tag{8}$$

Using Young's inequality,

$$2e^{\mathrm{T}}\check{P}\Xi \le \eta_0^2\|e\|^2 + \frac{1}{\eta_0^2}\left(m_2^2 + m_4^2\right)\left\|\check{P}\right\|^2\|e\|^2 \tag{9}$$

$$2e^{\mathrm{T}}\check{P}\tilde{\sigma} \le \eta_0^2\|e\|^2 + \frac{2}{\eta_0^2}\left\|\check{P}\right\|^2\sigma_0^2 \tag{10}$$

where η_0 is a positive constant. Let $\check{P}_0 = \lambda_{min}(\check{Q}) - 2\eta_0^2 - \frac{1}{\eta_0^2}\left(m_2^2 + m_4^2\right)\left\|\check{P}\right\|^2 > 0$, where $\lambda_{min}(\check{Q})$ is the minimum eigenvalue of $\check{Q}$,then

$$\dot{V}_0 \le -\check{P}_0\|e\|^2 + R_0 \tag{11}$$

where $R_0 = \frac{2}{\eta_0^2}\left\|\check{P}\right\|^2\sigma_0^2$.

IV. Controller design and Stability analysis

The following error variables are defined as: $z_1 = \varsigma_1 - y_d$, $z_v = \hat{\varsigma}_v - \varsigma_{v-1,c}$, $v = 2,3,4$, $\varkappa_j = z_j - \xi_j$, $j = 1,2,3,4$, where $\varsigma_{v-1,c} = [\varsigma_{v-1,c,1}, \varsigma_{v-1,c,2}, ..., \varsigma_{v-1,c,n}]^T$ is the output of the filter. ξ_j is the error compensation signal, $\varkappa_j = [\varkappa_{j,1}, \varkappa_{j,2}, ..., \varkappa_{j,n}]^T$, $\xi_j = [\xi_{j,1}, \xi_{j,2}, ..., \xi_{j,n}]^T$. Then, choosing the control laws and the adaptive laws as

$$\alpha_1 = -k_1 z_1 + \dot{y}_d - \frac{1}{2} z_1 + S_1, \tag{12}$$

$$\begin{aligned}\alpha_2 =&(h_2)^{-1}(-k_2 z_2 - \frac{1}{2} z_2 - \hat{\vartheta}_2^T \varphi_2(\hat{\varsigma}_2) \\ &+ \dot{\varsigma}_{1,c} - \bar{d}_2 e_1 - \eth),\end{aligned} \tag{13}$$

$$\alpha_3 = -k_3 z_3 - \frac{1}{2} z_3 + \dot{\varsigma}_{2,c} - h_2 z_2, \tag{14}$$

$$\begin{aligned}u =&(h_4)^{-1}(-k_4 z_4 - z_3 - \frac{1}{2} z_4 - \hat{\vartheta}_4^T \varphi_4(\hat{\varsigma}_4) \\ &+ \dot{\varsigma}_{3,c} - \bar{d}_4 e_3),\end{aligned} \tag{15}$$

$$\dot{\hat{\vartheta}}_2 = -n_2 \hat{\vartheta}_2 + \lambda_2 \varkappa_2 \varphi(\hat{\varsigma}_2), \tag{16}$$

$$\dot{\hat{\vartheta}}_4 = -n_4 \hat{\vartheta}_4 + \lambda_4 \varkappa_\varphi(\hat{\varsigma}_4) \tag{17}$$

where $S_1 = \frac{k_b^T I_i \dot{k}_b}{k_b^T I_i k_b - v_1^T I_i v_1} v_1$, $\eth = [\frac{\varkappa_{1,1}}{k_{b,1}^2 - \varkappa_{1,1}^2}, ..., \frac{\varkappa_{1,n}}{k_{b,n}^2 - \varkappa_{1,n}^2}]^T$. $\lambda_i (i = 2,4)$, $n_i (i = 2,4)$, k_1, k_2, k_3 and k_4 are designed as positive constants.

Error compensation signals are designed as

$$\begin{cases} \dot{\xi}_1 = -k_1 \xi_1 + \xi_2 + \varsigma_{1,c} - \alpha_1 - \xi_1, \\ \dot{\xi}_2 = -k_2 \xi_2 - \xi_1 + h_2(\xi_3 + \varsigma_{2,c} - \alpha_2) - \frac{1}{2}\xi_2, \\ \dot{\xi}_3 = -k_3 \xi_3 + \xi_4 + \varsigma_{3,c} - \alpha_3 - h_2 \xi_2 - \frac{1}{2}\xi_3, \\ \dot{\xi}_4 = -k_4 \xi_4 - \xi_3 - \frac{1}{2}\xi_4 \end{cases} \tag{18}$$

Step 1: Choosing a TVBLF as $V_1 = V_0 + \sum_{i=1}^n \frac{1}{2} log \frac{k_{b,i}^2(t)}{k_{b,i}^2(t) - \varkappa_{1,i}^2(t)}$, where $k_b = [k_{b,1}, k_{b,2}, ..., k_{b,n}]^T$ is the constraints of $\varkappa_1$, $|\varkappa_{1,i}(t)| < k_{b,i}(t)$, it follows that

$$\begin{aligned}\dot{V}_1 =&\dot{V}_0 + \sum_{i=1}^n \frac{\varkappa_{1,i}}{k_{b,i}^2 - \varkappa_{1,i}^2}(\varkappa_{2,i} + \xi_{2,i} + \varkappa_{1,c,i} \\ &+ e_{2,i} - \varsigma_{d,i} - \xi_{1,i} - S_1)\end{aligned} \tag{19}$$

Exploring the Young's inequality gives

$$\varkappa_1^T e_2 \leq \frac{1}{2} \varkappa_1^T \varkappa_1 + \frac{1}{2} \|e\|^2. \tag{20}$$

By combining (12), (18)-(20), one has

$$\dot{V}_1 \leq \dot{V}_0 - \sum_{i=1}^n k_{1,i} (\frac{\varkappa_{1,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2}) + \sum_{i=1}^n \frac{\varkappa_{1,i} \varkappa_{2,i}}{k_{b,i}^2 - \varkappa_{1,i}^2} + \frac{1}{2} \|e\|^2. \tag{21}$$

Remark 2. Choosing log-type TVBLF in Step 1, the logarithmic function term has been eliminated when deducing this step, it is easier to combine with the ordinary Lyapunov function in Steps 2-4 to facilitate the following stability analysis.

Step 2: Choosing the Lyapunov function (LF) as $V_2 = V_1 + \frac{1}{2} \varkappa_2^T \varkappa_2 + \frac{1}{2\lambda_2} \tilde{\vartheta}_2^T \tilde{\vartheta}_2$, where $\lambda_2 > 0$, $\tilde{\vartheta}_2 = \hat{\vartheta}_2 - \vartheta_2$, $\hat{\vartheta}_2$ is the estimate of ϑ_2, then, it obtains

$$\begin{aligned}\dot{V}_2 \leq&\dot{V}_0 - \sum_{i=1}^n k_{1,i} (\frac{\varkappa_{1,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2}) + \sum_{i=1}^n \frac{\varkappa_{1,i} \varkappa_{2,i}}{k_{b,i}^2 - \varkappa_{1,i}^2} \\ &+ \varkappa_2^T (h_2 \varsigma_3 + \hat{\vartheta}_2^T \varphi(\hat{\varsigma}_2) + \bar{d}_2 e_1 - \dot{\varsigma}_{1,c} - \dot{\xi}_2) \\ &+ \frac{1}{\lambda_2} \tilde{\vartheta}_2^T (\lambda_2 \varkappa_2 \varphi(\hat{\varsigma}_2) - \dot{\hat{\vartheta}}_2) - \varkappa_2 \tilde{\vartheta}_2^T \varphi(\hat{\varsigma}_2) + \frac{1}{2} \|e\|^2.\end{aligned} \tag{22}$$

Using Young's inequality, we have

$$-\varkappa_2 \tilde{\vartheta}_2^T \varphi(\hat{\varsigma}_2) \leq \frac{1}{2} \varkappa_2^T \varkappa_2 + \frac{1}{2} \tilde{\vartheta}_2^T \tilde{\vartheta}_2. \tag{23}$$

Putting (13), (16), (18) and (23) into (22), the following inequalities hold:

$$\begin{aligned}\dot{V}_2 \leq&\dot{V}_0 - \sum_{i=1}^n k_{1,i} (\frac{\varkappa_{1,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2}) - k_2 \varkappa_2^T \varkappa_2 + h_2 \varkappa_3^T \varkappa_2 \\ &+ \Delta_2 + \frac{1}{2} \|e\|^2\end{aligned} \tag{24}$$

where $\Delta_j = \frac{n_j}{\lambda_j} \tilde{\vartheta}_j^T \hat{\vartheta}_2 + \frac{1}{2} \tilde{\vartheta}_j^T \tilde{\vartheta}_j$, $(j = 2,4)$.

Step 3: Selecting the LF as $V_3 = V_2 + \frac{1}{2} \varkappa_3^T \varkappa_3$, then, it obtains

$$\begin{aligned}\dot{V}_3 =&\dot{V}_2 + \varkappa_3^T (\dot{\varsigma}_3 - \dot{\varsigma}_{2,c} - \dot{\xi}_3) \\ \leq&\dot{V}_0 - \sum_{i=1}^n k_{1,i} (\frac{\varkappa_{1,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2}) - k_2 \varkappa_2^T \varkappa_2 + \varkappa_3^T (h_2 \varkappa_2 \\ &+ \varkappa_4 + \xi_4 + \varsigma_{3,c} + e_4 - \dot{\varsigma}_{2,c} - \dot{\xi}_3) + \Delta_2 + \frac{1}{2} \|e\|^2.\end{aligned} \tag{25}$$

Using Young's inequality, it obtains

$$\varkappa_3^T e_4 \leq \frac{1}{2} \varkappa_3^T \varkappa_3 + \frac{1}{2} \|e\|^2. \tag{26}$$

Then, substituting (14), (18) and (26) into (25) results in

$$\begin{aligned}\dot{V}_3 \leq&\dot{V}_0 - \sum_{i=1}^n k_{1,i} (\frac{\varkappa_{1,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2}) - k_2 \varkappa_2^T \varkappa_2 - k_3 \varkappa_3^T \varkappa_3 \\ &+ \varkappa_4^T \varkappa_3 + \Delta_2 + \|e\|^2.\end{aligned} \tag{27}$$

Step 4: Selecting the LF as $V_4 = V_3 + \frac{1}{2} \varkappa_4^T \varkappa_4 + \frac{1}{2\lambda_4} \tilde{\vartheta}_4^T \tilde{\vartheta}_4$, where $\lambda_4 > 0$, $\tilde{\vartheta}_4 = \hat{\vartheta}_4 - \vartheta_4$, $\hat{\vartheta}_4$ is the estimate of ϑ_4, then

$$\begin{aligned}\dot{V}_4 =&\dot{V}_3 + \varkappa_4^T (\dot{\varsigma}_4 - \dot{\varsigma}_{3,c} - \dot{\xi}_4) \\ \leq&\dot{V}_0 - \sum_{i=1}^n k_{1,i} (\frac{\varkappa_{1,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2}) - \sum_{i=2}^3 k_i \varkappa_i^T \varkappa_i + \Delta_2 + \|e\|^2 \\ &+ \varkappa_4^T (h_4 u + \varkappa_3 + \hat{\vartheta}_4^T \varphi(\hat{\varsigma}_4) + \bar{d}_4 e_3 - \dot{\varsigma}_{3,c} - \dot{\xi}_4) \\ &- \varkappa_4 \tilde{\vartheta}_4^T \varphi(\hat{\varsigma}_4) + \frac{1}{\lambda_4} \tilde{\vartheta}_4^T (\lambda_4 \varkappa_4 \varphi(\hat{\varsigma}_4) - \dot{\hat{\vartheta}}_4).\end{aligned} \tag{28}$$

Exploring the Young's inequality gives

$$-\varkappa_4 \tilde{\vartheta}_4^T \varphi(\hat{\varsigma}_4) \leq \frac{1}{2} \varkappa_4^T \varkappa_4 + \frac{1}{2} \tilde{\vartheta}_4^T \tilde{\vartheta}_4. \tag{29}$$

Then, substituting (15), (17), (18) and (29) into (28), we have

$$\dot{V}_4 \leq \dot{V}_0 - \sum_{i=1}^n k_{1,i} (\frac{\varkappa_{1,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2}) - \sum_{i=2}^4 k_i \varkappa_i^T \varkappa_i + \Delta_2 + \Delta_4 + \|e\|^2. \tag{30}$$

Remark 3. The fuzzy observer used in this paper can be applied together with the controller, and the adaptive law designed can be applied together in the two parts to better deal with the uncertainties in the system, while the high-gain observer [20], the finite-time observer [21] are independent of the controller.

Theorem 1. Based on Assumption 1, the system (1), command filter (3), control law (12)-(17) and compensation signal (18) are taken into account. If $|\varsigma_{1,i}(0)|<k_{c,i}(0)$, then

(1) $\varkappa_{j,i}$ is bounded and $z_{j,i}$ can converge to a small neighborhood close to the origin by choosing the parameters appropriately.

(2) All signals are bounded in the FJR system .

(3) The output variables are constrained in the presupposed space: $\varsigma_{1,i}(t)<k_{c,i}(t)$.

Stability analysis : Letting $V = V_4$, from (30), we can get

$$\dot{V} \leq \dot{V}_0 - \sum_{i=1}^{n} k_{1,i}\left(\frac{\varkappa_{1,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2}\right) - \sum_{i=2}^{4} k_i \varkappa_i^T \varkappa_i + \Delta_2 + \Delta_4 + \|e\|^2 \tag{31}$$

It follows from Young's inequality that

$$\frac{n_2}{\lambda_2}\tilde{\vartheta}_2^T\hat{\vartheta}_2 \leq -\frac{n_2}{2\lambda_2}\tilde{\vartheta}_2^T\tilde{\vartheta}_2 + \frac{n_2}{2\lambda_2}\vartheta_2^T\vartheta_2 \tag{32}$$

$$\frac{n_4}{\lambda_4}\tilde{\vartheta}_4^T\hat{\vartheta}_4 \leq -\frac{n_4}{2\lambda_4}\tilde{\vartheta}_4^T\tilde{\vartheta}_4 + \frac{n_4}{2\lambda_4}\vartheta_4^T\vartheta_4. \tag{33}$$

By [18], when $|\varkappa_{1,i}| \leq k_{b,i}$,

$$\log\frac{k_{b,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2} < \frac{\varkappa_{1,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2} \tag{34}$$

then,

$$\dot{V} \leq -\bar{a}V + \bar{b} \tag{35}$$

where $\bar{a} = \min\{2k_1, 2k_2, 2k_3, 2k_4, \frac{\check{P}_0+1}{\lambda_{\max}(\check{P})}, 2\lambda_2(\frac{n_2}{2\lambda_2} - \frac{1}{2}), 2\lambda_4(\frac{n_4}{2\lambda_4} - \frac{1}{2})\}$, $\bar{b} = \frac{n_2}{2\lambda_2}{\vartheta_2}^T\vartheta_2 + \frac{n_4}{2\lambda_4}{\vartheta_4}^T\vartheta_4 + R_0$. For $\bar{a} > 0, \bar{b} > 0$, the selection of control parameters should satisfy $\lambda_{\min}(\check{Q}) - 3 - \left(m_2^2 + m_4^2\right)\left\|\check{P}\right\|^2 > 0$, $\frac{n_2}{2\lambda_2} - \frac{1}{2} > 0$, $\frac{n_4}{2\lambda_4} - \frac{1}{2} > 0$.

Multiply both sides of equation (35) by $e^{\bar{a}t}$ and integrate:

$$\begin{aligned} V(t) \leq & (V(t_0) - \frac{\bar{b}}{\bar{a}})e^{-\bar{a}(t-t_0)} \\ \leq & V(t_0) + \frac{\bar{b}}{\bar{a}}, (\forall t \geq t_0) \end{aligned} \tag{36}$$

then, (36) becomes $|\varkappa_{1,i}| \leq k_{b,i}\sqrt{1 - e^{-2V(0) - \frac{2\bar{b}}{\bar{a}}}}, |\varkappa_{2,i}| \leq \sqrt{\frac{2\bar{b}}{\bar{a}}}, |\varkappa_{3,i}| \leq \sqrt{\frac{2\bar{b}}{\bar{a}}}, |\varkappa_{4,i}| \leq \sqrt{\frac{2\bar{b}}{\bar{a}}}$, since $z_{j,i} = \varkappa_{j,i} + \xi_{j,i}$ and according to literature [16], $\lim_{t\to\infty}\|\xi_i\| \leq \frac{\varpi\varrho}{2k_0}$, where $k_0 = \frac{1}{2}\min_i(k_i)$, then,$z_{1,i} \leq k_{b,i}\sqrt{1 - e^{-2V(0) - \frac{2\bar{b}}{\bar{a}}}} + \frac{\varpi\varrho}{2k_0}$. Based on (34), $log\frac{k_{b,i}^2}{k_{b,i}^2 - \varkappa_{1,i}^2}$ is bounded. Furthermore, $\varkappa_{1,i} = z_{1,i} - \xi_{1,i}, \varkappa_{1,i}<k_{b,i}$, one can get that $|z_{1,i}| \leq |\varkappa_{1,i}| + |\xi_{1,i}| \leq k_{b,i} + |\xi_{1,i}|$. Based on $z_{1,i} = \varsigma_{1,i} - y_{d,i}$ and $|y_{d,i}| \leq Y_0, |\varsigma_{1,i}|<k_{b,1,i} + |\xi_{1,i}| + Y_0 = k_{c,1,i}$, so $\varsigma_{1,c,i}$ is bounded. Supposing $\varsigma_{1,c,i}$ satisfies $|\varsigma_{1,c,i}| \leq \bar{\varsigma}_{1,c,i}$, where $\bar{\varsigma}_{1,c,i} > 0$, furthermore, because $z_{2,i} = \varkappa_{2,i} + \xi_{2,i}$, it has $|z_{2,i}| \leq |\varkappa_{2,i}| + |\xi_{2,i}| \leq \sqrt{\frac{2\bar{b}}{\bar{a}}} + \frac{\varpi\varrho}{2k_0}$. Due to $z_{2,i} = \varsigma_{2,i} - e_2 - \varsigma_{1,c,i}$, then $|\varsigma_{2,i}| \leq \sqrt{\frac{2\bar{b}}{\bar{a}}} + \frac{\varpi\varrho}{2k_0} + \bar{\varsigma}_{1,c,i} + |e|$. Similarly, $|\varsigma_{3,i}| \leq \sqrt{\frac{2\bar{b}}{\bar{a}}} + \frac{\varpi\varrho}{2k_0} + \bar{\varsigma}_{2,c,i}$, $|\varsigma_{4,i}| \leq \sqrt{\frac{2\bar{b}}{\bar{a}}} + \frac{\varpi\varrho}{2k_0} + \bar{\varsigma}_{3,c,i} + |e|$.

In summary, we can get that all variables of the FJR are bound and Theorem 1 can be demonstrated.

V. Simulation

To show the advantages of the presented method, simulation analysis is verified by MATLAB/Simulink on a 2-link FJR system whose matrices and parameters in model (1) are given below: $M(q) = [M_{11}\ M_{12};\ M_{21}\ M_{22}]$, $C(q,\dot{q}) = [C_{11}\ C_{12};\ C_{21}\ 0]$, $G(q) = [G_1,\ G_2]^T$, $J = diag[1,\ 1]$, $K = diag[2,\ 2]$.

where $M_{11} = I_1 + m_1L_{c1}^2 + m_2l_1^2 + I_2 + m_2L_{c2}^2 + 2m_2l_1L_{c2}(\cos(\varsigma_{12}))$, $M_{12} = M_{21} = I_2 + m_2L_{c2}^2 + m_2l_1L_{c2}(\cos(\varsigma_{12}))$, $M_{22} = I_2 + m_2L_{c2}^2$, $C_{11} = -m_2l_1L_{c2}(\sin(\varsigma_{12}))\varsigma_{22}$, $C_{12} = -m_2l_1L_{c2}(\sin(\varsigma_{12}))(\varsigma_{21} + \varsigma_{22})$, $C_{21} = -m_2l_1L_{c2}(\sin(\varsigma_{12}))\varsigma_{21}$, $G_1 = -(m_1 + m_2)gl_1(\sin(\varsigma_{11}))$, $G_2 = -m_2gl_2(\sin(\varsigma_{12}))$.

TABLE I: Parameters of the two-link FJR

Contents	Description	Values	Unit
l_1	Length of link 1	0.4	m
l_2	Length of link 2	0.4	m
L_{c1}	Centroid distance about link 1	0.2	m
L_{c2}	Centroid distance about link 2	0.2	m
m_1	Mass about link 1	1.2	kg
m_2	Mass about link 2	1	kg
I_1	Inertia of link 1	0.048	kg·m^2
I_2	Inertia of link 2	0.04	kg·m^2
g	Acceleration of gravity	9.8	m/s^2

The system parameters are given in TABLE I. The desired trajectories $y_d = [0.5\sin(2t); 0.4\sin(2t) + 0.2\sin(t)]$, the initial state are $\varsigma_1(0) = \varsigma_3(0) = \hat{\varsigma}_1(0) = \hat{\varsigma}_3(0) = [0,0]^T$, $\varsigma_2(0) = \hat{\varsigma}_2(0) = [1,1]^T$, $\varsigma_4(0) = [0.8, 1.2]^T$, $\hat{\varsigma}_4(0) = [1.2, 0.8]^T$, the uncertainties are defined as $F(\dot{q}) = [0.01\cos(\varsigma_{21}); 0.01\cos(\varsigma_{21})], B =$diag$[0.1, 0.1]$.

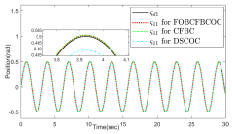
1)The fuzzy observer-based command filtering backstepping controllers with output constraint (FOBCFBCOC) are constructed. Select the design parameters as $k_1 = k_2 = k_3 = k_4 = 10, \lambda_2 = \lambda_4 = 0.5, n_2 = n_4 = 10, k_b = [0.2 + 0.1\sin(t); 0.2 + 0.1\sin(t)], \bar{d}_1 = 40, \bar{d}_2 = 80000, \bar{d}_3 = 500, \bar{d}_4 = 80000, \omega_n = 1100, \zeta = 0.8$.

2) The CFBC (command filtering backstepping controller) without output constraint will be designed as a comparison. The control parameters are the same as 1).

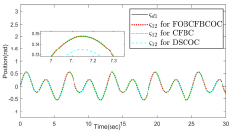
3) The DSCOC (dynamic surface controllers with output constraint) without ECM will be introduced as a comparison. The control parameters are the same as 1), $\zeta = 0.05$.

Remark 4. From the simulation, the larger k_i and $\bar{d}_i$ are, the better the tracking effect and estimated effect are, but the more energy will be consumed. Considering the above situation comprehensively, the control parameters are selected by trial and error method.

As can be seen from Figs.1-2, contrasting CFBC method and DSCOC method, the proposed control algorithm has a more outstanding tracking effect and can get smaller tracking errors. From Figs.3-4 that the TVOC bound is not been violated by the output y, the method proposed can estimate

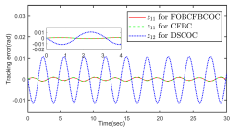


(a) the 1-th of the model

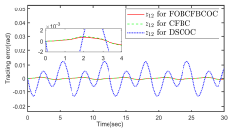


(b) the 2-th of the model

Fig. 1: Output signal y.

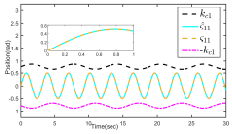


(a) the 1-th link

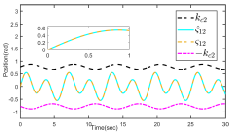


(b) the 2-th link

Fig. 2: Tracking error z_1.

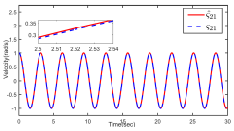


(a)the 1-th link

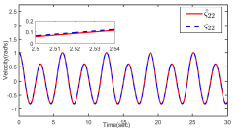


(b) the 2-th link

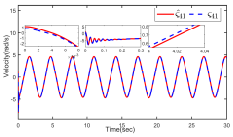
Fig. 3: Trajectories of ς_1 and $\hat{\varsigma}_1$ of the two links.



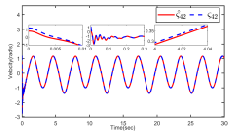
(a1)ς_2 and $\hat{\varsigma}_2$ of 1-th link



(b1)ς_2 and $\hat{\varsigma}_2$ of 2-th link

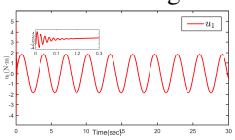


(a2)ς_4 and $\hat{\varsigma}_4$ of 1-th link

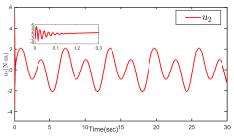


(b2)ς_4 and $\hat{\varsigma}_4$ of 2-th link

Fig. 4: Estimation effect.



(a)the 1-th link



(b)the 2-th link

Fig. 5: Control input u.

the velocities of FJR effectively and the estimation errors are small, which proves that the designed fuzzy observer has a good estimation effect. Fig.5 displays that the control input is smooth and bounded.

VI. Conclusion

For FJR system with TVOC, a fuzzy observer-based CFC method has been proposed in the article. A fuzzy state observer has been projected to effectively estimate the link's angle velocity and motor angle velocity of FJR, as well as resolve the uncertainties of the system. The combination of CFC method and ECM solves the problem of EOC in the process of the controller design and eliminates the influence of filtering errors. In addition, the output has been constrained in the preset area. In the end, the devised approach has been validated by simulation results. In the future research, the full-state time-varying asymmetric constrains will be considered.

References

[1] M. W. Spong, "Adaptive control of flexible-joint manipulators," *Automatica*, vol. 31, no. 4, pp. 585-590, 1995.

[2] T. Hao, H. Wang, F. Xu, J. Wang, and Y. Miao, " Uncalibrated visual servoing for a planar two link rigid-flexible manipulator without joint-space-velocity measurement," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 52, no. 3, pp. 1935–1947, Mar. 2022.

[3] H. Wang, W. Peng, X. Tan, J. Sun, X. Tang, and I. -M. Chen, " Robust output feedback tracking control for flexible-joint robots based on CTSMC technique," *IEEE Trans. Circuits Syst. II-Exp. Briefs*, vol. 68, no. 6, pp. 1982–1986, Jun. 2021.

[4] Khan, K. Rsetam, Z. Cao, and Z. Man, " Singular perturbation-based adaptive integral sliding mode control for flexible joint robots," *IEEE Trans. Ind. Electron.*, vol. 70, no. 10, pp. 10516–10525, Oct. 2023.

[5] D. Shang, X. Li, M. Yin, and F. Li, " Dynamic modeling and fuzzy compensation sliding mode control for flexible manipulator servo system," *Appl. Math. Model.*, vol. 170, pp. 530–556, Jun. 2022.

[6] G. Rigatos and M. Abbaszadeh, " Nonlinear optimal control for multi-DOF robotic manipulators with flexible joints," *Optimal Control Appl. Methods*, vol. 42, pp. 1708–1733, Nov. 2021.

[7] L. Wang, Q. Shi, J. Liu, and D. Zhang, " Backstepping control of flexible joint manipulator based on hyperbolic tangent function with control input and rate constraints," *Asian J. Control*, vol. 22, no. 3, pp. 1268–1279, May. 2020.

[8] R. Datouo, J. J. B. M. Ahanda, A. Melingui, F. Biya-Motto, and B. E. Zobo, " Adaptive fuzzy finite-time command-filtered back-stepping control of flexible-joint robots," *Robotica*, vol. 39, no. 6, pp. 1081–1100, Jun. 2021.

[9] S. Y. Dian, Y. Hu, T. Zhao, and J. X. Han, " Adaptive backstepping control for flexible-joint manipulator using interval type-2 fuzzy neural network approximator," *Nonlinear Dyn.*, vol. 97, no. 2, pp. 1567–1580, Jul. 2019.

[10] S. Sui, C. L. P. Chen, and S. Tong, " A novel full errors fixed-time control for constraint nonlinear systems," *IEEE Trans. Autom. Control*, vol. 68, no. 4, pp. 2568–2575, Apr. 2023.

[11] L. Zhao, J. Yu, and C. Lin, " Distributed adaptive output consensus tracking of nonlinear multi-agent systems via state observer and command filtered backstepping," *Inf. Sci.*, vol. 478, no. 4, pp. 355-374, 2019.

[12] W. Chang, Y. Li, and S. Tong, " Adaptive fuzzy backstepping tracking control for flexible robotic manipulator," *IEEE/CAA J. Autom. Sinica*, vol. 8, no. 12, pp. 1923–1930, 2021.

[13] S. Ling, H. Q. Wang, and P. X. Liu, " Adaptive fuzzy tracking control of flexible-joint robots based on command filtering," *IEEE Trans. Ind. Electron.*, vol. 67, no. 5, pp. 4046-4055, 2020.

[14] J. Yu, L. Zhao, H. Yu, and C. Lin, " Barrier Lyapunov functions-based command filtered output feedback control for full-state constrained nonlinear systems," *Automatica*, vol. 105, pp. 71-79, 2019.

[15] H. Xu, D. Yu and Y. -J. Liu, " Observer-based fuzzy adaptive predefined time control for uncertain nonlinear systems with full-state error constraints," *IEEE Trans. Fuzzy Syst.*, early access, Oct. 13, 2023, doi: 10.1109/TFUZZ.2023.3321669.

[16] H. Ma, H. Ren, Q. Zhou, H. Li and Z. Wang, " Observer-Based Neural Control of N -Link Flexible-Joint Robots," " *IEEE Trans. Neural Netw. Learn. Syst.*, early access, Sep. 2022.

[17] Z. Zhang, Y. Song, and K. Zhao, " Neuroadaptive cooperative control without velocity measurement for multiple humanoid robots under full-state constraints," *IEEE Trans. Ind. Electron.*, vol. 66, no. 4, pp. 2956–2964, Apr. 2019.

[18] C. Si, Q. Wang, G. Cui and J. Yu, " Event-triggered adaptive output feedback control for stochastic nonlinear systems with time-varying full-state constraints," *IEEE Trans. Circuits Syst. II-Exp. Briefs*, vol. 70, no. 1, pp. 251-255, 2023.

[19] K. Zhao, Y. Song, and Z. Shen, "Neuroadaptive fault-tolerant control of nonlinear systems under output constraints and actuation faults," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 2, pp. 286–298, Feb. 2018.

[20] D. M. Jeong, Y. H. Choi, and S. J. Yoo, " Adaptive output-feedback control of a class of nonlinear systems with unknown sensor sensitivity and its experiment for flexible-joint robots," *J. Elect. Eng. Technol.*, vol. 15, no. 2, pp. 907-918, 2020.

[21] D. Wu, M. Chen, and H. Gong, " Robust control of post-stall pitching maneuver based on finite-time observer," *ISA Trans.*, vol. 70, pp. 53–63, 2017.