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Model Reduction of Discrete-time Interval Type-2 T-S Fuzzy Systems

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Abstract—This paper is concerned with the model reduction problem of discrete-time interval type-2 Takaga-Sugeno (T-S) fuzzy systems which represent the discrete-time nonlinear systems subject to uncertainty. With the use of interval type-2 fuzzy sets, the uncertainty of the discrete-time nonlinear system can be captured by the lower and upper membership functions. For a given high-order discrete-time interval type-2 T-S fuzzy system, the purpose is to find a lower dimensional system to approximate the original system. To achieve the approximation performance, an \mathcal{H}_∞ norm is used to suppress the error between the original system and its simplified system. By introducing a membership-functions-dependent technique and applying a convex linearization method, a membership-functions-dependent condition, which takes the information of membership functions into account, is obtained to reduce the dimensions of system matrices and the number of fuzzy rules of the system. All the obtained theorems are represented as in the form of linear matrix inequalities (LMIs). Finally, simulation results are demonstrated to show the effectiveness of the derived results.

Index Terms—model reduction, discrete-time interval type-2 Takagi-Sugeno (T-S) fuzzy system, convex linearization method, membership-functions-dependent technique.

I. INTRODUCTION

NONLINEARITY is ubiquitous in various industrial systems such as chemical industry systems [1], aerospace systems [2] and electric systems [3]. In support of analyzing and controlling of nonlinear systems, researchers introduced T-S fuzzy model (type-1) to represent the system dynamics by Takaga and Sugeno [4] which broadens the applications of fuzzy control theory and techniques. Since then, many studies on fuzzy control are based on this model such as in [5]–[9] for stability analysis and in [10] for distributed fuzzy optimal control law using adaptive dynamic programming algorithm. However, with the need for modeling accuracy, uncertainty of the system has also become one of the factors to be considered. Due to the difficulty in representing uncertainty when using type-1 fuzzy model, type-2 or interval type-2 (IT2) fuzzy model draws the attention with its capability of capturing uncertainty using type-2 or IT2 fuzzy sets [11]–[13]. Type-2 fuzzy sets were first proposed by Zadeh in 1975 [14]. After that various studies on type-2 fuzzy-model-based (FMB) control have been carried out (e.g. [15], [16]). In 2000, Liang

and Mendel proposed IT2 fuzzy logic systems [17] which use lower and upper membership functions to represent the uncertainty in realistic system. Since then, the study of type-2 fuzzy logic systems has been a promising research topic. For example, in the context of interval type-2 fuzzy-model-based (IT2FMB) control, Lam *et al.* introduced an IT2 fuzzy model for nonlinear system subject to uncertainty and studied the stability analysis and control synthesis of IT2FMB control systems in [18]–[20]. O Castillo *et al.* studied the type-2 fuzzy logic controller in [21]–[23]. Sanchez *et al.* studied IT2 fuzzy sets based information granule formation in [24] and Li *et al.* studied the sliding mode control and filter design for IT2 fuzzy systems in [25], [26]. In addition to the research on control theory, type-2 fuzzy control systems have also been widely studied in industrial applications in recent years (e.g. [27]–[31]). It is worth mentioning that Bustince *et al.* presented a wide view on the relationship between interval-valued fuzzy sets and IT2 fuzzy sets in [32], in which they pointed out that interval-valued fuzzy sets are a special case of the IT2 fuzzy sets, while Mendel *et al.* mentioned that the methods or systems in the early published work about IT2 fuzzy sets are only valid when the IT2 fuzzy sets are equivalent to interval-valued fuzzy sets in [33].

Besides the nonlinearity and uncertainty of the system, another issue in control engineering is that the mathematical models established are mostly complex high-order models which are very difficult to analyze and synthesize. Model reduction is introduced to deal with such problem, which aims to find a simplified reduced-order model to approximate the original complex high-order model. Model reduction has been widely used in many engineering fields such as power systems [34], filters design [35], [36], circuit simulations [37], [38]. Besides, in the past few decades, many methods have been introduced for solving the problem of model reduction such as Hankel norm based methods [39], [40], \mathcal{H}_∞ norm based methods [41], [42], \mathcal{H}_2 norm based methods [43], and \mathcal{H}_2 - \mathcal{H}_∞ based methods [44]. In addition, some other methods such as balanced truncation approach [45], [46] and frequency response matching [47] have also been proposed. For model reduction of type-1 FMB systems, there have been some research results reported in recent years. For example, Wu *et al.* studied the Hankel norm model reduction and \mathcal{H}_∞ model reduction for time-delay T-S fuzzy system [48], [49], and Su *et al.* obtained the approach of model reduction for T-S fuzzy stochastic systems [50] and T-S fuzzy switched systems [51]. However, in numerous studies of model reduction for T-S fuzzy systems, the reduced-order systems are considered as linear systems (for example [50]) or fuzzy systems with the

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same membership functions as original systems (for example [51]), which demonstrate limitation in choosing reduced-order systems. Besides, Type-1 T-S fuzzy models show difficulty in representing system uncertainty.

In consideration of the superiority of type-2 fuzzy systems in expressing uncertainties, showing a higher degree of stability, and better performance in optimization problems [21], [52], type-2 fuzzy system can be widely used in various industrial systems (e.g. [27], [28]). To the best of the authors' knowledge, for model reduction of IT2 T-S fuzzy systems, there are only some preliminary studies such as [53]. Besides, the existing work seldom consider that the simplification of the number of fuzzy rules, which limits the design flexibility of the reduced model. Therefore, the research of model reduction of IT2 T-S fuzzy systems is of significance in theoretical research and practical applications, which motivates this research.

In this work, the problem of model reduction for discrete-time IT2 fuzzy systems is investigated. The objective is to find a simplified discrete-time IT2 fuzzy system such that the error system is asymptotically stable with an \mathcal{H}_∞ error performance. Based on convex linearization procedure, the condition of \mathcal{H}_∞ model reduction is represented as a form of linear matrix inequalities (LMIs). In order to further relax the result, a membership-functions-dependent technique is applied. The method proposed in this paper can reduce the order of system matrices and number of membership functions of original systems which have more flexibility in choosing reduced-order systems than [48]–[51]. Besides, unlike [53], the method proposed in this paper designs the membership functions of the reduced-order model independent of the state variables of the original system, which is more reasonable in practical applications.

The rest of this paper is organized as follows. Section II shows the mathematical description of the discrete-time IT2 T-S fuzzy systems and some definitions. Section III shows the main results of this paper. Section IV demonstrates two numerical examples to show the effectiveness of the analysis results. Section V concludes the article.

Notations: Some of the superscripts or symbols used in this paper are defined as follows. “ T ” stands for matrix transposition. “ $P > 0$ ” or “ $P \geq 0$ ” stand for matrix “ P ” being a symmetric positive matrix or positive semi-definite matrix. “ $*$ ” stand for a symmetric term in a symmetric matrix. “diag(...)” stands for a diagonal matrix whose diagonal elements are in the bracket. “ $\|\cdot\|$ ” stands for Euclidean norm. “ $\ell_2[0, \infty)$ ” stands for signals that are square integrable over $[0, \infty)$ with the norm $\|\cdot\|_2$. $\min(\dots)$ and $\max(\dots)$ stand for minimum and maximum.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this section, some preliminaries are presented, which help define the problem to be investigated.

A. Original System Model

In this paper, we consider the following discrete-time IT2 T-S fuzzy model (1) that represents a nonlinear system subject to uncertainty with r rules:

Plant Rule i : IF $f_1(x(k))$ is \tilde{M}_1^i AND ... AND $f_p(x(k))$ is \tilde{M}_p^i THEN:

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k), \\ y(k) &= C_i x(k) + D_i u(k), \end{aligned} \quad (1)$$

where $f_\alpha(x(k))$ is the premise variable and \tilde{M}_α^i is an IT2 fuzzy set, $\alpha = 1, 2, \dots, p$; $i = 1, 2, \dots, r$; p is a positive integer; $x(k) \in \mathbb{R}^n$ is the system state vector; $y(k) \in \mathbb{R}^l$ is the output vector; $u(k) \in \mathbb{R}^q$ is the system input vector; A_i , B_i , C_i and D_i are the known system matrix, input matrix, output matrix and feedforward matrix with appropriate dimensions, respectively; $k = 1, 2, \dots$ is the discrete time instant. The firing strength of the i th fuzzy rule which is represented by interval sets is shown as follows:

$$W_i(x(k)) = [\underline{w}_i(x(k)), \bar{w}_i(x(k))], i = 1, 2, \dots, r, \quad (2)$$

where $\underline{w}_i(x(k)) = \prod_{\alpha=1}^p \underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(x(k))) \geq 0$, $\bar{w}_i(x(k)) = \prod_{\alpha=1}^p \bar{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(x(k))) \geq 0$, $\underline{w}_i(x(k))$ and $\bar{w}_i(x(k))$ are the lower and upper grades of membership, respectively. $\underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(x(k))) \geq 0$ and $\bar{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(x(k))) \geq 0$ are the lower and upper membership functions, respectively. Also, $\bar{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(x(k))) \geq \underline{\mu}_{\tilde{M}_\alpha^i}(f_\alpha(x(k)))$ and $\bar{w}_i(x(k)) \geq \underline{w}_i(x(k))$ for all i . Then, we can describe discrete-time IT2 T-S fuzzy systems with a more compact form:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^r \tilde{w}_i(x(k)) [A_i x(k) + B_i u(k)], \\ y(k) &= \sum_{i=1}^r \tilde{w}_i(x(k)) [C_i x(k) + D_i u(k)], \end{aligned} \quad (3)$$

where

$$\begin{aligned} \tilde{w}_i(x(k)) &= \frac{a_i(x(k)) \underline{w}_i(x(k)) + \bar{a}_i(x(k)) \bar{w}_i(x(k))}{\sum_{\bar{k}=1}^r (a_{\bar{k}}(x(k)) \underline{w}_{\bar{k}}(x(k)) + \bar{a}_{\bar{k}}(x(k)) \bar{w}_{\bar{k}}(x(k)))}, \\ \forall i, \sum_{i=1}^r \tilde{w}_i(x(k)) &= 1. \end{aligned}$$

The nonlinear functions $a_i(x(k))$ and $\bar{a}_i(x(k))$ satisfy: 1) $0 \leq a_i(x(k)) \leq 1$ and $0 \leq \bar{a}_i(x(k)) \leq 1$. 2) $a_i(x(k)) + \bar{a}_i(x(k)) = 1$.

Remark 1: In order to avoid the conceptual confusion, in view of [32] and [33], we consider a special case when interval-valued fuzzy sets are equivalent to IT2 fuzzy sets. The systems studied in this paper can also be called interval-valued fuzzy systems.

B. Reduced-order Model

Our aim is to approximate the original system (3) by a low-order discrete-time IT2 T-S fuzzy system, which is described by the rules of following format:

Rule j : IF $g_1(\hat{x}(k))$ is \tilde{N}_1^j AND ... AND $g_l(\hat{x}(k))$ is \tilde{N}_l^j , THEN:

$$\begin{aligned} \hat{x}(k+1) &= \hat{A}_j \hat{x}(k) + \hat{B}_j u(k), \\ \hat{y}(k) &= \hat{C}_j \hat{x}(k) + \hat{D}_j u(k), \end{aligned} \quad (4)$$

where $g_\beta(\hat{x}(k))$ is the premise variable and \tilde{N}_β^j is an IT2 fuzzy set, $j = 1, 2, \dots, c$; $\beta = 1, 2, \dots, l$; l is a positive integer; $\hat{x}(k) \in \mathbb{R}^m$ is the state vector of the simplified system in which $m < n$; $\hat{y} \in \mathbb{R}^{\bar{t}}$ is the output of the simplified system, \bar{t} is a positive integer; \hat{A}_j , \hat{B}_j , \hat{C}_j and \hat{D}_j are system matrix, input matrix, output matrix and feedforward matrix, respectively, which have appropriate dimensions and to be determined. The firing strength of j th fuzzy rule which is represented by interval sets is shown as follows:

$$M_j(\hat{x}(k)) = [\underline{m}_j(\hat{x}(k)), \overline{m}_j(\hat{x}(k))], j = 1, 2, \dots, c \quad (5)$$

where $\underline{m}_j(\hat{x}(k)) = \prod_{\beta=1}^l \underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\hat{x}(k))) \geq 0$, $\overline{m}_j(\hat{x}(k)) = \prod_{\beta=1}^l \overline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\hat{x}(k))) \geq 0$, $\underline{m}_j(\hat{x}(k))$ and $\overline{m}_j(\hat{x}(k))$ are the lower and upper grades of membership, respectively. $\underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\hat{x}(k))) \geq 0$ and $\overline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\hat{x}(k))) \geq 0$ are the lower and upper membership functions, respectively. From $\overline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\hat{x}(k))) \geq \underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(\hat{x}(k)))$, we can obtain $\overline{m}_j(\hat{x}(k)) \geq \underline{m}_j(\hat{x}(k))$. Then, we can describe the simplified discrete-time IT2 T-S fuzzy systems with a more compact form:

$$\begin{aligned} \hat{x}(k+1) &= \sum_{i=1}^c \tilde{m}_j(\hat{x}(k)) [\hat{A}_j \hat{x}(k) + \hat{B}_j u(k)], \\ \hat{y}(k) &= \sum_{j=1}^c \tilde{m}_j(\hat{x}(k)) [\hat{C}_j \hat{x}(k) + \hat{D}_j u(k)], \end{aligned} \quad (6)$$

where

$$\begin{aligned} \tilde{m}_j(\hat{x}(k)) &= \frac{\underline{b}_j(\hat{x}(k))\underline{m}_j(\hat{x}(k)) + \overline{b}_j(\hat{x}(k))\overline{m}_j(\hat{x}(k))}{\sum_{k=1}^c (\underline{b}_k(\hat{x}(k))\underline{m}_k(\hat{x}(k)) + \overline{b}_k(\hat{x}(k))\overline{m}_k(\hat{x}(k)))} \\ &\geq 0, \forall j \text{ and } \sum_{j=1}^c \tilde{m}_j(\hat{x}(k)) = 1. \end{aligned}$$

Here, $\underline{b}_j(\hat{x}(k))$ and $\overline{b}_j(\hat{x}(k))$, which are functions to be determined by users, satisfy: 1) $0 \leq \underline{b}_j(\hat{x}(k)) \leq 1$ and $0 \leq \overline{b}_j(\hat{x}(k)) \leq 1$. 2) $\underline{b}_j(\hat{x}(k)) + \overline{b}_j(\hat{x}(k)) = 1$. This IT2 T-S fuzzy system (4) has lower order of system matrices and less number of membership functions compared with the original system (3).

C. Error System

Then construct the error system by combining the original system (3) and reduced-order system (6):

$$\begin{aligned} \bar{x}(k+1) &= \sum_{i=1}^r \sum_{j=1}^c \tilde{w}_i(x(k)) \tilde{m}_j(\hat{x}(k)) [\overline{A}_{ij} \bar{x}(k) + \overline{B}_{ij} u(k)], \\ e(k) &= \sum_{i=1}^r \sum_{j=1}^c \tilde{w}_i(x(k)) \tilde{m}_j(\hat{x}(k)) [\overline{C}_{ij} \bar{x}(k) + \overline{D}_{ij} u(k)], \end{aligned} \quad (7)$$

where $\sum_{i=1}^r \tilde{w}_i(x(k)) = \sum_{j=1}^c \tilde{m}_j(\hat{x}(k)) = 1$, $\bar{x}(k) = [x^T(k), \hat{x}^T(k)]^T$, $e(k) = y(k) - \hat{y}(k)$, $\overline{A}_{ij} = \begin{bmatrix} A_i & 0 \\ 0 & \hat{A}_j \end{bmatrix}$, $\overline{B}_{ij} = \begin{bmatrix} B_i \\ \hat{B}_j \end{bmatrix}$, $\overline{C}_{ij} = [C_i \quad -\hat{C}_j]$, $\overline{D}_{ij} = D_i - \hat{D}_j$.

From (7), we can obtain the following error system:

$$\begin{aligned} \bar{x}(k+1) &= \sum_{i=1}^r \sum_{j=1}^c h_{ij}(\bar{x}(k)) [\overline{A}_{ij} \bar{x}(k) + \overline{B}_{ij} u(k)], \\ e(k) &= \sum_{i=1}^r \sum_{j=1}^c h_{ij}(\bar{x}(k)) [\overline{C}_{ij} \bar{x}(k) + \overline{D}_{ij} u(k)], \end{aligned} \quad (8)$$

where $h_{ij}(\bar{x}(k)) = \tilde{w}_i(x(k)) \tilde{m}_j(\hat{x}(k))$ and $\sum_{i=1}^r \sum_{j=1}^c h_{ij}(\bar{x}(k)) = 1$.

D. Definitions and Assumption

To facilitate the analysis, the following definitions are given as follows.

Definition 1: The error system (8) is said to be asymptotically stable under $u(k) = 0$ if the following is achieved:

$$\lim_{k \rightarrow \infty} |\bar{x}(k)| = 0.$$

For an asymptotically stable error system (8), we have $e \equiv e(k) \in \ell_2[0, \infty)$ when $u \equiv u(k) \in \ell_2[0, \infty)$.

Definition 2: Given a scalar $\gamma > 0$, the error system (8) is said to be asymptotically stable with an \mathcal{H}_∞ error performance γ if it is asymptotically stable and $\|e\|_2 < \gamma^2 \|u\|_2$ for all non-zero $u \in \ell_2[0, \infty)$, where

$$\|e\|_2 \triangleq \sqrt{\sum_{k=0}^{\infty} e^T(k)e(k)}.$$

Throughout this paper, we make the following assumption.

Assumption 1: System (3) is asymptotically stable.

The original system (3) to be asymptotically stable is a prerequisite for error system (7) to be asymptotically stable, which plays a key role in the following analysis.

III. MAIN RESULTS

This section will show the main results of this paper.

A. \mathcal{H}_∞ Performance

Before analyzing the method of model reduction, we first introduce the following theorem.

Theorem 1: Consider the error system in (8). It is asymptotically stable with an \mathcal{H}_∞ error performance index γ if there exist matrices P , \underline{R}_{ij} and \overline{R}_{ij} with appropriate dimensions satisfying the following inequalities for $i = 1, 2, \dots, r$; $j = 1, 2, \dots, c$:

$$\begin{aligned} \Theta_{ij} + \underline{R}_{ij} - \overline{R}_{ij} + \sum_{s=1}^r \sum_{t=1}^c \left(\underline{h}_{st}(\bar{x}(k)) (-\underline{R}_{st}) \right. \\ \left. + \overline{h}_{st}(\bar{x}(k)) \overline{R}_{st} \right) < 0, \end{aligned} \quad (9)$$

$$P > 0, \underline{R}_{ij} \geq 0, \overline{R}_{ij} \geq 0, \quad (10)$$

where

$$\Theta_{ij} = \begin{bmatrix} \overline{A}_{ij}^T P \overline{A}_{ij} - P + \overline{C}_{ij}^T \overline{C}_{ij} & \overline{A}_{ij}^T P \overline{B}_{ij} + \overline{C}_{ij}^T \overline{D}_{ij} \\ * & \overline{B}_{ij}^T P \overline{B}_{ij} - \gamma^2 I + \overline{D}_{ij}^T \overline{D}_{ij} \end{bmatrix},$$

$\underline{h}_{ij}(\bar{x}(k))$ and $\bar{h}_{ij}(\bar{x}(k))$ are the lower bound and upper bound functions of $h_{ij}(\bar{x}(k))$ to be determined:

$$0 \leq \underline{h}_{ij}(\bar{x}(k)) \leq h_{ij}(\bar{x}(k)) \leq \bar{h}_{ij}(\bar{x}(k)) \leq 1, \quad (11)$$

Proof 1: Choose a Lyapunov function as

$$V(k) = \bar{x}^T(k)P\bar{x}(k), \quad (12)$$

where P is a symmetric and positive definite matrix to be determined.

Then calculate the difference of $V(k)$:

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &\leq \sum_{i=1}^r \sum_{j=1}^c h_{ij}(\bar{x}(k)) \left\{ [\bar{A}_{ij}\bar{x}(k) + \bar{B}_{ij}u(k)]^T P \right. \\ &\quad \left. \times [\bar{A}_{ij}\bar{x}(k) + \bar{B}_{ij}u(k)] - \bar{x}^T(k)P\bar{x}(k) \right\}. \quad (13) \end{aligned}$$

Next, it is obtained that

$$\begin{aligned} \Delta V(k) &+ e^T(k)e(k) - \gamma^2 u^T(k)u(k) \\ &\leq \sum_{i=1}^r \sum_{j=1}^c h_{ij}(\bar{x}(k)) \xi^T(k) \left(\Theta_{ij} + \underline{R}_{ij} - \bar{R}_{ij} + \right. \\ &\quad \left. \sum_{s=1}^r \sum_{t=1}^c ((\underline{h}_{st}(\bar{x}(k))(-\underline{R}_{st}) \right. \\ &\quad \left. + \bar{h}_{st}(\bar{x}(k))(\bar{R}_{st})) \right) \xi(k), \quad (14) \end{aligned}$$

where $\xi = \begin{bmatrix} \bar{x}(k) \\ u(k) \end{bmatrix}$.

It follows from (9) that it indicates

$$\Delta V(k) + e^T(k)e(k) - \gamma^2 u^T(k)u(k) < 0. \quad (15)$$

For all non-zero $u = \{u(k)\} \in \ell_2[0, \infty)$, $e = \{e(k)\} \in \ell_2[0, \infty)$. Then summing both sides of (15) from 0 to ∞ , we can get $\|e\|_2 < \gamma^2 \|u\|_2$. Also, when $u(k) \equiv 0$, it can be seen from (14) that $\Delta V(k) < 0$, which means that system (8) with $u = 0$ is asymptotically stable. The proof is completed.

Remark 2: For getting a reduced-order system such that the error system has an \mathcal{H}_∞ performance γ , users can choose the value of γ they want, or take γ as a decision variable and find the feasible minimum value by formulating GEVP.

Remark 3: Due to the existence of $\bar{h}_{ij}(\bar{x}(k))$ and $\underline{h}_{ij}(\bar{x}(k))$, (9) is not a strict LMI. For solving this, one can choose $\bar{h}_{ij}(\bar{x}(k))$ and $\underline{h}_{ij}(\bar{x}(k))$ as constant functions or staircase functions to make (9) in an LMI form.

Remark 4: For a model reduction problem, due to the \hat{A}_j , \hat{B}_j , \hat{C}_j and \hat{D}_j , the conditions in Theorem 1 are not convex. For solving this, convex conditions will be derived based on the conditions in Theorem 1 in the following section.

B. \mathcal{H}_∞ Model Reduction

Based on the above result about the \mathcal{H}_∞ performance of error system, next we introduce the method of model reduction for discrete-time IT2 T-S fuzzy model.

Theorem 2: Considering the discrete-time IT2 T-S fuzzy system (3), there is a low dimensional system (6) such that the error system (8) is asymptotically stable with an \mathcal{H}_∞ norm

error performance index γ if there exist matrices \mathcal{P} , \mathcal{Q} , \hat{A}_j , \hat{B}_j , \hat{C}_j , \hat{D}_j , $\hat{\underline{R}}_{ij}$ and $\hat{\bar{R}}_{ij}$ with appropriate dimensions satisfying the following inequalities (16) to (18) for $i = 1, 2, \dots, r$; $j = 1, 2, \dots, c$:

$$\begin{aligned} \hat{\Theta}_{ij} + \hat{\underline{R}}_{ij} - \hat{\bar{R}}_{ij} + \sum_{s=1}^r \sum_{t=1}^c \left(\underline{h}_{st}(\bar{x}(k))(-\hat{\underline{R}}_{st}) \right. \\ \left. + \bar{h}_{st}(\bar{x}(k))\hat{\bar{R}}_{st} \right) < 0, \quad (16) \end{aligned}$$

$$\begin{bmatrix} \mathcal{P} & \mathcal{H}\mathcal{Q} \\ * & \mathcal{Q} \end{bmatrix} > 0, \quad (17)$$

$$\hat{\underline{R}}_{ij} \geq 0, \hat{\bar{R}}_{ij} \geq 0, \quad (18)$$

where

$$\hat{\Theta}_{ij} = \begin{bmatrix} \hat{\Theta}_{ij}^{11} & \hat{\Theta}_{ij}^{12} & \hat{\Theta}_{ij}^{13} & 0 \\ * & \hat{\Theta}_{ij}^{22} & 0 & \hat{\Theta}_{ij}^{24} \\ * & * & -\gamma^2 I & \hat{\Theta}_{ij}^{34} \\ * & * & * & -I \end{bmatrix}, \quad (19)$$

$$\mathcal{H} = \begin{bmatrix} I_{m \times m} & 0_{m \times (n-m)} \end{bmatrix}^T, \quad (20)$$

$$\hat{\Theta}_{ij}^{11} = \hat{\Theta}_{ij}^{22} = \begin{bmatrix} -\mathcal{P} & -\mathcal{H}\mathcal{Q} \\ * & -\mathcal{Q} \end{bmatrix}, \quad (21)$$

$$\hat{\Theta}_{ij}^{12} = \begin{bmatrix} \mathcal{P}A_i & \mathcal{H}\hat{A}_j \\ \mathcal{Q}\mathcal{H}^T A_i & \hat{A}_j \end{bmatrix}, \quad (22)$$

$$\hat{\Theta}_{ij}^{13} = \begin{bmatrix} \mathcal{P}B_i + \mathcal{H}\hat{B}_j \\ \mathcal{Q}\mathcal{H}^T B_i + \hat{B}_j \end{bmatrix}, \quad (23)$$

$$\hat{\Theta}_{ij}^{24} = [C_i \quad -\hat{C}_j]^T, \quad (24)$$

$$\hat{\Theta}_{ij}^{34} = D_i^T - \hat{D}_j^T. \quad (25)$$

$\underline{h}_{ij}(\bar{x}(k))$ and $\bar{h}_{ij}(\bar{x}(k))$ are defined in (11). The matrix parameters of reduced-order model (6) can be obtained by:

$$\begin{bmatrix} \hat{A}_j & \hat{B}_j \\ \hat{C}_j & \hat{D}_j \end{bmatrix} = \begin{bmatrix} \mathcal{Q}^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{A}_j & \tilde{B}_j \\ \tilde{C}_j & \tilde{D}_j \end{bmatrix}. \quad (26)$$

Proof 2: First, from (17) we can get $\mathcal{Q} > 0$. Then define $G \triangleq J\mathcal{Q}^{-1}J^T$, where J is a nonsingular matrix and $J \in \mathbb{R}^{m \times m}$. Applying Schur complement, from (17) we know $\mathcal{P} - \mathcal{H}\mathcal{Q}\mathcal{H}^T > 0$. Consider $Q = J^T G^{-1}J$, then we have $\mathcal{P} - \mathcal{H}J^T G^{-1}J\mathcal{H}^T > 0$. Through Schur complement, we obtain

$$\begin{bmatrix} \mathcal{P} & \mathcal{H}J^T \\ * & G \end{bmatrix} > 0. \quad (27)$$

Define $\bar{P} \triangleq \begin{bmatrix} \mathcal{P} & \mathcal{H}J^T \\ * & G \end{bmatrix}$ and $K \triangleq \begin{bmatrix} I & 0 \\ 0 & J^{-1}G \end{bmatrix}$. It obtains that

$$\begin{aligned} \bar{P} &= \begin{bmatrix} I & 0 \\ 0 & G^T J^{-T} \end{bmatrix} \begin{bmatrix} \mathcal{P} & \mathcal{H}\mathcal{Q} \\ * & \mathcal{Q} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & J^{-1}G \end{bmatrix} \\ &= K^T \begin{bmatrix} \mathcal{P} & \mathcal{H}\mathcal{Q} \\ * & \mathcal{Q} \end{bmatrix} K. \end{aligned}$$

Then define

$$\begin{bmatrix} \hat{A}_j & \hat{B}_j \\ \hat{C}_j & \hat{D}_j \end{bmatrix} = \begin{bmatrix} J^T & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{A}_j & \tilde{B}_j \\ \tilde{C}_j & \tilde{D}_j \end{bmatrix} \begin{bmatrix} G^{-1}J & 0 \\ 0 & I \end{bmatrix}, \quad (28)$$

$$\hat{\underline{R}}_{ij} = \text{diag}(K^T, K^T, I, I) \hat{\underline{R}}_{ij} \text{diag}(K, K, I, I), \quad (29)$$

$$\hat{\bar{R}}_{ij} = \text{diag}(K^T, K^T, I, I) \hat{\bar{R}}_{ij} \text{diag}(K, K, I, I), \quad (30)$$

where $\underline{\mathcal{R}}_{ij}$ and $\overline{\mathcal{R}}_{ij}$ are matrices with appropriate dimensions. Performing congruence transformation to (16) and (17) by $\text{diag}(K, K, I, I)$ and K respectively, we obtain $\Pi_{ij} + \underline{\mathcal{R}}_{ij} - \overline{\mathcal{R}}_{ij} + \sum_{s=1}^r \sum_{t=1}^c \left(\underline{h}_{st}(\bar{x}(k))(-\underline{\mathcal{R}}_{st}) + \overline{h}_{st}(\bar{x}(k))(\overline{\mathcal{R}}_{st}) \right) < 0$, $\begin{bmatrix} \mathcal{P} & \mathcal{H}J^T \\ * & G \end{bmatrix} > 0$, where

$$\Pi_{ij} = \begin{bmatrix} \Pi_{ij}^{11} & \Pi_{ij}^{12} & \Pi_{ij}^{13} & 0 \\ * & \Pi_{ij}^{22} & 0 & \Pi_{ij}^{24} \\ * & * & -\gamma^2 I & \Pi_{ij}^{34} \\ * & * & * & -I \end{bmatrix}, \quad (31)$$

$$\Pi_{ij}^{11} = \Pi_{ij}^{22} = \begin{bmatrix} -\mathcal{P} & -\mathcal{H}J^T \\ * & -G \end{bmatrix} = -\overline{P}, \quad (32)$$

$$\Pi_{ij}^{12} = \begin{bmatrix} \mathcal{P}A_i & \mathcal{H}J^T \hat{A}_j \\ J\mathcal{H}^T A_i & G\hat{A}_j \end{bmatrix} = \overline{P}A_{ij}, \quad (33)$$

$$\Pi_{ij}^{13} = \begin{bmatrix} \mathcal{P}B_i + \mathcal{H}J^T \hat{B}_j \\ J\mathcal{H}^T B_i + G\hat{B}_j \end{bmatrix} = \overline{P}B_{ij}, \quad (34)$$

$$\Pi_{ij}^{24} = [C_i \quad -\tilde{C}_j]^T = [C_i \quad -\hat{C}_j]^T, \quad (35)$$

$$\Pi_{ij}^{34} = D_i^T - \tilde{D}_j^T = D_i^T - \hat{D}_j^T. \quad (36)$$

Then replace \overline{P} , $\underline{\mathcal{R}}_{ij}$ and $\overline{\mathcal{R}}_{ij}$ with P , $\underline{\mathcal{R}}_{ij}$ and $\overline{\mathcal{R}}_{ij}$, we can obtain the inequality below:

$$\hat{\Theta}_{ij} + \underline{\mathcal{R}}_{ij} - \overline{\mathcal{R}}_{ij} + \sum_{s=1}^r \sum_{t=1}^c \left(\underline{h}_{st}(\bar{x}(k))(-\underline{\mathcal{R}}_{st}) + \overline{h}_{st}(\bar{x}(k))(\overline{\mathcal{R}}_{st}) \right) < 0, \quad (37)$$

where

$$\hat{\Theta}_{ij} = \begin{bmatrix} -P & P\overline{A}_{ij} & P\overline{B}_{ij} & 0 \\ * & -P & 0 & \overline{C}_{ij}^T \\ * & * & -\gamma^2 I & \overline{D}_{ij}^T \\ * & * & * & -I \end{bmatrix}, \quad (38)$$

$\underline{\mathcal{R}}_{ij}$ and $\overline{\mathcal{R}}_{ij}$ are matrices with appropriate dimensions. According to Schur complement, (9) is obtained by (37), which means that the error system (8) is asymptotically stable with an \mathcal{H}_∞ norm error performance γ .

According to (28), we obtain

$$\begin{bmatrix} \hat{A}_j & \hat{B}_j \\ \hat{C}_j & \hat{D}_j \end{bmatrix} = \begin{bmatrix} (J^{-1}G)^{-1}Q^{-1} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \tilde{A}_j & \tilde{B}_j \\ \tilde{C}_j & \tilde{D}_j \end{bmatrix} \times \begin{bmatrix} J^{-1}G & 0 \\ 0 & I \end{bmatrix}. \quad (39)$$

For the simplicity of calculation, we set $J^{-1}G = I$. Then we can get (26). The proof is completed.

Remark 5: Similar to Theorem 1, γ can be manually picked. Also, when GEVP is formulated, γ can be minimised.

Remark 6: Similar to Theorem 1, Theorem 2 is also not a strict form of LMIs duo to $\overline{h}_{ij}(\bar{x}(k))$ and $\underline{h}_{ij}(\bar{x}(k))$. The comment in Remark 2 is applicable to Theorem 2.

Remark 7: The membership-functions-dependent technique applied in theorem reduces the conservativeness. Also, Theorem 2 allows the membership functions of the reduced order system to be different from the original system, which offers more freedom to choose the type of reduced-order system's

than those approaches in which the reduced-order model is designed as linear system such as [49], [50] or fuzzy system sharing the same membership functions as its original system such as [51].

Remark 8: Different from [53], there is no D-stability constraint in the above theorems. Also, the reduced-order system's membership functions depend on system state variables of reduced-order system rather than those of original system, which is more reasonable in practice.

C. Zero-order Model Reduction

The problem of zero-order model reduction has also been received attention in the previous studies, for example [41], [49], [54]. Zero-order model reduction is a special case of model reduction. In addition to obtaining a zero-order model to approximate the original system, it can help to analyze other general cases. The zero-order model reduction for discrete-time IT2 T-S fuzzy system is to approximate system (3) by a zero-order system described as follows.

$$\hat{y}(k) = \hat{D}u(k) \quad (40)$$

where \hat{D} is the feedforward matrix with appropriate dimensions to be determined. The error system is defined as

$$\begin{aligned} x(k+1) &= \sum_{i=1}^r \tilde{w}_i(x(k)) [A_i x(k) + B_i u(k)], \\ e_0(k) &= \sum_{i=1}^r \tilde{w}_i(x(k)) [C_i x(k) + \overline{D}_i u(k)], \end{aligned} \quad (41)$$

where $\overline{D}_i = D_i - \hat{D}$ for $i = 1, 2, \dots, r$. By applying a similar derivation process, the method of \mathcal{H}_∞ zero-order model reduction for discrete-time IT2 fuzzy systems is shown in the following corollary.

Corollary 1: Considering the discrete-time IT2 T-S fuzzy system (3), there is a zero-order system (40) such that the error system (41) is asymptotically stable with an \mathcal{H}_∞ norm error performance index γ if there exist matrices P , \hat{D} , $\tilde{\mathcal{R}}_i$ and $\overline{\mathcal{R}}_i$ with appropriate dimensions satisfying the following inequalities (42) and (43) for $i = 1, 2, \dots, r$:

$$\tilde{\Theta}_i + \tilde{\mathcal{R}}_i - \overline{\mathcal{R}}_i + \sum_{s=1}^r \left(\tilde{w}_s(x(k))(-\tilde{\mathcal{R}}_{st}) + \overline{w}_s(x(k))(\overline{\mathcal{R}}_s) \right) < 0, \quad (42)$$

$$P > 0, \tilde{\mathcal{R}}_i \geq 0, \overline{\mathcal{R}}_i \geq 0 \quad (43)$$

where

$$\tilde{\Theta}_i = \begin{bmatrix} -P & PA_i & PB_i & 0 \\ * & -P & 0 & C_i^T \\ * & * & -\gamma^2 I & D_i^T - \hat{D}^T \\ * & * & * & -I \end{bmatrix}, \quad (44)$$

$\tilde{w}_i(x(k))$ and $\overline{w}_i(x(k))$ are defined in (45),

$$0 \leq \underline{w}_i(x(k)) \leq \tilde{w}_i(x(k)) \leq \overline{w}_i(x(k)) \leq 1, \quad (45)$$

IV. SIMULATION EXAMPLE

In this section, two numerical examples are presented to demonstrate the effectiveness of the proposed results.

Example 1: Consider a discrete-time IT2 fuzzy model in the form of (3) with following parameters:

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0.1599 & 0.0521 & -0.0142 & 0.1852 & 0.0312 \\ 0.0423 & -0.3881 & 0.5037 & -0.0344 & 0.0421 \\ -0.0726 & -0.3046 & -0.0503 & -0.1103 & 0.1345 \\ -0.1625 & 0.4304 & 0.0332 & 0.2799 & 0.1213 \\ -0.1701 & 0.4223 & 0.0281 & 0.2288 & 0.4213 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0.1723 & 0.0356 & -0.0482 & 0.2052 & -0.1412 \\ 0.0398 & -0.2441 & 0.4934 & -0.0364 & -0.0211 \\ -0.0599 & -0.2137 & -0.1903 & -0.1603 & 0.1462 \\ -0.1715 & 0.4379 & 0.0232 & 0.3329 & 0.1913 \\ -0.1435 & 0.5314 & 0.0314 & 0.1799 & 0.5856 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} 0.2675 & 0.1024 & -0.6210 & 0.1528 & -0.1345 \\ 0.0901 & -0.3493 & 0.4032 & -0.0525 & -0.0346 \\ -0.0702 & -0.1694 & -0.2208 & -0.1945 & 0.1198 \\ -0.1462 & 0.2474 & 0.0743 & 0.2637 & 0.1045 \\ -0.1286 & 0.4741 & 0.0431 & 0.1475 & 0.4993 \end{bmatrix}, \\
B_1 &= [0.2034 \quad -0.2337 \quad -0.1122 \quad 0.1257 \quad 0.3621]^T, \\
B_2 &= [0.2724 \quad -0.2323 \quad -0.1773 \quad 0.1769 \quad 0.4904]^T, \\
B_3 &= [0.3421 \quad -0.1653 \quad -0.1829 \quad 0.2382 \quad 0.3992]^T, \\
C_1 &= [1.1512 \quad 0.6691 \quad 0.1408 \quad -0.8611 \quad 0.1334], \\
C_2 &= [1.3974 \quad 0.9692 \quad 0.1982 \quad -0.9758 \quad 0.2959], \\
C_3 &= [0.3954 \quad 0.3281 \quad 0.3388 \quad -0.3921 \quad 0.3812], \\
D_1 &= [1.1], D_2 = [0.5], D_3 = [0.8].
\end{aligned}$$

The lower and upper membership functions of system (3) are shown in Table I. Besides, we set $\underline{a}_i(x_1(k)) = 0.5(\sin(3x_1(k)))^2$, $\bar{a}_i(x_1(k)) = 1 - \underline{a}_i(x_1(k))$ for demonstration purposes. We aim to find reduced-order systems with the form of (6) whose lower and upper membership functions are shown in Table II. Here, $r = 3$ and $c = 2$. $\underline{b}_j(\hat{x}(k)) = \bar{b}_j(\hat{x}(k)) = 0.5$. Then, $\underline{h}_{ij}(\bar{x}(k))$ and $\bar{h}_{ij}(\bar{x}(k))$ are set as $\underline{h}_{ij}(\bar{x}(k)) = \min(h_{ij}(\bar{x}(k)))$, $\forall i, j$ and $\bar{h}_{ij}(\bar{x}(k)) = \max(h_{ij}(\bar{x}(k)))$, $\forall i, j$, where the values of $\underline{h}_{ij}(\bar{x}(k))$ and $\bar{h}_{ij}(\bar{x}(k))$ are denoted as \underline{h}_{ij} and \bar{h}_{ij} . The value of \underline{h}_{ij} and \bar{h}_{ij} are shown as follows: $\underline{h}_{11} = 0$, $\underline{h}_{12} = 0.0808$, $\underline{h}_{21} = 0$, $\underline{h}_{22} = 0.2424$, $\underline{h}_{31} = 0$, $\underline{h}_{32} = 0$, $\bar{h}_{11} = 0.1364$, $\bar{h}_{12} = 0.5000$, $\bar{h}_{21} = 0.1370$, $\bar{h}_{22} = 0.5023$, $\bar{h}_{31} = 0.1515$ and $\bar{h}_{32} = 0.5556$.

It can be verified that the original discrete-time IT2 T-S fuzzy system in (3) is asymptotically stable when the parameters are set as above. Here our aim is to find different 3rd order systems in the form of (6) with different \mathcal{H}_∞ performance index γ to approximate the above-mentioned original system and show how \mathcal{H}_∞ performance γ can affect the performance of model reduction. By applying Theorem 2, the obtained results for different cases are demonstrated as

TABLE I
THE LOWER AND UPPER MEMBERSHIP FUNCTIONS OF SYSTEM (3)

| The lower membership functions | The upper membership functions |
|---|--|
| $\underline{u}_{M_1}(x_1) = 1 - e^{-\frac{x_1^2}{2}}$ | $\bar{u}_{M_1}(x_1) = 1 - 0.8e^{-\frac{x_1^2}{0.7}}$ |
| $\underline{u}_{M_2}(x_1) = 1 - e^{-\frac{x_1^2}{1.4}}$ | $\bar{u}_{M_2}(x_1) = 1 - 0.4e^{-\frac{x_1^2}{0.3}}$ |
| $\underline{u}_{M_3}(x_1) = 0.4e^{-\frac{x_1^2}{0.3}}$ | $\bar{u}_{M_3}(x_1) = e^{-\frac{x_1^2}{1.4}}$ |

TABLE II
THE LOWER AND UPPER MEMBERSHIP FUNCTIONS OF SYSTEM (6)

| The lower membership functions | The upper membership functions |
|--|--|
| $\underline{u}_{N_1}(\hat{x}_1) = 0.1e^{-\frac{\hat{x}_1^2}{0.6}}$ | $\bar{u}_{N_1}(\hat{x}_1) = 0.2e^{-\frac{\hat{x}_1^2}{3}}$ |
| $\underline{u}_{N_2}(\hat{x}_1) = 1 - e^{-\frac{\hat{x}_1^2}{3}}$ | $\bar{u}_{N_2}(\hat{x}_1) = 1 - 0.2e^{-\frac{\hat{x}_1^2}{0.6}}$ |

follows.

Case 1: γ is set as 0.5.

$$\begin{aligned}
\hat{A}_1 &= \begin{bmatrix} 0.0967 & -0.0144 & -0.1349 \\ 0.0840 & -0.1427 & 0.3726 \\ 0.0600 & -0.2479 & -0.1167 \end{bmatrix}, \\
\hat{A}_2 &= \begin{bmatrix} 0.1078 & -0.0047 & -0.2550 \\ 0.0997 & -0.1650 & 0.3687 \\ 0.0540 & -0.2624 & -0.1003 \end{bmatrix}, \\
\hat{B}_1 &= [-0.2530 \quad 0.2320 \quad 0.1535]^T, \\
\hat{B}_2 &= [-0.2734 \quad 0.2175 \quad 0.1611]^T, \\
\hat{C}_1 &= [-0.7032 \quad -0.2616 \quad -0.4368], \\
\hat{C}_2 &= [-0.6259 \quad -0.2513 \quad -0.3438], \\
\hat{D}_1 &= [0.7246], \hat{D}_2 = [0.7715].
\end{aligned}$$

Case 2: γ is set as 0.6.

$$\begin{aligned}
\hat{A}_1 &= \begin{bmatrix} 0.1010 & -0.0206 & -0.1248 \\ 0.0668 & -0.1278 & 0.3072 \\ 0.0694 & -0.2217 & -0.1169 \end{bmatrix}, \\
\hat{A}_2 &= \begin{bmatrix} 0.1193 & -0.0228 & -0.2058 \\ 0.0845 & -0.1501 & 0.3147 \\ 0.0637 & -0.2326 & -0.1116 \end{bmatrix}, \\
\hat{B}_1 &= [-0.2577 \quad 0.2314 \quad 0.1552]^T, \\
\hat{B}_2 &= [-0.2753 \quad 0.2192 \quad 0.1620]^T, \\
\hat{C}_1 &= [-0.6270 \quad -0.2581 \quad -0.3185], \\
\hat{C}_2 &= [-0.5507 \quad -0.2550 \quad -0.2318], \\
\hat{D}_1 &= [0.7227], \hat{D}_2 = [0.7685].
\end{aligned}$$

Case 2: γ is set as 0.7.

$$\hat{A}_1 = \begin{bmatrix} 0.1004 & -0.0208 & -0.1229 \\ 0.0636 & -0.1251 & 0.2957 \\ 0.0700 & -0.2184 & -0.1149 \end{bmatrix},$$

$$\hat{A}_2 = \begin{bmatrix} 0.1201 & -0.0249 & -0.1990 \\ 0.0817 & -0.1475 & 0.3038 \\ 0.0644 & -0.2287 & -0.1111 \end{bmatrix},$$

$$\hat{B}_1 = [-0.2581 \quad 0.2312 \quad 0.1552]^T,$$

$$\hat{B}_2 = [-0.2763 \quad 0.2192 \quad 0.1626]^T,$$

$$\hat{C}_1 = [-0.6183 \quad -0.2573 \quad -0.3078],$$

$$\hat{C}_2 = [-0.5412 \quad -0.2567 \quad -0.2223],$$

$$\hat{D}_1 = [0.7202], \hat{D}_2 = [0.7649].$$

In order to show the effectiveness of obtained theorems, the simulation results of \mathcal{H}_∞ model reduction performance by using Theorem 2 for discrete-time IT2 T-S fuzzy system is given graphically where the impulse input as follows:

$$u(k) = \begin{cases} 0 & k \leq 20 \\ \sin(0.15(k-20))e^{-0.15(k-20)} & 20 < k. \end{cases}$$

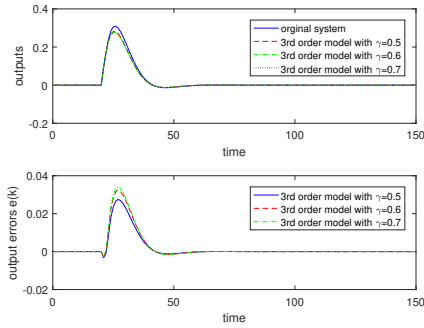


Fig. 1. Output trajectories of original system and its 3rd order reduced models and output errors $e(k)$ of the 3rd order reduced models.

The top graph in Fig. 1 illustrates the output trajectories of the original 5th order system (blue solid line), 3rd order system with $\gamma = 0.5$ (red dashed line), 3rd order system with $\gamma = 0.6$ (green dotted-dashed line) and 3rd order system with $\gamma = 0.7$ (black dotted line). The bottom graph in Fig. 1 illustrates the output error between original 5th order system and 3rd order system with $\gamma = 0.5$ (blue solid line), the output error between original 5th order system and 3rd order system with $\gamma = 0.6$ (red dashed line) and the output error between original 5-order system and 3rd order system with $\gamma = 0.7$ (green dotted-dashed line).

It can be seen from the simulation results that as the \mathcal{H}_∞ performance index γ reduces, the error between the original system and its reduced order system reduces, which means the suppression of \mathcal{H}_∞ performance index γ can make reduced-order system approximate original system successfully.

Example 2: Consider a discrete-time IT2 fuzzy model in the form of (3) with the same parameters as in Example 1.

Here, our aim is to find the 2nd order, 3rd order and 4th order systems in the form of (6) which approximate

the above-mentioned original system with an minimized \mathcal{H}_∞ performance index γ by formulating GEVP and show how different orders can affect the performance of model reduction method in Theorem 2. By applying Theorem 2, the obtained results for different cases are demonstrated as follows.

Case 1: $m = 4$ and the minimum feasible γ is 0.2484

$$\hat{A}_1 = \begin{bmatrix} 0.2201 & 0.1716 & -0.3829 & -0.0737 \\ 0.0361 & -0.2999 & 0.5128 & 0.0465 \\ -0.0867 & -0.4280 & 0.0243 & 0.1094 \\ -0.1377 & 0.2520 & 0.0326 & 0.4144 \end{bmatrix},$$

$$\hat{A}_2 = \begin{bmatrix} 0.2153 & 0.1655 & -0.3479 & -0.0397 \\ 0.0348 & -0.3077 & 0.5384 & 0.0563 \\ -0.0684 & -0.4102 & -0.0228 & 0.0654 \\ -0.1616 & 0.2282 & 0.1395 & 0.5084 \end{bmatrix},$$

$$\hat{B}_1 = [-0.2661 \quad 0.2147 \quad 0.1573 \quad -0.1711]^T,$$

$$\hat{B}_2 = [-0.2676 \quad 0.2145 \quad 0.1583 \quad -0.1724]^T,$$

$$\hat{C}_1 = [-1.0343 \quad -0.4224 \quad -0.4348 \quad 0.3245],$$

$$\hat{C}_2 = [-1.0285 \quad -0.4159 \quad -0.4338 \quad 0.3217],$$

$$\hat{D}_1 = [0.7701], \hat{D}_2 = [0.7716].$$

Case 2: $m = 3$ and the minimum feasible γ is 0.2496

$$\hat{A}_1 = \begin{bmatrix} 0.2031 & 0.1618 & -0.3181 \\ 0.0352 & -0.3209 & 0.4934 \\ -0.0500 & -0.4026 & -0.0668 \end{bmatrix},$$

$$\hat{A}_2 = \begin{bmatrix} 0.2124 & 0.1739 & -0.3316 \\ 0.0365 & -0.3276 & 0.5098 \\ -0.0505 & -0.4054 & -0.0610 \end{bmatrix},$$

$$\hat{B}_1 = [-0.2674 \quad 0.2151 \quad 0.1590]^T,$$

$$\hat{B}_2 = [-0.2687 \quad 0.2148 \quad 0.1595]^T,$$

$$\hat{C}_1 = [-0.9412 \quad -0.4367 \quad -0.6058],$$

$$\hat{C}_2 = [-0.9372 \quad -0.4318 \quad -0.6018],$$

$$\hat{D}_1 = [0.7722], \hat{D}_2 = [0.7739].$$

Case 3: $m = 2$ and the minimum feasible γ is 0.3009

$$\hat{A}_1 = \begin{bmatrix} 0.3780 & 0.2117 \\ -0.1485 & -0.2460 \end{bmatrix},$$

$$\hat{A}_2 = \begin{bmatrix} 0.4521 & 0.2720 \\ -0.1886 & -0.2731 \end{bmatrix},$$

$$\hat{B}_1 = [-0.2707 \quad 0.2260]^T,$$

$$\hat{B}_2 = [-0.2700 \quad 0.2241]^T,$$

$$\hat{C}_1 = [-0.8841 \quad -0.7335],$$

$$\hat{C}_2 = [-0.8881 \quad -0.7300],$$

$$\hat{D}_1 = [0.7616], \hat{D}_2 = [0.7655].$$

Similar to Example 1, we show the above-mentioned simulation results graphically by using the same impulse input as in Example 1.

The top graph in Fig. 2 illustrates the output trajectories of the original 5th order system (blue solid line), 4th order system (red dashed line), 3rd order system (green dotted-dashed line) and 2nd order system (black dotted line). The bottom graph

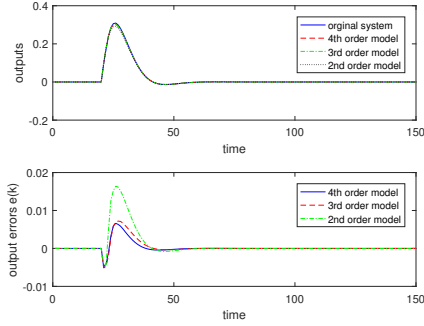


Fig. 2. Output trajectories of original system and different reduced-order systems and output errors $e(k)$ of different reduced-order systems.

in Fig. 2 illustrates the output error between the original 5th order system and 4th order simplified system (blue solid line), the output error between the original 5th order system and 3rd order simplified system (red dashed line) and the output error between the original 5th order system and 2nd order simplified system (green dotted-dashed line).

It can be seen from the simulation results that in each case, the reduced-order can approximate the 5th order original system within a certain margin of error, which demonstrates the obtained model reduction method is feasible. Also, from different cases ($\gamma = 0.2484, 0.2496, 0.3009$), it can be seen that the lower the reduced-order system's order is the larger the value of minimized γ is, which means, in general, the closer the order of the reduced-order system is to the original system, the better the approximation is.

To make a comparison with membership-function-independent conditions, we remove all the terms associate with $\underline{\mathcal{R}}, \bar{\mathcal{R}}, \underline{h}_{ij}(\bar{x}(k)), \bar{h}_{ij}(\bar{x}(k))$ in (16), so that the conditions become membership function independent. Then the conditions become the follows.

Considering the discrete-time IT2 T-S fuzzy system (3), there is a low dimensional system (6) such that the error system (8) is asymptotically stable with an \mathcal{H}_∞ norm error performance γ if there exist matrices $\mathcal{P}, \tilde{A}_j, \tilde{B}_j, \tilde{C}_j$ and \tilde{D}_j with appropriate dimensions satisfying the following inequalities (46) for $i = 1, 2, \dots, r; j = 1, 2, \dots, c$:

$$\hat{\Theta}_{ij} < 0, \quad (46)$$

where $\hat{\Theta}_{ij}$ is defined in (19) to (25).

Here we use the above-mentioned membership-function-independent approach to find the 3rd order reduced-order system of the original 5th order system and compare with the above results using membership-function-dependent technique. According to Remark 5, by formulating GEVP, the minimum feasible γ for the membership-function-independent approach is 0.4813. The obtained reduced-order system matrices are shown as follows:

$$\begin{aligned} \hat{A}_1 = \hat{A}_2 &= \begin{bmatrix} 0.2383 & 0.1810 & -0.3893 \\ 0.1068 & -0.3080 & 0.5351 \\ -0.0599 & -0.4328 & -0.0358 \end{bmatrix}, \\ \hat{B}_1 = \hat{B}_2 &= \begin{bmatrix} -0.2551 & 0.2237 & 0.1527 \end{bmatrix}^T, \\ \hat{C}_1 = \hat{C}_2 &= \begin{bmatrix} -0.8457 & -0.4772 & -0.5444 \end{bmatrix}, \\ \hat{D}_1 = \hat{D}_2 &= \begin{bmatrix} 0.7616 \end{bmatrix}. \end{aligned}$$

By using the same impulse input as in Example 1, for

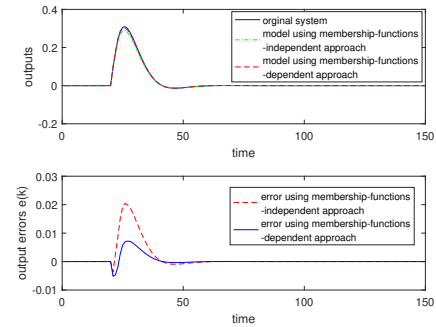


Fig. 3. Output trajectories of original system and the models using membership-functions-dependent and independent approach and output errors $e(k)$ of different approaches.

the 3rd order models, the comparison graphs between the membership-functions-dependent and independent approach are shown in Fig. 3. The top graph in Fig. 3 illustrates the output trajectories of the original 5th order system (blue solid line), 3rd order system using membership-functions-independent approach (green dotted-dashed line) and 3rd order system using membership-functions-dependent approach (red dashed line). The bottom graph in Fig. 3 illustrates the output error between the original 5th order system and 3rd order system using the membership-functions-independent approach (red dashed line), the output error between the original 5th order system and 3rd order system using the membership-functions-dependent approach (blue solid line).

It can be seen from (46) that for each j the LMIs are the same, and that is why $\hat{A}_1 = \hat{A}_2, \hat{B}_1 = \hat{B}_2, \hat{C}_1 = \hat{C}_2$ and $\hat{D}_1 = \hat{D}_2$. Besides, the feasible minimum $\gamma = 0.4813$ obtained by membership-functions-independent approach is larger than $\gamma = 0.2496$ obtained by membership-functions-dependent approach, which shows that the membership-functions-dependent technique demonstrates a better \mathcal{H}_∞ performance and reduces the conservativeness.

V. CONCLUSION

In this research, a membership-function-dependent \mathcal{H}_∞ model reduction of discrete-time IT2 T-S fuzzy model is studied. First, a condition of \mathcal{H}_∞ performance for error system which is constituted by the original system and the reduced-order system is obtained. Then, based on convex linearization, we propose a method to obtain the reduced-order system for discrete-time IT2 T-S fuzzy system. This method introduces a membership-functions-dependent approach which make the

theorems have less conservativeness and more freedom to choose the reduced-order system. In the end, two numerical examples are presented to illustrate the effectiveness of the obtained approaches.

The results proposed in this paper are mainly LMI-based, which limits the selection of the lower bound and the upper bound of the membership functions to be constant or piecewise linear. However, by applying Sum of Squares (SOS) technique, the selection of parameters can be more extensive. Besides, in addition to \mathcal{H}_∞ norm model reduction, the results proposed in this paper can be further extended to other types of model reduction such as Hankel norm-based model reduction or \mathcal{H}_2 norm-based model reduction.

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