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Dissipativity-Based Filtering of Time-Varying Delay Interval Type-2 Polynomial Fuzzy Systems under Imperfect Premise Matching

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***Abstract*—This paper investigates the dissipativity-based filtering problem for the nonlinear systems subject to both uncertainties and time-varying delay in the time-delay interval type-2 (IT2) polynomial fuzzy framework. Filter design is a challenging issue for complex nonlinear systems especially when the uncertainties and time delay exist. IT2 polynomial fuzzy model is an effective and powerful approach to analyze and synthesize uncertain nonlinear systems. This is the first attempt to design both the full-order and reduced-order IT2 polynomial fuzzy filters to ensure that the filtering error system is asymptotically stable under the dissipativity constraint. The design of filtering is based on the imperfect premise matching scheme where the number of fuzzy rules and shapes of membership functions of the designed fuzzy filter can differ from those of IT2 polynomial fuzzy model, to provide greater design flexibility and lower implementation burden. By utilizing the Lyapunov-Krasovskii functional based approach, the information of membership functions, time delay and system states is taken into account in the design process to develop the relaxed membership-function-dependent (MFD) and delay-dependent filtering existence criteria. Finally, simulation results are presented to illustrate the effectiveness of the filtering algorithm reported in this paper.**

***Index Terms*—Interval type-2 fuzzy sets, polynomial fuzzy model, time-varying delay, filter design, membership-function-dependent approach, sum-of-squares.**

I. INTRODUCTION

THE Takagi-Sugeno (T-S) fuzzy model [1] offers a powerful framework to conduct system analysis and control synthesis of nonlinear systems that are pervasive in a wide range of control applications. It effectively characterizes the dynamics of nonlinear systems by connecting a family of local linear subsystems described by IF-THEN rules through the membership functions. Thanks to its favorable model structure to facilitate system analysis and synthesis, T-S fuzzy systems have caught considerable concerns of researchers and a large body of achievements have been established, see, e.g., [2]-[5]. Among these studies, the filtering problem is a research hotspot owing to its vital theoretical and practical significance in signal processing and control applications. To date, various methodologies have been reported such as H_∞ filtering [5], H_2 filtering [6] and dissipative filtering [7]. It is worth pointing out that dissipativity [8], is a more generic criterion which unifies some important system performance indices including H_∞ index and passivity index. There are enormous outcomes related to dissipativity theory published [9]-[11].

Polynomial fuzzy model serves as an effective generalization and expansion of the T-S fuzzy model to construct, analyze and control nonlinear systems [12]. It not only inherits the superiority possessed by T-S fuzzy model, but has more potential to depict nonlinear dynamics by permitting polynomial terms to emerge in the consequent part. Due to the existence of polynomial terms, the linear matrix inequalities (LMI) approaches [13] adopted for T-S fuzzy systems are inapplicable for polynomial fuzzy systems. Instead, the sum-of-squares (SOS) based method is employed for polynomial fuzzy systems with the third-party Matlab toolbox SOSTOOLS [14]. In [12], the SOS-based approaches were proposed to tackle the model and control design of polynomial fuzzy systems, which set up the foundation research on polynomial fuzzy systems. Although some crucial results such as [15]-[18] are published on polynomial fuzzy systems, there are still very limited research works to be found.

As discussed before, both polynomial fuzzy model and T-S fuzzy model are equipped with the ability to address nonlinearities of the concerned systems by resorting to the type-1 fuzzy sets. However, type-1 fuzzy sets lack the capability to represent and capture uncertainties directly which are frequently encountered in realistic systems. To endow the fuzzy sets with the potential to handle uncertainties compromising on computational complexity, interval type-2 (IT2) fuzzy sets [19] are put forward to capture uncertainties. Different from the crisp values owned by type-1 membership functions, the IT2 membership functions are indeterminate and vary in a specific interval characterized by upper and lower bounds of membership functions. Generally speaking, the IT2 fuzzy systems can be deemed as a set of infinite type-1 systems [20]. An effective construction approach of IT2 fuzzy model was first developed in [20] in favor of further research on this class of systems. In the pioneering work [21], by virtue of the imperfect premise matching (IPM) concept which grants the fuzzy controller freedom to select the number of fuzzy rules and the membership functions not restricted to the counterparts of the fuzzy model, the novel membership-function-dependent (MFD) control techniques [22] are developed for IT2 fuzzy model to enhance design flexibility and decrease result con-

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servatism. Other related works are reported in [23]-[25].

Apart from uncertainties incorporated in the systems, time delay is another primary factor to be considered in practical systems which poses a threat to system performance and even stability [3]. Thereby, research on time-delay systems is essential and meaningful and a great deal of literature on investigation of the T-S fuzzy systems perturbed by time delay has been published [3], [4], [10], [26]. In contrast, there are relatively much less works reported on time-delay IT2 polynomial systems. In [27] and [28], the time delay issues concerning tracking and stabilization control for IT2 polynomial fuzzy systems are handled. To the authors' best knowledge, so far no attempt has been made to design filters for the IT2 polynomial fuzzy systems with time-varying delay. In addition, most studied filtering techniques are limited to the assumption that the membership functions of the designed fuzzy filter and the fuzzy model are perfectly matched with each other [10], [16] or there is a constrained relationship between them [26], [29], which restrains the design flexibility and increases the implementation costs. Thus, how to design a filter for time-delay IT2 polynomial fuzzy systems using IPM concept and MFD analysis is an interesting and challenging topic, which motivates this work.

Responding to the discussion above, the problem of filtering design with strictly dissipative performance for delayed IT2 polynomial fuzzy systems will be addressed in this paper. Firstly, a delay-dependent sufficient criterion of dissipativity analysis is developed based on Lyapunov-Krasovskii functional approach to ensure that the filter error system is asymptotically stable under the predefined dissipative constraint index. In order to obtain more relaxed results, the MFD technique is applied which brings the information of membership functions into analysis and allows the designed fuzzy filter to freely select membership functions. And then, the existence criteria of IT2 polynomial fuzzy filters are obtained in terms of SOS. Finally, the simulation is given to exhibit the validity of the filter design approach developed. The main contributions of this paper can be summarized as follows.

1) In this paper, a more generic category of fuzzy systems considering both uncertainties and time-varying delay, i.e. time-delay IT2 polynomial fuzzy system is investigated, which enlarges the application range of the developed filtering strategy.
2) The imperfectly matching membership functions between the designed filter and IT2 polynomial fuzzy systems are adopted, which grants the fuzzy filter design flexibility to select membership functions according to practical requirements and reduces the implementation burden.
3) To further relax the resultant conditions, the whole operating domain is partitioned into a series of subdomains and much richer information related to membership functions are involved in the analysis and design. From the simulation results, it can be found that the filtering performance can be enhanced along with richer information of membership functions being utilized.
4) The strictly dissipative full-order and reduced-order filtering design approaches in terms of SOS are proposed for the delayed IT2 polynomial fuzzy systems, which cover the design of H_∞ filtering and passive filtering as the special cases.

The remainder of this paper is organized as follows. The problem formulation and some preliminaries are depicted in Section II. The main results are shown in Section III. Section IV presents an example to demonstrate the effectiveness of the filtering technique reported. Our conclusions are summarized in Section V.

Notation: $P > 0 (\geq 0)$ signifies that P is real symmetric and positive definite (semidefinite); $diag(\ldots)$ represents a block-diagonal matrix; $\star$ is used to denote an element induced by symmetry; $g(x(t))$ is called as a polynomial of $x(t)$ if $g(x(t))$ can be represented as finite linear combination of monomials with real coefficients; $g(x(t)) = \sum_{i=1}^{m} z_i^2(x(t))$ signifies $g(x(t))$ can be depicted in the form of SOS as well as $g(x(t)) \geq 0$. Matrices without explicit definition of the dimensions, are assumed to be compatible for algebraic operations.

II. PROBLEM FORMULATION AND PRELIMINARIES

A. IT2 Polynomial Fuzzy Model with Time-varying Delay

Consider a nonlinear system subject to uncertainties and time-varying delay which can be characterized by the following IT2 polynomial fuzzy model.

♦ **Plant Form:**

Rule i: IF $\theta_1(x(t))$ is $\tilde{M}_1^i$ and $\theta_2(x(t))$ is $\tilde{M}_2^i$ and $\cdots$ and $\theta_\Psi(x(t))$ is $\tilde{M}_\Psi^i$, THEN

$$\begin{cases} \dot{x}(t) = A_i(x(t))\hat{x}(x(t)) + A_{di}(x(t))\hat{x}(x(t-d(t))) \\ \qquad\quad +B_i(x(t))\omega(t), \\ y(t) = C_i(x(t))\hat{x}(x(t)) + C_{di}(x(t))\hat{x}(x(t-d(t))) \\ \qquad\quad +D_i(x(t))\omega(t), \\ z(t) = E_i(x(t))\hat{x}(x(t)) + E_{di}(x(t))\hat{x}(x(t-d(t))) \\ x(t) = \varphi(t), \quad t \in \left[-\bar{d}, 0\right] \end{cases}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $\hat{x}(x(t)) \in \mathbb{R}^n$ is a vector of monomials in $x(t)$; $y(t) \in \mathbb{R}^p$ is the measurement output; $z(t) \in \mathbb{R}^q$ is the signal to be estimated; $\omega(t) \in \mathbb{R}^l$ denotes the disturbance signal belonging to $\mathcal{L}_2[0,\infty)$; $\theta_\alpha(x(t))$ and $\tilde{M}_\alpha^i$, $i = 1, 2, \ldots, r$, $\alpha = 1, 2, \ldots, \Psi$, signify the premise variables and IT2 fuzzy sets, respectively; $d(t)$ signifies the time-varying delay satisfying $d(t) \in (0, \bar{d}]$, $\dot{d}(t) \leq \tau$; $\varphi(t)$ is the initial condition. $A_i(x(t))$, $A_{di}(x(t))$, $B_i(x(t))$, $C_i(x(t))$, $C_{di}(x(t))$, $D_i(x(t))$, $E_i(x(t))$, $E_{di}(x(t))$ are known polynomial matrices. The firing strength of rule i is determined by the following interval sets

$$\mathcal{W}_i(x(t)) = [\underline{w}_i(x(t)),\ \bar{w}_i(x(t))], \quad i = 1, 2, \ldots, r$$

where $\underline{w}_i(x(t)) = \prod_{\alpha=1}^{\Psi} \underline{\mu}_{\tilde{M}_\alpha^i}(\theta_\alpha(x(t)))$ and $\bar{w}_i(x(t)) = \prod_{\alpha=1}^{\Psi} \bar{\mu}_{\tilde{M}_\alpha^i}(\theta_\alpha(x(t)))$ signify the lower and upper grades of membership respectively. $\underline{\mu}_{\tilde{M}_\alpha^i}(\theta_\alpha(x(t)))$ and $\bar{\mu}_{\tilde{M}_\alpha^i}(\theta_\alpha(x(t)))$ represent the lower and upper membership functions respectively and possess the property of $\bar{\mu}_{\tilde{M}_\alpha^i}(\theta_\alpha(x(t))) \geq \underline{\mu}_{\tilde{M}_\alpha^i}(\theta_\alpha(x(t))) \geq 0$, resulting in $\bar{w}_i(x(t)) \geq \underline{w}_i(x(t)) \geq 0$.

The inferred delayed IT2 polynomial fuzzy model is depicted as follows

$$\begin{cases} \dot{x}(t)=\sum_{i=1}^{r} w_i(x(t))\{A_i(x(t))\hat{x}(x(t))+A_{di}(x(t))\hat{x}(x(t-d(t))) \\ \qquad +B_i(x(t))\omega(t)\}, \\ y(t)=\sum_{i=1}^{r} w_i(x(t))\{C_i(x(t))\hat{x}(x(t))+C_{di}(x(t))\hat{x}(x(t-d(t))) \\ \qquad +D_i(x(t))\omega(t)\}, \\ z(t)=\sum_{i=1}^{r} w_i(x(t))\{E_i(x(t))\hat{x}(x(t))+E_{di}(x(t))\hat{x}(x(t-d(t)))\} \\ x(t)=\varphi(t), \quad t\in\left[-\bar{d},0\right] \end{cases} \tag{1}$$

where $w_i(x(t)) = \underline{\lambda}_i(x(t))\underline{w}_i(x(t)) + \bar{\lambda}_i(x(t))\bar{w}_i(x(t)) \geq 0$, $\sum_{i=1}^{r} w_i(x(t)) = 1$, in which $\underline{\lambda}_i(x(t)) \in [0,1]$ and $\bar{\lambda}_i(x(t)) \in [0,1]$ are nonlinear weighting functions with the property of $\underline{\lambda}_i(x(t)) + \bar{\lambda}_i(x(t)) = 1$ for all i.

B. IT2 Polynomial Fuzzy Filter

In order to prompt design flexibility, the IT2 polynomial fuzzy filters are considered under the IPM concept. The details of the IT2 polynomial fuzzy filter are presented by

♦ **Filter Form:**

Rule j: IF $g_1(x(t))$ is $\tilde{N}_1^j$ and $g_2(x(t))$ is $\tilde{N}_2^j$ and $\cdots$ and $g_\Omega(x(t))$ is $\tilde{N}_\Omega^j$, THEN

$$\begin{cases} \dot{x}_f(t) = A_{fj}(x_f(t))x_f(t) + B_{fj}(x_f(t))y(t) \\ z_f(t) = E_{fj}(x_f(t))x_f(t) \end{cases}$$

where $x_f \in \mathbb{R}^k$ denotes the filter's state vector with $k \leq n$; $z_f \in \mathbb{R}^q$ represents the estimate signal of $z(t)$; $g_\beta(x(t))$ and $\tilde{N}_\beta^j$, $j = 1,2,\dots,c$, $\beta = 1,2,\dots,\Omega$ signify the premise variables and IT2 fuzzy sets respectively; $A_{fj}(x_f(t))$, $B_{fj}(x_f(t))$, $E_{fj}(x_f(t))$ are the filter gain matrices to be determined. The firing strength of rule j is determined by the following interval sets

$$\mathcal{M}_j(x(t)) = [\underline{m}_j(x(t)),\ \bar{m}_j(x(t))], \quad j = 1,2,\dots,c$$

where $\underline{m}_j(x(t)) = \prod_{\beta=1}^{\Omega} \underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(x(t))$ and $\bar{m}_j(x(t)) = \prod_{\beta=1}^{\Omega} \bar{\mu}_{\tilde{N}_\beta^j}(g_\beta(x(t))$ stand for the lower and upper grades of membership respectively. $\underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(x(t)))$ and $\bar{\mu}_{\tilde{N}_\beta^j}(g_\beta(x(t)))$ represent the lower and upper membership functions respectively and equip with the property of $\bar{\mu}_{\tilde{N}_\beta^j}(g_\beta(x(t))) \geq \underline{\mu}_{\tilde{N}_\beta^j}(g_\beta(x(t))) \geq 0$, resulting in $\bar{m}_j(x(t)) \geq \underline{m}_j(x(t)) \geq 0$. The overall IT2 polynomial fuzzy filter is depicted as

$$\begin{cases} \dot{x}_f(t)=\sum_{j=1}^{c} m_j(x(t))\left\{A_{fj}(x_f(t))x_f(t) + B_{fj}(x_f(t))y(t)\right\}, \\ z_f(t)=\sum_{j=1}^{c} m_j(x(t))\left\{E_{fj}(x_f(t))x_f(t)\right\}, \end{cases} \tag{2}$$

where $m_j(x(t)) = \frac{\underline{\beta}_j(x(t))\underline{m}_j(x(t))+\bar{\beta}_j(x(t))\bar{m}_j(x(t))}{\sum_{k=1}^{c}\underline{\beta}_k(x(t))\underline{m}_k(x(t))+\bar{\beta}_k(x(t))\bar{m}_k(x(t))}$, $\sum_{j=1}^{c} m_j(x(t)) = 1$, in which $\underline{\beta}(x(t)) \in [0,1]$ and $\bar{\beta}(x(t)) \in [0,1]$ are nonlinear weighting functions serving as the type-reduction purpose, which have the property of $\underline{\beta}_j(x(t)) + \bar{\beta}_j(x(t)) = 1$ for all j [20], [29].

C. IT2 Polynomial Fuzzy Filtering Error System

Considering the plant (1) to include the filter system (2) with the property of $\sum_{i=1}^{r}\sum_{j=1}^{c} w_i(x(t))m_j(x(t)) = \sum_{i=1}^{r} w_i(x(t)) = \sum_{j=1}^{c} m_j(x(t)) = 1$, we obtain the IT2 polynomial fuzzy filtering error system as follows

$$\begin{cases} \dot{\bar{x}}(t)=\sum_{i=1}^{r}\sum_{j=1}^{c} h_{ij}(x(t))\{\bar{A}_{ij}\hat{\bar{x}}(\bar{x}(t))+\bar{A}_{dij}\hat{\bar{x}}(\bar{x}(t-d(t)))+\bar{B}_{ij}\omega(t)\} \\ e(t)=\sum_{i=1}^{r}\sum_{j=1}^{c} h_{ij}(x(t))\{\bar{E}_{ij}\hat{\bar{x}}(\bar{x}(t)) + \bar{E}_{di}\hat{\bar{x}}(\bar{x}(t-d(t)))\} \end{cases} \tag{3}$$

where $\bar{x}(t) = [x^T(t), x_f^T(t)]^T$, $\hat{\bar{x}}(t) = [\hat{x}^T(x(t)), x_f^T(t)]^T$, $e(t) = z(t) - z_f(t)$, $h_{ij}(x(t)) = w_i(x(t))m_j(x(t))$ and

$$\bar{A}_{ij} = \begin{bmatrix} A_i(x(t)) & 0 \\ B_{fj}(x_f(t))C_i(x(t)) & A_{fj}(x_f(t)) \end{bmatrix},$$
$$\bar{A}_{dij} = \begin{bmatrix} A_{di}(x(t)) & 0 \\ B_{fj}(x_f(t))C_{di}(x(t)) & 0 \end{bmatrix}, \bar{B}_{ij} = \begin{bmatrix} B_i(x(t)) \\ B_{fj}(x_f(t))D_i(x(t)) \end{bmatrix},$$
$$\bar{E}_{ij} = \begin{bmatrix} E_i(x(t)) & -E_{fj}(x_f(t)) \end{bmatrix}, \bar{E}_{di} = \begin{bmatrix} E_{di}(x(t)) & 0 \end{bmatrix}.$$

In order to facilitate the performance analysis, we partition the whole state space of interest Ψ into $\mathcal{L}$ connected substate space expressed by Ψ_l, $l = 1,2,\dots,\mathcal{L}$, such that $\Psi = \cup_{l=1}^{\mathcal{L}}\Psi_l$. Besides, $h_{ij}(x(t))$ is reconstructed as follows [21]

$$h_{ij}(x(t)) = \underline{\gamma}_{ij}(x(t))\underline{h}_{ij}(x(t)) + \bar{\gamma}_{ij}(x(t))\bar{h}_{ij}(x(t)) \tag{4}$$

where $\underline{\gamma}_{ij}(x(t)) \in [0,1]$ and $\bar{\gamma}_{ij}(x(t)) \in [0,1]$ are nonlinear weighting functions which are unnecessary to be known but exist, equipping with the characteristic of $\underline{\gamma}_{ij}(x(t)) + \bar{\gamma}_{ij}(x(t)) = 1$ for all i,j. The functions $\underline{h}_{ij}(x(t))$ and $\bar{h}_{ij}(x(t))$ are denoted as the lower and upper bounds of $h_{ij}(x(t))$ and possess the property that $\underline{h}_{ij}(x(t)) \leq h_{ij}(x(t)) \leq \bar{h}_{ij}(x(t))$ for all i,j. They are defined as follows referring to [21].

$$\underline{h}_{ij}(x(t))=\sum_{l=1}^{\mathcal{L}}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2}\prod_{g=1}^{n}\upsilon_{gi_gl}(x_g(t))\underline{\delta}_{iji_1i_2\dots i_nl} \tag{5}$$

$$\bar{h}_{ij}(x(t))=\sum_{l=1}^{\mathcal{L}}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2}\prod_{g=1}^{n}\upsilon_{gi_gl}(x_g(t))\bar{\delta}_{iji_1i_2\dots i_nl} \tag{6}$$

in which $0 \leq \underline{\delta}_{iji_1i_2\dots i_nl} \leq \bar{\delta}_{iji_1i_2\dots i_nl} \leq 1$ are constant scalars to be determined; $\upsilon_{gi_gl}(x_g(t)) \in [0,1]$ and $\upsilon_{g1l}(x_g(t)) + \upsilon_{g2l}(x_g(t)) = 1$ for all g,l, $x(t) \in \Psi_l$. Otherwise, $\upsilon_{gi_gl}(x_g(t)) = 0$. Therefore, it is easy to obtain the property of $\sum_{l=1}^{\mathcal{L}}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2}\prod_{g=1}^{n}\upsilon_{gi_gl}(x_g(t)) = 1$, which is useful in the following deduction. For simplification, we remove t, $x(t)$, $x_f(t)$ from the notations for the situation without ambiguity. For instance, $\bar{x}(t)$, $A_i(x(t))$ and $A_{fj}(x_f(t))$ are represented as $\bar{x}$, A_i and A_{fj} respectively.

Before stepping into the next section, we introduce the following definition and lemma first.

Definition 1. *[30] Given a scalar α and constant matrices $\mathcal{X} \leq 0$, $\mathcal{Y}$ and $\mathcal{Z} = \mathcal{Z}^T$, the IT2 polynomial filtering error system (3) is called strictly $(\mathcal{X},\mathcal{Y},\mathcal{Z})$-$\alpha$ dissipative if for any $t_f > 0$ under zero initial condition, the following inequality holds*

$$\int_0^{t_f}\left[e^T(t)\mathcal{X}e(t) + 2e^T(t)\mathcal{Y}w(t) + w^T(t)\mathcal{Z}w(t)\right]dt \\ \geq \alpha\int_0^{t_f} w^T(t)w(t)dt \tag{7}$$

Without loss of generality, it is assumed that $\mathcal{X}=-\bar{\mathcal{X}}^T\bar{\mathcal{X}}\le 0$ for some $\bar{\mathcal{X}}\ge 0$.

Lemma 1. *[31] For any matrix $\begin{bmatrix} R & S \\ \star & R\end{bmatrix}\ge 0$, scalar $d(t)\in(0,\bar{d}]$ and a vector function $\dot{\hat{\bar{x}}}:[-\bar{d},0)\longrightarrow\mathbb{R}^{n+k}$ such that the integration in the following inequality is well defined, then*

$$-\bar{d}\int_{t-\bar{d}}^{t}\dot{\hat{\bar{x}}}^T(w)R\dot{\hat{\bar{x}}}dw\le v^T(t)\triangle v(t) \tag{8}$$

in which

$$\triangle=\begin{bmatrix} -R & R-S & S \\ \star & -2R+S+S^T & R-S \\ \star & \star & -R\end{bmatrix}$$
$$v(t)=\begin{bmatrix}\hat{\bar{x}}^T(t) & \hat{\bar{x}}^T(t-d(t)) & \hat{\bar{x}}^T(t-\bar{d})\end{bmatrix}^T$$

The purpose of this paper is to develop IT2 polynomial fuzzy filters to satisfy the following conditions.

1) The IT2 polynomial fuzzy filtering error system in (3) is asymptotically stable with $w(t)=0$.
2) The IT2 polynomial fuzzy filtering error system in (3) satisfies strictly dissipativity in the sense of Definition 1.

III. MAIN RESULTS

A. Strictly Dissipative Analysis

Based on Lyapunov-Krasovskii functional method, the asymptotical stability and strict dissipativity performance for the delayed IT2 polynomial filter error system in (3) are discussed in this section. The mismatched membership functions and time-varying delay are taken into account in the deduction of sufficient conditions. In order to facilitate analysis, let us first consider the following IT2 polynomial dynamics:

$$\begin{cases}\dot{\hat{\bar{x}}}(\bar{x}(t))=\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Big\{\tilde{A}_{ij}\hat{\bar{x}}(\bar{x}(t))+\tilde{A}_{dij}N\hat{\bar{x}}(\bar{x}(t-d(t))) \\ \qquad +\tilde{B}_{ij}\omega(t)\Big\} \\ e(t)=\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Big\{\tilde{E}_{ij}\hat{\bar{x}}(\bar{x}(t))+\tilde{E}_{di}\hat{\bar{x}}(\bar{x}(t-d(t)))\Big\}\end{cases} \tag{9}$$

where

$$\tilde{A}_{ij}=\begin{bmatrix}\tilde{A}_i & 0 \\ B_{fj}C_i & A_{fj}\end{bmatrix},\tilde{A}_{dij}=\begin{bmatrix}\tilde{A}_{di} \\ B_{fj}C_{di}\end{bmatrix},\tilde{B}_{ij}=\begin{bmatrix}\tilde{B}_i \\ B_{fj}D_i\end{bmatrix},$$
$$\tilde{E}_{ij}=\bar{E}_{ij},\quad \tilde{E}_{di}=E_{di}N,\quad N=[I,0],\quad \tilde{A}_i=\mathcal{T}A_i,$$
$$\tilde{A}_{di}=\mathcal{T}A_{di},\quad \tilde{B}_i=\mathcal{T}B_i,$$

and the ij-th element of polynomial matrix $\mathcal{T}\in\mathbb{R}^{n\times n}$ is defined as:

$$\mathcal{T}_{ij}=\frac{\partial\hat{x}_i(x)}{\partial x_j},i=1,2,\ldots,n,j=1,2,\ldots,n.$$

To lighten notations in the derivation, we rewrite the above IT2 polynomial dynamics as the following equivalent form

$$\begin{cases}\dot{\hat{\bar{x}}}(t)=\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Gamma_{1ij}z(t), \\ e(t)=\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Gamma_{2ij}z(t),\end{cases} \tag{10}$$

in which $z(t)=[\hat{\bar{x}}^T(\bar{x}(t))\ \hat{\bar{x}}^T(\bar{x}(t-d(t))\ \hat{\bar{x}}^T(\bar{x}(t-\bar{d}))\ w^T(t)]^T$, $\Gamma_{1ij}=[\tilde{A}_{ij}\ \tilde{A}_{dij}N\ 0\ \tilde{B}_{ij}]$ and $\Gamma_{2ij}=[\tilde{E}_{ij}\ \tilde{E}_{di}\ 0\ \ 0]$.

Theorem 1. *Consider the time-varying delay IT2 polynomial fuzzy system in (1) and suppose the filter parameter matrices in (2) are known in advance. Given scalars α, $\bar{d}$, τ, $\underline{\delta}_{iji_1i_2\ldots i_nl}$, $\bar{\delta}_{iji_1i_2\ldots i_nl}$ and matrices $\mathcal{X}=-\bar{\mathcal{X}}^T\bar{\mathcal{X}}\le 0$, $\mathcal{Y}$ and $\mathcal{Z}=\mathcal{Z}^T$, the IT2 polynomial fuzzy filtering error system in (3) is asymptotically stable and satisfies strict $(\mathcal{X},\mathcal{Y},\mathcal{Z})$-$\alpha$ dissipativity, if there exist matrices $P>0$, $Q>0$, $R>0$, S, $M(\bar{x})=M^T(\bar{x})$, $W_{ij}(\bar{x})=W_{ij}^T(\bar{x})\ge 0$ and $T(x)=T^T(x)\ge 0$ with appropriate dimensions satisfying the following SOS-based conditions for $i=1,2,\ldots,r$, $j=1,2,\ldots,c$.*

$$v_1^T(P-\epsilon_1 I)v_1 \quad \textit{is SOS} \tag{11}$$
$$v_2^T(Q-\epsilon_2 I)v_2 \quad \textit{is SOS} \tag{12}$$
$$v_3^T\left(\begin{bmatrix} R & S \\ \star & R\end{bmatrix}-\epsilon_3 I\right)v_3 \quad \textit{is SOS} \tag{13}$$
$$v_4^T(T(x)-\epsilon_4(x)I)v_4 \quad \textit{is SOS} \tag{14}$$
$$v_4^T(W_{ij}(\bar{x})-\epsilon_5(\bar{x})I)v_4 \quad \textit{is SOS} \tag{15}$$
$$v_4^T\big(\Omega_{ij}+W_{ij}(\bar{x})+M(\bar{x})-\epsilon_6(\bar{x})I\big)v_4 \quad \textit{is SOS} \tag{16}$$
$$\begin{aligned}&-v_4^T\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}(\bar{\delta}_{iji_1i_2\ldots i_nl}\Omega_{ij}-(\underline{\delta}_{iji_1i_2\ldots i_nl}-\bar{\delta}_{iji_1i_2\ldots i_nl})W_{ij}(\bar{x}) \\ &\quad+\bar{\delta}_{iji_1i_2\ldots i_nl}M(\bar{x}))-M(\bar{x})+(x-\underline{x}_l)^T\Lambda(\bar{x}_l-x)T(x) \\ &\qquad+\epsilon_7(\bar{x})I\Big)v_4 \quad \textit{is SOS},\quad \forall i_1,i_2,\ldots,i_n,l\end{aligned} \tag{17}$$

where v_1, v_2, v_3 and v_4 are arbitrary vectors independent of $\bar{x}$; ϵ_1, ϵ_2, ϵ_3, $\epsilon_4(x)$, $\epsilon_5(\bar{x})$, $\epsilon_6(\bar{x})$ and $\epsilon_7(\bar{x})$ are predefined and nonnegative scalars. In addition,

$$\Omega_{ij}=\begin{bmatrix}\bar{\Phi}_{ij} & \bar{d}\Sigma_{1ij}^T & \Sigma_{2ij}^T \\ \star & -R & 0 \\ \star & \star & -I\end{bmatrix},\bar{\Phi}_{ij}=\begin{bmatrix}\Phi_{ij}^{11} & \Phi_{ij}^{12} & N^TS & P\tilde{B}_{ij}-\tilde{E}_{ij}^T\mathcal{Y} \\ \star & \Phi^{22} & R-S & -E_{di}^T\mathcal{Y} \\ \star & \star & -R & 0 \\ \star & \star & \star & \alpha I-\mathcal{Z}\end{bmatrix},$$
$$\Phi_{ij}^{11}=P\tilde{A}_{ij}+\tilde{A}_{ij}^TP+N^T(Q-R)N,$$
$$\Phi_{ij}^{12}=P\tilde{A}_{dij}+N^T(R–S),\ \Phi^{22}=(\tau-1)Q–2R+S+S^T,$$
$$\Sigma_{1ij}=\big[RN\tilde{A}_{ij}\ RN\tilde{A}_{dij}\ 0\ RN\tilde{B}_{ij}\big],\Sigma_{2ij}=\big[\ \bar{\mathcal{X}}\tilde{E}_{ij}\ \ \bar{\mathcal{X}}E_{di}\ \ 0\ \ 0\big].$$

Proof. Considering time-varying delay existing in the plant system, we construct the Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}V(t)&=V_1(t)+V_2(t)+V_2(t), \\ V_1(t)&=\hat{\bar{x}}^T(\bar{x}(t))P\hat{\bar{x}}(\bar{x}(t)), \\ V_2(t)&=\int_{t-d(t)}^{t}\hat{\bar{x}}^T(\bar{x}(w))N^TQN\hat{\bar{x}}(\bar{x}(w))dw, \\ V_3(t)&=\bar{d}\int_{-\bar{d}}^{0}\int_{t+w}^{t}\dot{\hat{\bar{x}}}^T(\bar{x}(v))N^TRN\dot{\hat{\bar{x}}}(\bar{x}(v))dvdw,\end{aligned} \tag{18}$$

in which $P>0$, $Q>0$, $R>0$. To make stability analysis more succinct, we introduce an augment vector $\xi(t)=[\xi_1^T(t)\ \xi_2^T(t)\ \xi_3^T(t)\ \xi_4^T(t)]^T$, where $\xi_1(t)=\hat{\bar{x}}(\bar{x}(t))$, $\xi_2(t)=N\hat{\bar{x}}(\bar{x}(t-d(t)))$, $\xi_3(t)=N\hat{\bar{x}}(\bar{x}(t-\bar{d}))$, $\xi_4(t)=w(t)$. Then, it is easy to obtain the derivative of $V_1(t)$, $V_2(t)$ and $V_3(t)$ as follows:

$$\begin{aligned}\dot{V}_1(t)&=\dot{\hat{\bar{x}}}^T(\bar{x}(t))P\hat{\bar{x}}(\bar{x}(t))+\hat{\bar{x}}^T(\bar{x}(t))P\dot{\hat{\bar{x}}}(\bar{x}(t)) \\ &=\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}(z^T(t)\Gamma_{1ij}^TP\hat{\bar{x}}(\bar{x}(t))+\hat{\bar{x}}^T(\bar{x}(t))P\Gamma_{1ij}z(t)),\end{aligned}$$

$$\begin{aligned}\dot{V}_2(t)\leq&\hat{\bar{x}}^T(\bar{x}(t))N^TQN\hat{\bar{x}}(\bar{x}(t))\\&-(1-\tau)\hat{\bar{x}}^T(\bar{x}(t-d(t)))N^TQN\hat{\bar{x}}(\bar{x}(t-d(t)))\\=&\xi_1^T(t)N^TQN\xi_1(t)-(1-\tau)\xi_2^T(t)Q\xi_2(t),\\\dot{V}_3(t)=&\bar{d}^2\dot{\hat{\bar{x}}}^T(\bar{x}(t))N^TRN\dot{\hat{\bar{x}}}(\bar{x}(t))\\&-\bar{d}\int_{t-\bar{d}}^{t}\dot{\hat{\bar{x}}}^T(\bar{x}(v))N^TRN\dot{\hat{\bar{x}}}(\bar{x}(v))dv.\end{aligned}$$

Applying Lemma 1 to the integral term in $\dot{V}_3(t)$, we have

$$\begin{aligned}\dot{V}_3(t)\leq&\bar{d}^2(\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}z^T(t)\Gamma_{1ij}^T)N^TRN(\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Gamma_{1ij}z(t))\\&+\xi^T(t)\mathcal{J}\xi(t)\end{aligned}$$

in which

$$\mathcal{J}=\begin{bmatrix}-N^TRN & N^T(R-S) & N^TS & 0\\ \star & -2R+S+S^T & R-S & 0\\ \star & \star & -R & 0\\ \star & \star & \star & 0\end{bmatrix}.$$

Based on the deduction above, the time derivative of $V(t)$ can be derived as follows:

$$\begin{aligned}\dot{V}(t)=&\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)\\\leq&\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Big(z^T(t)\Gamma_{1ij}^TP\hat{\bar{x}}(\bar{x}(t))+\hat{\bar{x}}^T(\bar{x}(t))P\Gamma_{1ij}z(t)\Big)\\&+\xi_1^T(t)N^TQN\xi_1(t)-(1-\tau)\xi_2^T(t)Q\xi_2(t)+\xi^T(t)\mathcal{J}\xi(t)\\&+\bar{d}^2\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}z^T(t)\Gamma_{1ij}^T\Big)N^TRN\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Gamma_{1ij}z(t)\Big)\\=&\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\xi^T(t)\Phi_{ij}\xi(t)\\&+\bar{d}^2\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\xi^T(t)\Sigma_{1ij}^T\Big)R^{-1}\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Sigma_{1ij}\xi(t)\Big),\end{aligned}\tag{19}$$

where

$$\Phi_{ij}=\begin{bmatrix}\Phi_{ij}^{11} & \Phi_{ij}^{12} & N^TS & P\tilde{B}_{ij}\\ \star & \Phi^{22} & R-S & 0\\ \star & \star & -R & 0\\ \star & \star & \star & 0\end{bmatrix},$$

in which

$$\Phi_{ij}^{11}=P\tilde{A}_{ij}+\tilde{A}_{ij}^TP+N^T(Q-R)N,\ \Phi^{22}=(\tau-1)Q-2R+S+S^T,$$
$$\Phi_{ij}^{12}=P\tilde{A}_{dij}+N^T(R-S),\ \Sigma_{1ij}=\big[RN\tilde{A}_{ij}\ RN\tilde{A}_{dij}\ 0\ RN\tilde{B}_{ij}\big].$$

In order to set up the strict dissipativity performance of IT2 polynomial filtering error system in (3) based on Definition 1, we introduce the following performance index

$$H(t)=\dot{V}(t)-e^T(t)\mathcal{X}e(t)-2e^T(t)\mathcal{Y}w(t)-w^T(t)(\mathcal{Z}-\alpha I)w(t).\tag{20}$$

Then, by taking (9), (19) and (20) into account, one can get

$$\begin{aligned}H(t)\leq&\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\xi^T(t)\bar{\Phi}_{ij}\xi(t)\\&+\bar{d}^2\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\xi^T(t)\Sigma_{1ij}^T\Big)R^{-1}\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Sigma_{1ij}\xi(t)\Big)\\&+\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\xi^T(t)\Sigma_{2ij}^T\Big)\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Sigma_{2ij}\xi(t)\Big)\end{aligned}$$

where

$$\bar{\Phi}_{ij}=\begin{bmatrix}\Phi_{ij}^{11} & \Phi_{ij}^{12} & N^TS & P\tilde{B}_{ij}-\tilde{E}_{ij}^T\mathcal{Y}\\ \star & \Phi^{22} & R-S & -E_{di}^T\mathcal{Y}\\ \star & \star & -R & 0\\ \star & \star & \star & \alpha I-\mathcal{Z}\end{bmatrix},\ \Sigma_{2ij}=\big[\bar{\mathcal{X}}\tilde{E}_{ij}\ \bar{\mathcal{X}}E_{di}\ 0\ 0\big].$$

The other symbols are the same as defined in Theorem 1.

The index $H(t)\leq 0$ can be ensured by the following condition with the help of Schur complement [32]

$$\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Omega_{ij}\leq 0\tag{21}$$

where

$$\Omega_{ij}=\begin{bmatrix}\bar{\Phi}_{ij} & \bar{d}\Sigma_{1ij}^T & \Sigma_{2ij}^T\\ \star & -R & 0\\ \star & \star & -I\end{bmatrix}.$$

To reduce the conservatism, the information of IT2 membership functions are employed to deduce the stability conditions in terms of (4) and the slack polynomial matrices $M(\bar{x})=M^T(\bar{x})$ and $W_{ij}(\bar{x})=W_{ij}^T(\bar{x})\geq 0$, $i=1,2,\ldots,r$, $j=1,2,\ldots,c$ are introduced via the following expression

$$\begin{cases}\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}(\underline{\gamma}_{ij}\underline{h}_{ij}+\bar{\gamma}_{ij}\bar{h}_{ij})-1\Big)M(\bar{x})=0,\\ -\sum_{i=1}^{r}\sum_{j=1}^{c}(1-\underline{\gamma}_{ij})(\underline{h}_{ij}-\bar{h}_{ij})W_{ij}(\bar{x})\geq 0.\end{cases}\tag{22}$$

Combining (21) and (22), we have

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Omega_{ij}\\\leq&\sum_{i=1}^{r}\sum_{j=1}^{c}\Big(\underline{\gamma}_{ij}\underline{h}_{ij}+(1-\underline{\gamma}_{ij})\bar{h}_{ij}\Big)\Omega_{ij}\\&+\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}(\underline{\gamma}_{ij}\underline{h}_{ij}+(1-\underline{\gamma}_{ij})\bar{h}_{ij})-1\Big)M(\bar{x})\\&-\sum_{i=1}^{r}\sum_{j=1}^{c}(1-\underline{\gamma}_{ij})(\underline{h}_{ij}-\bar{h}_{ij})W_{ij}(\bar{x})\\=&\sum_{i=1}^{r}\sum_{j=1}^{c}\underline{\gamma}_{ij}(\underline{h}_{ij}-\bar{h}_{ij})\big(\Omega_{ij}+W_{ij}(\bar{x})+M(\bar{x})\big)\\&+\sum_{i=1}^{r}\sum_{j=1}^{c}\Big(\bar{h}_{ij}\Omega_{ij}-(\underline{h}_{ij}-\bar{h}_{ij})W_{ij}(\bar{x})+\bar{h}_{ij}M(\bar{x})\Big)-M(\bar{x}).\end{aligned}\tag{23}$$

By virtue of the result (23) and the fact $\underline{\gamma}_{ij}(\underline{h}_{ij}-\bar{h}_{ij})\leq 0$, $\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\Omega_{ij}\leq 0$ is guaranteed if the following inequalities hold

$$\Omega_{ij}+M(\bar{x})+W_{ij}(\bar{x})>0,\ \forall i,j,\tag{24}$$

$$\sum_{i=1}^{r}\sum_{j=1}^{c}\big(\bar{h}_{ij}\Omega_{ij}-(\underline{h}_{ij}-\bar{h}_{ij})W_{ij}(\bar{x})+\bar{h}_{ij}M(\bar{x})\big)-M(\bar{x})<0.\tag{25}$$

The inequality (24) can be directly solved by SOSTOOLS, but the inequality (25) cannot owing to the existence of the lower and upper bounds of the membership functions

$\underline{h}_{ij}(x)$ and $\bar{h}_{ij}(x)$. Therefore, rewriting the above mentioned condition (25) in light of (5) and (6), one can get the following equivalent inequality

$$\sum_{l=1}^{\mathcal{L}}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2}\prod_{g=1}^{n}\upsilon_{gi_gl}(x_g(t))\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}\big(\bar{\delta}_{iji_1i_2\ldots i_nl}\Omega_{ij} -(\underline{\delta}_{iji_1i_2\ldots i_nl}-\bar{\delta}_{iji_1i_2\ldots i_nl})W_{ij}(\bar{x})+\bar{\delta}_{iji_1i_2\ldots i_nl}M(\bar{x})\big) -M(\bar{x})\Big)<0. \tag{26}$$

Recalling the property that $\upsilon_{gi_gl}(x_g(t)) \in [0,1]$ and $\sum_{l=1}^{\mathcal{L}}\sum_{i_1=1}^{2}\sum_{i_2=1}^{2}\cdots\sum_{i_n=1}^{2}\prod_{g=1}^{n}\upsilon_{gi_gl}(x_g(t))=1$, the inequality (26) can be achieved if the following inequality holds

$$\sum_{i=1}^{r}\sum_{j=1}^{c}\big(\bar{\delta}_{iji_1i_2\ldots i_nl}\Omega_{ij}-(\underline{\delta}_{iji_1i_2\ldots i_nl}-\bar{\delta}_{iji_1i_2\ldots i_nl})W_{ij}(\bar{x}) +\bar{\delta}_{iji_1i_2\ldots i_nl}M(\bar{x})\big)-M(\bar{x})<0,\ \forall i_1,i_2,\ldots,i_n,l, \tag{27}$$

where $\underline{\delta}_{iji_1i_2\ldots i_nl}$ and $\bar{\delta}_{iji_1i_2\ldots i_nl}$ are constant scalars, which allow the inequality (27) to be solved by SOSTOOLS.

In addition, in order to further relax the criteria obtained, the state information stemmed from each subregion is also considered into stability analysis. Define slack matrices $T(x)=T^T(x)\geq 0$ and assume that $\underline{x}_l$ and $\bar{x}_l$, $l=1,2,\ldots,\mathcal{L}$ denote the minimal and maximal bounds of the system state $x(t)$ in the l-th subregion. Hence, we have

$$(x-\underline{x}_l)^T\Lambda(\bar{x}_l-x)T(x)\geq 0,\quad \forall l, \tag{28}$$

where $\Lambda=diag\{\Lambda_1,\ldots,\Lambda_i,\ldots,\Lambda_n\}$, $i=1,2,\ldots,n$, represents a diagonal matrix whose elements are either 0 or 1 to determine whether the ith state information of the lth subspace is included in the stability analysis. That is, when $\Lambda_i=1$, the information of state x_i corresponding to the lth subspace is involved into stability analysis, otherwise not. Considering (28), the inequality (27) is guaranteed if one can get

$$\sum_{i=1}^{r}\sum_{j=1}^{c}\big(\bar{\delta}_{iji_1i_2\ldots i_nl}\Omega_{ij}-(\underline{\delta}_{iji_1i_2\ldots i_nl}-\bar{\delta}_{iji_1i_2\ldots i_nl})W_{ij}(\bar{x}) +\bar{\delta}_{iji_1i_2\ldots i_nl}M(\bar{x})\big)-M(\bar{x})+(x-\underline{x}_l)^T\Lambda(\bar{x}_l-x)T(x)<0.$$

Hence, it can be concluded that if the conditions (11)-(17) of Theorem 1 are feasible, we have $H(t)\leq 0$, which means

$$\dot{V}(t)-e^T(t)\mathcal{X}e(t)-2e^T(t)\mathcal{Y}w(t)-w^T(t)(\mathcal{Z}-\alpha I)w(t)\leq 0. \tag{29}$$

Integrating both sides of the aforementioned inequality from 0 to t_f yields

$$V(t_f)-V(0)+\int_0^{t_f}\big(-e^T(t)\mathcal{X}e(t)-2e^T(t)\mathcal{Y}w(t) -w^T(t)(\mathcal{Z}-\alpha I)w(t)\big)dt\leq 0.$$

According to $V(0)=0$ and $V(t_f)\geq 0$, for nonzero $w(t)$, we can deduce the following inequality

$$\int_0^{t_f}(e^T(t)\mathcal{X}e(t)+2e^T(t)\mathcal{Y}w(t)+w^T(t)\mathcal{Z}w(t))dt \geq \alpha\int_0^{t_f}w^T(t)w(t)dt. \tag{30}$$

Therefore, on the basis of Definition 1, we can make a conclusion that the IT2 polynomial filtering error system in (3) is strictly $(\mathcal{X},\mathcal{Y},\mathcal{Z})$-$\alpha$ dissipative. In addition, when $w(t)=0$, considering (29), we have $\dot{V}(t)<0$ for all $\hat{\bar{x}}\neq 0$, which means the IT2 polynomial filtering error system in (3) is asymptotically stability. The proof is completed. □

Remark 1. *When the whole operating domain of interest is partitioned into more subdomains, the approximating membership functions will be more accurate as well as ampler information of membership functions would be incorporated in the stability analysis conditions, which leads to more relaxed results. However, the number of SOS conditions will also increase resulting in a higher computational cost. In a consequence, there is a tradeoff between conservatism reduction and computational complexity.*

B. IT2 Polynomial Fuzzy Filter Design with Strict Dissipativity

The filter parameter matrices cannot be directly obtained from Theorem 1 due to the existing non-convex terms in the conditions (16) and (17). To solve this problem, we develop the following theorem to design the IT2 polynomial fuzzy filter based on the results obtained from Theorem 1 where all the conditions are convex to be solved. First, the full-order dissipative filter will be designed.

Theorem 2. *Consider the time-varying delay IT2 polynomial fuzzy system in (1). Given scalars α, a, $\bar{d}$, τ, $\underline{\delta}_{iji_1i_2\ldots i_nl}$, $\bar{\delta}_{iji_1i_2\ldots i_nl}$ and matrices $\mathcal{X}=-\bar{\mathcal{X}}^T\bar{\mathcal{X}}\leq 0$, $\mathcal{Y}$ and $\mathcal{Z}=\mathcal{Z}^T$, there exists an IT2 polynomial fuzzy filter in (2) to ensure the filtering error system in (3) is asymptotically stable with strictly $(\mathcal{X},\mathcal{Y},\mathcal{Z})$-$\alpha$ dissipative performance, if there exist matrices $U>0$, $V>0$, $Q>0$, $R>0$, S and polynomial matrices $\breve{A}_{fj}\in\mathbb{R}^{n\times n}$, $\breve{B}_{fj}\in\mathbb{R}^{n\times p}$, $\breve{E}_{fj}\in\mathbb{R}^{q\times n}$, $M(\bar{x})=M^T(\bar{x})$, $W_{ij}(\bar{x})=W_{ij}^T(\bar{x})\geq 0$ and $T(x)=T^T(x)\geq 0$ with appropriate dimensions satisfying the following SOS-based conditions for $i=1,2,\ldots,r$, $j=1,2,\ldots,c$.*

$$v_1^T\left(\begin{bmatrix}U & aV\\ \star & V\end{bmatrix}-\epsilon_1 I\right)v_1\ \text{ is SOS} \tag{31}$$

$$v_1^T\left(\begin{bmatrix}R & S\\ \star & R\end{bmatrix}-\epsilon_2 I\right)v_1\ \text{ is SOS} \tag{32}$$

$$v_2^T(Q-\epsilon_3 I)v_2\ \text{ is SOS} \tag{33}$$

$$v_3^T(T(x)-\epsilon_4(x)I)v_3\ \text{ is SOS} \tag{34}$$

$$v_3^T(W_{ij}(\bar{x})-\epsilon_5(\bar{x})I)v_3\ \text{ is SOS} \tag{35}$$

$$v_3^T\big(\tilde{\Omega}_{ij}+W_{ij}(\bar{x})+M(\bar{x})-\epsilon_6(\bar{x})I\big)v_3\ \text{ is SOS} \tag{36}$$

$$-v_3^T\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}(\bar{\delta}_{iji_1i_2\ldots i_nl}\tilde{\Omega}_{ij}-(\underline{\delta}_{iji_1i_2\ldots i_nl}-\bar{\delta}_{iji_1i_2\ldots i_nl})W_{ij}(\bar{x}) +\bar{\delta}_{iji_1i_2\ldots i_nl}M(\bar{x}))-M(\bar{x})+(x-\underline{x}_l)^T\Lambda(\bar{x}_l-x)T(x) +\epsilon_7(\bar{x})I\Big)v_3\text{ is SOS},\quad \forall i_1,i_2,\ldots,i_n,l \tag{37}$$

where v_1, v_2 and v_3 are arbitrary vectors independent of $\bar{x}$; ϵ_1, ϵ_2, ϵ_3, $\epsilon_4(x)$, $\epsilon_5(\bar{x})$, $\epsilon_6(\bar{x})$ and $\epsilon_7(\bar{x})$ are predefined and nonnegative scalars. In addition,

$$\tilde{\Omega}_{ij}=\begin{bmatrix}\tilde{\Phi}_{ij} & \bar{d}\tilde{\Sigma}_{1ij}^T & \tilde{\Sigma}_{2ij}^T\\ \star & -R & 0\\ \star & \star & -I\end{bmatrix},\ \tilde{\Phi}_{ij}=\begin{bmatrix}\tilde{\Phi}_{ij}^{11} & \tilde{\Phi}_{ij}^{12} & \tilde{\Phi}_{ij}^{13} & \tilde{\Phi}_{ij}^{14}\\ \star & \Phi^{22} & R-S & -E_{di}^T\mathcal{Y}\\ \star & \star & -R & 0\\ \star & \star & \star & \alpha I-\mathcal{Z}\end{bmatrix},$$

$$\tilde{\Phi}^{11}_{ij}=\begin{bmatrix}\begin{pmatrix}U\tilde{A}_i+a\breve{B}_{fj}C_i+\tilde{A}_i^TU\\+aC_i^T\breve{B}_{fj}^T+Q-R\end{pmatrix} & a\breve{A}_{fj}+a\tilde{A}_i^TV+C_i^T\breve{B}_{fj}^T\\ \star & \breve{A}_{fj}+\breve{A}_{fj}^T\end{bmatrix},$$

$$\tilde{\Phi}^{12}_{ij}=\begin{bmatrix}U\tilde{A}_{di}+a\breve{B}_{fj}C_{di}+R-S\\ aV\tilde{A}_{di}+\breve{B}_{fj}C_{di}\end{bmatrix},\quad \tilde{\Phi}^{13}_{ij}=\begin{bmatrix}S\\0_{n\times n}\end{bmatrix},$$

$$\tilde{\Phi}^{14}_{ij}=\begin{bmatrix}U\tilde{B}_i+a\breve{B}_{fj}D_i-E_i^T\mathcal{Y}\\ aV\tilde{B}_i+\breve{B}_{fj}D_i+\breve{E}_{fj}^T\mathcal{Y}\end{bmatrix},$$

$$\tilde{\Sigma}_{1ij}=\left[\,R\tilde{A}_i\;\;0_{n\times n}\,|\;R\tilde{A}_{di}\;\;0_{n\times n}\;\;R\tilde{B}_i\,\right],$$

$$\tilde{\Sigma}_{2ij}=\left[\,\bar{\mathcal{X}}E_i\;\;-\bar{\mathcal{X}}\breve{E}_{fj}\,|\;\bar{\mathcal{X}}E_{di}\;\;0_{q\times n}\;\;0_{q\times l}\,\right].$$

The other variables are the same as depicted in Theorem 1. Then, the gain matrices of the IT2 polynomial fuzzy filter in (3) are designed as

$$\begin{bmatrix}A_{fj_{n\times n}} & B_{fj_{n\times p}}\\ E_{fj_{q\times n}} & 0\end{bmatrix}=\begin{bmatrix}V^{-1} & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}\breve{A}_{fj} & \breve{B}_{fj}\\ \breve{E}_{fj} & 0\end{bmatrix}. \quad (38)$$

Proof. In Theorem 1, there exist nonconvex terms tangled by Lyapunov matrix variable P and unknown filter gain matrices. Therefore, a decoupling technique is required to compute the filter gain matrices A_{fj}, B_{fj} and E_{fj}. Based on Theorem 1, since $P>0$, we can partition P in the form of

$$P=\begin{bmatrix}U_{n\times n} & aV\\ \star & V_{n\times n}\end{bmatrix}, \quad (39)$$

where U and V are both positive definite matrices. a is an arbitrary scalar determined by the user. Now, we utilize the matrix P denoted in the format of (39), $A_{fj}=V^{-1}\breve{A}_{fj}$ and $B_{fj}=V^{-1}\breve{B}_{fj}$ to rewrite $P\tilde{A}_{ij}$, $P\tilde{A}_{dij}$ and $P\tilde{B}_{ij}$ in Theorem 1. Then we have

$$P\tilde{A}_{ij}=\begin{bmatrix}U\tilde{A}_i+a\breve{B}_{fj}C_i & a\breve{A}_{fj}\\ aV\tilde{A}_i+\breve{B}_{fj}C_i & \breve{A}_{fj}\end{bmatrix},$$

$$P\tilde{A}_{dij}=\begin{bmatrix}U\tilde{A}_{di}+a\breve{B}_{fj}C_{di}\\ aV\tilde{A}_{di}+\breve{B}_{fj}C_{di}\end{bmatrix},\; P\tilde{B}_{ij}=\begin{bmatrix}U\tilde{B}_i+a\breve{B}_{fj}D_i\\ aV\tilde{B}_i+\breve{B}_{fj}D_i\end{bmatrix}.$$

According to the derivation process of Theorem 1, we can see that $H(t)\le 0$ is satisfied by requiring

$$\sum_{i=1}^{r}\sum_{j=1}^{c}h_{ij}\tilde{\Omega}_{ij}\le 0.$$

The corresponding symbols are defined in Theorem 2. Following the similar deduction built in Theorem 1, we can get Theorem 2. The proof is completed. □

Next, we will propose a reduced-order IT2 polynomial fuzzy filtering design approach equipped with strictly dissipative performance. The reduced-order filter is beneficial to simplifying structural complexity and reduce implementation costs in practical applications.

Theorem 3. *Consider the time-varying delay IT2 polynomial fuzzy system in (1). Given scalars α, a, $\bar{d}$, τ $\underline{\delta}_{iji_1i_2\ldots i_nl}$, $\bar{\delta}_{iji_1i_2\ldots i_nl}$ and matrices $\mathcal{X}=-\bar{\mathcal{X}}^T\bar{\mathcal{X}}\le 0$, $\mathcal{Y}$ and $\mathcal{Z}=\mathcal{Z}^T$, there exists a reduced-order IT2 polynomial fuzzy filter in (2) to ensure the filtering error system in (3) is asymptotically stable with strictly $(\mathcal{X},\mathcal{Y},\mathcal{Z})$-$\alpha$ dissipative performance, if there exist matrices $U>0$, $V>0$, $Q>0$, $R>0$, S and polynomial matrices $\breve{A}_{fj}\in\mathbb{R}^{k\times k}$, $\breve{B}_{fj}\in\mathbb{R}^{k\times p}$, $\breve{E}_{fj}\in\mathbb{R}^{q\times k}$, $M(\bar{x})=M^T(\bar{x})$, $W_{ij}(\bar{x})=W_{ij}^T(\bar{x})\ge 0$ and $T(x)=T^T(x)\ge 0$ with appropriate dimensions satisfying the following SOS-based conditions for $k<n$, $i=1,2,\ldots,r$, $j=1,2,\ldots,c$.*

$$v_1^T\left(\begin{bmatrix}U & aHV\\ \star & V\end{bmatrix}-\epsilon_1I\right)v_1 \;\text{ is SOS} \quad (40)$$

$$v_2^T\left(\begin{bmatrix}R & S\\ \star & R\end{bmatrix}-\epsilon_2I\right)v_2 \;\text{ is SOS} \quad (41)$$

$$v_3^T(Q-\epsilon_3I)v_3 \;\text{ is SOS} \quad (42)$$

$$v_4^T(T(x)-\epsilon_4(x)I)v_4 \;\text{ is SOS} \quad (43)$$

$$v_4^T(W_{ij}(\bar{x})-\epsilon_5(\bar{x})I)v_4 \;\text{ is SOS} \quad (44)$$

$$v_4^T\big(\breve{\Omega}_{ij}+W_{ij}(\bar{x})+M(\bar{x})-\epsilon_6(\bar{x})I\big)v_4 \;\text{ is SOS} \quad (45)$$

$$-v_4^T\Big(\sum_{i=1}^{r}\sum_{j=1}^{c}(\bar{\delta}_{iji_1i_2\ldots i_nl}\breve{\Omega}_{ij}-(\underline{\delta}_{iji_1i_2\ldots i_nl}-\bar{\delta}_{iji_1i_2\ldots i_nl})W_{ij}(\bar{x}) + \bar{\delta}_{iji_1i_2\ldots i_nl}M(\bar{x}))-M(\bar{x})+(x-\underline{x}_l)^T\Lambda(\bar{x}_l-x)T(x) + \epsilon_7(\bar{x})I\Big)v_4 \text{ is SOS}, \quad \forall i_1,i_2,\ldots,i_n,l \quad (46)$$

where v_1, v_2, v_3 and v_4 are arbitrary vectors independent of $\bar{x}$; ϵ_1, ϵ_2, ϵ_3, $\epsilon_4(x)$, $\epsilon_5(\bar{x})$, $\epsilon_6(\bar{x})$ and $\epsilon_7(\bar{x})$ are predefined and nonnegative scalars. In addition,

$$\breve{\Omega}_{ij}=\begin{bmatrix}\breve{\Phi}_{ij} & \bar{d}\tilde{\Sigma}_{1ij}^T & \tilde{\Sigma}_{2ij}^T\\ \star & -R & 0\\ \star & \star & -I\end{bmatrix},\breve{\Phi}_{ij}=\begin{bmatrix}\breve{\Phi}^{11}_{ij} & \breve{\Phi}^{12}_{ij} & \breve{\Phi}^{13}_{ij} & \breve{\Phi}^{14}_{ij}\\ \star & \Phi^{22} & R-S & -E_{di}^T\mathcal{Y}\\ \star & \star & -R & 0\\ \star & \star & \star & \alpha I-\mathcal{Z}\end{bmatrix},$$

$$\breve{\Phi}^{11}_{ij}=\begin{bmatrix}\begin{pmatrix}U\tilde{A}_i+aH\breve{B}_{fj}C_i+\tilde{A}_i^TU\\+aC_i^T\breve{B}_{fj}^TH^T+Q-R\end{pmatrix} & \begin{pmatrix}aH\breve{A}_{fj}+a\tilde{A}_i^THV\\+C_i^T\breve{B}_{fj}^T\end{pmatrix}\\ \star & \breve{A}_{fj}+\breve{A}_{fj}^T\end{bmatrix},$$

$$\breve{\Phi}^{12}_{ij}=\begin{bmatrix}U\tilde{A}_{di}+aH\breve{B}_{fj}C_{di}+R-S\\ aVH^T\tilde{A}_{di}+\breve{B}_{fj}C_{di}\end{bmatrix},\quad \breve{\Phi}^{13}_{ij}=\begin{bmatrix}S\\0_{k\times n}\end{bmatrix},$$

$$\breve{\Phi}^{14}_{ij}=\begin{bmatrix}U\tilde{B}_i+aH\breve{B}_{fj}D_i-E_i^T\mathcal{Y}\\ aVH^T\tilde{B}_i+\breve{B}_{fj}D_i+\breve{E}_{fj}^T\mathcal{Y}\end{bmatrix},H=\begin{bmatrix}I_{k\times k}\\0_{(n-k)\times k}\end{bmatrix},$$

$$\tilde{\Sigma}_{1ij}=\left[\,R\tilde{A}_i\;\;0_{n\times k}\,|\;R\tilde{A}_{di}\;\;0_{n\times n}\;\;R\tilde{B}_i\,\right],$$

$$\tilde{\Sigma}_{2ij}=\left[\,\bar{\mathcal{X}}E_i\;\;-\bar{\mathcal{X}}\breve{E}_{fj}\,|\;\bar{\mathcal{X}}E_{di}\;\;0_{q\times n}\;\;0_{q\times l}\,\right].$$

The other variables are the same as depicted in Theorem 1. Then, the gain matrices of the IT2 polynomial fuzzy filter in (3) are designed as

$$\begin{bmatrix}A_{fj_{k\times k}} & B_{fj_{k\times p}}\\ E_{fj_{q\times k}} & 0\end{bmatrix}=\begin{bmatrix}V^{-1} & 0\\ 0 & I\end{bmatrix}\begin{bmatrix}\breve{A}_{fj} & \breve{B}_{fj}\\ \breve{E}_{fj} & 0\end{bmatrix}. \quad (47)$$

Proof. Consider the filter's state $x_f\in\mathbb{R}^k$, $k<n$, hence we partition P in the form of

$$P=\begin{bmatrix}U_{n\times n} & aHV\\ \star & V_{k\times k}\end{bmatrix}, \quad (48)$$

where U and V are both positive definite matrices. a is an arbitrary scalar determined by the user. The following proof can resort to the same lines in the proving process of Theorem 2. This fulfills the proof. □

Remark 2. *Note that due to the inconsistent state dimensions between the concerned system and the designed filter, the* *matrix H adopted in Theorem 3 plays an important role in dealing with the reduced-order filter design, which acts as an order reduction factor to facilitate analysis.*

IV. NUMERICAL EXAMPLE

In this section, we will give a simulation example to clarify the validity of the full-order and reduced-order IT2 polynomial fuzzy filtering algorithm developed in this paper. Also, we demonstrate that Theorem 2 adopting the information of membership functions earns less conservative results with regard to those independent of membership functions.

Consider an uncertain nonlinear system with time-varying delay represented by a three-rule IT2 polynomial fuzzy model in the form of (1) with $\hat{x}(x(t)) = x(t) = [x_1(t)\ x_2(t)]^T$,

$$A_1 = \begin{bmatrix} -2.1 - 0.001x_1^2 & 0.1 \\ 1.0 & -2.0 \end{bmatrix}, A_2 = \begin{bmatrix} -1.9 - 0.002x_1^2 & 0 \\ -0.2 & -1.1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -1.6 - 0.001x_1^2 & 0.2 \\ 0.7 & -1.0 \end{bmatrix}, A_{d1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}, \quad A_{d3} = \begin{bmatrix} 0.6 & 0.2 \\ 0.5 & -0.8 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1.0 \\ -0.2 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \ B_3 = \begin{bmatrix} 0.1 \\ 1.0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1.0 & 0 \end{bmatrix}, \ C_2 = \begin{bmatrix} 0.5 & -0.6 \end{bmatrix}, \ C_3 = \begin{bmatrix} 0 & 1.0 \end{bmatrix},$$

$$C_{d1} = \begin{bmatrix} -0.8 & 0.6 \end{bmatrix}, C_{d2} = \begin{bmatrix} -0.2 & 1.0 \end{bmatrix}, C_{d3} = \begin{bmatrix} 0.1 & 0 \end{bmatrix},$$

$$D_1 = 0.3, \ D_2 = -0.6, \ D_3 = -0.1,$$

$$E_1 = \begin{bmatrix} 1 & -0.5 \end{bmatrix}, \ E_2 = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}, \ E_3 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},$$

$$E_{d1} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \ E_{d2} = \begin{bmatrix} 0 & 0.2 \end{bmatrix}, \ E_{d3} = \begin{bmatrix} 0 & 0.1 \end{bmatrix}.$$

The lower and upper membership functions of the IT2 polynomial fuzzy model are selected as $\underline{w}_1(x_1) = 1 - 1/(1 + e^{(-x_1-3.5)})$, $\underline{w}_3(x_1) = 1/(1 + e^{(-x_1+3.5)})$, $\bar{w}_2(x_1) = 1 - \underline{w}_1(x_1) - \underline{w}_3(x_1)$, $\bar{w}_1(x_1) = 1 - 1/(1 + e^{(-x_1-2.5)})$, $\bar{w}_3(x_1) = 1/(1+e^{(-x_1+2.5)})$, $\underline{w}_2(x_1) = 1-\bar{w}_1(x_1)-\bar{w}_3(x_1)$. It is assumed the system's operating domain for x_1 is set as $x_1 \in [-10, 10]$. Partition the domain of interest of x_1 to be 15 uniform subdomains where each interval is limited in $[-\frac{34}{3}+\frac{4}{3}l, -10+\frac{4}{3}l]$, $l = 1, 2, \ldots, 15$. We set the upper bound of the time-varying delay $\bar{d}$ as 0.55 and τ as 0.1.

In this simulation, we choose different number of fuzzy rules and membership functions between the designed filter and the IT2 polynomial fuzzy model to exhibit the design flexibility. Here, the two-rule IT2 polynomial fuzzy filters in (2) are built whose lower and upper membership functions are selected as $\underline{m}_1(x_1) = \{1$, when $x_1 < -5.2$; $(-x_1 + 4.8)/10$, when $-5.2 \leq x_1 \leq 4.8$; 0, when $x_1 > 4.8\}$. $\bar{m}_1(x_1) = \{1$, when $x_1 < -4.8$; $(-x_1 + 5.2)/10$, when $-4.8 \leq x_1 \leq 5.2$; 0, when $x_1 > 5.2\}$. $\bar{m}_2(x_1) = 1 - \underline{m}_1(x_1)$, $\underline{m}_2(x_1) = 1 - \bar{m}_1(x_1)$. Let $\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4(x) = \epsilon_5(\bar{x}) = \epsilon_6(\bar{x}) = \epsilon_7(\bar{x}) = 0.001$, $M(x_1)$ and $W_{ij}(x_1)$, $i = 1, 2, 3$, $j = 1, 2$ as the polynomial of degree 2 in x_1, $T(x_1)$ as a polynomial of degree 0 and $\Lambda = diag(1, 0)$.

In order to perform simulation of the designed filter, we assume weighting functions $\underline{\lambda}_1(x_1) = (\sin(5x_1) + 1)/2$, $\bar{\lambda}_1(x_1) = 1 - \underline{\lambda}_1(x_1)$, $\underline{\lambda}_3(x_1) = (\cos(5x_1)+1)/2$, $\bar{\lambda}_3(x_1) = 1 - \underline{\lambda}_3(x_1)$. It is lucid to obtain $w_2(x_1)$ by resorting to the relationship $w_2(x_1) = 1-w_1(x_1)-w_3(x_1)$ without providing the concrete expression of $\underline{\lambda}_2(x_1)$ and $\bar{\lambda}_2(x_1)$ as long as $w_1(x_1)$ and $w_3(x_1)$ are defined. In addition, the weighting functions of the filter are defined as $\underline{\beta}_j(x_1) = \bar{\beta}_j(x_1) = 0.5$, $j = 1, 2$. By employing Theorem 2 and Theorem 3, the results achieved by H_∞ filter and strictly dissipative filter are illustrated as follows:

1) H_∞ performance case: According to Definition 1, set $\mathcal{X} = -I$, $\mathcal{Y} = 0$, $\mathcal{Z} = (\alpha + \alpha^2)I$ with $\alpha = 0.765$. Employing SOSTOOLS to solve the SOS-based conditions (31)-(37) with the assumption of the IT2 polynomial filter gain matrices of degree 0 in x_f, the full-order filter gain matrices referring to (38) are calculated as follows:

$$A_{f1} = \begin{bmatrix} -25.0193 & -1.4927 \\ -1.7271 & -19.8659 \end{bmatrix}, B_{f1} = \begin{bmatrix} -3.1631 \\ 0.6885 \end{bmatrix},$$

$$E_{f1} = \begin{bmatrix} -0.2050 & 0.0726 \end{bmatrix},$$

$$A_{f2} = \begin{bmatrix} -23.1472 & -0.2374 \\ 3.0248 & -16.8178 \end{bmatrix}, B_{f2} = \begin{bmatrix} -8.5804 \\ -5.0775 \end{bmatrix},$$

$$E_{f2} = \begin{bmatrix} -0.0577 & -0.1794 \end{bmatrix}.$$

Employing SOSTOOLS to solve the SOS conditions (40)-(46) with the assumption of the IT2 polynomial filter gain matrices of degree 0 in x_f, the corresponding reduced-order filter gain matrices referring to (47) are computed as follows:

$$A_{f1} = -25.3983, \quad B_{f1} = -3.3075, \quad E_{f1} = -0.2036,$$
$$A_{f2} = -23.8586, \quad B_{f2} = -9.1185, \quad E_{f2} = -0.1368.$$

To investigate the filtering performance, we assume the initial condition $x(0) = 0$ and $x_f(0) = 0$. Consider the external disturbance to be

$$\omega(t) = \begin{cases} 0.3\sin(0.3t), & t \leqslant 42, \\ 0, & \text{otherwise}. \end{cases} \tag{49}$$

The simulation results of the designed full-order and reduced-order filters with the prescribed H_∞ performance index are presented in Fig. 1, which depicts the trajectories of the signal to be estimated (solid line), its full-order estimation signal (dashed line) and reduced-order estimation signal (dashed-dotted line) in presence of the aforementioned disturbance input. Based on the idea of mismatched membership functions, both full-order and reduced-order 2-rule IT2 polynomial fuzzy filters are designed to estimate the unknown signal stemming from a 3-rule IT2 polynomial fuzzy system. By employing fewer number of fuzzy rules and simpler structure of membership functions in fuzzy filtering design, the implementation cost can be eased. From the figures displayed, one can see that the designed IT2 polynomial fuzzy filter can produce an effective estimation for the unknown signal. Besides, the full-order H_∞ filter provides a more superior estimation performance contrast with the reduced one.

2) Strictly dissipative performance case: In light of Definition 1, let $\mathcal{X} = -0.2025I$, $\mathcal{Y} = -0.18I$, $\mathcal{Z} = 2\alpha I$ with $\alpha = 0.8$. Utilizing SOSTOOLS to solve the SOS conditions (31)-(37) with the assumption of the IT2 polynomial filter gain matrices of degree 0 in x_f, the full-order filter gain matrices referring to (38) are given as follows:

$$A_{f1} = \begin{bmatrix} -27.5400 & 2.4207 \\ -1.1090 & -22.2539 \end{bmatrix}, B_{f1} = \begin{bmatrix} 2.7205 \\ -1.3820 \end{bmatrix},$$

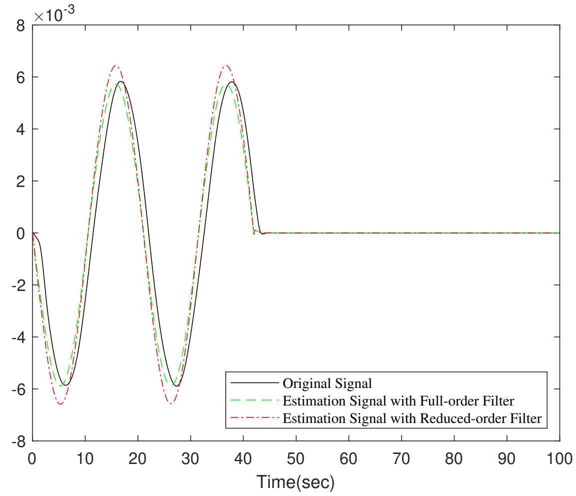

Fig. 1. Original signal and its estimations generated by the full-order and reduced-order H_∞ filters

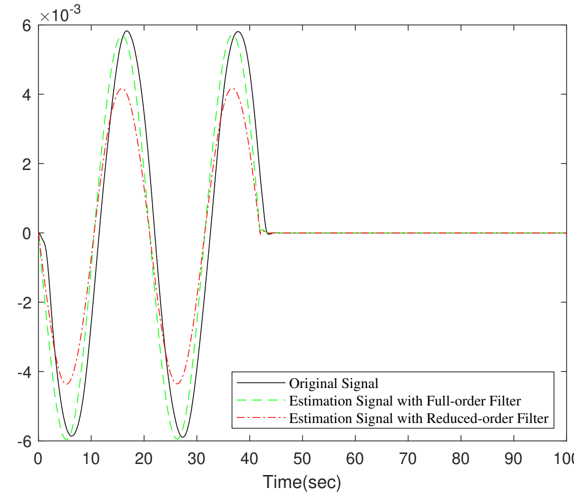

Fig. 2. Original signal and its estimations generated by the full-order and reduced-order dissipative filters

$$
\begin{aligned}
E_{f1} &= \begin{bmatrix} -0.2221 & 0.0700 \end{bmatrix}, \\
A_{f2} &= \begin{bmatrix} -25.5751 & -0.0824 \\ 5.6942 & -19.1592 \end{bmatrix}, B_{f2} = \begin{bmatrix} -9.9196 \\ -2.3082 \end{bmatrix}, \\
E_{f2} &= \begin{bmatrix} -0.1507 & -0.3055 \end{bmatrix}.
\end{aligned}
$$

Calculate the SOS conditions (40)-(46) with the assumption of the IT2 polynomial filter gain matrices of degree 0 in x_f, the corresponding reduced-order filter gain matrices referring to (47) are denoted as follows:

$$
\begin{aligned}
A_{f1} &= -28.2034, \quad B_{f1} = 2.6354, \quad E_{f1} = -0.2280, \\
A_{f2} &= -26.0669, \quad B_{f2} = -9.1274, \quad E_{f2} = -0.2423.
\end{aligned}
$$

To investigate the strictly dissipative filtering performance for the concerned system, let the initial conditions and disturbance input be the same as above. The simulation results of the designed filter with the predefined dissipativity performance index are presented in Fig. 2, which draws the trajectories of the signal to be estimated (solid line), its full-order estimation signal (dashed line) and reduced-order estimation signal (dashed-dotted line). It can be seen that the estimation performance is decent and the validity of filtering algorithm investigated is confirmed again.

Remark 3. *To illustrate the advantages of the membership functions dependent filtering technique proposed, we drop the information of membership functions when coping with filtering design by utilizing the similar line of the derivation of Theorem 1 and Theorem 2. Then the obtained filtering existence conditions independent on membership function are described by (31)-(33), and (34) with $T(x)$ replaced by $\tilde{\Omega}_{ij}$, $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, c$. Under the same settings aforementioned, there is no feasible solution found by using SOSTOOLS. It manifests that the filtering design algorithm equipped with the information of membership functions yields less conservative results with regard to the counterparts independent of membership functions.*

In addition, in order to clarify the estimation performance influenced by different number of subdomains, we divide the working region of x_1 into 15, 10 and 5 intervals. Based on H_∞ performance case, the simulation results are shown in Fig. 3, which exhibits the trajectories of the original signal (solid line), its full-order estimation signal with 15 subdomains (dashed line), its full-order estimation signal with 10 subdomains (dashed-dotted line) and its full-order estimation signal with 5 subdomains (dotted line). It can be observed that the filtering estimation performance becomes more preferable along with the number of partitioned subdomains increasing because more useful information of membership functions is involved in the resultant conditions, which verifies the effectiveness and merits again of our membership functions dependent filtering algorithm.

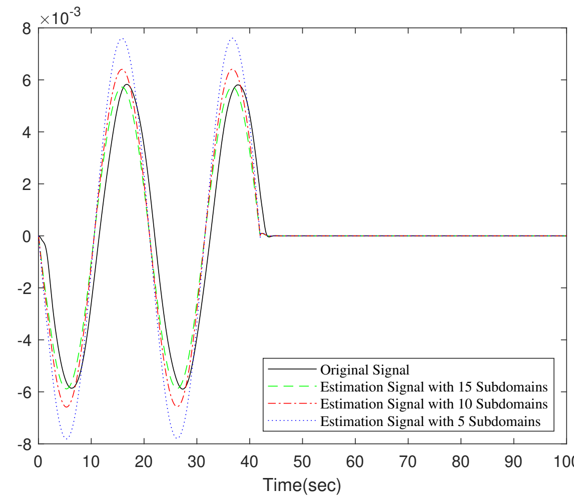

Fig. 3. Original signal and its estimations generated by the H_∞ filters with different number of subdomains

Remark 4. *Because of the uncertainties included within the IT2 membership functions, the type-1 fuzzy filtering ap-*

proaches cannot be applied for IT2 fuzzy systems directly. Furthermore, up to now, no related result has been reported concerning filter design for the time-delay IT2 polynomial fuzzy systems. Therefore, there is no comparison simulation made in this paper.

V. CONCLUSIONS

The problem of strict dissipativity based IT2 polynomial fuzzy filtering for continuous-time IT2 polynomial fuzzy systems subject to time-varying delay has been studied in this paper. Firstly, a delay-dependent sufficient criterion is developed by adopting Lyapunov-Krasovskii functional based approach such that the filtering error system is asymptotically stable restricted to the predefined dissipative performance index. In order to achieve more relaxed results, the MFD techniques are utilized where the information of membership functions is injected into the resultant conditions. Then, on the basis of the results derived, both the full-order and reduced-order IT2 polynomial fuzzy filters whose membership functions are not required to coincide with those of the concerned fuzzy system have been designed under the SOS-based conditions. Finally, simulation results have been presented to demonstrate the validity of the IT2 polynomial fuzzy filtering approach proposed. In addition, the method investigated in this paper can be extended to the event-triggered fault detection for the IT2 polynomial systems subject to cyber attacks, which could be our future work.

REFERENCES

[1] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modelling and control," *IEEE Transactions on Cybernetics*, vol. SMC-15, no. 1, pp. 116-132, 1985.

[2] K. Tanaka and M. Sano, "A robust stabilization problem of fuzzy control systems and its application to backing up control of a truck-trailer," *IEEE Transactions on Fuzzy Systems*, vol. 2, no. 2, pp. 119-134, 1994.

[3] X. Su, H. Zhou and Y.-D. Song, "An optimal divisioning technique to stabilization synthesis of T–S fuzzy delayed systems," *IEEE Transactions on Cybernetics*, vol. 47, no. 5, pp. 1147-1156, 2016.

[4] Y.-D. Song, H. Zhou, X. Su and L. Wang, "Pre-specified performance based model reduction for time-varying delay systems in fuzzy framework," *Information Sciences*, vol. 328, pp. 206-221, 2016.

[5] X.-L. Wang and G.-H. Yang, "Event-triggered H_∞ filtering for discrete-time T–S fuzzy systems via network delay optimization technique," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 10, pp. 2026-2035, 2019.

[6] X.-H. Chang, Z.-M. Li and J. Park, "Fuzzy generalized H_2 filtering for nonlinear discrete-time systems with measurement quantization," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 12, pp. 2419-2430, 2018.

[7] M. Zhang, C. Shen, Z.-G. Wu and D. Zhang, "Dissipative filtering for switched fuzzy systems with missing measurements," *IEEE Transactions on Cybernetics*, vol. 50, no. 5, pp. 1931-1940, 2020.

[8] J. C. Willems, "Dissipative dynamical systems—Part I: General theory," *Archive for Rational Mechanics and Analysis*, vol. 45, no. 5, pp. 321–351, 1972.

[9] L. Wu, X. Yang and H.K. Lam, "Dissipativity analysis and synthesis for discrete-time T–S fuzzy stochastic systems with time-varying delay," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 2, pp. 380-394, 2014.

[10] X, Su, P. Shi, L. Wu and M. Basin, "Reliable filtering with strict dissipativity for T-S fuzzy time-delay systems," *IEEE Transactions on Cybernetics*, vol. 44, no. 12, pp. 2470-2483, 2014.

[11] Y Liu, B.-Z. Guo, J. Park and S. Lee, "Event-based reliable dissipative filtering for T–S fuzzy systems with asynchronous constraints," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 4, pp. 2089-2098, 2018.

[12] K. Tanaka, H. Yoshida, H. Ohtake and H. O. Wang, "A sum-of-squares approach to modeling and control of nonlinear dynamical systems with polynomial fuzzy systems," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 4, pp. 911-922, 2009.

[13] J. VanAntwerp and R. Braatz, "A tutorial on linear and bilinear matrix inequalities," *Journal of Process Control*, vol. 10, no. 4, pp. 363-385, 2000.

[14] S. Prajna, A. Papachristodoulou and P. A. Parrilo, "Introducing SOSTOOLS: a general purpose sum of squares programming solver," *Proceedings of the 41st IEEE Conference on Decision and Control*, vol. 1, pp. 741-746, 2002.

[15] H.K. Lam and S.-H. Tsai, "Stability analysis of polynomial-fuzzy-model-based control systems with mismatched premise membership functions," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 1, pp. 223-229, 2014.

[16] Y. Sun, J. Yu, Z. Chen and X. Xing, "Event-triggered filtering for nonlinear networked discrete-time systems," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 11, pp. 7163-7170, 2015.

[17] D. Ye and X. Li, "Event-triggered fault detection for continuous-time networked polynomial-fuzzy-model-based systems," *Applied Mathematics and Computation*, DOI: 10.1016/j.amc.2019.124729, 2020.

[18] B. Xiao, H.K. Lam, H. Zhou and J. Gao, "Analysis and Design of Interval Type-2 Polynomial-Fuzzy-Model-Based Networked Tracking Control Systems," *IEEE Transactions on Fuzzy Systems*, DOI:10.1109/TFUZZ.2020.3006587.

[19] Q. Liang and J. Mendel, "Interval type-2 fuzzy logic systems: Theory and design," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 5, pp. 535–550, 2000.

[20] H.K. Lam and L. D. Seneviratne, "Stability analysis of interval type-2 fuzzy-model-based control systems," *IEEE Transactions on Systems, Man, and Cybernetics Part B: Cybernetics*, vol. 38, no. 3, pp. 617-628, 2008.

[21] H.K. Lam, H. Li, C. Deters, E. L. Secco, H. A. Wurdemann and K. Althoefer, "Control design for interval type-2 fuzzy systems under imperfect premise matching," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 2, pp. 956–968, 2014.

[22] H.K. Lam, "A review on stability analysis of continuous-time fuzzy-model-based control systems: From membership-function-independent to membership-function-dependent analysis," *Engineering Applications of Artificial Intelligence*, vol. 67, pp. 390–408, 2018.

[23] Q. Zhou, D. Liu, Y. Gao, H. K. Lam and R. Sakthivel, "Interval type-2 fuzzy control for nonlinear discrete-time systems with time-varying delays," *Neurocomputing*, vol. 157, pp. 22-32, 2015.

[24] Z. Zhang, Y. Niu, Z. Cao and J. Song, "Security sliding mode control of interval type-2 fuzzy systems subject to cyber attacks: the stochastic communication protocol case," *IEEE Transactions on Fuzzy Systems*, DOI: 10.1109/TFUZZ.2020.2972785, 2020.

[25] Y. Pan and G.-H. Yang, "Event-based reduced-order fuzzy filtering for networked control systems with time-varying delays," *Applied Mathematics and Computation*, vol. 359, pp. 71–83, 2019.

[26] J. Li, X. Huang and Z. Li, "Extended dissipative filtering of fuzzy systems with time-varying delay under imperfect premise matching," *International Journal of Systems Science*, vol. 48, no. 8, pp. 1731-1743, 2017.

[27] B. Xiao, H.K. Lam, X. Yang, Y. Yu and H. Ren, "Tracking control design of interval type-2 polynomial-fuzzy-model-based systems with time-varying delay," *Engineering Applications of Artificial Intelligence*, vol. 75, pp. 76-87, 2018.

[28] Y. Li, H.K. Lam and L. Zhang, "Control design for interval type-2 polynomial fuzzy-model-based systems with time-varying delay," *IET Control Theory and Applications*, vol. 11, no. 14, pp. 2270-2278, 2017.

[29] Z. Zhang, Q. Zhou, C. Wu and H. Li, "Dissipativity-Based Reliable Interval Type-2 Fuzzy Filter Design for Uncertain Nonlinear Systems," *International Journal of Fuzzy Systems*, vol. 20, no. 2, pp. 390-402, 2018.

[30] R. Ma, P. Shi, and L. Wu, "Dissipativity-based sliding-mode control of cyber-physical systems under denial-of-service attacks," *IEEE Transactions on Cybernetics*, DOI: 10.1109/TCYB.2020.2975089, 2020.

[31] F. Yang, H. Zhang, G Hui and S Wang, "Mode-independent fuzzy fault-tolerant variable sampling stabilization of nonlinear networked systems with both time-varying and random delays," *Fuzzy Sets and Systems*, vol. 207, pp. 45-63, 2012.

[32] C. Wang and Y. Shen, "Improved delay-dependent robust stability criteria for uncertain time delay systems," *Applied Mathematics and Computation*, vol. 218, no. 6, pp. 2880-2888, 2011.