Adaptive Active Anti-vibration Control for a Three-dimensional Helicopter Flexible Slung-load System with Input Saturations and Backlash

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Abstract—This study investigates active anti-vibration control for a three-dimensional helicopter flexible slung-load system (HFSLS) subject to input saturations and backlash. The first target of the study is to establish a model for the three-dimensional HFSLS. The second target is to develop an adaptive control law for the HFSLS by analyzing its ability to compensate for the effects of input saturations, input backlash, and external disturbances, while achieving the goal of vibration reduction. Simulation results of the numerical show that the proposed adaptive active control technology is effective in solving the oscillation suppression problem for the three-dimensional HFSLS with input saturations and backlash.

Index Terms—Adaptive active control, anti-vibration, three-dimensional HFSLS, input saturation, input backlash

I. INTRODUCTION

The helicopter slung-load system plays an increasingly important role in modern society. Current research is aimed at the rigid-body model of a helicopter slung-load system [1]-[9] with little focus on helicopter flexible slung-load systems (HFSLS) [10]-[13]. However, these studies were meant for a horizontal model of the HFSLS. Thus, vibration suppression control for three-dimensional HFSLS is required.

Numerous effective vibration reduction control methods have been proposed [14]-[25]. In [26], a new adaptive active control scheme was introduced to investigate the anti-swing control design problem of a flexible Timoshenko system. The effects of compensating for actuator failures, backlash-like hysteresis, and external disturbances were remedied simultaneously. For a flexible wind turbine tower, the problems of vibration and disturbance rejection were studied using active control technology in [27]. Under the developed control design, the scope of the vibration can remain in a small compact set; in addition, to solve the asymmetric input-output constraint, the authors constructed new auxiliary systems and barrier Lyapunov functions for flexible rotary crane systems in [28]. The authors in [29] adopted active feedback control to suppress the vibration of a flexible air-breathing hypersonic vehicle system described by a set of distributed parameter systems. Based on the above analysis, the active control strategy was efficient for stability analysis of flexible structure systems. Thus, an active control method is adopted to investigate the control problem for a three-dimensional HFSLS.

Constraint problems are common phenomena in engineering, and several outstanding research goals have been achieved [30]-[41]. Based on the auxiliary design and disturbance observer method, the problems of input saturation and external disturbances were addressed for hypersonic flight vehicles in [42]. In [43], output feedback control was studied for a flexible manipulator modeled by a partial differential equation model, in which an adaptive backlash inverse function was utilized to handle the unknown input backlash. In [44], the trajectory tracking control problem was researched for robot manipulators, and a neural network technique was employed to handle input saturation. The authors proved the uniform ultimate boundedness of nonlinear systems using a fuzzy robust constrained control in [45], in which a sigmoid function and fuzzy logic systems were used to address input saturation. However, to the best of the author’s knowledge, the mixed impacts of input saturation and input backlash were not considered for a three-dimensional HFSLS.

In this study, a three-dimensional HFSLS subjected to input saturations and input backlash is established using a set of partial differential equations. To reduce the number of calculations resulting from addressing input saturation and input backlash separately, the two functions are transformed into a new saturation function. Subsequently, the method of auxiliary system design is adopted to compensate for the effect of the new saturation function. A novel adaptive active control scheme using the designed auxiliary systems is introduced to ensure that the closed-loop system of the three-dimensional HFSLS is semi-uniformly ultimately bounded.
These innovations are reflected in the following three aspects:

- A three-dimensional model for a HFSLS is proposed, which is defined by a set of partial differential equations;
- The functions of input saturation and backlash are transformed into a new saturation function in order to reduce computation complexity;
- Finally, a novel adaptive active control law is designed to compensate for the effects of input saturations, input backlash, and external disturbances for the HFSLS. Meanwhile, the objective of reducing vibration is came true.

Notations: \( \min \{s_1, s_2, \ldots, s_n\} \) is used to calculate the minimum value among \( s_1, s_2, \ldots, s_n \), \( \max \{s_1, s_2, \ldots, s_n\} \) is employed to acquire the maximum value among \( s_1, s_2, \ldots, s_n \). For simplicity, the following definitions are given below: \( (\cdot) = \frac{\partial (\cdot)}{\partial t}, (\cdot)' = \frac{\partial^2 (\cdot)}{\partial t^2}, (\cdot)'' = \frac{\partial^2 (\cdot)}{\partial t^2}, \forall r \in [0, t], t \in [0, \infty) \).

II. PROBLEM FORMULATION AND PRELIMINARIES

A three-dimensional diagram of a HFSLS is shown in Fig. 1. Here, \( \mu(\tau, t) \) and \( \nu(\tau, t) \) denote the lateral displacements in the \( X_b \) and \( Y_b \) coordinates, respectively, and \( \omega(\tau, t) \) represents the longitudinal displacement in the \( Z_b \) coordinate. A mathematical model for the HFSLS is established using the extended Hamiltonian principle and the detailed modeling process is as follows.

The kinetic energy \( W_e(t) \) of the HFSLS is described by:

\[
W_e(t) = \frac{\bar{\omega}}{2} \int_0^t \left[ \dot{\mu}^2(r, t) + \dot{\nu}^2(r, t) + \dot{\omega}^2(r, t) \right] dr + \frac{M_p}{2} \left[ \ddot{\mu}(l, t) + \ddot{\nu}(l, t) + \ddot{\omega}(l, t) \right],
\]

where \( M_p \) and \( \bar{\omega} \) denote the mass of the load and mass per unit length of the slung string, respectively. By applying the variation operator \( \delta \) to Equation (1), it obtains:

\[
\int_{t_1}^{t_2} \delta W_e(t) dt = \int_{t_1}^{t_2} \delta \left\{ \frac{EA}{2} \int_0^t \left[ \frac{\mu'(r, t)^2}{2} + \frac{\nu'(r, t)^2}{2} \right] + \frac{T}{2} \int_0^t \left[ \mu'(r, t)^2 + \nu'(r, t)^2 \right] dr \right\} dt
\]

\[
\int_{t_1}^{t_2} \delta \left\{ \frac{M_p}{2} \left[ \ddot{\mu}(l, t) + \ddot{\nu}(l, t) + \ddot{\omega}(l, t) \right] \right\} dt
\]

\[
+ \int_{t_1}^{t_2} \left\{ T \mu''(r, t) + \frac{3EA}{2} \left[ \mu'(r, t)^2 \right] \right\} dr
\]

\[
+ EA \left[ \mu'(r, t) \omega''(r, t) + \mu''(r, t) \omega'(r, t) + \frac{EA}{2} \left[ \mu'(r, t) \omega'(r, t) \right] \right]
\]

\[
+ \int_{t_1}^{t_2} \left\{ T \nu''(r, t) + \frac{3EA}{2} \left[ \nu'(r, t)^2 \right] \right\} dr
\]

\[
+ EA \left[ \nu'(r, t) \omega''(r, t) + \nu''(r, t) \omega'(r, t) + \frac{EA}{2} \left[ \nu'(r, t) \omega'(r, t) \right] \right]
\]

\[
+ \int_{t_1}^{t_2} \left\{ \mu''(r, t) \mu'(r, t)^2 + 2 \nu'(r, t) \nu'(r, t) \mu''(r, t) \right\}
\]

\[
+ \int_{t_1}^{t_2} \left\{ \nu''(r, t) \mu'(r, t)^2 + 2 \nu'(r, t) \nu'(r, t) \mu''(r, t) \right\}
\]

\[
+ \int_{t_1}^{t_2} \delta \left( \frac{M_p}{2} \left[ \ddot{\mu}(l, t) + \ddot{\nu}(l, t) + \ddot{\omega}(l, t) \right] \right) dt
\]
\[ \times \delta \nu(r, t) + \left\{ E \omega''(r, t) + E \mu'(r, t) \mu''(r, t) \right\} \delta \omega(r, t) dr dt + \int_{t_1}^{t_2} \left\{ T \mu'(l, t) \right\} d\mu(l, t) \right\} \delta \mu(l, t) + \left\{ T \nu'(l, t) \right\} \delta \nu(l, t) dt + \left\{ E \omega''(l, t) + E \mu'(l, t) \omega''(l, t) \right\} \delta \omega(l, t) \right\} \delta \omega(l, t) dt. \]

Virtual work done by the distributed loads \( u_{dx}(r, t), u_{dy}(r, t), \) and \( u_{dz}(r, t) \) on the string and the external disturbances \( d_x(t), d_y(t), \) and \( d_z(t) \) on the load is expressed as follows:

\[ \delta W_1(t) = \int_0^1 \left[ u_{dx}(r, t) \delta \mu(r, t) + u_{dy}(r, t) \delta \nu(r, t) + u_{dz}(r, t) \delta \omega(r, t) \right] dr + d_x(t) \delta \mu(l, t) + d_y(t) \delta \nu(l, t) + d_z(t) \delta \omega(l, t). \] (5)

The work performed by the control inputs is as follows:

\[ \delta W_2(t) = u_x(t) \delta \mu(l, t) + u_y(t) \delta \nu(l, t) + u_z(t) \delta \omega(l, t). \] (6)

Combining (5) and (6), the total virtual work performed on the HFSLS is described as follows:

\[ \delta W(t) = \delta W_1(t) + \delta W_2(t) = \int_0^1 \left[ u_{dx}(r, t) \delta \mu(r, t) + u_{dy}(r, t) \delta \nu(r, t) + u_{dz}(r, t) \delta \omega(r, t) \right] dr + \left[ u_x(t) + d_x(t) \right] \delta \mu(l, t) + \left[ u_y(t) + d_y(t) \right] \delta \nu(l, t) + \left[ u_z(t) + d_z(t) \right] \delta \omega(l, t). \] (7)

From the extended Hamiltonian principle, \( \int_{t_1}^{t_2} \delta [W(x, t) - W\mu(t)] dt = 0 \) [46], defining \( \mu = \mu(r, t), \nu = \nu(r, t), \omega = \omega(r, t), \mu_t = \mu_l(l, t), \nu_t = \nu_l(l, t), \omega_t = \omega_l(l, t), \) and \( u_{dx} = u_{dx}(r, t), k = x, y, z, \) it derives the three-dimensional HFSLS as follows:

\[ \varpi \ddot{\mu} = T \mu''' + \frac{3}{2} \varpi \right[ \mu'' \right] \right] + u_{dx}, \]

\[ \varpi \ddot{\nu} = T \nu''' + \frac{3}{2} \varpi \right[ \nu'' \right] \right] + u_{dy}, \]

\[ \varpi \ddot{\omega} = E \omega''' + E \mu' \mu'' + E A \nu' \omega'' + u_{dz}, \] under the following boundary conditions:

\[ \mu(0, t) = \nu(0, t) = \omega(0, t) = 0, \] (11)

\[ M_p \ddot{u}_x = -T \mu' - \frac{E A}{2} \left[ \mu'' \right]^2 - E A \mu' \omega'' + \frac{E A}{2} \left[ \mu'' \right]^2 + u_{dx}, \] (12)

\[ M_p \ddot{u}_y = -T \nu' - \frac{E A}{2} \left[ \nu'' \right]^2 - E A \nu' \omega'' + \frac{E A}{2} \left[ \nu'' \right]^2 + u_{dy}, \] (13)

\[ M_p \ddot{u}_z = -E A \omega' + \frac{E A}{2} \left[ \nu'' \right]^2 - E A \nu' \omega'' + u_{dz}. \] (14)

Input saturation and backlash, which are common in mechanical actuators, are considered in this study and the saturation function is given by [30]:

\[ u_{kb}(t) = S(\vartheta_k(t)) = \begin{cases} \vartheta_{kM}, & \text{if } \vartheta_k(t) \geq \vartheta_{kM}, \\ \vartheta_k(t), & \text{if } \vartheta_{kM} < \vartheta_k(t) < \vartheta_{km}, \\ \vartheta_{km}, & \text{if } \vartheta_k(t) \leq \vartheta_{km}, \end{cases} \] (15)

where \( S(\cdot) \) is the saturation function and \( \vartheta_{kM} > 0 \) and \( \vartheta_{km} < 0 \) denote the saturation levels of the control input \( u_{kb}(t), k = x, y, z \). The backlash function is described by [43]:

\[ u_k(t) = B(u_{kb}(t)) = \begin{cases} \xi_k(u_{kb}(t) - \iota_{kr}), & \text{if } \dot{u}_{kb}(t) > 0 \\ \xi_k(u_{kb}(t) + \iota_{kl}), & \text{if } \dot{u}_{kb}(t) < 0 \\ u_k(t), & \text{otherwise}, \end{cases} \] (16)

where \( B(\cdot) \) is the input backlash function, \( u_{kb}(t) \) denotes the desired control input, \( \xi_k > 0 \) represents the slope of \( B(\cdot) \), \( \iota_{kr} > 0 \) and \( \iota_{kl} > 0 \) indicate backlash spacing, and \( u_k(t) = u_k(t_{-}), t = x, y, z \).

According to [47], the following right inverse function \( B^+(\cdot) \) of \( B(\cdot) \) is defined as:

\[ \vartheta_k(t) = B^+(\chi_k(t)) = \begin{cases} \chi_k(t)/\xi_k + \iota_{kr}, & \text{if } \dot{u}_{kb}(t) > 0 \\ \chi_k(t)/\xi_k - \iota_{kl}, & \text{if } \dot{u}_{kb}(t) < 0 \\ \vartheta_k(t_{-}), & \text{otherwise}, \end{cases} \] (17)

where \( \vartheta_k(t) = \vartheta_k(t_{-}), t = x, y, z \).

Based on the above definitions, it can conclude that

From Fig. 2, the mixed input nonlinearity, which is constituted by input saturation and input backlash, is depicted by:

\[ u_k(t) = B(S(B^+(\chi_k(t)))) = \begin{cases} \xi_k(\vartheta_{kM} - \iota_{kr}), & \text{if } \chi_k \geq \xi_k(\vartheta_{kM} - \iota_{kr}), \\ \xi_k(\vartheta_{km} + \iota_{kl}), & \text{if } \chi_k \leq \xi_k(\vartheta_{km} + \iota_{kl}), \\ u_k(t), & \text{if } \xi_k(\vartheta_{km} + \iota_{kl}) < \chi_k < \xi_k(\vartheta_{kM} - \iota_{kr}). \end{cases} \] (18)

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Hence, the input nonlinearity function, which includes a saturation function and a input backlash function, can be transformed into a new saturation function.

The following assumptions and lemma are required for the stability proof.

Assumption 1 [14]: The distributed loads \( u_{d_k}, u_{d_y}, \) and \( u_{d_z} \) satisfy \( |u_{d_k}| \leq \bar{u}_k \) where \( \bar{u}_k > 0 \) denotes a constant, \( k = x, y, z \). Meanwhile, the disturbance \( d_k(t) \) acting on the load is bounded, that is, \( |d_k(t)| \leq D_k \) where \( D_k > 0 \) denotes a constant, \( k = x, y, z \).

Assumption 2 [10]: The new saturation function in (18) satisfies \( \Delta u_k(t) \leq \varepsilon_k \) where \( \Delta u_k(t) = \chi_k(t) - u_k(t), \varepsilon_k > 0 \) is a constant, \( k = x, y, z \).

Lemma 1 [48]: For a function \( \alpha \in C^1 \) on \( r \) with \( \alpha(0,t) = 0 \), it has that

\[
\alpha^2 \leq \int_0^t \alpha'(r)^2 dr,
\]

\[
\alpha = \alpha(r, t), \quad \beta = \beta(r, t), \quad \forall r \in [0, l], \quad \epsilon > 0
\]

where \( \alpha = \alpha(r, t), \beta = \beta(r, t), \forall r \in [0, l], \epsilon > 0 \) represents a constant.

Remark 1: Addressing input saturation and backlash directly will be a complex task for a three-dimensional HFSLS. Thus, we transform the input saturation and backlash functions into a new saturation function by defining the right-inverse function \( B^+(\cdot) \) of the input backlash function \( B(\cdot) \).

Remark 2: The difficulty points in the modeling of the three-dimensional HFSLS are listed as follows:

- The first thorny issue is to determine the forms of the overall kinetic energy, potential energy, and virtual work.
- The second thorny issue is to calculate the system model by using the extended Hamiltonian principle

\[
\int_0^t \delta [W_x(t) - W_p(t) + W(t)] dt = 0
\]

and selecting effective calculation methods.

III. ADAPTIVE ACTIVE CONTROLLER DESIGN AND STABILITY ANALYSIS

First, we adopt the strategy of designing auxiliary systems to address the newly constructed saturation function in (18). Subsequently, an adaptive active control scheme is developed for three-dimensional HFSLS based on the proposed auxiliary systems. The control diagram is given by Fig. 3 for a three-dimensional HFSLS. Under the developed control strategy, a stability analysis is performed for the three-dimensional HFSLS using the direct Lyapunov method.

A. Controller Design

To compensate for the deviation between the control input and actuator output, the auxiliary systems are designed as follows:

\[
\delta_x(t) = \frac{1}{M_p} \left\{ -\gamma_x \delta x(t) + \Delta u_x(t) + T \mu'_x + T \mu_l \right\},
\]

\[
\delta_y(t) = \frac{1}{M_p} \left\{ -\gamma_y \delta y(t) + \Delta u_y(t) + T \mu'_y + T \mu_l \right\},
\]

\[
\delta_z(t) = \frac{1}{M_p} \left\{ -\gamma_z \delta z(t) + \Delta u_z(t) + E \omega'_z + T \mu_l \right\},
\]

\[
\delta(t) = \frac{1}{M_p} \left\{ -\gamma \delta(t) + \Delta u(t) + E \omega'_z \right\}.
\]
and
\[ \dot{\zeta}_z(t) = \ddot{\omega}_z + \dot{\zeta}_z(t) = \frac{1}{M_p} \left\{ -EA\omega'_z - \frac{EA}{2} [\mu'_z]^2 - \frac{EA}{2} [\nu'_z]^2 + u_z(t) + d_z(t) + M_p \dot{\omega}_z - \gamma_z \delta_z(t) + \Delta u_z(t) + EA\omega'_z + \frac{EA}{2} [\mu'_z]^2 + \frac{EA}{2} [\nu'_z]^2 \right\} = \frac{1}{M_p} \left\{ \chi_z(t) + d_z(t) + M_p \dot{\omega}_z - \gamma_z \delta_z(t) \right\}. \] (27)

**Remark 3:** The technology for developing auxiliary systems in (19)-(21), which is a forward compensation method, addresses the input saturation problem.

### B. Stability Analysis

To decrease the vibration of a three-dimensional HFSLS with input saturations and backlash, the following adaptive active control laws are designed below:

\[ \begin{align*}
\dot{D}_x(t) &= \psi_x \left\{ \beta_1 \zeta_x(t) \tanh \left( \frac{\zeta_x(t)}{\sigma_x} \right) - \lambda_x \dot{D}_x(t) \right\} \quad (28) \\
\dot{D}_y(t) &= \psi_y \left\{ \beta_1 \zeta_y(t) \tanh \left( \frac{\zeta_y(t)}{\sigma_y} \right) - \lambda_y \dot{D}_y(t) \right\} \quad (29) \\
\dot{D}_z(t) &= \psi_z \left\{ \beta_1 \zeta_z(t) \tanh \left( \frac{\zeta_z(t)}{\sigma_z} \right) - \lambda_z \dot{D}_z(t) \right\} \quad (30)
\end{align*} \]

\[ \begin{align*}
\chi_x(t) &= -M_p \mu'_t - T \mu_t - g_x \zeta_x(t) - \frac{EA}{2} [\mu'_z]^3 - EA\mu'_z \omega'_z - \frac{EA}{2} \mu'_z [\nu'_z]^2 + \gamma_x \delta_x(t) - \tanh \left( \frac{\zeta_x(t)}{\sigma_x} \right) \dot{D}_x(t), \quad (31) \\
\chi_y(t) &= -M_p \nu'_t - T \nu_t - g_y \zeta_y(t) - \frac{EA}{2} [\nu'_z]^3 - EA\nu'_z \omega'_z - \frac{EA}{2} \nu'_z [\mu'_z]^2 + \gamma_y \delta_y(t) - \tanh \left( \frac{\zeta_y(t)}{\sigma_y} \right) \dot{D}_y(t), \quad (32) \\
\chi_z(t) &= -M_p \mu'_z - g_z \zeta_z(t) - \frac{EA}{2} [\mu'_z]^2 - \frac{EA}{2} [\nu'_z]^2 + \gamma_z \delta_z(t) - \tanh \left( \frac{\zeta_z(t)}{\sigma_z} \right) \dot{D}_z(t), \quad (33)
\end{align*} \]

where \( \psi_k, \beta_1, \sigma_k, \lambda_k, \) and \( g_k \) are positive constants, \( k = x, y, z \).

In the following section, the semi-uniform boundedness of the HFSLS is proven by employing Lemma 2. First, the following Lyapunov candidate function is chosen:

\[ Y(t) = Y_1(t) + Y_2(t) + Y_3(t), \] (34)

where,

\[ \begin{align*}
Y_1(t) &= \frac{\beta_1 \varpi}{2} \int_0^t \dot{\mu}^2 + \dot{\nu}^2 + \omega^2 \mathrm{d}r + \frac{\beta_1 T}{2} \int_0^t \left[ \dot{\mu}^2 + \dot{\nu}^2 \right] \mathrm{d}r, \quad (35) \\
Y_2(t) &= \frac{\beta_1 M_p \mu'_z(t) + \delta'_z(t)}{2} + \frac{1}{2 \psi_x} D_x^2(t) + \frac{\beta_1 M_p \mu'_z(t)}{2} \\
&+ \frac{1}{2 \psi_y} D_y^2(t) + \frac{\beta_1 M_p \mu'_z(t)}{2} \left[ \delta'_z(t) + \frac{1}{2 \psi_z} D_z^2(t) \right], \quad (36) \\
Y_3(t) &= \beta_2 \varpi \int_0^t r \dot{\mu}' \lambda' + \dot{\nu}' \omega' + \dot{\omega}' \nu' \mathrm{d}r, \quad (37)
\end{align*} \]

where \( D_k(t) = D_k - \Delta D_k(t), \) \( k = x, y, z, \) and \( \beta_1 > 0 \) and \( \beta_2 > 0 \) are constants that satisfy \( \beta_2 \varpi \varrho < \Gamma_1 \) with the detailed form of \( \Gamma_1 \) given below in (42).

Next, we verify the positive definiteness of the Lyapunov candidate function (34). First, Equation (35) can be represented as:

\[ Y_1(t) = \frac{\beta_1 \varpi}{2} \int_0^t \dot{\mu}^2 + \dot{\nu}^2 + \omega^2 \mathrm{d}r + \frac{\beta_1 T}{2} \int_0^t \left[ \dot{\mu}^2 + \dot{\nu}^2 \right] \mathrm{d}r + \frac{\beta_1 \mu'_z(t)}{2} \int_0^t \left[ \delta'_z(t) + \frac{1}{2 \psi_z} D_z^2(t) \right] \mathrm{d}r, \]

According to [49], it derives that \( 2[\omega']^2 \leq [\mu']^2 \) and \( 2[\nu']^2 \leq [\nu']^2 \). By invoking Lemma 1, the cross terms in (38) satisfy the following inequalities:

\[ \begin{align*}
&\frac{1}{2 \sigma_1} \int_0^t \mu'_z(t)^2 \mathrm{d}r - \varsigma \int_0^t \mu'_z(t)^4 \mathrm{d}r \leq \int_0^t \omega' \mu'_z(t)^2 \mathrm{d}r \quad (39) \\
&\frac{1}{2 \sigma_1} \int_0^t \mu'_z(t)^2 \mathrm{d}r + \sigma_1 \int_0^t \mu'_z(t)^4 \mathrm{d}r \leq \int_0^t \omega' \mu'_z(t)^2 \mathrm{d}r \quad (39) \\
&\frac{1}{2 \sigma_1} \int_0^t \nu'_z(t)^2 \mathrm{d}r - \varsigma \int_0^t \nu'_z(t)^4 \mathrm{d}r \leq \int_0^t \omega' \nu'_z(t)^2 \mathrm{d}r \quad (40) \\
&\frac{1}{2 \sigma_1} \int_0^t \nu'_z(t)^2 \mathrm{d}r + \sigma_1 \int_0^t \nu'_z(t)^4 \mathrm{d}r \leq \int_0^t \omega' \nu'_z(t)^2 \mathrm{d}r \quad (40)
\end{align*} \]

where \( \varsigma_1 \) denotes a positive constant satisfying \( \frac{EA}{T} \leq \varsigma_1 \leq \frac{1}{4} \). Then, we have:

\[ \frac{\beta_1 \varpi}{2} \int_0^t \dot{\mu}^2 + \dot{\nu}^2 + \omega^2 \mathrm{d}r + \frac{\beta_1 T}{2} \int_0^t \left[ \dot{\mu}^2 + \dot{\nu}^2 \right] \mathrm{d}r + \frac{\beta_1 \mu'_z(t)}{2} \int_0^t \left[ \delta'_z(t) + \frac{1}{2 \psi_z} D_z^2(t) \right] \mathrm{d}r \]

\[ \leq Y_1(t) \]

\[ \leq \frac{\beta_1 \varpi}{2} \int_0^t \dot{\mu}^2 + \dot{\nu}^2 + \omega^2 \mathrm{d}r + \frac{\beta_1 T + \frac{EA}{2 \varsigma_1}}{2} \int_0^t \mu'_z(t)^2 \mathrm{d}r \]

...
Furthermore, function $\Upsilon_1(t)$ satisfies the following inequality:

$$\Gamma_1 \int_0^l \Pi(r,t)dr \leq \Upsilon_1(t) \leq \Gamma_2 \int_0^l \Pi(r,t)dr,$$  \hspace{1cm} (42)

where $\Pi(r,t) = \dot{\mu}^2 + \dot{\nu}^2 + \dot{\omega}^2 + [\mu']^2 + [\nu']^2 + [\mu''']^2 + [\nu''']^2$, $\Gamma_1 = \frac{\beta_1}{2} \min \{\omega, T - \frac{E_A}{\omega^2}, E_A(1/4 - \varsigma_1), E_A\}$, and $\Gamma_2 = \frac{\beta_1}{2} \max \{\omega, T - \frac{E_A}{\omega^2}, E_A(1/4 + \varsigma_1), E_A\}$.

In addition, by invoking $\dot{\phi}_1 \dot{\phi}_2 \leq \frac{\sigma_1^2}{2} + \sigma_2^2$, function $\Upsilon_3(t)$ satisfies the following relation:

$$|\Upsilon_3(t)| \leq \frac{\beta_2 \pi l}{2} \int_0^l \Pi(r,t)dr.$$  \hspace{1cm} (43)

Then, it has:

$$|\Upsilon_3(t)| \leq \frac{\beta_2 \pi l}{2} \int_0^l \Pi(r,t)dr.$$  \hspace{1cm} (44)

From (42) and (44), we have that:

$$(\Gamma_1 - \frac{\beta_2 \pi l}{2}) \int_0^l \Pi(r,t)dr \leq \Upsilon_1(t) + \Upsilon_3(t) \leq (\Gamma_2 + \frac{\beta_2 \pi l}{2}) \int_0^l \Pi(r,t)dr.$$  \hspace{1cm} (45)

By quoting (36), (45), and $\beta_2 \pi l < \Gamma_1$, we obtain:

$$c_1 \left( \int_0^l \Pi(r,t)dr + \Upsilon_2(t) \right) \leq \Upsilon(t) \leq c_2 \left( \int_0^l \Pi(r,t)dr + \Upsilon_2(t) \right),$$  \hspace{1cm} (46)

where $c_1 = \min \{\{\Gamma_1 - \frac{\beta_2 \pi l}{2}\}, 1\}$ and $c_2 = \max \{\{\Gamma_2 + \frac{\beta_2 \pi l}{2}\}, 1\}$. Thus, function (34) can be selected as a Lyapunov candidate function.

In the following section, the stability of the three-dimensional HFSLS is proceeded under the Lyapunov candidate function (34).

Considering (8)-(11) and (35), the derivative of $\Upsilon_1(t)$ can be written as:

$$\begin{align*}
\dot{\Upsilon}_1(t) &= \beta_1 \pi l \int_0^l \dot{\mu} \dot{\mu} + \dot{\nu} \dot{\nu} + \dot{\omega} \dot{\omega}dr + \beta_1 T \int_0^l \mu'' \mu'' + \nu'' \nu'' + \frac{[\mu']^2}{2} + \frac{[\nu']^2}{2} \times \left( \dot{\omega} + \mu'' \nu' + \nu'' \mu' \right)dr \\
&= \beta_1 \pi l \int_0^l \dot{\mu} \left( \frac{3E_A}{2} [\mu']^2 \mu'' + EA \left( \mu'' \omega'' + \mu'' \right) \right) \times \dot{\omega}' + \frac{E_A}{2} \left( \mu'' [\nu']^2 + 2 \mu' \nu' \nu'' \right) + ud_\theta \\
&\quad + \dot{\nu} \left( \frac{3E_A}{2} [\nu']^2 \nu'' + EA \left( \nu'' \omega'' + \nu'' \omega' \right) \right) + \frac{E_A}{2} \left( \nu'' [\mu']^2 + 2 \mu' \mu' \nu'' \right) + ud_\theta + \dot{\omega} \left( EA \omega'' \right)
\end{align*}$$

$$+ EA \mu'' \nu'' + du'' \right)dr + \beta_1 T \int_0^l \mu'' \nu'' + \nu'' \mu'' \right)dr + \beta_1 T \int_0^l \mu'' \nu'' + \nu'' \mu'' \right)dr + \beta_1 T \int_0^l \mu'' \nu'' + \nu'' \mu'' \right)dr$$
where \( \beta_1 = \frac{1}{S_1} \). The derivative of the cost function \( \tilde{Y}(t) \) can be written as:

\[
\frac{d}{dt} \tilde{Y}(t) = \tilde{Y}_1(t) + \tilde{Y}_2(t) + \tilde{Y}_3(t)
\]

(48)

where \( S_5 - S_{10} > 0 \) denote constants.

Quoting (8)-(11) and (37), and considering inequation \( 2[\omega']^2 \leq [\mu']^2 \) and \( 2[\omega']^2 \leq [\nu']^2 \), the derivative of \( \tilde{Y}_3(t) \) is calculated as follows:

\[
\tilde{Y}_3(t) = \beta_2 \int_0^t r[\mu'\mu + \mu'\nu + \nu'\nu + \omega'\omega + \omega'\omega'] \, dr
\]

= \[ \beta_2 \int_0^t \{ T \mu'' + \frac{3E A}{2} [\mu']^2 \mu'' + E A [\mu' \omega'' + \mu'' \omega'] + E A [\mu' \nu'' + \mu'' \nu'] + u_{dx} \} + \nu_r \{ T \nu'' \}
\]

+ \frac{3E A}{2} [\nu']^2 \nu'' + E A [\nu' \omega'' + \nu'' \omega'] + \frac{E A}{2} [\nu'' \mu']^2 + 2 \mu'' \nu' + u_{dy} + \nu_r \{ E A \omega'' + E A \mu'' \}

\leq -\beta_2 \int_0^t \mu' \nu' + \omega' \omega' + \omega' \omega' \, dr - \beta_2 \left( \frac{T}{2} - \frac{E A}{2S_{12}} - \frac{l}{S_{14}} \right) \int_0^t [\nu']^2 \, dr
\]

(49)

where \( S_1 - S_1 > 0 \) denote constants.

Synthesizing the above results in (47)-(49), the derivative of \( \tilde{Y}(t) \) can be written as:

\[
\frac{d}{dt} \tilde{Y}(t) = \tilde{Y}_1(t) + \tilde{Y}_2(t) + \tilde{Y}_3(t)
\]

(50)

where

\[
v_1 = \min \left\{ \frac{\beta_2 \omega}{2} - \frac{\beta_1}{S_2}, \frac{\beta_2 \omega}{2} - \frac{\beta_1}{S_3}, \frac{\beta_2 \omega}{2} - \frac{\beta_1}{S_4}, \beta_2 \left( \frac{T}{2} - \frac{E A}{2S_{12}} - \frac{l}{S_{14}} \right), \beta_2 \left( \frac{T}{2} - \frac{E A}{2S_{12}} - \frac{l}{S_{14}} \right), \beta_2 \left( \frac{E A}{2} - \frac{l}{S_{15}} \right), \beta_2 E A \left( \frac{3}{8} - \frac{l}{S_{11}} \right), \beta_2 \frac{3eA}{4} \right\}
\]
\[ \begin{align*}
\beta_2 EA \left( \frac{3}{8} - \varsigma_{12} \right), & \quad 2g_x - T, \quad 2\gamma_x + T - \frac{2}{\varsigma_8}, \\
2g_y - T, & \quad 2\gamma_y + T - \frac{2}{\varsigma_8}, \quad 2g_z - EA, \\
2\gamma_z + EA - 2EA\varsigma_{16} - \frac{2}{\varsigma_{10}}, & \quad 2\lambda_x\psi_x \left( 1 - \frac{1}{\varsigma_5} \right), \\
2\lambda_y\psi_y \left( 1 - \frac{1}{\varsigma_7} \right), & \quad 2\lambda_z\psi_z \left( 1 - \frac{1}{\varsigma_5} \right), \\
v_2 = (\beta_1 \varsigma_{21} + \beta_2 l^2 \varsigma_{13}) \psi_x^2 + \lambda_x \varsigma_5 \psi_x^2 + \lambda_y \varsigma_7 \psi_v^2 + (\beta_1 \varsigma_{51} + \beta_2 l^2 \varsigma_{14}) \psi_x^2 + 0.2785 \beta_1 \varsigma_1 \psi_x \psi_x + 0.2785 \beta_1 \sigma \psi_y \psi_y + (\beta_1 \varsigma_{14} + \beta_2 l^2 \varsigma_{15}) \psi_x^2 + \lambda_x \varsigma_3 \psi_x^2 + 0.2785 \beta_1 \varsigma_x \psi_x + \beta_1 \varsigma_{10} \psi_x^2, \\
\text{under} & \quad \beta_2 \omega - \beta_1 \varsigma_3 > 0, \quad \beta_2 \omega - \beta_1 \varsigma_4 > 0, \quad \beta_2 \omega - \beta_1 > 0, \\
T - \frac{2}{\varsigma_{11}} & \quad \varsigma_3 > 0, \quad T - \frac{2}{\varsigma_{11}} - \varsigma_3 > 0, \\
EA - \frac{1}{\varsigma_{15}} & \quad > 0, \quad 3 - \varsigma_{11} > 0, \quad 3 - \varsigma_{12} > 0, \\
2g_x - T > 0, & \quad 2\gamma_x + T - \frac{2}{\varsigma_8} > 0, \quad 2g_y - T > 0, \\
2\gamma_y + T - \frac{2}{\varsigma_8} > 0, & \quad 2g_z - EA > 0, \quad 1 - \frac{1}{\varsigma_5} > 0, \\
2\gamma_z + EA - 2EA\varsigma_{16} - \frac{2}{\varsigma_{10}} > 0, & \quad 1 - \frac{1}{\varsigma_7} > 0, \\
1 - \frac{1}{\varsigma_9} > 0, & \quad \beta_1 T - \beta_2 l \omega \geq 0, \\
\beta_1 EA - \beta_2 l \omega & \quad \geq 0, \\
\beta_1 l & \quad \geq 0, \\
\beta_1 EA - 3\beta_2 l \omega & \quad \geq 0, \\
\beta_1 EA & \quad \geq 0, \\
\beta_1 EA - \frac{3}{4} \beta_2 l & \quad \geq 0, \\
\beta_1 EA & \quad \geq 0, \\
\beta_1 EA - 3\beta_2 l \omega & \quad \geq 0, \\
\beta_1 EA - \frac{3}{4} \beta_2 l & \quad \geq 0, \\
\beta_1 EA & \quad \geq 0,
\end{align*}\]
with \( \varsigma_{16} - \varsigma_{18} > 0 \) being constants.

According to the above analysis and [50], the closed-loop system of the three-dimensional HFSLS is confirmed to be semi-uniformly bounded. The convergence of the vibration values \( \mu(r, t), \nu(r, t), \omega(r, t) \), and \( r \in [0, l] \) for the three-dimensional HFSLS are analyzed in the theorem as follows.

**Theorem 1:** Consider a three-dimensional HFSLS subject to input saturations and backlash described by (8)-(14) with bounded initial values and the adaptive active control scheme given by (28)-(33), the vibration values \( \mu(r, t), \nu(r, t), \omega(r, t) \), and \( r \in [0, l] \), are proved to be semi-uniformly ultimately bounded. In other words, the vibration values \( \mu(r, t), \nu(r, t), \) and \( \omega(r, t) \) eventually converge to the following compact sets: \( \Phi_1 = \left\{ \mu \left| \left| \mu \right| \leq \sqrt{\frac{v_{2l}}{c_1 c_3}} \right. \right\} \), \( \Phi_2 = \left\{ \nu \left| \left| \nu \right| \leq \sqrt{\frac{v_{2l}}{c_1 c_3}} \right. \right\} \), and \( \Phi_3 = \left\{ \omega \left| \left| \omega \right| \leq \sqrt{\frac{v_{2l}}{c_1 c_3}} \right. \right\} \).

**Proof:** Letting \( v_3 = \frac{v_{2l}}{c_1 c_3} \), we obtain that \( \Upsilon(t) \leq -v_3 \Upsilon(t) + v_2 \). The above inequality (50) is calculated as follows:

\[ \Upsilon(t) \leq \Upsilon(0) \exp(-v_3 t) + \frac{v_2}{v_3} \]

where \( \Upsilon(0) \) is the initial value of the function \( \Upsilon(t) \).

Considering (36) and (46), we know that

\[ \int_0^l \Pi(r, t) dr \leq \frac{T(t)}{c_1} \]

Moreover, quoting \( \Pi(r, t) = \mu^2 + \nu^2 + \omega^2 + [\mu']^2 + [\nu']^2 + [\omega']^2 + [\mu']^2 + [\nu']^2 + [\omega']^2 \), it holds that

\[ \int_0^l \mu^2 dr \leq \int_0^l \Pi(r, t) dr, \]

\[ \int_0^l \nu^2 dr \leq \int_0^l \Pi(r, t) dr, \]

\[ \int_0^l \omega^2 dr \leq \int_0^l \Pi(r, t) dr. \]

Thus, invoking Lemma 1 and (52)-(55), it follows that:

\[ \frac{\mu^2}{T} \leq \int_0^l \mu^2 dr \leq \int_0^l \Pi(r, t) dr \leq \frac{\Upsilon(t)}{c_1}, \]

\[ \frac{\nu^2}{T} \leq \int_0^l \nu^2 dr \leq \int_0^l \Pi(r, t) dr \leq \frac{\Upsilon(t)}{c_1}, \]

\[ \frac{\omega^2}{T} \leq \int_0^l \omega^2 dr \leq \int_0^l \Pi(r, t) dr \leq \frac{\Upsilon(t)}{c_1}. \]

Subsequently, by quoting (51) and (56)-(58), we have

\[ |\mu| \leq \sqrt{\frac{\Upsilon(0) \exp(-v_3 t)}{c_1} + \frac{v_{2l}}{c_1 c_3}}, \]

\[ |\nu| \leq \sqrt{\frac{\Upsilon(0) \exp(-v_3 t)}{c_1} + \frac{v_{2l}}{c_1 c_3}}, \]

\[ |\omega| \leq \sqrt{\frac{\Upsilon(0) \exp(-v_3 t)}{c_1} + \frac{v_{2l}}{c_1 c_3}}. \]

When \( t \to +\infty \), it derives that:

\[ |\mu| \leq \sqrt{\frac{v_{2l}}{c_1 c_3}}, \quad |\nu| \leq \sqrt{\frac{v_{2l}}{c_1 c_3}}, \quad |\omega| \leq \sqrt{\frac{v_{2l}}{c_1 c_3}}. \]

Thus, the vibration values \( \mu(r, t), \nu(r, t), \) and \( \omega(r, t), r \in [0, l] \), of the three-dimensional HFSLS are proved to be semi-uniformly ultimately bounded. That is, the vibration values \( \mu(r, t), \nu(r, t), \) and \( \omega(r, t) \) eventually converge to the following compact sets: \( \Phi_1 = \left\{ \mu \left| \left| \mu \right| \leq \sqrt{\frac{v_{2l}}{c_1 c_3}} \right. \right\} \), \( \Phi_2 = \left\{ \nu \left| \left| \nu \right| \leq \sqrt{\frac{v_{2l}}{c_1 c_3}} \right. \right\} \), and \( \Phi_3 = \left\{ \omega \left| \left| \omega \right| \leq \sqrt{\frac{v_{2l}}{c_1 c_3}} \right. \right\} \).

**Remark 4:** The eventually convergent compact sets of the vibration values \( \mu(r, t), \nu(r, t), \) and \( \omega(r, t) \) can be regulated by choosing reasonable control parameters \( g_k, \gamma_k, \psi_k, \) and \( \sigma_k, k = x, y, z \). The above compact sets will become smaller ones by selecting bigger parameters \( g_k, \gamma_k, \psi_k, \) and a smaller parameter \( \sigma_k, k = x, y, z, \) appropriately.
Remark 5: In the papers [51]-[53], the asymptotic tracking control problems have been investigated and the effective control performance has been obtained for the ordinary differential system models. However, when input saturations, input backlash, distributed disturbance, and boundary disturbance are considered simultaneously, the asymptotic tracking control for the three-dimensional HFSLS depicted by the infinite dimensional systems cannot be achieved, thus the uniform ultimate boundedness is studied in this paper under the compromise. Moreover, although the composite problem, caused by the input saturations, input backlash, distributed disturbance, and boundary disturbance, is a tough problem, a good control result will be shown in the following Simulation section of this paper under the proposed control scheme given by (28)-(33). The similar results on the uniform ultimate boundedness have been obtained in the articles [54]-[56].

IV. NUMERICAL SIMULATION

Consider the three-dimensional HFSLS with input saturations and backlash, given by (8)-(14) and the system parameters below: \( l = 1.0 \text{ m}, \) \( \varpi = 0.1 \text{ kg/m}, \) \( T = 16 \text{ N}, \) \( EA = 1.2 \text{ N}, \) and \( m_l = 3.0 \text{ kg}. \) The boundary disturbances and distributed loads are as follows:

\[
d_x(t) = (1.0 + 0.5 \sin(0.5t) + 0.3 \sin(0.3t) + 0.2 \sin(0.2t)) \times 10, \quad d_y(t) = (1.0 + 0.5 \sin(0.5t) + 0.3 \sin(0.3t) + 0.2 \sin(0.2t)) \times 10, \quad d_z(t) = (1.5 + 0.5 \sin(0.5t) + 0.3 \sin(0.3t) + 0.2 \sin(0.2t)) \times 10, \quad u_{dx} = (1.0 + 0.4 \sin(0.1\pi t) + 0.2 \sin(0.2\pi t) + 0.1 \sin(0.4\pi t))r/5, \quad u_{dy} = (1.0 + 0.4 \sin(0.1\pi t) + 0.2 \sin(0.2\pi t) + 0.1 \sin(0.4\pi t))r/5, \quad \text{and} \quad u_{dz} = (0.5 + 0.4 \sin(0.1\pi t) + 0.2 \sin(0.2\pi t) + 0.1 \sin(0.4\pi t))r/10. \]

The backlash spacings are shown as: \( \epsilon_{xr} = \epsilon_{yr} = 4, \) \( \epsilon_{zl} = \epsilon_{yl} = 4, \) and \( \epsilon_{zl} = 2. \) The slop parameters of the backlash function are \( \xi_x = \xi_y = \xi_z = 1 \) and the saturation levels are given by \( \vartheta_{xM} = 24, \) \( \vartheta_{ym} = -34, \) \( \vartheta_{yM} = 24, \) \( \vartheta_{ym} = -34, \) \( \vartheta_{zM} = 12, \) and \( \vartheta_{zm} = -12. \)

Here, the stability of a three-dimensional HFSLS is discussed under two circumstances.

Circumstance I: Without control.

The response curves of the open-loop system of the three-dimensional HFSLS are shown in Fig. 4-9. Figs. 4-6 display the simulation curves \( \mu(r, t), \nu(r, t), \) and \( \omega(r, t) \) for the three-dimensional HFSLS without control. When \( r = l \), the corresponding curves \( \mu(l, t), \nu(l, t), \) and \( \omega(l, t) \) are as shown in Figs. 7-9. According to Figs. 4-9, the swing amplitude of the slung-load system is large. Therefore, an efficient control strategy must be proposed to reduce vibration.

Circumstance II: With control.

The design parameters of the control scheme given by (28)-(33) are as follows: \( \beta_r = 1, \) \( \psi_x = \psi_y = \psi_z = 0.1, \) \( \lambda_x = \lambda_y = \lambda_z = 0.1, \) \( \sigma_x = \sigma_y = \sigma_z = 0.1, \) \( \gamma_x = \gamma_y = \gamma_z = 1.0 \times 10^2, \) and \( g_x = g_y = g_z = 1.0 \times 10^4. \) Under the introduced control laws given by (28)-(33), the simulation curves \( \mu(r, t), \nu(r, t), \) and \( \omega(r, t) \) of the closed-
loop system of the three-dimensional HFSLS are shown in Figs. 10-12.

To verify the performance of the proposed control scheme, the following PD control is introduced by: $\chi_x = -\kappa_{px}\mu(l, t) - \kappa_{dx}\dot{\mu}(l, t)$, $\chi_y = -\kappa_{py}\nu(l, t) - \kappa_{dy}\dot{\nu}(l, t)$, and $\chi_z = -\kappa_{pz}\omega(l, t) - \kappa_{dz}\dot{\omega}(l, t)$, where $\kappa_{px} = \kappa_{py} = 300$, $\kappa_{dx} = \kappa_{dy} = 200$, $\kappa_{pz} = 100$, and $\kappa_{dz} = 200$. With the PD control, the simulation curves $\mu(r, t)$, $\nu(r, t)$, and $\omega(r, t)$ are shown in Figs. 13-15.

Comparing the simulation graphs of the vibration amplitudes $\mu(r, t)$, $\nu(r, t)$, and $\omega(r, t)$ at $r = l$ in Figs. 16-18, it is clear that the proposed control strategy has a better anti-vibration control performance than PD control. Figs. 19-21 show that the phenomena of input nonlinearities appear in the process of control design, which suggests that proposed control strategy effectively addresses input nonlinearities.

Based on the above discussion, the developed adaptive active control scheme enhances performance robustness against input saturations, input backlash, and external disturbances. Furthermore, it also allows for effective anti-vibration control of three-dimensional HFSLS.
Fig. 14. The simulation curve of $\nu(r, t)$ with PD control.

Fig. 15. The simulation curve of $\omega(r, t)$ with PD control.

Fig. 16. (a) The simulation curve of $\mu(l, t)$ with proposed control; (b) The simulation curve of $\mu(l, t)$ with PD control.

Fig. 17. (a) The simulation curve of $\nu(l, t)$ with proposed control; (b) The simulation curve of $\nu(l, t)$ with PD control.

Fig. 18. (a) The simulation curve of $\omega(l, t)$ with proposed control; (b) The simulation curve of $\omega(l, t)$ with PD control.
includes the reinforcement learning approach of the proposed control scheme. The prospective study was conducted to demonstrate the control performance guaranteed control \[58, 59\] for the three-dimensional HFSLS.

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YONG REN ET AL.: Adaptive Active Anti-vibration Control for a Three-dimensional Helicopter Flexible Slung-load System


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