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DOI:

[10.1007/978-3-662-46675-9_8](https://doi.org/10.1007/978-3-662-46675-9_8)

Document Version

Peer reviewed version

[Link to publication record in King's Research Portal](#)

Citation for published version (APA):

Landsberg, D., Chockler, H., Kroening, D., & Lewis, M. (2015). Evaluation of Measures for Statistical Fault Localisation and an Optimising Scheme. In A. Egyed, & I. Schaefer (Eds.), *Fundamental Approaches to Software Engineering: 18th International Conference, FASE 2015, Held as Part of the European Joint Conferences on Theory and Practice of Software, ETAPS 2015, London, UK, April 11-18, 2015, Proceedings* (Vol. 9033, pp. 115-129). (Lecture Notes in Computer Science; Vol. 9033). Springer International Publishing. https://doi.org/10.1007/978-3-662-46675-9_8

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Evaluation of Measures for Statistical Fault Localisation and an Optimising Scheme

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Abstract. *Statistical Fault Localisation* (SFL) is a widely used method for localizing faults in software. SFL gathers coverage details of passed and failed executions over a faulty program and then uses a measure to assign a degree of *suspiciousness* to each of a chosen set of program entities (statements, predicates, etc.) in that program. The program entities are then inspected by the engineer in descending order of suspiciousness until the bug is found. The effectiveness of this process relies on the quality of the suspiciousness measure. In this paper, we compare 157 measures, 95 of which are new to SFL and borrowed from other branches of science and philosophy. We also present a new measure optimiser Lex_g , which optimises a given measure g according to a criterion of single bug optimality. An experimental comparison on benchmarks from the Software-artifact Infrastructure Repository (SIR) indicates that many of the new measures perform competitively with the established ones. Furthermore, the large-scale comparison reveals that the new measures Lex_{Ochiai} and Pattern-Similarity perform best overall.

1 Introduction

Software engineers use fault localization methods in order to focus their debugging efforts on a subset of program entities (such as statements or predicates) that are most likely to be causes of the error. Since the attempts to reduce the number of faults in software are estimated to consume 50 – 60% of the development and maintenance effort [6], accurate and efficient fault localization techniques have the potential to greatly reduce the overall effort of software development.

In statistical fault localisation (SFL), statistical information on passing and failing executions of a faulty program is gathered and analysed [1, 3, 14–16, 33]. Based on the resulting data, SFL assigns a degree of suspiciousness to each member of a chosen set of program entities of the program under test. Essentially, the degree of suspiciousness depends on the number of appearances of this entity in the passing and failing executions. There are many approaches to computing this degree. Naturally, entities that cause the error are hoped to have the highest degree of suspiciousness. The program entities are inspected by the user in descending order of suspiciousness until the bug is found. SFL has been considered a highly effective and efficient way for localising faults in software [31].

Our contributions The contributions of this paper are as follows:

1. We introduce and motivate 95 new measures (borrowed from other areas of science and philosophy) to SFL. These measures are divided into five categories: similarity, prediction, causation, confirmation and custom.
2. We formally prove that over 50 measures are equivalent to others for the purpose of ranking suspicious entities.
3. We experimentally compare the measures on the Siemens test suite along with five larger programs `space`, `grep`, `gzip`, `sed` and `flex`³. We show that many of the new measures perform competitively, with an optimised version of `PatternSimilarity` outperforming all pre-existing SFL measures on the benchmarks.
4. We introduce a new measure-optimising scheme Lex_g , and show Lex_{Ochiai} outperforms all other measures on the benchmarks.

Along with providing two new best performing measures, to the best of our knowledge, this research provides one of the largest scale SFL studies to date in three ways. Firstly, it contains the largest experimental study over C programs in SFL, consisting of the largest number (and largest sized) C programs. Secondly, it introduces and compares the largest number of measures. Thirdly, it contains results for the largest number of ranking equivalence proofs (see [21]).

Related work Research in SFL is largely driven by the construction or introduction of new suspiciousness measures. Experimental results assess the quality of measures by applying them to known benchmarks [1, 2, 17, 19, 21, 25, 26, 30]. Theoretical results have included formal properties and equivalence proofs of different measures [20, 21, 31].

A similar paper to ours is the paper by Lucia et al. [19], which compares association measures on C and Java Programs. However, Lucia et al. conclude that there is no measure which is clearly the best, whereas we show our new measure Lex_{Ochiai} is a robust overall top performer. Another similar paper is the paper by Naish [21], who set the standards for proving equivalences between suspiciousness measures, discuss optimal measures, and compares a (smaller) set of measures against a (smaller) set of benchmarks.

A recent paper by Yoo et al. [32] analyses fault localisation in conjunction with prioritisation. The problem studied in [32] is deciding the best course of action when a fault is found. Their approach is complementary to ours (and applied to a similar set of benchmarks) and can also be applied in conjunction with the measure we construct in this paper.

Xie et al. develop a theoretical approach to proving that some measures are better than others [31]. However, their proof relies on several critical simplifying assumptions (in particular, the bug must be contained in a single line of code or block). Bugs that are more realistic often break this assumption, invalidating the proof on realistic examples. We impose no such theoretical restrictions.

³ As maintained at the Software Artifact Infrastructure Repository at <http://sir.unl.edu>.

On a more general level, Parnin et al. [22] raise the question of whether fault localisation techniques are useful at all. The paper compared the efficiency of fault localisation with and without the automated tool Tarantula [14]. They reported that experts are faster at locating bugs using the tool for simple programs, but not for harder ones. However, their study is limited in its ability to generalize, as their experiments included only two small, single-bug programs (Tetris and NanoXML, 2K/4K LOC respectively), and is limited to the Tarantula tool.

Paper structure The rest of the paper is organised as follows. In Section 2, we present the informal ideas and formal definitions of SFL and discuss the 62 previously used suspiciousness measures. In Section 3, we discuss 95 measures that have not yet been applied to SFL and demonstrate that these new measures are well suited to SFL. We briefly outline proofs of equivalence of many of these measures when applied to SFL. Section 4 presents the experimental results of applying the non-equivalent measures to the benchmarks. We summarize our results in Section 5. Due to lack of space, the complete ranking equivalence proofs, results tables and tables containing definitions of measures are deferred to the appendix. The data set and code used to perform the experiments are available from <http://www.cprover.org/sfl/>.

2 Definitions and Notations

In this section, we introduce the basic definitions and notations of SFL, and survey the established measures. We also present a small motivating example.

2.1 Definitions

Let a program under test (PUT) \mathbf{P} be an ordered set of *program entities*, such that $\mathbf{P} = \langle C_1, \dots, C_n \rangle$, where $n \in \mathbb{N}$. Program entities can be statements, branches, paths, or blocks of code (see, for example, [15, 17, 29]). Let a test suite \mathbf{T} be an ordered set of test cases $\mathbf{T} = \langle t_1, \dots, t_m \rangle$, where m is the size of the test suite. Each test case t_i is a Boolean vector of length n (where n is the number of program entities) such that $t_i = \langle b_1^i, \dots, b_n^i \rangle$, where $b_j^i \in \{0, 1\}$, where we have $b_j^i = 1$ iff C_j is covered by t_i . We represent each program entity C_i by the set of test cases where C_i is 1. The last program entity C_n is the error statement E , which is 1 if the test case *fails* and 0 if it *passes*. A convenient way to store this information is using *coverage matrices*, in which the i -th row of the j -th column represents whether test case t_i covers program entity C_j , an example of which is given in in Table 1.

For each program \mathbf{P} , test suite \mathbf{T} and program entity C_i we can construct this program entity’s *contingency table* [24]. This table can be symbolically represented as a vector of four elements denoted as $\langle a_{ef}^i, a_{ep}^i, a_{nf}^i, a_{np}^i \rangle$, where a_{ef}^i is the number of failing test cases in \mathbf{T} that cover C_i , a_{ep}^i is the number of passing test cases in \mathbf{T} that cover C_i , a_{nf}^i is the number of failing test cases in \mathbf{T} that do not cover C_i , and a_{np}^i is the number of passing test cases in \mathbf{T} that do not

cover C_i . For each program entity, we can calculate its contingency table for a test suite. See Table 2 for an example. We let $F_i = a_{ef}^i + a_{nf}^i$, $P_i = a_{ep}^i + a_{np}^i$, and $T_i = F_i + P_i$. For each test suite and C_i and C_j , $F_i = F_j$ and $P_i = P_j$. When the context is clear we drop numerical indices, writing, for instance, C and a_{ef} instead of C_i and a_{ef}^i .

A *suspiciousness measure* m maps a contingency table $\langle a_{ef}^i, a_{ep}^i, a_{nf}^i, a_{np}^i \rangle$ to a real number [21]. Roughly speaking, for a test suite and faulty program, the higher the output of the measure the more suspicious the program entity C_i is assumed to be with respect to containing a bug. The output of each suspiciousness measure is the *suspiciousness score* that is assigned to each program entity (we also say that a program entity is *ranked* according to its suspiciousness score). The program entities are then ordered according to their degree of suspiciousness and are investigated in descending order by the user until the bug is found.

A probability space for each test suite is defined as follows. Given a program \mathbf{P} and test suite \mathbf{T} , we identify a probability space (Ω, S, Pr) , where the sample space $\Omega = \{t_1, \dots, t_n\}$ is the set of test cases, the set of events S is the power-set of the set of program entities, where $Pr: S \rightarrow [0, 1]$ is a probability function with the usual signature. Assuming the axioms and language of classical probabilistic calculus and given the definitions of $a_{ef}^i, a_{nf}^i, a_{ep}^i, a_{np}^i$ above, we can identify $Pr(C_i \cap E)$, $Pr(\neg C_i \cap E)$, $Pr(C_i \cap \neg E)$ and $Pr(\neg C_i \cap \neg E)$ with $\frac{a_{ef}^i}{T_i}$, $\frac{a_{ep}^i}{T_i}$, $\frac{a_{nf}^i}{T_i}$ and $\frac{a_{np}^i}{T_i}$ respectively. Using probabilistic calculus, this is sufficient to generate the other probabilistic expressions we need. Probabilistic expressions may also be translated into algebraic form in the obvious way. For example, $P(E|C_i)$ is equal to $\frac{a_{ef}^i}{a_{ep}^i + a_{ef}^i}$.

Naish's notion of *single-bug optimality* [20] is based on the observation that if a program contains only a single bug, then all failing traces cover that bug. Formally, a measure m is *single-bug optimal* if (1) when $a_{ef} < F$, then the value returned is less than any value returned when $a_{ef} = F$, and (2) when $a_{ef} = F$ and $a_{np} = k$, then the value returned is greater any value returned when $a_{np} < k$ [20].

We use Naish's notion of *ranking equivalence* between suspiciousness measures, defined as follows. Two suspiciousness measures m_1 and m_2 are said to be *monotonically equivalent* if $(m_1(\mathbf{x}) < m_1(\mathbf{y})) \Leftrightarrow (m_2(\mathbf{x}) < m_2(\mathbf{y}))$ for all vectors \mathbf{x} and \mathbf{y} . Many suspiciousness measures turn out to be monotonically equivalent on domains in which the measures share the same program and test suite [21]. In other words, they are monotonically equivalent on domains in which the number of failing test cases F and the number of passing test cases P is the same for the vectors \mathbf{x} and \mathbf{y} . This property is called *ranking equivalence* [21].

2.2 Motivating Example

In this section, we present a simple motivating example to illustrate a typical instance of SFL. Consider the faulty program *minmax.c* in Figure 1 (taken from [12]). The program has six program entities $\langle C_1, C_2, C_3, C_4, C_5, E \rangle$, where E is the specification. The program fails to satisfy the specification $least \leq most$. The reason for the failure is the bug at C_4 , which should be an assignment to $least$ instead of an assignment to $most$. To locate the fault, we collected

```

int main () { // C1
  int inp1, inp2, inp3;
  int least = inp1;
  int most = inp1;

  if (most < inp2)
    most = inp2; // C2
  if (most < inp3)
    most = inp3; // C3
  if (least > inp2)
    most = inp2; // C4 (Bug!)
  if (least > inp3)
    least = inp3; // C5
  assert(least <= most);
  // E (Specification) }

```

Fig. 1: minmax.c

	C_1	C_2	C_3	C_4	C_5	E
t_1	1	0	1	1	0	1
t_2	1	0	0	1	1	1
t_3	1	0	0	1	0	1
t_4	1	1	0	0	0	0
t_5	1	0	1	0	0	0
t_6	1	0	0	0	1	0
t_7	1	0	0	1	1	0
t_8	1	0	0	0	0	0
t_9	1	1	0	0	1	0
t_{10}	1	1	1	0	0	0

Table 1: Coverage matrix for minmax.c

coverage data from ten test cases t_1 to t_{10} . Three of the test cases fail and seven pass. The coverage matrix for these test cases is given in Table 1. We compute contingency tables for each program entity using the coverage matrix and give the contingency table for C_4 in Table 2 as an example. We then apply a suspiciousness measure to assign a degree of suspiciousness to each of the program entities given that program entity's contingency table. We use the Wong-II measure [30] $a_{ef} - a_{ep}$ as example.

	E	$\neg E$
C_4	3	1
$\neg C_4$	0	6

Table 2: Contingency table for C_4

The user then investigates the program entities in descending order of suspiciousness until the fault is found (ignoring E). In this example, Wong-II ranks C_4 the highest with a score of 2 and thereby successfully identifies the bug within the most suspicious program entity.

2.3 Established Measures

We include 62 measures selected by Naish [21] and Lo [19] in our comparison. To motivate many of these measures to SFL, Naish et al. discuss desirable formal properties [20]. One important property that has been discussed is *monotonicity*: for a fixed number of passed and failed tests, a measure should strictly increase as a_{ef} increases, and strictly decrease as a_{ep} decreases [?, 14, 20]. Examples of some prominent measures from [21] are Wong-I = a_{ef} , Wong-II = $a_{ef} - a_{ep}$, Naish = $a_{ef} - \frac{a_{ep}}{a_{ep} + a_{np} + 1}$, Zoltar = $\frac{a_{ef}}{a_{ef} + a_{nf} + a_{ep} + \frac{10000a_{nf}a_{ep}}{a_{ef}}}$, Jaccard = $\frac{a_{ef}}{a_{ef} + a_{nf} + a_{ep}}$, Ochiai = $\frac{a_{ef}}{\sqrt{(a_{ef} + a_{nf})(a_{ef} + a_{ep})}}$ [1], Tarantula = $\frac{a_{ef}}{a_{ef} + a_{np}}$ [14]⁴, Kulczynski-II = $\frac{1}{2}(\frac{a_{ef}}{a_{ef} + a_{nf}} + \frac{a_{ef}}{a_{ef} + a_{ep}})$, and M2 = $\frac{a_{ef}}{a_{ef} + a_{np} + 2(a_{nf} + a_{ep})}$.

3 New SFL Measures

In this section we introduce 95 new suspiciousness measures that have not yet been applied in SFL. We organise them into five different groups: *similarity*, *predictive*, *causal*, *confirmation*, and *custom* measures. Their application to SFL is motivated in terms of different proposed criteria about what a suspiciousness measure should exactly capture. We discuss such criteria at the beginning of each paragraph, and the reader is referred to the appendix for the full definitions of the new measures. We identify over 50 measures which are ranking equivalent and summarise interesting monotonic simplifications of some measures. The introduction of many measures has the benefit of consolidating the results concerning top performing measures. Furthermore, we introduce a new measure optimiser, which we later show can be used to construct the best performing measure on our benchmark suite.

3.1 New SFL measures from the literature

Similarity measures. The first proposed criterion is that a suspiciousness measure should measure how similar a program entity C is to the error E . This motivates the use of similarity measures in SFL, and has been discussed in the literature [20, 21]. Indeed, many of the measures of the previous section are similarity measures (such as Jaccard) that were originally used in different domains. The new similarity measures we include in our experiments are available in the survey of [5] and are as follows: 3w-Jaccard, Baroni-Urbani-Buser-I and II, Braun-Blanquet, Bray-Curtis, Cosine, Cole, Chord, Dennis, Dispersion, Driver&Kroebner, F&M, Faith, Forbes-I, Forbes-II, Fossum, Gower, Gower-Legendre, Hellinger, Johnson, Lance-Williams, MCconnaughey, Michael, Mountford, Nei-Li, Otsuka, PatternSimilarity, ShapeSimilarity, SizeSimilarity, Vari, Simpson, Sorgenfrei, and Sokal&Sneath-I, II, III, IV, V and Tarwid.⁵ The

⁴ Strictly, this is an algebraic simplification of the original Tarantula measure.

⁵ In some cases, a distance measure m has been converted to a similarity measure for our purposes, using the convention of $-\text{distance} \equiv \text{similarity}$.

measure $\text{PatternSimilarity} = -\frac{4(a_{ep}a_{nf})}{(a_{ef}+a_{ep}+a_{nf}+a_{np})^2}$, which is used in clustering [5], is of particular interest, and we discuss it later in more detail.

Prediction measures. The second proposed criterion is that a suspiciousness measure should measure the degree by which the execution of a program entity C predicts the error E . This motivates the use of what we loosely call *prediction* measures. Many of these measures are commonly used in epidemiology and diagnosis to estimate how well a test result predicts a disease or successful treatment [9, 11]. The prediction measures we include in our experiments are as follows: Positive predictive value (PPV) = $P(E|C)$, Negative predictive value (NPV) = $P(\neg E|\neg C)$, Sensitivity = $P(C|E)$, Specificity = $P(\neg C|\neg E)$, Youden’s J, Positive Likelihood, Tetrachoric, Relative risk, Z-ratio, Peirce, Pearson-I, II and III [24], Pearson-Heron I and II, Anderberg-II, Tanimoto, Mutual Info, Simpson, Gilbert&Wells and Goodman&Kruskal (see the surveys [5, 9, 11, 27]).

Causal measures. The third proposed criterion is that a suspiciousness measure should measure the degree by which the execution of a program entity C has the power to cause the error E . This motivates the use of measures of causal power/strength to SFL. Such measures are principally found in the domain of philosophy of science [10] and artificial intelligence [23] and many of their formal properties have been shown [10]. Examples of causal measures are Suppes = $P(E|C) - P(E|\neg C)$, Eels = $P(E|C) - P(E)$, Lewis = $\log \frac{P(E|C)}{P(E|\neg C)}$, Fitelson = $\log \frac{P(E|C)}{P(E)}$ [10]. The other causal measures considered are: Pearl-I, II, III and IV Fitelson II and III Korb I, II and III, Cheng and Good.

Confirmation Measures. The fourth proposed criterion is that a suspiciousness measure should measure the degree by which the execution of a program entity H is a hypothesis which explains the error E . This motivates the use of measures of explanation (sometimes called evidential/inductive/confirmation measures) to the domain of SFL. Such measures have been developed in the domain of philosophy of science [13] and many of their formal properties have been proven [8]. Some example confirmation measures are Earman = $P(H|E) - P(H)$, Joyce = $P(H|E) - P(H|\neg E)$, Milne = $\log \frac{P(H|E)}{P(H)}$, and Good-II = $\log \frac{P(H|E)}{P(H|\neg E)}$ [13]. The other measures considered are Carnap-I and II, Crupi, Rescher, Kemeny, Popper-I, II, and III, Levi, Finch-I, Gaifman and Rips.

3.2 Ranking equivalent measures

Naish proved that many different suspiciousness measures are in fact equivalent for the purposes of ranking suspiciousness entities [21]. We extend Naish’s work by providing many of the remaining equivalence proofs (over 50) for the measures in this paper (see Section B in the appendix for the proofs). Proving ranking equivalences is essential in determining a maximal set of inequivalent measures to investigate and allows us to ignore the remainder in experimentation. Furthermore, using equivalence proofs we can find some elegant monotonic simplifications which identify the underlying “essence” of some of the new

suspiciousness measures, which may be used to guide future development. For instance, of our new measures (established measures are bracketed), we have found that Sensitivity is ranking equivalent to a_{ef} (as is Wong-I), Specificity to a_{np} , PPV to $\frac{a_{ef}}{a_{ep}}$ (as is Tarantula), NPV to $\frac{a_{np}}{a_{nf}}$, YulesQ to $\frac{a_{ef}a_{np}}{a_{ep}a_{nf}}$, F1 to $\frac{a_{ef}}{a_{ep}+a_{nf}}$ (as is Jaccard), SizeSimilarity to $a_{ef}-a_{ep}$ (as is Wong-II) and PatternSimilarity to $-a_{nf}a_{ep}$.

3.3 A new custom measure

We propose the fifth criterion for suspiciousness measures: that a measure should be tailored to particular features concerning software errors (similar ideas had been proposed in [21] and Wong-III [30]).

We motivate our measure as follows. Firstly, following work by Naish [21], we state that our measure should be *single bug optimal* as defined in Section 2.1, because of deeper reasoning pertaining to Occam’s razor. That is, we think the simplest hypothesis for explaining the error should be investigated first, and as the simplest hypothesis is that the program contains a single bug, the measure should be single bug optimal.

Secondly, although we state that the measure should be single bug optimal, we diverge from Naish [21] insofar as we do not make the *single fault assumption* – that the program contains only a single bug. This is because there exist programs with multiple bugs and our goal is to construct a measure that provides a complete solution to the problem of SFL. Consequently, it still remains to work out how to rank the suspiciousness of entities when no bug is covered by all bad traces. In this case, the suspiciousness of each program entity is determined by an existing measure g which is chosen on its ability to deal with multiple bugs and by success in experimentation. We account for this in the second condition of our measure below. For a measure g , we define our *measure optimising scheme* Lex_g as follows. Let \mathbf{x} be the vector $\langle a_{ef}, a_{ep}, a_{nf}, a_{np} \rangle$. Then,

$$Lex_g(\mathbf{x}) = \begin{cases} a_{np} + 2 & \text{if } a_{ef} = F \\ g(\mathbf{x}) & \text{otherwise.} \end{cases} \quad (1)$$

In Equation 1, g stands for an internal measure, and can represent any measure appropriately scaled from 0 to 1. Based on its performance in our experiments, we choose Ochiai as the internal measure g , hence called Lex_{Ochiai} .

The following theorem states the optimality of our scheme (see Section A in the appendix for the proof).

Theorem 1. *Lex_g is single bug optimal.*

Lex_g can be considered as an optimising scheme which “converts” an appropriately scaled measure g into a single-bug optimal measure. The name of our scheme is derived from its underlying idea — to lexically order two different classes of entities in terms of suspiciousness.

4 Experiments

In this section we describe the results of empirical evaluation of the measures. First we describe the experimental setup; then, we discuss our two means of assessment – an average scoring method and a Wilcoxon rank sum significance test. We conclude the section with the presentation and the analysis of the results.

Program	Vs	LOC	TC	FTC	PE	FPV	Program	Vs	LOC	TC	FTC	PE	FPV
tcas	41	173	1608	38	53	1.61	replace	32	563	5542	96	218	1.79
schedule	9	410	2650	80	146	1.45	gzip	3	7996	214	126	1223	3.33
schedule2	10	307	2710	27	126	1.10	space	38	9126	13585	1439	976	5.21
tot_info	23	406	1052	83	116	1.04	sed	2	11990	360	210	2378	5.50
print_tks	7	563	4130	69	179	1.14	grep	3	13229	750	304	1785	17.33
print_tk2	10	508	4115	206	196	1.00	flex	2	14230	567	71	3092	29.50

Table 3: Table of benchmarks

4.1 Experimental setup

The benchmarks are listed in Table 3. For each program, the table specifies the number of faulty versions (Vs), the number of lines of code in the original version of the program (LOC), the number of test cases (TC), the average number of failing test cases per version (FTC), the number of program entities of the original version of the program (PE), and the average number of faulty lines of code per version (FPV)⁶.

The benchmarks are obtained from the Software Information Repository [7]. The versions of *sed*, *grep*, *flex*, and *gzip* used are the same as ones used in Lo [18], the versions of the Siemens and *space* test suites are the same as the ones used in Naish [21]. The Siemens test suite consists of *tcas*, *schedule*, *schedule2*, *totinfo*, *print tokens*, *print tokens2*, and *replace*. Overall, the experimental setup consists of 180 program versions, with over a million lines of code in total, and an average of 2.88 buggy lines per version.

The Siemens test suite is a widely used set of benchmarks in the domain of SFL [1, 2, 17, 25, 26, 30]. *Space* was additionally included by Naish [21]. The second set of benchmarks, consisting of versions of *gzip*, *grep*, *flex*, and *sed*, has been used to assess SFL in [18]. In this paper, we demonstrate experimental results on the union of the sets of benchmarks used in these papers, making our evaluation, to the best of our knowledge, the largest set of C programs used to evaluate measures for SFL. Moreover, the set of measures in our evaluation is,

⁶ Note that some program versions contain no faults. This happens when the fault appears in a non-executable line of code, such as a macro definition. These versions are removed from the experiment following [21].

to the best of our knowledge, the largest set of measures ever compared over any set of benchmarks in SFL.

Each test case was executed for each of the faulty programs and the result (pass or fail) recorded together with the set of the lines of code that were executed during this test (this data was extracted using GCOV). The pass or fail result was decided based on the output of the program and its comparison with the original program on the same input. Crashes were recorded as failures. The collected coverage data was used as an input to the measures, which assigned a suspiciousness score to each program entity (statements) in the (mutant) program and sorted the lines of code in the descending order of suspiciousness. To assign a score, we added a small *prior constant* (0.5) to each cell of each program entity’s contingency table in order to avoid divisions by zero, as is convention [21].

We experimented over a range of different prior constants (PC) in between 0 to 1, and did not discover any significant or noteworthy differences in results. The exception was for the PatternSimilarity measure (for which we used the ranking equivalent measure $-a_{nf}a_{ep}$ in our implementation). We discovered that this measure was optimised if we set the PC to $a_{nf} = 0.1$ and $a_{ep} = 0.5$. The optimised version of PatternSimilarity is henceforth called PattSim2, and the unoptimised PattSim1.

4.2 Methods of assessment

We use two means of assessment: an average scoring method and a Wilcoxon rank-sum significance test. We discuss the details here.

To score how well a measure performs on a benchmark, we introduce the *best*, *worst* and *average* scoring methods [1, 21, 30]. Formally, where m is a measure, n is the number of program entities in the program, b is a bug with the highest degree of suspiciousness of any bug and $bugs$ is the number of faulty lines in the program, we define $best(m) = (|\{x|m(x) > m(b)\}|/n) \times 100$ and $worst(m) = (|\{x|m(x) > m(b)\}|/n) \times 100$ and $avg(m) = best(m) + ((worst(m) - best(m))/(bugs + 1))$. For our evaluation we use the *avg* scoring method [21], which gives us the percentage of non-buggy program entities which we’d expect an engineer to examine before locating a bug, given the number of faulty lines in the program. To get the *avg* score for a benchmark, we take the mean *avg* of the scores of all the versions in that benchmark. To get the overall *avg* score, we take the mean of the 12 benchmark scores.

We performed a second means of assessment using Wilcoxon rank-sum tests. The Wilcoxon rank-sum test is a non-parametric statistical test which tests whether one population of values is significantly larger than another population [28]. Using this test, we were able to establish which measures were significantly better than others, by comparing each measure’s 12 average scores for each benchmark. To establish a baseline for localisation efficiency, we included a measure (Rand) which assigns each program entity a random suspiciousness score.

4.3 Results

Name	Score	Name	Score	Name	Score	Name	Score
LexOchiai	13.74	Ample2	16.48	Keynes	18.22	InfoGain	19.93
PattSim2	13.88	Dennis	16.66	Good2	18.22	JMeasure	19.95
Zoltar	13.92	Popper1	16.87	Finch1	18.22	Ochiai2	20.40
Naish	14.01	Korb3	16.93	Forbes1	18.22	SokSneath5	20.40
PattSim1	14.21	2WaySupport	16.93	Tarantula	18.22	MI	20.53
WongIII	14.23	YulesQ	17.11	Interest	18.22	Peirce	22.37
Kulc2	14.41	NPV	17.15	AddedValue	18.22	Leverage	23.20
M2	14.52	Rescher	17.15	SebagSch	18.22	BinaryNaish	23.34
Ochiai	15.25	Lewis	17.16	OddMultiplier	18.22	WongI	23.43
Conviction	15.65	AMean	17.17	Example	18.22	Confidence	23.43
Certainty	15.65	Stiles	17.27	Zhang	18.22	Fleiss	23.61
Crupi	15.88	GMean	17.27	Korb2	18.24	Scott	23.86
Michael	16.00	Phi	17.27	1WaySupport	18.24	Faith	24.49
Klosgen	16.31	Jaccard	17.40	Laplace	18.42	LeastCont.	25.83
Mountford	16.41	CBISqrt	17.68	Suppes	18.62	WongII	25.85
YoudensJ	16.48	Popper2	17.74	Pearson1	18.62	Gower	26.41
Earnan	16.48	Cohen	17.98	SokSneath4	18.65	GoodKrus	26.64
Carnap1	16.48	Kappa	17.98	HMean	18.65	Specificity	27.20
Carnap2	16.48	CBILog	18.11	Good	19.26	Anderberg2	27.25
Levi	16.48	Likelihood	18.22	PearIII	19.26	FagerMc	29.10
Dispersion	16.48	GilbertW	18.22	Cheng	19.26	Rand	31.74

Table 4: Overall *avg* scores for measures

We now present our experimental data and quantify their significance. We first discuss Table 4. The average scores for those suspiciousness measure with a higher score than the random measure are listed in Table 4. Equivalent measures are represented by one measure per equivalence class (with preference given to measures already established in SFL), and the new measures are in bold. Note that (thirteen) additional potential sets of equivalences are suggested by the equal scores in the table.

We first make some general observations about the table. Some prominent established measures appear quite low on the list, such as Jaccard and Tarantula. It is interesting that Tarantula (which is equal to $P(E|C)$), performs worse than NPV (which is equal to $P(\neg E|\neg C)$). Also, some established measures appear quite high on the list, such as Zoltar and Naish. Thus, our larger-scale comparison accentuates the successes and failures of established measures. The top performing measures from each of our newly introduced categories of similarity, confirmation, predictive, causal and custom measures are PattSim2, Crupi, NPV, Lewis and *LexOchiai*, respectively. As we can see, many new measures are competitive with established ones. Finally, Rand’s average score was consistently between 30-38% on reruns, which is what one might expect given an average of 2.88 bugs per program version.

We now discuss PattSim2. We saw that PattSim2 has an elegant monotonic reduction to $-a_{nf}a_{ep}$, and performs particularly well despite its relative simplicity, coming in the second place. Note that the difference between the results for PattSim2 and PattSim1 was a consequence of changing the prior constant (PC) (the details of which are discussed in the experimental setup section) in order to try and optimise the PatternSimilarity measure. We experimented in this way with this measure, because we noticed (as a theoretical observation) that by lowering the prior constant for a_{nf} it became a measure that converged to being single bug optimal. PattSim2 is a statistically significant improvement over PattSim1 using $p = 0.02$. We emphasise that we did not observe that changing the PC for our other top measures resulted in improvements in terms of their relative position in Table 4. We believe this is because the simplified ranking equivalent version of PatternSimilarity = $-a_{nf}a_{ep}$ used in our experiments is an extremely simple measure, and is consequently altered significantly by small adjustments (such as PC), where other measures are not.

We now discuss *LexOchiai*. This is our new optimising scheme Lex_g with Ochiai as the internal measure g , and it is the top performer. Most of the measures can be used as a submeasure g for Lex_g and achieve a better score than all other measures below *LexOchiai* in the table. To this end, *LexStiles*, *LexM2*, and *LexWong-III* are the runners up. Thus, Lex_g can be viewed as a good measure optimiser on our benchmarks. *LexOchiai* achieved the best score for three of the twelve benchmarks (Tcas, Totinfo, Schedule2, Replace, Gzip, Flex, Grep), the second best score for two (PrintTokens, PrintTokens2) and performed less well (towards the bottom end) in the remaining three, but never went below a score of 18.77 for such benchmarks (meaning it still has a good score in cases where other measures are ranked higher). *LexOchiai* still maintains the top overall average score if the test is run on the small programs alone (i.e. the Siemens test suite, with a score of 17.83) and comes a close third with a score of 8.02 on the larger programs alone (after PattSim2 and Zoltar which score 7.53 and 7.76, respectively). If the worst score is used instead of the average, *LexOchiai* still has the top overall score (22.32). Overall, these results support the claim that *LexOchiai* is a robust and top-performing suspiciousness measure.

We now discuss our Wilcoxon significance tests. Firstly, Rand was significantly better than Loevinger, TwoWaySupportVariation, CollectiveStrength, and GiniIndex, using $p = 0.05$. We believe this is sufficient to conclude that these measures are ineffective in SFL. *LexOchiai* was significantly better than all measures using $p = 0.29$. Using $p = 0.05$, it was significantly better than everything that scored below and including Peirce in the fourth column of Table 4, and was additionally significantly better than Good, PearlIII, Cheng and Infogain. Thirdly, PattSim2 was significantly better than everything below and including Leverage in the fourth column (apart from Fleiss and Scott), and was additionally better than MI, Jmeasure, Infogain, Cheng, PearlIII, Good, Hmean, SokSneath4, Pearson1, Suppes, Mountford and PattSim1. In general, PattSim2 was significantly better than all the measures below it on Table 4 using $p = 0.21$ (apart from Zoltar $p = 0.92$).

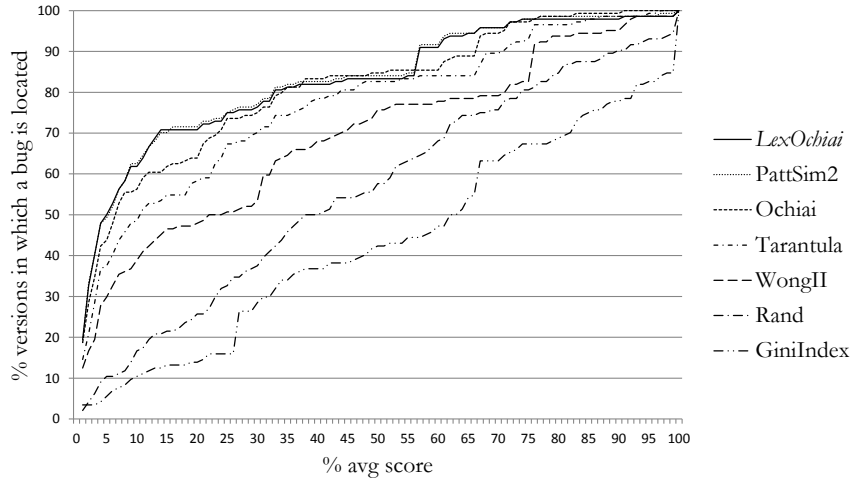


Fig. 2: Graphical comparison of prominent measures

Finally, we discuss Fig. 2, which compares the performance of some prominent measures graphically. For each measure, a line is plotted as a function of the avg scores for each of the 180 program versions. If $y\%$ of those versions have an average score $\leq x\%$, a point is plotted on the graph at (x, y) . For example, for the Rand measure, 50% of the versions have an average score lower than 58%. Intuitively, this means that if we used a random measure we might expect to find a bug around half the time if one had investigated half of the code. After investigating 5% of the code, the list of measures in Fig. 2 respectively find a bug 50, 49.31, 43.75, 37.5, 29.86, 10.42, 5.56 per cent of the time. After 10% of the code, 61.81, 62.5, 56.25, 48.61, 38.89, 16.67, 10.42 per cent of the time. *LexOchiai* and *PattSim2* have an almost aligned performance.

5 Conclusions and Future Work

We presented what is, to the best of our knowledge, the largest comparative analysis of suspiciousness measures on C programs for SFL to date, comparing 157 different measures on 12 C programs, constituting over a million lines of examined code. Out of these measures 95 are new to the domain of SFL. We taxonomised these measures into five different classes: similarity, association, causation, confirmation and custom measures. We demonstrated that each class is applicable to SFL, and that many measures are in fact equivalent in terms of ranking, thus reducing the space of measures for experimental consideration. We defined a new custom measure optimiser *Lex_g* that can admit any other measure g as its inner measure, and used *LexOchiai* in our experiments. Our experimental results demonstrate that our new measures *LexOchiai* and *PattSim2*

achieve the best average scores over other measures and are significantly better than many of them with $p = 0.05$. Lex_g can be viewed as a measure optimiser, improving the performance of any other measure g . We conjecture that the top performance of Lex_{Ochiai} is owed to a strong a priori component (single bug optimality), together with an experimentally vindicated a posteriori component (using the Ochiai measure as a submeasure). Our second best performer, PattSim2, is ranking equivalent to $-a_{nf}a_{ep}$, demonstrating the success of an extremely simple measure.

We will extend our work in several directions. On the experimental side, we wish to perform further experiments with the measures in order to consolidate our results. We would additionally like to perform experiments on benchmarks that emphasise multiple bugs. Given publicly available multiple-bug benchmarks are rare, this includes the creation of such benchmarks. On the theoretical side, we would like to investigate conditions for multiple-bug optimality, and develop measures that satisfy those conditions. Finally, we would like to create an easy-to-use tool that implements the measures discussed in this paper.

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Appendix

A Proof of Theorem 1

Where g is a measure scaled from 0 to 1, and where \mathbf{x} is the vector $\langle a_{ef}, a_{ep}, a_{nf}, a_{np} \rangle$, then Lex_g is defined as follows:

$$Lex_g(\mathbf{x}) = \begin{cases} a_{np} + 2 & \text{if } a_{ef} = F \\ g(\mathbf{x}) & \text{otherwise} \end{cases} \quad (2)$$

Theorem 1. *Lex_g is single bug optimal.*

Proof. We first show Lex_g satisfies the first condition of single bug optimality. Thus it suffices to prove that if $a_{ef} < F$, then the value returned by Lex_g is less than any value returned when $a_{ef} = F$. This holds because if $a_{ef} = F$, the value returned is greater or equal to 2, and if $a_{ef} < F$, the value returned is between 0 and 1 (by definition of the scale of g). We now show it satisfies the second condition. Thus it suffices to prove that if $a_{ef} = F$ and $a_{np} = k$, then the value returned is greater than any value returned when $a_{np} < k$. Assume $a_{ef} = F$. Now if $a_{np} = k$, then the value returned by Lex_g is $k + 2$ (by substitution). However, if $a_{np} < k$, then there is an $n > 0$ such that $k - n = a_{np}$, and the value returned by Lex_g is $(k - n) + 2$ (by substitution), which is less.

B Complete Ranking Equivalence Proofs

B.1 Outline

Two suspiciousness measures m_1 and m_2 are said to be monotonically equivalent if and only if $m_1(\mathbf{x}) < m_1(\mathbf{y}) \Leftrightarrow m_2(\mathbf{x}) < m_2(\mathbf{y})$ for all vectors \mathbf{x} and \mathbf{y} . Naish proved that many groups of suspiciousness measures are monotonically equivalent on domains in which the measures share the same program and test suite [21]. In other words, he proved they were monotonically equivalent on domains in which the number of failing test cases F and the number of passing test cases P are the same for the vectors \mathbf{x} and \mathbf{y} . This property is called *ranking equivalence* [21]. We extend this work by providing many of the remaining equivalence proofs for the measures in this paper.

Note that proving equivalences is essential in determining a maximal set of inequivalent measures to investigate and allows us to ignore the remainder in experimentation. We have summarised the results of the proofs in the table 5. We provide groups of the equivalence classes by (Eq.), the person who proved the equivalences (Proved), the measures which are equivalence in that class. In some cases, where there is a nice monotonic reduction available for the measures in a class, we have provided one such reduction in the final column (redux). We have included the results of Naish for the sake of completeness.

Eq	Proved	Measures	redux
	Naish Us	Wong-I, Russell&Rao Sensitivity, Support	a_{ef}
	Us	Specificity	$\frac{a_{np}}{a_{ef}}$
	Naish Us	Tarantula, Qe, CBI-Inc, Naish-III PPV, Eels, Fitelson-I, Fitelson-III, Pearl-IV, Cosine, Tarwid, Fossum, Popper-III	$\frac{a_{np}}{a_{ep}}$
	Us	NPV, Fitelson-II, Pearl-III, Gaifman, Rips	$\frac{a_{np}}{a_{nf}}$
	Us	Lewis, RelativeRisk, Korb-I, Pearl-I, Kemeny	$\frac{a_{ef} a_{nf} + a_{ef} a_{np}}{a_{ef} a_{nf} + a_{nf} a_{ep}}$
	Us	OddsRatio, YulesQ, YulesY, Tetrachoric, P-Heron-II	$\frac{a_{ef} a_{np}}{a_{ep} a_{nf}}$
	Naish Us	Jaccard, Anderberg, S-Dice, Goodman, Kulc-I 3w-Jaccard, SokalSneath-I, F1 Tanimoto, Lance&Williams, Nei&Lei	$\frac{a_{ef}}{a_{nf} + a_{ep}}$
	Naish Us	Wong-II, Hamann, SimpleMatching, Sokal Rogers & Tanimoto, Hamming, Euclid, M1 Sokal-Sneath-II, Sokal-Sneath-III, Gower&Legendre, Vari, Size-Similarity	$a_{ef} - a_{ep}$
	Us	Kulczynski-II, Johnson, MCconnaughey	$\frac{a_{ef}}{a_{ef} + a_{nf}} - \frac{a_{ef}}{a_{ef} + a_{ep}}$
	Us	GeometricMean, Pearson-III Pearson-Heron-I	-
	Us	Pearson-I, Pearson-II, Eyrraud	-
	Us	Ochiai, Otsuka, Sorgenfrei, Fager, Hellinger, Chord	$\frac{a_{ef}}{\sqrt{(a_{ef} + a_{nf})(a_{ef} + a_{ep})}}$
	Us	Joyce, Ample-II, YoudensJ	$\frac{a_{ef}}{a_{ef} + a_{nf}} - \frac{a_{ep}}{a_{ep} + a_{np}}$
	Us	Simpson, Confidence	-
	Us	Piatetsky-Shapiro, Carnap-I	-
	Naish Us	Scott, Rogot-II Sokal	-
	Us	PatternSimilarity	$-a_{nf} a_{ep}$

Table 5: Table of some ranking equivalences

B.2 Outline of the proof method

For the benefit of the reader, we outline the proof method in this section. To prove equivalence between measures we make use of Lee’s Lemma [21].

Lemma 1. *A measure m is monotonically equivalent to $f \circ m$ if f is a monotonically increasing function.*

Thus, to show that two measures m_1 and m_2 are equivalent, it suffices to find a monotonic function f such that $f \circ m_1 = m_2$, and then apply Lee’s Lemma. There then remains the first step of proving that f is indeed monotonic, and the second step of proving the aforementioned equality. The general method for these two steps are described below.

Step 1. In our proofs, demonstrating the monotonicity of f trivial affair. Together with the identification of relatively simple monotonically increasing functions (such as $x + 1$, 2^x etc) and decreasing functions (such as $1/x$ etc), f can easily be identified as monotonic on a single argument x using some trivial rules:

1. if f is increasing on x , and g is increasing, then $g \circ f$ is increasing on x
2. if f is decreasing on x , and g is decreasing, then $g \circ f$ is increasing on x
3. if f is decreasing on x , and g is increasing, then $g \circ f$ is decreasing on x
4. if f is increasing on x , and g is decreasing, then $g \circ f$ is decreasing on x

For instance, we can easily show that $\frac{1}{(1/x)+1}$ is monotonic on x using the above rules. $1/x$ is decreasing on x , and $y + 1$ is increasing, thus $(1/x) + 1$ is decreasing (by 3). $(1/x) + 1$ is decreasing on x (by our last result), and $1/y$ is decreasing, thus $\frac{1}{(1/x)+1}$ is increasing (by 2).

Step 2. For our proofs, demonstrating $f \circ m_1 = m_2$ is another trivial affair, when f is found to be monotonic on x . For instance, where $x = \frac{a_{ef}}{a_{ep}}$, we can easily show $\frac{1}{(1/x)+1} = \frac{a_{ef}}{a_{ep}+a_{ef}}$ using basic algebra.

Given the space consumption and triviality of steps 1 and 2, we have forgone their explicit inclusion in this appendix. This is an accepted convention established by Naish [21]. In some cases the proof is not entirely obvious so we have added extra steps to the proof to make it obvious. Verification of steps 1 and 2 is easily obtained by appeal to automatic methods.

Note that, in many cases, we have tried to find the most syntactically simple monotonically equivalent expression for a given a measure. For instance Pearl-III is simply equivalent to $\frac{a_{np}}{a_{nf}}$. We think these monotonic reductions do much to identify the essence of the measure which determines its performance in SFL.

B.3 Proofs

We follow the proof method in the previous section. We detail the monotonic functions f which transform a measure m in tables 6 and 7. In the tables, Eq. denotes groups of equivalent measures (note that the number of the class in

the table also corresponds to the number of the proposition in the text which expresses the proof), Name gives the name of the measure, m is the measure, and f gives a function, which when composed with m , evaluates to the first measure in the group (usually a redux, which is the largest monotonic simplification we could find), or alternatively evaluates to a measure in the group which itself evaluates to the first measure in the group. When the latter is the case we have been explicit. For example, in class 1, if m is the measure denoted by PPV, then $\frac{1}{(1/m)-1}$ reduces to the measure denoted by the Redux. We have organised the tables such that proof of proposition n corresponds to Eq n .

Proposition 1. *PPV, Qe, Eels, Fitelson-I, Fitelson-III, Pearl-IV, Cosine, Tarwid, Fossum, Popper-III, and Tarantula are all equivalent to $\frac{a_{ef}}{a_{ep}}$ for ranking.*

Proof. We show that each measure is ranking equivalent to a redux $\frac{a_{ef}}{a_{ep}}$. We do this by finding a monotonic function f for each of the measures m which transforms m into $\frac{a_{ef}}{a_{ep}}$, and then applying Lee's lemma. This is summarised in Eq 1 of table 6, in which we give a function f which reduces PPV to redux, and then functions which reduce the other expressions to PPV.

Note that Tarantula, Qe, and PPV are equal, and Pearl-IV is equal to Fitelson-III. Thus we have not included additional details about them in the table. Note that Eels, Fitelson-I, Fitelson-III, and Popper-III each have an equivalent algebraic form corresponding to their expression in probabilistic calculus.

The monotonic reduction of Tarwid is slightly complex, and can be elucidated as follows. Tarwid equals $\frac{T a_{ef} - (a_{ef} + a_{ep})F}{T a_{ef} + (a_{ef} + a_{ep})F}$. This is equal to $2 \frac{T a_{ef}}{T a_{ef} + (a_{ef} + a_{ep})F} - 1$. Add 1, then divide by 2 to get $\frac{T a_{ef}}{T a_{ef} + (a_{ef} + a_{ep})F}$. Apply $1/((1/x) - 1)$ to get $T \frac{a_{ef}}{(a_{ef} + a_{ep})F}$. Divide by T , then multiply by F to get $\frac{a_{ef}}{a_{ef} + a_{ep}}$. The process is equivalent to the expression $((1/((1/((m + 1)/2)) - 1))/T) * F$, which in turn simplifies to the function f for Tarwid in the table.

Proposition 2. *Lewis, RelativeRisk, Korb-I, Pearl-I and Kemeny are equivalent for ranking.*

Proof. We show they are equivalent to RelativeRisk, in a similar fashion as the previous proof. See tables 6 and 7.

Proposition 3. *NPV, Fitelson-II, Pearl-III, Gaifman and Rips are equivalent to $\frac{a_{np}}{a_{nf}}$ for ranking.*

Proof. See tables 6 and 7. We give a function which reduces NPV to redux, and then functions which reduce the other expressions to NPV.

Proposition 4. *Sensitivity, Support and Wong-I are equivalent for ranking.*

Proof. See tables 6 and 7.

Proposition 5. *OddsRatio, YulesQ, YulesY, Tetrachoric and Pearson-Heron-II are equivalent for ranking.*

Eq	Name	m	f
1	Redux	$\frac{a_{ef}}{a_{ep}}$	m
	Tarantula	$\frac{\frac{a_{ef}}{a_{ef}+a_{nf}}}{\frac{a_{ef}}{a_{ef}+a_{nf}} + \frac{a_{ep}}{a_{ep}+a_{np}}}$	$\frac{1}{(1/m)-1}$
	PPV	$\frac{a_{ef}}{a_{ef}+a_{ep}}$	-
	Qe	-	-
	Eels	$P(E C) - P(E)$	$m + \frac{F}{T}$
	Fitelson-I	$\log \frac{P(E C)}{P(E)}$	$2^m \times \frac{F}{T}$
	Fitelson-III	$\frac{P(E C)-P(E)}{P(\neg E)}$	$m \frac{P}{T} + \frac{F}{T}$
	Pearl-IV	-	-
	Cosine	$\frac{a_{ef}}{\sqrt{(a_{ef}+a_{ep})(a_{ef}+a_{nf})^2}}$	mF
	Tarwid	$\frac{T a_{ef} - (a_{ef}+a_{ep})(a_{ef}+a_{nf})}{T a_{ef} + (a_{ef}+a_{ep})(a_{ef}+a_{nf})}$	$\frac{F(m+1)}{T(1-m)}$
	Fossum	$\frac{T(a_{ef}-0.5)^2}{(a_{ef}+a_{ep})(a_{ef}+a_{nf})}$	$\frac{mF}{T}$
	Popper-III	$\frac{P(E C)-P(E)}{P(E C)+P(E)}$	$m \frac{F}{T}$
	2	RelativeRisk	$\frac{P(E C)}{P(E \neg C)}$
Lewis		$\log \frac{P(E C)}{P(E \neg C)}$	2^m
Korb-I		$\frac{P(E C)-P(E \neg C)}{P(E \neg C)}$	$m + 1$
Pearl-I		$\frac{P(E C)-P(E \neg C)}{P(E C)}$	$\frac{1}{(1/m)-1} + 1$
Kemeny		$\frac{P(E C)-P(E \neg C)}{P(E C)+P(E \neg C)}$	$\frac{-2}{m-1} - 1$
3		Redux	$\frac{a_{np}}{a_{nf}}$
	NPV	$\frac{a_{np}}{a_{np}+a_{nf}}$	$\frac{1}{(1/m)-1}$
	Fitelson-II	$\frac{P(E) - P(E \neg C)}{P(C)(P(E C)-P(E \neg C))}$	$(m - \frac{F}{T}) + 1$
	Pearl-III	$\frac{P(E)}{P(C)(P(E C)-P(E \neg C))}$	$(m \frac{F}{T}) + \frac{P}{T}$
	Gaifman	$\frac{P(\neg C)}{P(\neg C E)}$	$(\frac{m}{F} \cdot T) - 1$
	Rips	$1 - \frac{P(\neg C E)}{P(\neg C)}$	$(\frac{1/(1-m)}{F} \cdot T) - 1$
4	Wong-I	$\frac{a_{ef}}{a_{ef}}$	m
	Sensitivity	$\frac{a_{ef}}{a_{ef}+a_{nf}}$	mF
	Support	$\frac{a_{ef}}{a_{ef}+a_{nf}+a_{ep}+a_{np}}$	mT
5	OddsRatio	$\frac{a_{ef}a_{np}}{a_{ep}a_{nf}}$	m
	YulesQ	$\frac{a_{ef}a_{np}-a_{ep}a_{nf}}{a_{ef}a_{np}+a_{ep}a_{nf}}$	$\frac{1}{(2/(m+1))-1}$
	YulesY	$\frac{\sqrt{a_{ef}a_{np}}-\sqrt{a_{ep}a_{nf}}}{\sqrt{a_{ef}a_{np}}+\sqrt{a_{ep}a_{nf}}}$	see proof
	Tetrachoric	$\cos\left(\frac{\pi}{1+\sqrt{\frac{a_{ef}a_{np}}{a_{ep}a_{nf}}}}\right)$	$\cos\left(\frac{\pi}{\sqrt{x+1}}\right)$
	Pearson-Heron-II	$\cos\left(\frac{\pi\sqrt{a_{ep}a_{nf}}}{\sqrt{a_{ef}a_{np}}+\sqrt{a_{ep}a_{nf}}}\right)$	see proof

Table 6: Monotonic compositions part 1/2

Eq	Name	m	f
6	Kulczynski-I	$\frac{a_{ef}}{a_{nf}+a_{ep}}$	m
	3w-Jaccard	$\frac{3a_{ef}}{3a_{ef}+a_{ep}+a_{nf}}$	$\frac{1}{(3/m)-3}$
	SokalSneath-I	$\frac{a_{ef}}{a_{ef}+2a_{ep}+2a_{nf}}$	$\frac{2}{(1/m)-1}$
	Tanimoto	$\frac{a_{ef}+a_{ep}+a_{nf}}{a_{ep}+a_{nf}}$	$\frac{1}{(1/m)-1}$
	Lance&Williams	$-\frac{2a_{ef}+a_{ep}+a_{nf}}{2a_{ef}}$	$-\frac{m+1}{2m}$
	Nei&Lei	$\frac{2a_{ef}+a_{ep}+a_{nf}}{2}$	$\frac{1}{(2/m)-2}$
	F1	$\frac{\frac{1}{a_{ef}+a_{ep}} + \frac{1}{a_{ef}+a_{nf}}}{\frac{1}{a_{ef}+a_{ep}} + \frac{1}{a_{ef}+a_{nf}}}$	$\frac{1}{(2/m)-2}$
7	Wong-II	$\frac{a_{ef} - a_{ep}}{2a_{ef}+2a_{np}}$	m
	SokalSneath-II	$\frac{2a_{ef}+2a_{np}}{2a_{ef}+2a_{np}+a_{ep}+a_{nf}}$	$\frac{1}{(2/x)-1} \times T - P$
	Gower&Legendre	-	-
	SokSneath-III	$\frac{a_{ef}+a_{np}}{a_{ep}+a_{nf}}$	$\frac{1}{(1/x)+1} \times T - P$
	Vari	$\frac{(a_{ep}+a_{nf})}{4(a_{ef}+a_{ep}+a_{nf}+a_{np})}$	$(x4T) + F$
	SizeSimilarity	$-\frac{(a_{ep}+a_{nf})^2}{(a_{ef}+a_{ep}+a_{nf}+a_{np})^2}$	$\sqrt{x \times T^2} + F$
8	Johnson	$\frac{a_{ef}}{a_{ef}+a_{nf}} + \frac{a_{ef}}{a_{ef}+a_{ep}}$	m
	Kulczynski-II	$\frac{1}{2} \left(\frac{a_{ef}}{a_{ef}+a_{nf}} + \frac{a_{ef}}{a_{ef}+a_{ep}} \right)$	$m/2$
	MCconnaughey	$\frac{a_{ef}^2 - a_{ep}a_{nf}}{(a_{ef}+a_{ep})(a_{ef}+a_{nf})}$	m
9	GeometricMean	$\frac{a_{ef}a_{np} - a_{nf}a_{ep}}{\sqrt{(a_{ef}+a_{ep})(a_{np}+a_{nf})(a_{ef}+a_{nf})(a_{ep}+a_{np})}}$	m
	PearsonHeron-I	-	-
	Pearson-III	$\left(\frac{\rho}{T+\rho} \right)^{\frac{1}{2}}$	$\frac{1}{(1/m^2)-1} \times T$
10	Redux	$\frac{a_{ef}a_{np} - a_{ep}a_{nf}}{(a_{ef}+a_{ep})(a_{ef}+a_{nf})(a_{ep}+a_{np})(a_{nf}+a_{np})}$	m
	Pearson-I	χ^2	$\frac{m}{T}$
	Pearson-II	$\left(\frac{\chi^2}{T+\chi^2} \right)^{\frac{1}{2}}$	$\frac{1}{(1/m^2)-1} \times T$
	Eyraud	$\frac{T^2(a_{ef}a_{np} - a_{ep}a_{nf})}{(a_{ef}+a_{ep})(a_{ef}+a_{nf})(a_{ep}+a_{np})(a_{nf}+a_{np})}$	$\frac{m}{T^2}$
11	Ochiai	$\frac{a_{ef}}{\sqrt{(a_{ef}+a_{ep})(a_{ef}+a_{nf})}}$	m
	Otsuka	-	-
	Hellinger	$-2\sqrt{1 - \frac{a_{ef}}{\sqrt{(a_{ef}+a_{ep})F}}}$	$-2\sqrt{1-m}$
	Chord	$-\sqrt{2\left(1 - \frac{a_{ef}}{\sqrt{(a_{ef}+a_{ep})F}}\right)}$	$-\sqrt{2(1-m)}$
12	Redux	$-\frac{a_{ep}a_{nf}}{4(a_{ep}a_{nf})}$	m
	PatternSimilarity	$-\frac{4(a_{ep}a_{nf})}{(a_{ef}+a_{ep}+a_{nf}+a_{np})^2}$	$\frac{mT^2}{4}$

Table 7: Monotonic compositions part 2/2

Proof. See tables 6 and 7. We give monotonic functions f which transform YulesQ into Oddsratio, YulesY into YulesQ, Oddsratio into Tetrachoric and, PearsonHeronII into Oddsratio.

We do the proofs for YulesY and Pearson-Heron separately here. For YulesY, this is famously designed as a monotonic transformation of OddsRatio. For Pearson-Heron-II, we first note that $\cos(x)$ is monotonically decreasing when $0 \leq x \leq \pi$. Thus the expression $-\frac{\pi\sqrt{a_{ep}a_{nf}}}{\sqrt{a_{ef}a_{np}}+\sqrt{a_{ep}a_{nf}}}$ is monotonically equivalent to PearsonHeron-II, by substitution of order preserving functions. We then apply $\frac{1}{((1/x)-1)} + 1$ to get $\frac{\sqrt{a_{ef}a_{np}}+\sqrt{a_{ep}a_{nf}}}{\pi\sqrt{a_{ep}a_{nf}}}$. We then apply $(\pi x - 1)^2$ to get OddsRatio.

Proposition 6. *Kulczynski-I, Jaccard, 3w-Jaccard, SokalSneath-I, Tanimoto, Lance&Williams, Nei&Lei. and F1 are equivalent for ranking.*

Proof. See tables 6 and 7. Note that the unsimplified form of f for LanceWilliams is $1/(1/(((1/(1/(x) - 1)) * 2) + 1) - 1)$. Naish proved that Jaccard was equivalent to Kulczynski-I [21].

Proposition 7. *Wong-II, SokalSneath-II, SokalSneath-III, Gower&Legendre, Vari and SizeSimilarity are equivalent for ranking.*

Proof. See tables 6 and 7. Gower&Legendre is equal to SokalSneath-II.

Proposition 8. *Kulczynski-II, Johnson and MConnaughey are equivalent for ranking.*

Proof. See tables 6 and 7. Note that Johnson is equal to MConnaughey.

Proposition 9. *GeometricMean, Pearson-III and PearsonHeron-I are equivalent for ranking.*

Proof. See tables 6 and 7. PearsonHeron-I and ρ are equal to Geometric Mean.

Proposition 10. *Pearson-I, Pearson-II and Eyraud are equivalent for ranking.*

Proof. See tables 6 and 7. Note that Pearson-I is the chi-squared test for statistical significance $\chi^2 = \frac{T(a_{ef}a_{np} - a_{ep}a_{nf})}{(a_{ef} + a_{ep})(a_{ef} + a_{nf})(a_{ep} + a_{np})(a_{nf} + a_{np})}$.

Proposition 11. *Ochiai, Otsuka, Hellinger, Chord, Sorgenfrei and Fager are equivalent for ranking.*

Proof. See tables 6 and 7. Note that for Hellinger and Chord, the functions f map Ochiai onto Hellinger and Chord. For Sorgenfrei and Fager, the functions f map Sorgenfrei and Fager to Ochiai. This suffices to demonstrate the equivalence class.

Proposition 12. *PatternSimilarity is ranking equivalent to $-a_{ep}a_{nf}$.*

Proof. See tables 6 and 7.

Proposition 13. *Simpson and Confidence are equivalent for ranking.*

Proof. Simpson is $\frac{a_{ef}}{\min(a_{ef}+a_{ep}, a_{ef}+a_{nf})}$. This can be expressed as the condition if $a_{ef} + a_{ep} < a_{ef} + a_{nf}$ then $\frac{a_{ef}}{a_{ef}+a_{ep}}$, else $\frac{a_{ef}}{a_{ef}+a_{nf}}$. This is equivalent to the condition if $\frac{a_{ef}}{a_{ef}+a_{ep}} \geq \frac{a_{ef}}{a_{ef}+a_{nf}}$ then $\frac{a_{ef}}{a_{ef}+a_{ep}}$, else $\frac{a_{ef}}{a_{ef}+a_{nf}}$ (given $x < y$ if and only if $\frac{x}{z} \geq \frac{y}{z}$). This can be expressed as $\max(\frac{a_{ef}}{a_{ef}+a_{ep}}, \frac{a_{ef}}{a_{ef}+a_{nf}})$, which is equal to Confidence.

Proposition 14. *Joyce, Ample-II and Youden'sJ are equivalent for ranking.*

Proof. Joyce and Ample-II are equal. Youden'sJ + 1 is equal to both.

Proposition 15. *Rogot-II and Sokal&Sneath-IV are equal. Driver&Kroebner and Kulczynski-II are equal. Piatetsky-Shapiro is equal to Carnap-I. Wong-III and Wong-III' are equal if $a_{ep} + a_{ef} > 0$*

Proof. The proof is constructed by simplifying each of the respective measures.

C Complete Results Tables

In this section we provide the complete results tables for the measures in the main table of results in the main body of the paper. We have put the scores for *Lex Ochiai* in bold if it receives the highest score for that benchmark. For ease of presentation, the names of the benchmarks have been abbreviated.

We briefly discuss our particular appeal to the *avg* scoring method here. To calculate how well a measure performs at localising a fault on a particular program, we imagine an engineer beginning at the top of the list of program entities sorted by descending degree of suspiciousness and working down, examining each entity in turn until she encounters one containing a bug. The score for the measure is then just the percentage of program entities the engineer has to examine until a bug is found, known as the *rank percentage* [21]. A subtlety here is how to score a measure in cases where the bug has the same suspiciousness score as a set of non bugs. This leads to a *best* case score (the percentage of program entities with greater suspiciousness than the most suspicious bug), and a *worst* case score (the percentage of program entities with a greater or equal suspiciousness than the most suspicious bug). Formally, where m is a measure, n is the number of program entities in the program, and b is a bug with the highest degree of suspiciousness of any bug, we say $best(m) = (|\{x|m(x) > m(b)\}|/n) \times 100$, and $worst(m) = (|\{x|m(x) \geq m(b)\}|/n) \times 100$. Both scores have been used in the literature [1, 21, 30].⁷

We argue that the *best* score is inappropriate, since the optimal measure under this scoring method is the trivial measure assigning 1.0 to each line of

⁷ Note that we have analysed our rank percentages in terms of program entities, and not lines of code as per [21]. This is because it would otherwise have been easy to “improve” a measure’s score by empty lines or dead code, which we feel is an artificial way to improve the appearance of a measure’s performance.

code, which is unsatisfying (such a measure would always get a perfect score). The *worst* score is also inappropriate, since a measure can always increase its score by adding a small random number to otherwise equal suspiciousness scores, thereby breaking any ties. Consequently, we use an *avg* scoring method, which gives us the percentage of non-buggy program entities which we'd expect an engineer to examine before locating a bug, given the number of faulty lines in the program. Formally, where *bugs* is the number of bugs in the program, then $avg(m) = best(m) + ((worst(m) - best(m))/(bugs + 1))$ [21].

Note that we have used number of program entities in the denominator of our scoring functions, as opposed to the commonly used lines of code [21]. We feel that the former makes for more appropriate scoring functions, because a good score should not be a function of non executable lines of code (such as empty lines).

Note also that the scores for Tcas and Schedule2 are not as good as the others. We have checked this, and it is because the vast majority of program entities are covered by all the failing traces, which makes measures less effective at discerning faulty program entities according to the avg score.

D Complete Tables of Measures

The full tables of established similarity, established association, and new similarity measures are provided in this section.

The established similarity measures are studied in the context of fault localisation by Naish [21]. Naish proved that sets S1 to S6 were ranking equivalence classes.⁸ The established association measures are studied in the content of fault localisation by [19]

The new similarity measures are available in Choi [5]. The new causal and confirmation measures are largely found in the philosophy literature. The new confirmation measures are Joyce [43], Earman [36], Milne [49] Good-II [42], Carnap-I [4, 10], Carnap-II [46], Crupi [35], Rescher [52], Kemeny [44], Popper-I, II and III [46, 50, 51], Levi [46, 47], Finch-I [38], Gaifman [39], and Rips [53]. The new causal measures are Suppes [54], Eells [37], Lewis [48], Fitelson-I [10], Pearl-I,II,III, and IV [23], Fitelson-I, II and III [10], Korb I, II, and III [45], Cheng [34], Good [40, 41].

References for the Appendix

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⁸ slight exceptions are for Naish-II and Binary, which are only ranking equivalent to the other members of their respective sets given the single fault assumption

Name	AVG	TC	TI	S	S2	R	PT	PT2	SP	SD	GP	FL	GR
LexOchiai	13.74	32.81	9.44	17.72	42.95	4.9	3.82	13.17	11.88	18.77	7.27	0.9	1.29
PatSim2	13.88	32.81	9.78	20.63	42.95	5.02	3.82	13.93	8.41	18.77	7.27	1.45	1.74
Zoltar	13.92	33.06	9.54	20.63	42.95	5.04	3.82	13.17	8.36	18.77	7.27	2.64	1.74
Naish	14.01	32.81	9.44	20.63	42.95	4.9	3.82	12.98	11.88	18.77	7.27	0.9	1.74
PattSim1	14.21	33.31	9.87	20.63	42.95	5.33	4	13.93	8.58	18.77	7.27	4.07	1.76
WongIII	14.23	35.89	10.83	3.51	54.59	7.81	4.56	13.74	9.21	18.77	7.27	2.81	1.74
Kulc2	14.41	32.96	9.78	20.63	43.03	5.55	3.82	13.55	8.92	19.26	7.29	0.9	7.18
M2	14.52	34.35	12.08	5.74	49.56	5.63	3.82	15.91	8.98	19.65	7.27	2.81	8.48
Ochiai	15.25	35.59	16.44	4.88	50.44	6.45	4.56	19.67	6.46	19.32	7.29	3.82	8.11
Conviction	15.65	36.39	10.21	11.73	53.53	5.24	3.82	13.68	8.5	18.6	7.32	6.69	12.12
Certainty	15.65	36.39	10.21	11.73	53.53	5.24	3.82	13.68	8.5	18.6	7.32	6.69	12.12
Crupi	15.88	36.39	10.11	11.73	53.53	5.24	3.82	13.68	11.43	18.6	7.27	6.69	12.12
Michael	16.00	36.96	15.76	4.71	56.26	6.35	3.82	22.48	10.96	1.05	13.74	6.69	13.25
Klosgen	16.31	37.26	17.35	4.71	56.35	6.5	3.82	23.12	9.71	3.17	13.74	6.69	13.35
Mountford	16.41	36.88	19.02	5.05	50.88	7.21	4.75	23.37	7.99	19.32	7.29	6.65	8.52
YoudensJ	16.48	37.45	19.02	5.22	56.35	7.06	5.31	23.5	8.89	1.26	13.74	6.69	13.29
Earman	16.48	37.45	19.02	5.22	56.35	7.06	5.31	23.5	8.89	1.26	13.74	6.69	13.29
Carnap1	16.48	37.45	19.02	5.22	56.35	7.06	5.31	23.5	8.89	1.26	13.74	6.69	13.29
Carnap2	16.48	37.45	19.02	5.22	56.35	7.06	5.31	23.5	8.89	1.26	13.74	6.69	13.29
Levi	16.48	37.45	19.02	5.22	56.35	7.06	5.31	23.5	8.89	1.26	13.74	6.69	13.29
Dispersion	16.48	37.45	19.02	5.22	56.35	7.06	5.31	23.5	8.89	1.26	13.74	6.69	13.29
Ample2	16.48	37.45	19.02	5.22	56.35	7.06	5.31	23.5	8.89	1.26	13.74	6.69	13.29
Dennis	16.66	37.55	19.64	5.74	56.17	7.33	4.56	23.5	8.68	2.98	13.74	6.69	13.29
Popper1	16.87	37.45	20.27	6.42	56.35	7.43	7.36	23.5	8.65	1.26	13.74	6.69	13.29
Korb3	16.93	37.5	20.7	6.42	56.35	7.6	7.36	23.5	8.77	1.26	13.74	6.69	13.29
2WaySupport	16.93	37.5	20.7	6.42	56.35	7.6	7.36	23.5	8.77	1.26	13.74	6.69	13.29
YulesQ	17.11	36.88	10.83	11.56	54.23	5.9	3.82	13.74	8.78	18.7	7.27	8.03	25.54
NPV	17.15	36.51	10.11	11.73	53.7	5.24	3.82	13.81	11.43	18.6	7.27	8.03	25.54
Rescher	17.15	36.51	10.11	11.73	53.7	5.24	3.82	13.81	11.43	18.6	7.27	8.03	25.56
Lewis	17.16	36.78	10.5	11.56	54.23	5.84	3.82	13.55	10.2	18.6	7.27	8.03	25.54
AMean	17.17	37.63	19.6	6.42	56.08	7.6	5.12	24.52	8.78	3.52	15.42	8.03	13.29
Stiles	17.27	38.82	24.72	5.05	45.68	6.82	7.36	15.62	10.42	3.59	16.27	10.67	22.19
GMean	17.27	37.55	19.36	5.05	56.08	7.31	4.75	23.25	8.4	3.76	11.22	8.03	22.48
Phi	17.27	37.55	19.36	5.05	56.08	7.31	4.75	23.25	8.4	3.76	11.22	8.03	22.48
Jaccard	17.40	37.65	20.31	5.91	56.53	8.89	8.1	25.1	8.47	18.58	7.29	3.82	8.2
CBISqrt	17.68	37.75	22.28	6.42	56.53	9.25	11.08	25.29	9.82	3.82	11.98	5.89	12.1
Popper2	17.74	37.08	14.81	8.82	54.23	6.77	4.75	20.06	6.96	18.6	7.27	8.03	25.54
Cohen	17.98	37.75	22.61	6.42	56.53	9.23	10.52	25.35	9.99	2.98	14.34	6.69	13.29
Kappa	17.98	37.75	22.61	6.42	56.53	9.23	10.52	25.35	9.99	2.98	14.34	6.69	13.29
CBILog	18.11	37.75	22.76	6.42	56.53	9.28	11.45	25.35	12.14	2.98	13.74	6.69	12.21
Likelihood	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
GilbertW	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37

Table 8: Table of scores for measures for each benchmark

Name	AVG	TC	TI	S	S2	R	PT	PT2	SP	SD	GP	FL	GR
Keynes	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
Good2	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
Finch1	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
Forbes1	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
Tarantula	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
Interest	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
AddedValue	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
SebagSch	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
OddMult	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
Example	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
Zhang	18.22	37.75	23	6.42	56.53	9.3	11.45	25.54	13.82	1.05	13.74	6.69	13.37
Korb2	18.24	37.75	23	6.42	56.53	9.3	11.45	25.8	13.82	1.05	13.74	6.69	13.37
1WaySupport	18.24	37.75	23	6.42	56.53	9.3	11.45	25.8	13.82	1.05	13.74	6.69	13.37
Laplace	18.42	38	23.57	6.59	56.7	9.54	11.82	25.67	14.05	1.24	13.74	6.69	13.38
Suppes	18.62	37.68	18.26	5.91	54.41	7.65	6.98	24.14	8.54	18.81	7.27	8.03	25.79
Pearson1	18.62	37.68	18.26	5.91	54.41	7.65	6.98	24.14	8.54	18.81	7.27	8.03	25.79
Sok1Sneath4	18.65	37.45	18.97	5.05	56.26	7.2	5.49	23.12	8.32	18.91	8.3	8.03	26.64
HMean	18.65	37.45	18.97	5.05	56.26	7.2	5.49	23.12	8.32	18.91	8.3	8.03	26.64
Good	19.26	37.68	18.88	6.42	54.41	7.67	7.73	24.14	10.35	18.91	10.67	8.03	26.22
PearlII	19.26	37.68	18.88	6.42	54.41	7.67	7.73	24.14	10.35	18.91	10.67	8.03	26.22
Cheng	19.26	37.68	18.88	6.42	54.41	7.67	7.73	24.14	10.35	18.91	10.67	8.03	26.22
InfoGain	19.93	37.75	21.89	5.39	56.44	9.27	11.08	25.35	9.84	19.19	7.27	6.69	29.02
JMeasure	19.95	38.15	21.7	5.57	74.93	5.66	3.82	14.41	9.94	20.37	16.94	9.36	18.58
Ochiai2	20.40	38.32	30.76	6.42	64.37	8.11	4.75	18.46	11.81	4.68	16.31	14	26.86
SokSneath5	20.40	38.32	30.76	6.42	64.37	8.11	4.75	18.46	11.81	4.68	16.31	14	26.86
MI	20.53	37.06	21.22	12.59	56.26	6.48	3.82	16.61	21.8	6.08	18.32	14.18	32
Peirce	22.37	38.82	39.76	6.71	66.4	9.23	5.87	17.83	12.59	6.9	24.45	12.38	27.49
Leverage	23.20	38.75	30.37	6.76	61.46	10.22	11.82	27.33	22	6.08	21.97	13.36	28.32
BinaryNaish	23.34	45.31	23.44	40.24	34.89	18.76	20.11	34.02	38.53	6.29	13.24	1.65	3.62
WongI	23.43	45.31	23.44	40.53	34.89	18.76	20.11	35.65	38.53	6.29	13.24	1.65	2.7
Confidence	23.43	45.31	23.44	40.53	34.89	18.76	20.11	35.65	38.53	6.29	13.24	1.65	2.7
Fleiss	23.61	40.76	41.77	7.91	66.4	16.86	14.25	22.35	14.07	2.8	15.48	14	26.69
Scott	23.86	40.76	41.77	7.91	66.4	16.64	14.25	23.76	14.88	4.76	14.56	14	26.69
Faith	24.49	40.91	40.91	7.91	66.4	18.68	15.18	25.35	15.28	20.71	7.24	10.14	25.15
LeastCont	25.83	40.91	42.06	8.76	66.4	19.38	15.92	25.92	20.08	19.87	8.61	14.94	27.16
WongII	25.85	40.91	42.06	8.76	66.4	19.38	15.92	25.92	20.08	19.87	8.75	14.94	27.16
Gower	26.41	42.57	44.36	15.61	69.05	21.04	16.85	37.91	44.28	3.8	13.59	5.14	2.67
GoodKrus	26.64	41	44.12	9.79	66.67	21.06	16.29	26.05	26.6	7.27	16.22	14	30.61
Specificity	27.20	41	44.05	9.1	66.45	20.77	16.48	26.5	32.26	6.86	17.28	14	31.66
Anderberg2	27.25	43.64	54.63	31.96	59.79	16.03	2.7	23.79	23.5	7.51	18.28	14.86	30.32
FagerMc	29.10	41	45.51	9.1	66.49	21.31	17.6	28.03	38.78	7.59	30.66	14	29.13
Rand	31.74	57.64	38.88	28.54	47.03	31.25	48.56	53.19	45.94	2.63	13.65	4.97	8.65

Table 9: Continued table of scores for measures for each benchmark

Name	Measure
ϕ -coefficient	$\frac{P(E \cap C) - P(C)P(E)}{\sqrt{P(C)P(E)(1-P(C))(1-P(E))}}$
OddsRatio	$\frac{P(C \cap E)P(\neg C \cap \neg E)}{P(C \cap \neg E)P(\neg C \cap E)}$
Yule-Q	$\frac{P(C \cap E)P(\neg C \cap \neg E) - P(C, \neg E)P(\neg C \cap E)}{P(CE)P(\neg C \cap \neg E) + P(C \cap \neg E)P(\neg C \cap E)}$
Yule-Y	$\frac{\sqrt{P(C \cap E)P(\neg C \cap \neg E)} - \sqrt{P(C \cap \neg E)P(\neg C \cap E)}}{\sqrt{P(CE)P(\neg C \cap \neg E)} + \sqrt{P(C \cap \neg E)P(\neg C \cap E)}}$
Kappa	$\frac{P(C \cap E) + P(\neg C \cap \neg E) - P(C)P(E) - P(\neg C)P(\neg E)}{1 - P(C)P(E) - P(\neg C)P(\neg E)}$
J-Measure	
	$\max(P(C \cap E) \log(\frac{P(E C)}{P(E)}) + P(C \cap \neg E) \log(\frac{P(\neg E C)}{P(\neg E)}),$ $P(C \cap E) \log(\frac{P(C E)}{P(C)}) + P(\neg C \cap E) \log(\frac{P(\neg C E)}{P(\neg C)}))$
Gini Index	$\max(P(C)(P(E C)^2 + P(\neg E C)^2) +$ $P(\neg C)(P(E \neg C)^2 + P(\neg E \neg C)^2) - P(E)^2 - P(\neg E)^2,$ $P(E)(P(C E)^2 + P(\neg C E)^2) + P(\neg E)(P(C \neg E)^2$ $+ P(\neg C \neg E)^2 - P(C)^2 - P(\neg C)^2)$
Support	$P(C \cap E)$
Confidence	$\max((P(E C), P(C E)))$
Laplace	$\max(\frac{TP(C \cap E)+1}{TP(C)+2}, \frac{TP(C \cap E)+1}{TP(E)+2})$
Conviction	$\max(\frac{P(C)P(\neg E)}{P(C \cap \neg E)}, \frac{P(\neg C)P(E)}{P(\neg C \cap E)})$
Interest	$\frac{P(C \cap E)}{P(C)P(E)}$
Cosine	$\frac{P(C \cap E)}{\sqrt{P(C)P(E)}}$
Piatetsky-Shapiro	$\frac{P(C \cap E) - P(C)P(E)}{P(C \cap E)}$
Certainty	$\max(\frac{P(E C) - P(E)}{P(\neg E)}, \frac{P(C E) - P(C)}{P(\neg C)})$
Added-Value	$\max(P(E C) - P(E), P(C E) - P(C))$
Coll-Strength	$\frac{P(C \cap E) + P(\neg C \cap \neg E)}{P(C)P(E) + P(\neg C)P(\neg E)} \times$ $\frac{1 - P(C)P(E) - P(\neg C)P(\neg E)}{1 - P(C \cap E) + P(\neg C \cap \neg E)}$
Jaccard	$\frac{P(C, E)}{P(C) + P(E) - P(C, E)}$
Kloggen	$\sqrt{P(C \cap E)} \cdot \text{Added-Value}$
InformationGain	$(-P(E) \log P(E) - P(\neg E) \log P(\neg E)) -$ $(P(C)(-P(E C) \log P(E C)) - P(\neg E C) \log P(\neg E C) -,$ $P(\neg C)(-P(E \neg C) \log P(E \neg C)) - P(\neg E \neg C) \log P(\neg E \neg C))$

Table 10: 20 association measures established in SFL, full table

Eq	Name	Measure	Name	Measure
S1	Naish-I	-1 if $a_{ef} < F$ $P - a_{ep}$ if $a_{ef} = F$	Kulczynski-II	$\frac{1}{2} \left(\frac{a_{ef}}{a_{ef} + a_{nf}} + \frac{a_{ef}}{a_{ef} + a_{ep}} \right)$
	Naish-II	$a_{ef} - \frac{a_{ep}}{a_{ep} + a_{np} + 1}$	Ochiai	$\frac{a_{ef}}{\sqrt{(a_{ef} + a_{nf})(a_{ef} + a_{ep})}}$
S2	Jaccard	$\frac{a_{ef}}{a_{ef} + a_{nf} + a_{ep}}$	M2	$\frac{a_{ef} + a_{np} + 2(a_{nf} + a_{ep})}{a_{ef}}$
	Anderberg	$\frac{a_{ef} + 2(a_{nf} + a_{ep})}{2a_{ef}}$	Ample-I	$\left \frac{a_{ef} + a_{nf}}{a_{ef}} - \frac{a_{ep} + a_{np}}{a_{ep}} \right $
	Sorensen-Dice	$\frac{2a_{ef}}{2a_{ef} + a_{nf} + a_{ep}}$	Ample-II	$\frac{a_{ef} + a_{nf}}{a_{ef} + a_{ep}} - \frac{a_{ep} + a_{np}}{a_{ep}}$
	Dice	$\frac{2a_{ef}}{a_{ef} + a_{nf} + a_{ep}}$	CBI-Sqrt	$\frac{1}{\text{CBI-Inc}} + \frac{\sqrt{a_{ef} + a_{nf}}}{\sqrt{a_{ef}}}$
	Goodman	$\frac{2a_{ef} - a_{nf} - a_{ep}}{2a_{ef} + a_{nf} + a_{ep}}$	CBI-Log	$\frac{1}{\text{CBI-Inc}} + \frac{\log(a_{ef} + a_{nf})}{\log(a_{ef})}$
	Kulczynski-I	$\frac{a_{ef}}{a_{nf} + a_{ep}}$	Zoltar	$\frac{a_{ef}}{a_{ef} + a_{nf} + a_{ep} + \frac{10000a_{nf}a_{ep}}{a_{ef}}}$
S3	Tarantula	$\frac{\frac{a_{ef}}{a_{ef} + a_{nf}}}{\frac{a_{ef}}{a_{ef} + a_{nf}} + \frac{a_{ep}}{a_{ep} + a_{np}}}$	Ochiai-II	$\frac{a_{ef}a_{np}}{\sqrt{(a_{ef} + a_{ep})FP(a_{nf} + a_{np})}}$
	Qe	$\frac{a_{ef}}{a_{ef} + a_{ep}}$	Wong-III	$a_{ef} - h$, where
	CBI-Inc.	$\frac{a_{ef}}{a_{ef} + a_{ep}} - \frac{a_{ef} + a_{nf}}{a_{ef} + a_{ep} + a_{nf} + a_{np}}$		$h = a_{ep}$ if $a_{ep} \leq 2$
	Naish-III	$\frac{a_{ef}}{a_{ep}}$		$h = 2 + 0.1(a_{ep} - 2)$ if $2 < a_{ep} \leq 10$ $h = 2.8 + 001(a_{ep} - 10)$ if $a_{ep} > 10$
S4	Wong-II	$a_{ef} - a_{ep}$	Wong-III'	-1000 if $a_{ep} + a_{ef} = 0$ Wong-III otherwise
	Hamann	$\frac{a_{ef} + a_{np} - a_{nf} - a_{ep}}{a_{ef} + a_{nf} + a_{ep} + a_{np}}$	Rogot-II	$\frac{1}{4} \left(\frac{a_{ef}}{a_{ef} + a_{ep}} + \frac{a_{ef}}{a_{ef} + a_{nf}} + \frac{a_{np}}{a_{ep} + a_{np}} + \frac{a_{np}}{a_{nf} + a_{np}} \right)$
	SimpleMatching	$\frac{a_{ef} + a_{np}}{a_{ef} + a_{nf} + a_{np} + a_{ep}}$	ArithmeticMean	$\frac{2a_{ef}a_{np} - 2a_{nf}a_{ep}}{(a_{ef} + a_{ep})(a_{np} + a_{nf}) + (a_{ef} + a_{nf})(a_{ep} + a_{np})}$
	Sokal	$\frac{2(a_{ef} + a_{np})}{2a_{ef} + 2a_{nf} + a_{nf} + a_{ep}}$	GeometricMean	$\frac{a_{ef}a_{np} - a_{nf}a_{ep}}{\sqrt{(a_{ef} + a_{ep})(a_{np} + a_{nf})(a_{ef} + a_{nf})(a_{ep} + a_{np})}}$
	Rogers&Tanimoto	$\frac{a_{ef} + a_{np}}{a_{ef} + a_{np} + 2(a_{nf} + a_{ep})}$	HarmonicMean	$\frac{a_{ef}a_{np} - a_{nf}a_{ep}}{(a_{ef} + a_{ep})(a_{np} + a_{nf}) + (a_{ef} + a_{nf})(a_{ep} + a_{np})}$
	Hamming	$a_{ef} + a_{np}$	Cohen	$\frac{2a_{ef}a_{np} - 2a_{nf}a_{ep}}{(a_{ef} + a_{ep})(a_{np} + a_{ep}) + (a_{ef} + a_{nf})(a_{nf} + a_{np})}$
	Euclid	$\sqrt{a_{ef} + a_{np}}$	Fleiss	$\frac{4a_{ef}a_{np} - 4a_{nf}a_{ep} - (a_{nf} - a_{ep})^2}{(2a_{ef} + a_{nf} + a_{ep})(2a_{np} + a_{nf} + a_{ep})}$
	M1	$\frac{a_{ef} + a_{np}}{a_{nf} + a_{ep}}$	Rogot-I	$\frac{1}{2} \left(\frac{a_{ef}}{2a_{ef} + a_{nf} + a_{ep}} + \frac{a_{np}}{2a_{np} + a_{nf} + a_{ep}} \right)$
S5	Wong-I	a_{ef}		
	Russel&Rao	$\frac{a_{ef}}{a_{ef} + a_{nf} + a_{ep} + a_{np}}$		
	Binary	0 if $a_{ef} < F$ 1 if $a_{ef} = F$		
S6	Scott	$\frac{4a_{ef}a_{np} - 4a_{nf}a_{ep} - (a_{nf} - a_{ep})^2}{(2a_{ef} + a_{nf} + a_{ep})(2a_{np} + a_{nf} + a_{ep})}$		
	Rogot-I	$\frac{1}{2} \left(\frac{a_{ef}}{2a_{ef} + a_{nf} + a_{ep}} + \frac{a_{np}}{2a_{np} + a_{nf} + a_{ep}} \right)$		

Table 11: 42 Established SFL measures, from Naish, full table

Name	Measure
Nei&Li	$\frac{2a_{ef}}{2a_{ef}+a_{ep}+a_{nf}}$
3w-Jaccard	$\frac{3a_{ef}}{3a_{ef}+a_{ep}+a_{nf}}$
Sokal&Sneath-I	$\frac{a_{ef}}{a_{ef}+2a_{ep}+2a_{nf}}$
Sokal&Sneath-II	$\frac{2a_{ef}+2a_{np}}{2a_{ef}+2a_{np}+a_{ep}+a_{nf}}$
Sokal&Sneath-III	$\frac{a_{ef}+a_{np}}{a_{ep}+a_{nf}+0.5a_{np}}$
Faith	$\frac{a_{ef}+a_{np}+a_{ep}+a_{nf}}{a_{ef}+a_{np}}$
Gower&Legendre	$\frac{a_{ef}+a_{np}+0.5a_{ep}+0.5a_{nf}}{(a_{ep}+a_{nf})}$
Vari	$-\frac{(a_{ep}+a_{nf})}{4(a_{ef}+a_{ep}+a_{nf}+a_{np})^2}$
SizeSimilarity	$-\frac{(a_{ep}+a_{nf})^2}{(a_{ef}+a_{ep}+a_{nf}+a_{np})^2}$
ShapeSimilarity	$\frac{T(a_{ep}+a_{nf})-(a_{ep}+a_{nf})^2}{(a_{ef}+a_{ep}+a_{nf}+a_{np})^2}$
PatternSimilarity	$-\frac{4(a_{ep}a_{nf})}{(a_{ef}+a_{ep}+a_{nf}+a_{np})^2}$
Lance&Williams	$-\frac{(a_{ep}+a_{nf})}{2a_{ef}+a_{ep}+a_{nf}}$
Bray&Curtis	$-\frac{(a_{ep}+a_{nf})}{2a_{ef}+a_{ep}+a_{nf}}$
Cosine	$\frac{a_{ef}}{\sqrt{(a_{ef}+a_{ep})(a_{ef}+a_{nf})^2}}$
Forbes-I	$\frac{T a_{ef}}{(a_{ef}+a_{ep})(a_{ef}+a_{nf})}$
Fossum	$\frac{T(a_{ef}-0.5)^2}{(a_{ef}+a_{ep})(a_{ef}+a_{nf})}$
Sorgenfrei	$\frac{a_{ef}^2}{(a_{ef}+a_{ep})(a_{ef}+a_{nf})}$
Mountford	$\frac{0.5(a_{ef}a_{ep}+a_{ef}a_{nf})+a_{ep}a_{nf}}{a_{ef}}$
Otsuka	$\frac{a_{ef}}{((a_{ef}+a_{ep})(a_{ef}+a_{nf}))^{0.5}}$
Mcconnaughey	$\frac{a_{ef}^2-a_{ep}a_{nf}}{(a_{ef}+a_{ep})(a_{ef}+a_{nf})}$

Table 12: New similarity measures, part 1

Driver&Kroeber	$\frac{a_{ef}}{2} \left(\frac{1}{a_{ef}+a_{ep}} + \frac{1}{a_{ef}+a_{nf}} \right)$
Johnson	$\frac{\frac{1}{a_{ef}} + \frac{1}{a_{ef}+a_{nf}}}{a_{ef}+a_{ep} + a_{ef}+a_{nf}}$
Dennis	$\frac{a_{ef} a_{np} - a_{ep} a_{nf}}{\sqrt{T(a_{ef}+a_{ep})(a_{ef}+a_{nf})}}$
Braun-Blanquet	$\frac{a_{ef}}{\min(a_{ef}+a_{ep}, a_{ef}+a_{nf})}$
Dispersion	$\frac{a_{ef} a_{np} - a_{ep} a_{nf}}{(a_{ef}+a_{ep}+a_{nf}+a_{ep})^2}$
Michael	$\frac{4(a_{ef} a_{np} - a_{ep} a_{nf})}{(a_{ef}+a_{np})^2 + (a_{ep}+a_{np})^2}$
Baroni-Urbani-Buser-I	$\frac{\sqrt{a_{ef} a_{np} + a_{ef}}}{\sqrt{a_{ef} a_{np} + a_{ef} + a_{np} + a_{nf}}}$
Baroni-Urbani-Buser-II	$\frac{\sqrt{a_{ef} a_{np} + a_{ef}} - (a_{ep} + a_{nf})}{\sqrt{a_{ef} a_{np} + a_{ef} + a_{np} + a_{nf}}}$
Peirce	$\frac{a_{ef} a_{np} + a_{ep} a_{nf}}{a_{ef} a_{ep} + 2a_{ep} a_{nf} + a_{nf} a_{np}}$
Hellinger	$-2 \sqrt{1 - \frac{a_{ef}}{\sqrt{(a_{ef}+a_{ep})F}}}$
Chord	$-\sqrt{2 \left(1 - \frac{a_{ef}}{\sqrt{(a_{ef}+a_{ep})F}} \right)}$
Fager&McGowan	$\frac{a_{ef}}{\sqrt{(a_{ef}+a_{ep})F}} - \frac{\max(a_{ef}+a_{ep}, F)}{2}$
Forbes-II	$\frac{T a_{ef} - (a_{ef}+a_{ep})F}{T \min(a_{ef}+a_{ep}, F) - (a_{ef}+a_{ep})F}$
Gower	$\frac{\sqrt{(a_{ef}+a_{ep})FP(a_{nf}+a_{np})}}{a_{ef}+a_{ep} + \frac{a_{ef}}{F} + \frac{a_{np}}{P} + \frac{a_{nf}}{a_{nf}+a_{np}}}$
Sokal&Sneath-IV	$\frac{4 a_{ef} a_{np}}{((a_{ef}+a_{ep})FP(a_{nf}+a_{np}))^{0.5}}$
Sokal&Sneath-V	$\frac{a_{ef} a_{np}}{\sqrt{2(a_{ef} a_{np} - a_{ep} a_{nf})}}$
Cole	$\frac{1}{\sqrt{(a_{ef} a_{np} - a_{ep} a_{nf})^2 - (a_{ef}+a_{ep})FP(a_{nf}+a_{np})}}$
Stiles	$\log \frac{T(a_{ef} a_{np} - a_{ep} a_{nf} - \frac{T}{2})^2}{(a_{ef}+a_{ep})FP(a_{nf}+a_{np})}$
Eyraud	$\frac{T^2(T a_{ef} - (a_{ef}+a_{ep})F)}{(a_{ef}+a_{ep})FP(a_{nf}+a_{np})}$
Fager	$\frac{a_{ef}}{\sqrt{F(a_{ef}+a_{ep})}} - \frac{1}{2\sqrt{F}}$
F1	$\frac{\frac{1}{a_{ef}} + \frac{1}{a_{ef}}}{a_{ef}+a_{ep} + a_{ef}+a_{nf}}$
Simpson	$\frac{a_{ef}}{\min(a_{ef}+a_{ep}, a_{ef}+a_{nf})}$
Tarwid	$\frac{T a_{ef} - (a_{ef}+a_{ep})(a_{ef}+a_{nf})}{T a_{ef} + (a_{ef}+a_{ep})(a_{ef}+a_{nf})}$

Table 13: New similarity measures, part 2

Name	Measure
PPV	$\frac{a_{ef}}{a_{ef}+a_{ep}}$
NPV	$\frac{a_{np}}{a_{np}+a_{nf}}$
Sensitivity	$\frac{a_{ef}}{a_{ef}+a_{nf}}$
Specificity	$\frac{a_{np}}{a_{np}+a_{ep}}$
Youden's-J	$\frac{a_{ef}}{a_{ef}+a_{nf}} + \frac{a_{np}}{a_{np}+a_{ep}} - 1$
PosLikelihood	$\frac{a_{ep}a_{ef}+a_{ef}a_{np}}{a_{ep}a_{ef}+a_{ep}a_{nf}}$
Tetrachoric	$\cos\left(\frac{\pi}{1+\sqrt{\frac{a_{ef}a_{np}}{a_{ep}a_{nf}}}}\right)$
Relative risk	$\frac{P(E C)}{P(E \neg C)} = \frac{a_{ef}a_{nf}+a_{ef}a_{np}}{a_{ef}a_{nf}+a_{nf}a_{ep}}$
Z-ratio	$\frac{a_{ef}+a_{ep}}{a_{ef}} - \frac{a_{nf}+a_{np}}{a_{nf}}$
Peirce	$\frac{a_{ef}a_{np}+a_{ep}a_{nf}}{a_{ef}a_{ep}+2a_{ep}a_{nf}+a_{nf}a_{np}}$
Pearson-I	$\chi^2 = \frac{T(a_{ef}a_{np}-a_{ep}a_{nf})}{(a_{ef}+a_{ep})(a_{ef}+a_{nf})(a_{ep}+a_{np})(a_{nf}+a_{np})}$
Pearson-II	$\left(\frac{\chi^2}{T+\chi^2}\right)^{\frac{1}{2}}$
Pearson-III	$\left(\frac{\rho}{T+\rho}\right)^{\frac{1}{2}}$
Pearson&Heron-I	$\rho = \frac{a_{ef}a_{np}-a_{nf}a_{ep}}{\sqrt{(a_{ef}+a_{ep})(a_{ef}+a_{nf})(a_{ep}+a_{np})(a_{nf}+a_{np})}}$
Pearson&Heron-II	$\cos\left(\frac{\pi\sqrt{a_{ep}a_{nf}}}{\sqrt{a_{ef}a_{np}+a_{ep}a_{nf}}}\right)$
Anderberg-II	$\frac{\sigma-\sigma'}{2T}$
Tanimoto	$\frac{a_{ef}+a_{ep}+a_{nf}}{a_{ef}}$
Mutual-Info	$\frac{P(C,E) \log \frac{P(C,E)}{P(C)P(E)}}{\min(-P(C) \log P(C) - P(E) \log P(E))}$
Simpson	$\frac{a_{ef}}{\min(a_{ef}+a_{ep}, a_{ef}+a_{nf})}$
Gilbert&Wells	$\log a_{ef} - \log T - \log\left(\frac{a_{ef}+a_{ep}}{T}\right) - \log\left(\frac{a_{ef}+a_{nf}}{T}\right)$
Goodman&Kruskal	$\frac{\sigma-\sigma'}{2T-\sigma'}$, where $\sigma = \max(a_{ef}, a_{ep}) + \max(a_{nf}, a_{np})$ $+ \max(a_{ef}, a_{nf}) + \max(a_{ep}, a_{np})$, and $\sigma' = \max(a_{ef} + a_{nf}, a_{ep} + a_{np})$ $+ \max(a_{ef} + a_{ep}, a_{nf} + a_{np})$

Table 14: New prediction measures

Name	Causal Measure	Name	Confirmation Measure
Suppes	$P(E C) - P(E \neg C)$	Joyce	$P(H E) - P(H \neg E)$
Eels	$P(E C) - P(E)$	Earman	$P(H E) - P(H)$
Lewis	$\log \frac{P(E C)}{P(E \neg C)}$	Good-II	$\log \frac{P(H E)}{P(H \neg E)}$
Fitelson-I	$\log \frac{P(E C)}{P(E)}$	Milne	$\log \frac{P(H E)}{P(H)}$
Fitelson-II	$P(E) - P(E \neg C)$	Carnap-I	$P(H \cap E) - P(H)P(E)$
Fitelson-III	$\frac{P(E C) - P(E)}{P(\neg E)}$	Crupi	$\frac{P(H E) - P(H)}{P(\neg H)}$ if $\text{Erm} \geq 0$ $\frac{P(H E) - P(H)}{P(H)}$ otherwise
Carnap-II	$P(H) \frac{P(E H) - P(E)}{P(E)}$	Carnap-II	$P(H) \frac{P(E H) - P(E)}{P(E)}$
Pearl-I	$\frac{P(E C) - P(E \neg C)}{P(E C)}$	Rescher	$\frac{P(H)}{P(\neg H)} P(E H) - P(E)$
Pearl-II	$\frac{P(E C) - P(E \neg C)}{P(\neg E \neg C)}$	Kemeny	$\frac{P(E H) - P(E \neg H)}{P(E H) + P(E \neg H)}$
Pearl-III	$\frac{P(C)(P(E C) - P(E \neg C))}{P(E)}$	Popper-I	$P(H)P(H E) \frac{P(E H) - P(E)}{P(E H) + P(E)}$
Pearl-IV	$\frac{P(\neg C)(P(E C) - P(E \neg C))}{P(\neg E)}$	Popper-II	$\frac{P(E H) - P(E)}{P(\neg H)P(E H) + P(E)}$
Korb-I	$\frac{P(E C) - P(E \neg C)}{P(E \neg C)}$	Popper-III	$\frac{P(E H) - P(E)}{P(E H) + P(E)}$
Korb-II	$P(E C) \log \frac{P(E C)}{P(E)}$	Levi	$P(H)P(\neg H) \frac{P(E H) - P(E \neg H)}{P(E)}$
Korb-III	$P(C)P(E C) \log \frac{P(E C)}{P(E)}$	Finch-I	$\frac{P(H E) - P(H)}{P(H)}$
Good	$\log \frac{P(\neg E \neg C)}{P(\neg E C)}$	Gaifman	$\frac{P(\neg H)}{P(\neg H E)}$
Cheng	$\frac{P(E C) - P(E \neg C)}{1 - P(E \neg C)}$	Rips	$1 - \frac{P(\neg H E)}{P(\neg H)}$

Table 15: Causal and confirmation measures

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