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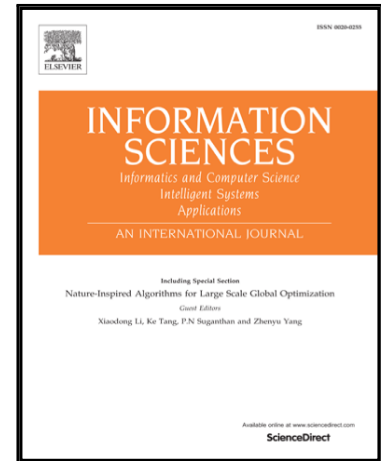
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Xiaojie Su, Xinxin Liu, Yong-Duan Song, Hak Keung Lam, Lei Wang

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Reduced-Order Model Approximation of Fuzzy Switched Systems with Pre-specified Performance[☆]

Xiaojie Su^{a,b,*}, Xinxin Liu^{a,b}, Yong-Duan Song^{a,b}, Hak Keung Lam^c, Lei Wang^{a,b}

^aKey Laboratory of Dependable Service Computing in Cyber Physical Society of Ministry of Education, Chongqing University, Chongqing 400044, China

^bCollege of Automation, Chongqing University, Chongqing 400044, China

^cDepartment of Informatics, King's College London, Strand, London WC2R 2LS, U.K

Abstract

The reduced-order model approximation problem for discrete-time hybrid switched nonlinear systems is addressed via Takagi-Sugeno (T-S) fuzzy modelling in this paper. For a high-dimension hybrid switched nonlinear system, our aim is on how to construct a reduced-dimension hybrid switched model, approximating its original high-order model well with a pre-specified system performance level. Firstly, the mean-square exponential stability analysis is provided, in which it guarantees the given weighted \mathcal{H}_∞ system performance level for the augmented error dynamic system by the average dwell time analysis approach and the hybrid switched Lyapunov stability theory. The solution of corresponding model reduction problem with pre-specified performance is given by using the projection Lemma, by which the algorithm of the reduced-order hybrid switched model parameters are designed by the cone complementary linearization technique. Finally, the advantage and effectiveness of the proposed reduced-order model approximation approach are shown by the simulation result.

Keywords: Model reduction, model approximation, reduced-order systems, switched systems, hybrid systems

1. Introduction

A finite number of independent control subsystems, including discrete-time or continuous-time dynamics, and a switching signal governing the activation of these concerned subsystems, form hybrid stochastic switched systems, which is a significant component of stochastic jump systems in [25, 26, 27, 28]. A large class of practical systems and processes, including advanced transportation managements systems, automated highway systems, communication systems and network control systems [40, 41], can be characterized as hybrid stochastic switched systems. Moreover, there are some intelligent control strategies with the idea of introducing hybrid switching controllers, which break the limitations of the traditionally adopted single controller effectively and greatly improve the resulted closed-loop control system performance level. By this, the corresponding closed-loop control systems are translated into a typical hybrid stochastic switched systems. Considerable efforts have been put on the hybrid stochastic switched systems

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*Corresponding author

Email addresses: suxiaojie@cqu.edu.cn (Xiaojie Su), liuxinxin@cqu.edu.cn (Xinxin Liu), ydsong@cqu.edu.cn (Yong-Duan Song), hak-keung.lam@kcl.ac.uk (Hak Keung Lam), leiwang08@cqu.edu.cn (Lei Wang)

recently, and many great achievements have been made in this research field. For example, the switched controller design problems of hybrid switched systems are addressed in [20, 35, 43], the model reduction/approximation approach for stochastic switched time delay systems is proposed in [36], the mixed \mathcal{H}_∞ and passive filtering issues are investigated in [29], and stability analysis and stabilization problems for switched linear systems are discussed in [24, 38].

Systems with complex nonlinear dynamics [6, 14, 22, 23] impose significant difficulty in theoretical analysis in the traditional way. The key point is how to find a credible solution to handle this sort of nonlinear dynamic systems. As fuzzy logic control systems progressed, T-S fuzzy modelling is proved to be an effective approach to represent these complex dynamic systems. By introducing IF-THEN logical statement, the resulted complex dynamic systems are converted into a number of local linear subsystems [8, 19, 15, 39]. Since the nonlinear system can be modelled as a weighted total of combined linear sub-models via T-S fuzzy modelling, researchers have put a great deal of efforts into T-S fuzzy systems and numerous achievements have been obtained in this research field. To mention a few, the receding horizon disturbance attenuation analysis and dynamic decoupling are studied in [2, 7], the fuzzy controllers are designed for nonlinear systems in [16, 32, 34, 42], the fault detection and \mathcal{H}_∞ synchronization problem are solved in [17, 30], the fuzzy-rule-dependent stability analysis and control problems are studied in [11, 12, 13, 18], and the fuzzy filtering problems with the given pre-specified performance are investigated in [1, 44].

On another research front line, the vast majority of real-time physical systems and complex industrial processes including 2-D dynamic systems [3, 4, 5], result in rather sophisticated high-order mathematical analysis model. It poses great challenge on system performance analysis and the corresponding control system design. Thus, how to get a suitable lower-dimension model to approximate the high-dimension one with minimum sacrifice of accuracy is a highly desirable solution to simplify the original high-order system. Extensive attention has been paid on this issue of reduced-order model approximation to simplify these models while achieve a pre-specified performance level. A sequence of effective approaches and strategies on the issue of model reduction have been proposed, such as the \mathcal{H}_∞ performance technique [33], the optimal Hankel-norm performance technique [31], the \mathcal{L}_2 - \mathcal{L}_∞ performance technique [37] and the optimal \mathcal{H}_2 performance technique [45]. However, to the best of our knowledge, there are few reports in the literature on how to construct reduced-order modelling with a pre-specified performance level for discrete-time nonlinear hybrid switched systems via T-S fuzzy modelling framework. In fact, some challenging technology hinder the development of this area, and it raises numerous crucial issues in implementing model reduction problem. For instance, 1) it is not easy to construct an appropriate reduced-order hybrid switched model, which could substitute for the original complex nonlinear hybrid switched model effectively, 2) how to find a feasible piecewise Lyapunov function, in which the parameter-dependent technique and the average dwell time research method can be applied to such nonlinear dynamic system effectively, and 3) how to further reduce the conservativeness introduced by reduced-order matrix constraints and release these constraints free as far as possible. Based on the above discussion and the motivation, this work is carried out to solve the theoretical and practical difficulties on model reduction problem.

Motivated by the recognition that fuzzy modelling can describe a dynamic nonlinear system effectively, in this paper, the reduced-order model approximation problem with a pre-specified performance level will be investigated for discrete-time nonlinear complex hybrid switched systems, which can be modelled as a hybrid stochastic switched systems via T-S fuzzy modelling. Since hybrid switched systems, as well as T-S fuzzy stochastic systems, are involved in the concerned hybrid switched systems, how to tackle the resulted model approximation issue should be interesting yet challenging. It should be emphasised that, heretofore, only limited works concern on different operation modes in the T-S fuzzy systems.

The main contributions of the proposed results are summarized as follows:

1. In the framework of linear matrix inequality techniques, the reduced-order model under a pre-specified error system performance level has been constructed.
2. For the nonlinear hybrid stochastic switched systems, the problem of reduced-order model approximation has been settled by the usage of the project technique and the conversion on cone complementary linearization algorithm.
3. To further reduce the conservativeness degree of the proposed model approximation problem, the piecewisely blending quadratic Lyapunov functional is constructed to the corresponding nonlinear hybrid stochastic switched systems.

2. System Description

Consider a series of high-order discrete-time nonlinear hybrid stochastic switched systems:

$$x(k+1) = \sum_{j=1}^N \rho_j(k) \left[\mathcal{C}_j(x(k), \omega(k)) + \mathcal{G}_j(x(k), \omega(k)) \varpi(k) \right], \quad (1a)$$

$$y(k) = \sum_{j=1}^N \rho_j(k) \left[\mathcal{H}_j(x(k), \omega(k)) + \mathcal{J}_j(x(k), \omega(k)) \varpi(k) \right], \quad (1b)$$

where the state variable $x(\bullet) \in \mathbb{R}^n$ is the vector; $\omega(\bullet) \in \mathbb{R}^m$ is known disturbance input which belongs to $\ell_2[0, \infty]$; The exogenous disturbance $\omega(\bullet)$ is assumed to be energy bounded, that is, $\|\omega(\bullet)\|_2 \triangleq \sqrt{\sum_{k=0}^{\infty} \omega^T(\bullet) \omega(\bullet)}$; $y(\bullet) \in \mathbb{R}^p$ is the measure output; $\varpi(\bullet)$ is a stochastic process on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ relating to an increasing family $(\mathcal{F}_k)_{k \in \mathbb{N}}$ of σ -algebras $\mathcal{F}_k \subset \mathcal{F}$ generated by $(\varpi(k))_{k \in \mathbb{N}}$. The stochastic process $\varpi(\bullet)$ is independent satisfying $\mathbf{E}\{\varpi(k)\} = 0$ and $\mathbf{E}\{\varpi(k)^2\} = \mu$; N is the positive integer, which denotes the number of sub-models;

$$\rho_j(k) : [0, \infty) \rightarrow \{0, 1\}, \text{ and } \sum_{j=1}^N \rho_j(k) = 1, k \in [1, \infty), j \in \mathcal{N} = \{1, 2, \dots, N\},$$

is the stochastic switching signal which implies which sub-model is accessible at the switching time, we use ρ_j for simplicity; $\mathcal{C}_j(\bullet)$, $\mathcal{G}_j(\bullet)$, $\mathcal{H}_j(\bullet)$ and $\mathcal{J}_j(\bullet)$ are a range of nonlinear regular functions.

At a discrete sampling time k , the value of $\rho_j(k)$, may be built by k or $x(\bullet)$, or both, or depend on any other hybrid scheme. As is mentioned in [21], here assume that the real-time value of ρ_j is accessible. For the switching signal ρ_j , the switching sequence

$$\{(j_0, k_0), (j_1, k_1), \dots, (j_k, k_k), \dots, | j_k \in \mathcal{N}, k = 0, 1, \dots\} \text{ with } k_0 = 0,$$

shows that the j_k th sub-model is enabled when $k \in [k_k, k_{k+1})$.

The high-dimension complex nonlinear model with a T-S fuzzy modelling is given here, and T-S fuzzy modelling is used here to handle the reduced-order model approximation for the nonlinear hybrid stochastic switched system.

Fuzzy Rule $\mathcal{R}_i^{[j]}$: IF $\vartheta_1^{[j]}(k)$ is $\mathcal{M}_{i1}^{[j]}$ and $\vartheta_2^{[j]}(k)$ is $\mathcal{M}_{i2}^{[j]}$ and \dots and $\vartheta_p^{[j]}(k)$ is $\mathcal{M}_{ip}^{[j]}$, THEN

$$\begin{aligned} x(k+1) &= A_i^{[j]} x(k) + B_i^{[j]} \omega(k) + E_i^{[j]} x(k) \varpi(k), \\ y(k) &= C_i^{[j]} x(k) + D_i^{[j]} \omega(k), \end{aligned}$$

where $i = 1, 2, \dots, r$, and r denotes the number of IF-THEN rules; $\mathcal{M}_{i1}^{[j]}, \dots, \mathcal{M}_{ip}^{[j]}$ are the fuzzy sets; $\vartheta_1^{[j]}(\bullet), \vartheta_2^{[j]}(\bullet), \dots, \vartheta_p^{[j]}(\bullet)$ are the premise variables, denoted by $\vartheta^{[j]}$; $\{(A_i^{[j]}, B_i^{[j]}, C_i^{[j]}, D_i^{[j]}, E_i^{[j]}) : j \in \mathcal{N}\}$ is a family of matrices parameterized by an index set $\mathcal{N} = \{1, 2, \dots, N\}$, and $A_i^{[j]}, B_i^{[j]}, C_i^{[j]}, D_i^{[j]}$ and $E_i^{[j]}$ are known matrices.

Assume that the premise variables are independent on the disturbance input $\omega(\bullet)$. Given a couple of $(x(\bullet), \omega(\bullet))$, the ultimate output of the corresponding hybrid switched fuzzy models is presented here:

$$x(k+1) = \sum_{j=1}^N \rho_j \sum_{i=1}^r h_i^{[j]}(\vartheta^{[j]}) \left[A_i^{[j]} x(k) + B_i^{[j]} \omega(k) + E_i^{[j]} x(k) \varpi(k) \right], \quad (2a)$$

$$y(k) = \sum_{j=1}^N \rho_j \sum_{i=1}^r h_i^{[j]}(\vartheta^{[j]}) \left[C_i^{[j]} x(k) + D_i^{[j]} \omega(k) \right], \quad (2b)$$

where $h_i^{[j]}(\vartheta^{[j]}) = \mathcal{M}_i^{[j]}(\vartheta^{[j]}) / \sum_{i=1}^r \mathcal{M}_i^{[j]}(\vartheta^{[j]})$, $\mathcal{M}_i^{[j]}(\vartheta^{[j]}) = \prod_{l=1}^p \mathcal{M}_{il}^{[j]}(\vartheta_l^{[j]})$, and $\mathcal{M}_{il}^{[j]}(\vartheta_l^{[j]})$ is the grade of membership of $\vartheta_l^{[j]}$ in $\mathcal{M}_{il}^{[j]}$. Suppose $\mathcal{M}_i^{[j]}(\vartheta^{[j]}) \geq 0$, $i = 1, 2, \dots, r$, $\sum_{i=1}^r \mathcal{M}_i^{[j]}(\vartheta^{[j]}) > 0$ for all k . Therefore, $h_i^{[j]}(\vartheta^{[j]}) \geq 0$ for $i = 1, 2, \dots, r$ and $\sum_{i=1}^r h_i^{[j]}(\vartheta^{[j]}) = 1$ for all k .

In our work, for the nonlinear hybrid switched model shown in (2), we shall approximate the high-order system by the following reduced-order system represented by

$$\tilde{x}(k+1) = \sum_{j=1}^N \rho_j \left[\tilde{A}_r^{[j]} \tilde{x}(k) + \tilde{B}_r^{[j]} \omega(k) + \tilde{E}_r^{[j]} x(k) \varpi(k) \right], \quad (3a)$$

$$\tilde{y}(k) = \sum_{j=1}^N \rho_j \left[\tilde{C}_r^{[j]} \tilde{x}(k) + \tilde{D}_r^{[j]} \omega(k) \right], \quad (3b)$$

where $\tilde{x}(k) \in \mathbb{R}^k$ is the desired reduced-order model's state vector, and the resulted dimension k is less than n ; $\tilde{y}(k) \in \mathbb{R}^p$ is the desired reduced-order model's output; $\tilde{A}_r^{[j]}$, $\tilde{B}_r^{[j]}$, $\tilde{C}_r^{[j]}$, $\tilde{D}_r^{[j]}$ and $\tilde{E}_r^{[j]}$ are proper dimensioned matrices to be determined.

Denoting $\check{x}(k) = \begin{bmatrix} x(k) \\ \tilde{x}(k) \end{bmatrix}$, $e_r(k) \triangleq y(k) - \tilde{y}(k)$, and augmenting the original nonlinear hybrid switched model of (2) to include the reduced-order state variables of (3), then the overall dynamics of corresponding error system are illustrated as

$$\check{x}(k+1) = \sum_{j=1}^N \rho_j \sum_{i=1}^r h_i^{[j]}(\vartheta^{[j]}) \left[\check{A}_i^{[j]} \check{x}(k) + \check{B}_i^{[j]} \omega(k) + \check{E}_i^{[j]} \check{x}(k) \varpi(k) \right], \quad (4a)$$

$$e_r(k) = \sum_{j=1}^N \rho_j \sum_{i=1}^r h_i^{[j]}(\vartheta^{[j]}) \left[\check{C}_i^{[j]} \check{x}(k) + \check{D}_i^{[j]} \omega(k) \right], \quad (4b)$$

where

$$\begin{cases} \check{A}_i^{[j]} \triangleq \begin{bmatrix} A_i^{[j]} & 0 \\ 0 & \tilde{A}_r^{[j]} \end{bmatrix}, & \check{B}_i^{[j]} \triangleq \begin{bmatrix} B_i^{[j]} \\ \tilde{B}_r^{[j]} \end{bmatrix}, & \check{E}_i^{[j]} \triangleq \begin{bmatrix} E_i^{[j]} & 0 \\ 0 & \tilde{E}_r^{[j]} \end{bmatrix}, \\ \check{C}_i^{[j]} \triangleq \begin{bmatrix} C_i^{[j]} & -\tilde{C}_r^{[j]} \end{bmatrix}, & \check{D}_i^{[j]} \triangleq D_i^{[j]} - \tilde{D}_r^{[j]}. \end{cases} \quad (5)$$

Definition 1. The error dynamic system in (4) with the exogenous disturbance $\omega(k)$ equaling 0 is said to be mean-square exponentially stable under $\rho_j(k)$ if its solution $\check{x}(k)$ satisfies

$$\mathbf{E} \left\{ \|\check{x}(k)\| \right\} \leq \eta \|\check{x}(k_0)\| \rho^{(k-k_0)}, \quad \forall k \geq k_0,$$

for parameters $\eta \geq 1$ and $0 < \rho < 1$.

Definition 2. For $\gamma > 0$ and $0 < \beta < 1$, the error dynamic system in (4) is said to guarantee the pre-specified \mathcal{H}_∞ performance level (γ, β) if it is mean-square exponentially stable with the exogenous disturbance $\omega(k)$ equaling 0 and, under $x(k) = 0$, then the following holds for all nonzero $\omega(k) \in \ell_2[0, \infty)$:

$$\mathbf{E} \left\{ \sum_{s=k_0}^{\infty} \beta^s e_r^T(s) e_r(s) \right\} < \gamma^2 \sum_{s=k_0}^{\infty} \omega^T(s) \omega(s). \quad (6)$$

Therefore, the model reduction problem with the pre-specified performance proposed in our work can be formulated as follows: given the high-order nonlinear hybrid switched system described in (2) and $0 < \beta < 1$, $\gamma > 0$, our aim is on how to construct a desired reduced-dimension hybrid switched model in (3) to ensure that the corresponding error dynamic system in (4) is satisfied to the pre-scribed \mathcal{H}_∞ performance.

3. Main Results

3.1. Pre-specified Performance Analysis

In this section, the proposed stability analysis problem will be solved by introducing the parameter-dependent matrix technique, and the sufficient condition of the weighted \mathcal{H}_∞ system performance will be given for the considered error dynamic system (4).

Theorem 1. For given positive scalars $0 < \beta < 1$, $\gamma > 0$ and $\sigma \geq 1$, suppose that there exists matrix $P^{[j]} \in \mathbb{R}^{(n+k) \times (n+k)}$ and $P^{[j]} > 0$ such that for $j \in \mathcal{N}$, $i = 1, 2, \dots, r$,

$$\Xi_i^{[j]} \triangleq \begin{bmatrix} -\beta P^{[j]} & 0 & (\check{A}_i^{[j]})^T & (\check{C}_i^{[j]})^T & (\check{E}_i^{[j]})^T \\ \star & -\gamma^2 I & (\check{B}_i^{[j]})^T & (\check{D}_i^{[j]})^T & 0 \\ \star & \star & -(P^{[j]})^{-1} & 0 & 0 \\ \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & -(\mu P^{[j]})^{-1} \end{bmatrix} < 0, \quad (7)$$

then, the hybrid switched error dynamic system in (4) has a weighted \mathcal{H}_∞ error system performance level (γ, β) with mean-square exponential stability for any stochastic switching signal with average dwell time satisfying $T_a > T_a^* = \frac{\ln \sigma}{\beta}$, where $\sigma \geq 1$ satisfies

$$P^{[j]} \leq \sigma P^{[s]}, \quad \forall j, s \in \mathcal{N}. \quad (8)$$

In addition, an upper bound of the estimate function of the state decay is shown as

$$\mathbf{E} \{ \|\check{x}(k)\| \} \leq \eta \|\check{x}(k_0)\| \rho^{(k-k_0)}, \quad (9)$$

where

$$\rho \triangleq \sqrt{\beta \sigma^{\frac{a}{T_a}}}, \quad \eta \triangleq \sqrt{\frac{b}{a}}, \quad a \triangleq \min_{j \in \mathcal{N}} \lambda_{\min}(P^{[j]}), \quad b \triangleq \max_{j \in \mathcal{N}} \lambda_{\max}(P^{[j]}). \quad (10)$$

Proof. Based on the fuzzy rule basis functions and the stochastic switching signal $\rho_j(k)$, from (7) we get

$$\sum_{j=1}^N \rho_j \sum_{i=1}^r h_i^{[j]}(\vartheta^{[j]}) \Xi_i^{[j]} < 0. \quad (11)$$

Form a class of piecewise smooth Lyapunov functional as

$$V(\check{x}(k), \rho_j) \triangleq \check{x}^T(k) \left(\sum_{j=1}^N \rho_j P^{[j]} \right) \check{x}(k), \quad (12)$$

where $\mathbb{R}^{(n+k) \times (n+k)} \ni P^{[j]} > 0$, $j \in \mathcal{N}$ are to be designed. For $k \in [k_l, k_{l+1})$, we define

$$\begin{aligned} & \mathbf{E} \{ \Delta V(\check{x}(k), \rho_j) \} \triangleq \mathbf{E} \{ V(\check{x}(k+1), \rho_j) - V(\check{x}(k), \rho_j) \} \\ & = \sum_{j=1}^N \rho_j \sum_{i=1}^r h_i^{[j]}(\vartheta^{[j]}) \check{x}^T(k) \left[(\check{A}_i^{[j]})^T P^{[j]} \check{A}_i^{[j]} - P^{[j]} + \mu (\check{E}_i^{[j]})^T P^{[j]} \check{E}_i^{[j]} \right] \check{x}(k). \end{aligned}$$

Thus, it follows that

$$\begin{aligned} & \mathbf{E} \{ \Delta V(\check{x}(k), \rho_j) \} + \mathbf{E} \{ (1 - \beta) V(\check{x}(k), \rho_j) \} \\ & = \sum_{j=1}^N \rho_j \sum_{i=1}^r h_i^{[j]}(\vartheta^{[j]}) \check{x}^T(k) \left[(\check{A}_i^{[j]})^T P^{[j]} \check{A}_i^{[j]} - \beta P^{[j]} + \mu (\check{E}_i^{[j]})^T P^{[j]} \check{E}_i^{[j]} \right] \check{x}(k). \end{aligned} \quad (13)$$

By (11), it follows

$$\mathbf{E} \{ \Delta V(\check{x}(k), \rho_j) + (1 - \beta) V(\check{x}(k), \rho_j) \} < 0, \quad \forall k \in [k_l, k_{l+1}), \quad \forall j \in \mathcal{N}. \quad (14)$$

For any switching signal and any $k > 0$, let

$$k_0 < k_1 < \dots < k_l < \dots < k_N, \quad l = 1, \dots, N$$

represent the piecewise transition points of ρ_j over the time interval $(0, k)$. As mentioned earlier, the j th sub-model is triggered when $k \in [k_l, k_{l+1})$. Therefore, for $k \in [k_l, k_{l+1})$, it suggests from (14) that

$$\mathbf{E}\left\{V(\check{x}(k), \rho_j)\right\} < \beta^{k-k_l} \mathbf{E}\left\{V(\check{x}(k_l), \rho_j(k_l))\right\}. \quad (15)$$

Using (8) and (12), we obtain

$$\mathbf{E}\left\{V(\check{x}(k_l), \rho_j(k_l))\right\} < \sigma \mathbf{E}\left\{V(\check{x}(k_l), \rho_j(k_{l-1}))\right\}. \quad (16)$$

Therefore, it concludes from (15)–(16) and the correlation $\vartheta = N_a(k_0, k) \leq \frac{k-k_0}{T_a}$ that

$$\mathbf{E}\left\{V(\check{x}(k), \rho_j)\right\} \leq (\beta\sigma^{\frac{1}{T_a}})^{k-k_0} \mathbf{E}\left\{V(\check{x}(k_0), \rho_j(k_0))\right\}. \quad (17)$$

It is noted from (12) that it is feasible to find two positive scalars a and b , and $a \leq b$, which are given in (10), then

$$\mathbf{E}\left\{V(\check{x}(k), \rho_j)\right\} \geq a \mathbf{E}\left\{\|x(k)\|^2\right\}, \quad \mathbf{E}\left\{V(\check{x}(k_0), \rho_j(k_0))\right\} \leq b \mathbf{E}\left\{\|x(k_0)\|^2\right\}. \quad (18)$$

Combining (17) and (18) yields

$$\mathbf{E}\left\{\|x(k)\|^2\right\} \leq \frac{1}{a} \mathbf{E}\left\{V(\check{x}(k), \rho_j)\right\} \leq \frac{b}{a} (\beta\sigma^{\frac{1}{T_a}})^{k-k_0} \mathbf{E}\left\{\|x(k_0)\|^2\right\}.$$

Defining $\varrho \triangleq \sqrt{\beta\sigma^{\frac{1}{T_a}}}$, then we obtain

$$\mathbf{E}\left\{\|x(k)\|\right\} \leq \sqrt{\frac{b}{a}} \varrho^{k-k_0} \|x(k_0)\|.$$

By Definition 1, we can get that if $0 < \varrho < 1$, on the other words, $T_a > T_a^* = \text{ceil}(-\frac{\ln \sigma}{\ln \beta})$, the considered error dynamic system in (4) with $\omega(k) = 0$ is mean-square exponentially stable, where the function $\text{ceil}(f)$ shows rounding real scalar f to the nearest integer greater than or equal to f .

Now, the weighted \mathcal{H}_∞ system performance level (γ, β) , which is defined in (6), is established as follows. Give an index of the form:

$$\mathcal{J}(k) \triangleq \mathbf{E}\left\{\Delta V(\check{x}(k), \rho_j) + (1 - \beta)V(\check{x}(k), \rho_j) + e_r^T(k)e_r(k) - \gamma^2 \omega^T(k)\omega(k)\right\}.$$

Thus, we can get

$$\mathcal{J}(k) = \sum_{j=1}^N \rho_j \sum_{i=1}^r h_i^{[j]}(\vartheta^{[j]}) \begin{bmatrix} \check{x}(k) \\ \omega(k) \end{bmatrix}^T \begin{bmatrix} \Xi_{11i}^{[j]} & \Xi_{12i}^{[j]} \\ \star & \Xi_{22i}^{[j]} \end{bmatrix} \begin{bmatrix} \check{x}(k) \\ \omega(k) \end{bmatrix},$$

where

$$\begin{aligned} \Xi_{11i}^{[j]} &\triangleq (\check{A}_i^{[j]})^T P^{[j]} \check{A}_i^{[j]} + (\check{C}_i^{[j]})^T \check{C}_i^{[j]} + \mu (\check{E}_i^{[j]})^T P^{[j]} \check{E}_i^{[j]} - \beta P^{[j]}, \\ \Xi_{12i}^{[j]} &\triangleq (\check{A}_i^{[j]})^T P^{[j]} \check{B}_i^{[j]} + (\check{C}_i^{[j]})^T \check{D}_i^{[j]}, \quad \Xi_{22i}^{[j]} \triangleq (\check{B}_i^{[j]})^T P^{[j]} \check{B}_i^{[j]} + (\check{D}_i^{[j]})^T \check{D}_i^{[j]} - \gamma^2 I. \end{aligned}$$

Considering (11) and Schur's complement, for $k \in [k_l, k_{l+1})$, we obtain $\mathcal{J}(k) < 0$. Let $\Omega(k) \triangleq e_r^T(k)e_r(k) - \gamma^2 \omega^T(k)\omega(k)$, then

$$\mathbf{E}\left\{\Delta V(\check{x}(k), \rho_j)\right\} < \mathbf{E}\left\{- (1 - \beta)V(\check{x}(k), \rho_j) - \Omega(k)\right\}. \quad (19)$$

Therefore, for $k \in [k_l, k_{l+1})$, it holds from (19) that

$$\mathbf{E}\left\{V(\check{x}(k), \rho_j)\right\} < \beta^{k-k_l} \mathbf{E}\left\{V(\check{x}(k_l), \rho_j(k_l))\right\} - \mathbf{E}\left\{\sum_{s=k_l}^{k-1} \beta^{k-1-s} \Omega(s)\right\}. \quad (20)$$

Thus, by (16) and (20), it follows that

$$\begin{aligned} \mathbf{E}\left\{V(\check{x}(k), \rho_j)\right\} &< \beta^{k-k_l} \mathbf{E}\left\{V(\check{x}(k_l), \rho_j(k_l))\right\} - \mathbf{E}\left\{\sum_{s=k_l}^{k-1} \beta^{k-1-s} \Omega(s)\right\}, \\ &\vdots \\ \mathbf{E}\left\{V(\check{x}(k_1), \rho_j(k_1))\right\} &< \beta^{k_1-k_0} \sigma \mathbf{E}\left\{V(\check{x}(k_0), \rho_j(k_0))\right\} - \sigma \mathbf{E}\left\{\sum_{s=k_0}^{k_1-1} \beta^{k_1-1-s} \Omega(s)\right\}. \end{aligned}$$

Thus, based on the inequalities presented above and the relationship $\vartheta = N_a(k_0, k) \leq \frac{k-k_0}{T_a}$ we obtain

$$\mathbf{E}\left\{V(\check{x}(k), \rho_j)\right\} < \beta^{k-k_0} \sigma^{N_a(k_0, k)} \mathbf{E}\left\{V(\check{x}(k_0), \rho_j(k_0))\right\} - \mathbf{E}\left\{\sum_{s=k_0}^{k-1} \beta^{k-1-s} \sigma^{N_a(s, k)} \Omega(s)\right\}. \quad (21)$$

Recalling the zero initial condition, (21) yields

$$\mathbf{E}\left\{\sum_{s=k_0}^{k-1} \beta^{k-1-s} \sigma^{N_a(s, k)} \left[e_r^T(s) e_r(s) - \gamma^2 \omega^T(s) \omega(s)\right]\right\} < 0.$$

Pre-and post-multiplying the above inequality with $\sigma^{-N_a(0, k)}$, it implies

$$\mathbf{E}\left\{\sum_{s=k_0}^{k-1} \beta^{k-1-s} \sigma^{-N_a(0, s)} \left[e_r^T(s) e_r(s) - \gamma^2 \omega^T(s) \omega(s)\right]\right\} < 0. \quad (22)$$

Notice that $N_a(0, s) \leq \frac{s}{T_a}$ and $T_a > -\frac{\ln \sigma}{\ln \beta}$, we have $N_a(0, s) \leq -s \frac{\ln \beta}{\ln \sigma}$. Thus, (22) implies

$$\mathbf{E}\left\{\sum_{s=k_0}^{k-1} \beta^{k-1-s} \sigma^{-s \frac{\ln \beta}{\ln \sigma}} e_r^T(s) e_r(s)\right\} < \gamma^2 \mathbf{E}\left\{\sum_{s=k_0}^{k-1} \beta^{k-1-s} \omega^T(s) \omega(s)\right\},$$

which yields that

$$\mathbf{E}\left\{\sum_{s=k_0}^{\infty} \beta^s e_r^T(s) e_r(s)\right\} < \mathbf{E}\left\{\sum_{s=k_0}^{\infty} \gamma^2 \omega^T(s) \omega(s)\right\}.$$

By Definition 2, we get that the hybrid switched error system in (4) has a weighted \mathcal{H}_∞ error performance level (γ, β) with mean-square exponential stability. Thus, the proof is completed. \square

Remark 1. By applying the piecewisely blending quadratic Lyapunov functions technique and the average dwell time analysis approach, a novel sufficient condition of pre-specified performance analysis is obtained for discrete-time nonlinear hybrid stochastic switched systems in T-S fuzzy modelling. The presented parameter-basis-dependent result for nonlinear switched systems further reduces the conservatism caused by the piecewisely blending quadratic Lyapunov functions which contains the parameter-basis-independent information as a typical form. Next, it shows that the proposed reduced-order model approximation problem can be resolved as sequential minimization algorithm that can be computed for the feasible solution very efficiently.

3.2. Model Approximation By Projection Technique

Now, the solution of the reduced-order approximation problem for the corresponding nonlinear hybrid switched systems is presented by projection technique.

Theorem 2. Consider the corresponding hybrid switched error system in (4). For given scalars $0 < \beta < 1$, $\gamma > 0$ and $\sigma \geq 1$, suppose there exist matrices $0 < P^{[j]} \in \mathbb{R}^{(n+k) \times (n+k)}$ and $0 < \mathcal{P}^{[j]} \in \mathbb{R}^{(n+k) \times (n+k)}$ such that for $j, s \in \mathcal{N}$, $i = 1, 2, \dots, r$,

$$\bar{\Xi}_i^{[j]} \triangleq \begin{bmatrix} -\beta P^{[j]} & 0 & (\check{A}_{i0}^{[j]})^T \mathcal{H}^T & (\check{E}_{i0}^{[j]})^T \mathcal{H}^T \\ \star & -\gamma^2 I & (\check{B}_{i0}^{[j]})^T \mathcal{H}^T & 0 \\ \star & \star & -\mathcal{H} \mathcal{P}^{[j]} \mathcal{H}^T & 0 \\ \star & \star & \star & -\frac{1}{\mu} \mathcal{H} \mathcal{P} \mathcal{H}^T \end{bmatrix} < 0, \quad (23a)$$

$$\hat{\Xi}_i^{[j]} \triangleq \begin{bmatrix} -\beta \mathcal{H} P^{[j]} \mathcal{H}^T & \mathcal{H} (\check{A}_{i0}^{[j]})^T & \mathcal{H} (\check{C}_{i0}^{[j]})^T & \mathcal{H} (\check{E}_{i0}^{[j]})^T \\ \star & -\mathcal{P}^{[j]} & 0 & 0 \\ \star & \star & -I & 0 \\ \star & \star & \star & -\frac{1}{\mu} \mathcal{P}^{[j]} \end{bmatrix} < 0, \quad (23b)$$

$$P^{[j]} \leq \sigma P^{[s]}, \quad (23c)$$

$$P^{[j]} \mathcal{P}^{[j]} = I, \quad (23d)$$

then, the hybrid switched error dynamic system in (4) has a weighted \mathcal{H}_∞ error performance level (γ, β) with mean-square exponential stability. In addition, the system matrices of an available weighted \mathcal{H}_∞ reduced-order model in (3) are shown by

$$\left\{ \begin{array}{l} \mathcal{G}^{[j]} \triangleq \begin{bmatrix} \check{D}_r^{[j]} & \check{C}_r^{[j]} \\ \check{B}_r^{[j]} & \check{A}_r^{[j]} \\ & \check{E}_r^{[j]} \end{bmatrix} = -\Pi^{-1} U^T \Lambda^{[j]} V^T (V \Lambda^{[j]} V^T)^{-1} + \Pi^{-1} (\Xi^{[j]})^{\frac{1}{2}} L (V \Lambda^{[j]} V^T)^{-1/2}, \\ \Lambda^{[j]} = (U \Pi^{-1} U^T - W^{[j]})^{-1} > 0, \\ \Xi^{[j]} = \Pi - U^T [\Lambda^{[j]} - \Lambda^{[j]} V^T (V \Lambda^{[j]} V^T)^{-1} V \Lambda^{[j]}] U > 0, \end{array} \right. \quad (24)$$

where $\Pi > 0$ and $\|L\| < 1$ are any proper dimensional matrices, and

$$\left\{ \begin{array}{l} W^{[j]} \triangleq \begin{bmatrix} -\beta P^{[j]} & 0 & (\check{A}_{i0}^{[j]})^T & (\check{C}_{i0}^{[j]})^T & (\check{E}_{i0}^{[j]})^T \\ \star & -\gamma^2 I & (\check{B}_{i0}^{[j]})^T & (\check{D}_{i0}^{[j]})^T & 0 \\ \star & \star & -(P^{[j]})^{-1} & 0 & 0 \\ \star & \star & \star & -I & 0 \\ \star & \star & \star & \star & -(\mu P^{[j]})^{-1} \end{bmatrix}, \\ U \triangleq \begin{bmatrix} 0_{(n+k) \times (p+2k)} \\ 0_{m \times (p+2k)} \\ \mathcal{X}_1 \\ \mathcal{X}_2 \\ \mathcal{X}_3 \end{bmatrix}, \quad \mathcal{X}_1 \triangleq \begin{bmatrix} 0_{n \times p} & 0_{n \times k} & 0_{n \times k} \\ 0_{k \times p} & I_{k \times k} & 0_{k \times k} \end{bmatrix}, \\ \mathcal{X}_3 \triangleq \begin{bmatrix} 0_{n \times p} & 0_{n \times k} & 0_{n \times k} \\ 0_{k \times p} & 0_{k \times k} & I_{k \times k} \end{bmatrix}, \\ V \triangleq \begin{bmatrix} \mathcal{Y}_1 & \mathcal{Y}_2 & 0_{(m+k) \times (n+k)} & 0_{(m+k) \times p} & 0_{(m+k) \times (n+k)} \end{bmatrix}, \\ \mathcal{Y}_1 \triangleq \begin{bmatrix} 0_{m \times n} & 0_{m \times k} \\ 0_{k \times n} & I_{k \times k} \end{bmatrix}, \quad \mathcal{Y}_2 \triangleq \begin{bmatrix} I_{m \times m} \\ 0_{k \times m} \end{bmatrix}, \quad \check{C}_{i0}^{[j]} \triangleq \begin{bmatrix} C_i^{[j]} & 0_{p \times k} \end{bmatrix}, \\ \mathcal{X}_2 \triangleq \begin{bmatrix} -I_{p \times p} & 0_{p \times k} & 0_{p \times k} \end{bmatrix}, \quad \mathcal{H} \triangleq \begin{bmatrix} I_{n \times n} & 0_{n \times k} \end{bmatrix}, \quad \check{D}_{i0}^{[j]} \triangleq D_i^{[j]}, \\ \check{A}_{i0}^{[j]} \triangleq \begin{bmatrix} A_i^{[j]} & 0_{n \times k} \\ 0_{k \times n} & 0_{k \times k} \end{bmatrix}, \quad \check{B}_{i0}^{[j]} \triangleq \begin{bmatrix} B_i^{[j]} \\ 0_{k \times m} \end{bmatrix}, \quad \check{E}_{i0}^{[j]} \triangleq \begin{bmatrix} E_i^{[j]} & 0_{n \times k} \\ 0_{k \times n} & 0_{k \times k} \end{bmatrix}. \end{array} \right. \quad (25)$$

Proof. Rewrite $\check{A}_i^{[j]}$, $\check{B}_i^{[j]}$, $\check{C}_i^{[j]}$, $\check{D}_i^{[j]}$ and $\check{E}_i^{[j]}$ in the following form:

$$\begin{cases} \check{A}_i^{[j]} \triangleq \check{A}_{i0}^{[j]} + \mathcal{X}_1 \mathcal{G}^{[j]} \mathcal{Y}_1, & \check{B}_i^{[j]} \triangleq \check{B}_{i0}^{[j]} + \mathcal{X}_1 \mathcal{G}^{[j]} \mathcal{Y}_2, \\ \check{C}_i^{[j]} \triangleq \check{C}_{i0}^{[j]} + \mathcal{X}_2 \mathcal{G}^{[j]} \mathcal{Y}_1, & \check{D}_i^{[j]} \triangleq \check{D}_{i0}^{[j]} + \mathcal{X}_2 \mathcal{G}^{[j]} \mathcal{Y}_2, \\ \check{E}_i^{[j]} \triangleq \check{E}_{i0}^{[j]} + \mathcal{X}_3 \mathcal{G}^{[j]} \mathcal{Y}_1, \end{cases} \quad (26)$$

where $\mathcal{G}^{[j]}$, $\check{A}_{i0}^{[j]}$, $\check{B}_{i0}^{[j]}$, $\check{C}_{i0}^{[j]}$, $\check{D}_{i0}^{[j]}$, $\check{E}_{i0}^{[j]}$, \mathcal{X}_1 , \mathcal{X}_2 , \mathcal{X}_3 , \mathcal{Y}_1 and \mathcal{Y}_2 are defined in (24) and (25). With (26), the proposed condition (7) in Theorem 1 can be transformed into

$$W^{[j]} + U \mathcal{G}^{[j]} V + (U \mathcal{G}^{[j]} V)^T < 0, \quad (27)$$

where the notations of $W^{[j]}$, U and V are given in (25). We select

$$V^{T\perp} \triangleq \begin{bmatrix} \mathcal{H} & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}, \quad U^\perp \triangleq \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & \mathcal{H} & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{H} \end{bmatrix},$$

where \mathcal{H} is shown in (25). Then, by projection Lemma, inequality (27) is feasible for $\mathcal{G}^{[j]}$ if and only if

$$U^\perp W U^{T\perp} < 0, \quad V^{T\perp} W V^\perp < 0,$$

which can be formulated specifically as

$$\begin{bmatrix} -\beta P^{[j]} & 0 & (\check{A}_{i0}^{[j]})^T \mathcal{H}^T & (\check{E}_{i0}^{[j]})^T \mathcal{H}^T \\ \star & -\gamma^2 I & (\check{B}_{i0}^{[j]})^T \mathcal{H}^T & 0 \\ \star & \star & -\mathcal{H} (P^{[j]})^{-1} \mathcal{H}^T & 0 \\ \star & \star & \star & -\mathcal{H} (\mu P^{[j]})^{-1} \mathcal{H}^T \end{bmatrix} < 0, \quad (28a)$$

$$\begin{bmatrix} -\beta \mathcal{H} P^{[j]} \mathcal{H}^T & \mathcal{H} (\check{A}_{i0}^{[j]})^T & \mathcal{H} (\check{C}_{i0}^{[j]})^T & \mathcal{H} (\check{E}_{i0}^{[j]})^T \\ \star & -(P^{[j]})^{-1} & 0 & 0 \\ \star & \star & -I & 0 \\ \star & \star & \star & -(\mu P^{[j]})^{-1} \end{bmatrix} < 0, \quad (28b)$$

By noting $\mathcal{P}^{[j]} \triangleq (P^{[j]})^{-1}$, it follows that (28a)–(28b) imply respectively (23a)–(23b). In addition, when the inequalities in (23) are satisfied, the reduced-order model parametrization in (24) corresponding to an available solution can be obtained by using projection technique. Thus, it completes the proof. \square

Remark 2. The proposed sufficient conditions in Theorem 2 are not all in linear matrix inequalities form because of the equation in (23d). Here, to solve this difficulty, we introduce the cone complementarity linearization algorithm as follows.

From the above discussion, the solution of the resulted nonconvex feasibility problem can be provided via the formulation of the following sequential optimization problem.

Reduced-order Model Approximation Problem:

$$\begin{aligned} \min \quad & \text{trace} \left(\sum_{j \in \mathcal{N}} P^{[j]} \mathcal{P}^{[j]} \right) \\ \text{subject to} \quad & (23a)-(23c), \quad \text{and for } j \in \mathcal{N}, \\ & \begin{bmatrix} P^{[j]} & I \\ I & \mathcal{P}^{[j]} \end{bmatrix} \geq 0. \end{aligned} \quad (29)$$

Remark 3. Note that the simulation result of Theorem 2 is feasible, if the aforementioned problem of min trace ($\sum_{j \in N} P^{[j]} \mathcal{P}^{[j]} = 2Nk$) can be solved effectively. In the following example, iteration approach is introduced to solve the proposed reduced-order model approximation problem. The iteration of the proposed solving condition will be terminated, when the matrices obtained satisfy the conditions of inequalities (23a)-(23c) with a pre-scribed performance level, or it reaches the maximum number of iterations.

4. Illustrative example

In this part, simulation result is represented to show the feasibility of the presented reduced-order model approximation approach.

Example 1. Consider the hybrid stochastic switched systems in (1) with $N = 2$ and the system parameters are shown here.

Subsystem 1.

$$\begin{aligned}
 A_1^{[1]} &= \begin{bmatrix} 0.57 & 0.45 & -0.42 & 0.86 \\ 0.48 & -0.32 & 0.54 & -0.26 \\ -0.34 & -0.22 & -0.38 & -0.40 \\ -0.42 & 0.60 & 0.36 & 0.46 \end{bmatrix}, & B_1^{[1]} &= \begin{bmatrix} 0.70 \\ -0.42 \\ -0.42 \\ 0.36 \end{bmatrix}, \\
 A_2^{[1]} &= \begin{bmatrix} 0.76 & 0.30 & -0.42 & 0.82 \\ 0.44 & -0.32 & 0.56 & -0.48 \\ -0.32 & -0.28 & -0.28 & -0.36 \\ -0.42 & 0.60 & 0.34 & 0.48 \end{bmatrix}, & B_2^{[1]} &= \begin{bmatrix} 0.62 \\ -0.76 \\ -0.42 \\ 0.34 \end{bmatrix}, \\
 E_1^{[1]} &= \begin{bmatrix} 0.07 & 0.04 & 0.02 & 0.05 \\ 0.02 & 0.06 & 0.08 & 0.10 \\ 0.04 & 0.02 & 0.06 & 0.02 \\ 0.02 & 0.08 & 0.02 & 0.04 \end{bmatrix}, & C_1^{[1]} &= \begin{bmatrix} 1.25 \\ 0.62 \\ 1.32 \\ 0.62 \end{bmatrix}^T, \\
 E_2^{[1]} &= \begin{bmatrix} 0.07 & 0.04 & 0.02 & 0.05 \\ 0.08 & 0.06 & 0.04 & 0.10 \\ 0.04 & 0.02 & 0.06 & 0.08 \\ 0.01 & 0.02 & 0.01 & 0.02 \end{bmatrix}, & C_2^{[1]} &= \begin{bmatrix} 1.15 \\ 0.66 \\ 1.36 \\ 0.64 \end{bmatrix}^T, \\
 D_1^{[1]} &= 1.90, & D_2^{[1]} &= 1.50
 \end{aligned} \tag{30}$$

Subsystem 2.

$$\begin{aligned}
 A_1^{[2]} &= \begin{bmatrix} 0.56 & 0.30 & -0.42 & 0.91 \\ 0.46 & -0.32 & 0.58 & -0.57 \\ -0.32 & -0.22 & -0.26 & -0.40 \\ -0.42 & 0.68 & 0.30 & 0.46 \end{bmatrix}, & B_1^{[2]} &= \begin{bmatrix} 0.64 \\ -0.46 \\ -0.48 \\ 0.32 \end{bmatrix}, \\
 A_2^{[2]} &= \begin{bmatrix} 0.52 & 0.12 & -0.08 & 0.76 \\ 0.48 & -0.32 & 0.54 & -0.24 \\ -0.30 & -0.22 & -0.24 & -0.28 \\ -0.42 & 0.60 & 0.36 & 0.42 \end{bmatrix}, & B_2^{[2]} &= \begin{bmatrix} 0.70 \\ -0.68 \\ -0.40 \\ 0.38 \end{bmatrix}, \\
 E_1^{[2]} &= \begin{bmatrix} 0.07 & 0.04 & 0.02 & 0.05 \\ 0.08 & 0.06 & 0.04 & 0.10 \\ 0.04 & 0.02 & 0.06 & 0.05 \\ 0.02 & 0.04 & 0.02 & 0.05 \end{bmatrix}, & C_1^{[2]} &= \begin{bmatrix} 1.15 \\ 0.52 \\ 1.34 \\ 0.64 \end{bmatrix}^T, \\
 E_2^{[2]} &= \begin{bmatrix} 0.07 & 0.04 & 0.02 & 0.06 \\ 0.02 & 0.06 & 0.08 & 0.10 \\ 0.04 & 0.08 & 0.06 & 0.02 \\ 0.02 & 0.04 & 0.02 & 0.08 \end{bmatrix}, & C_2^{[2]} &= \begin{bmatrix} 1.16 \\ 0.54 \\ 1.30 \\ 0.68 \end{bmatrix}^T,
 \end{aligned}$$

$$D_1^{[2]}=1.90, \quad D_2^{[2]} = 1.50, \quad (31)$$

and $\beta = 0.8$. Set $\sigma = 1.02$, it is easy to prove, from Theorem 1, that the concerned nonlinear hybrid stochastic switched system has mean-square exponential stability.

Here, the aim is to search for reduced-order hybrid switched systems in (3), that is, Case 1: $k = 1$; Case 2: $k = 2$; Case 3: $k = 3$, to simplify the above high-order system in the weighted \mathcal{H}_∞ error performance sense. Solve the sequential optimization problem in Theorem 2 by using the cone complementarity linearization algorithm, the different reduced-order model parameters are presented in the following.

Case 1. with $k = 1$, the minimized feasible error system performance level is 0.1391 and

$$\mathcal{G}^{[1]} = \left[\begin{array}{c|c} 1.5974 & -0.6204 \\ \hline -1.0057 & 0.4770 \\ \hline & 0.0556 \end{array} \right], \quad \mathcal{G}^{[2]} = \left[\begin{array}{c|c} 1.4117 & -0.6318 \\ \hline -1.2455 & -0.2945 \\ \hline & 0.0015 \end{array} \right]. \quad (32)$$

Case 2. with $k = 2$, the minimized feasible error system performance level is 0.1166 and

$$\mathcal{G}^{[1]} = \left[\begin{array}{c|cc} 1.4902 & -0.5756 & -0.4607 \\ \hline -0.5435 & 0.2649 & 0.0866 \\ -0.6248 & 0.3817 & 0.3742 \\ \hline & 0.0987 & 0.0549 \\ & 0.0313 & 0.0270 \end{array} \right], \quad \mathcal{G}^{[2]} = \left[\begin{array}{c|cc} 1.5044 & -0.5850 & -0.5215 \\ \hline -0.7510 & 0.0602 & -0.5025 \\ -0.4039 & 0.1989 & 0.1270 \\ \hline & 0.0576 & -0.0283 \\ & 0.0234 & 0.0202 \end{array} \right]. \quad (33)$$

Case 3. with $k = 3$, the minimized feasible error system performance level is 0.0836 and

$$\left. \begin{array}{l} \mathcal{G}^{[1]} = \left[\begin{array}{c|cccc} 1.4711 & -0.5484 & -0.4326 & -0.1501 \\ \hline -0.5445 & 0.2717 & 0.0053 & -0.0262 \\ -0.5441 & 0.4005 & 0.3705 & 0.1318 \\ -0.2047 & 0.1658 & 0.1751 & 0.0758 \\ \hline & 0.1070 & 0.0493 & 0.0122 \\ & 0.0236 & 0.0229 & 0.0085 \\ & -0.0003 & 0.0057 & 0.0028 \end{array} \right], \\ \mathcal{G}^{[2]} = \left[\begin{array}{c|cccc} 1.5189 & -0.5718 & -0.4762 & -0.1687 \\ \hline -0.7023 & 0.1416 & -0.5328 & -0.2735 \\ -0.3596 & 0.2357 & 0.1545 & 0.0449 \\ -0.0977 & 0.1000 & 0.1292 & 0.0607 \\ \hline & 0.0708 & -0.0287 & -0.0217 \\ & 0.0193 & 0.0207 & 0.0080 \\ & 0.0016 & 0.0132 & 0.0062 \end{array} \right]. \end{array} \right. \quad (34)$$

Additionally, to show the effectiveness of the weighted \mathcal{H}_∞ error performance for the designed reduced-order hybrid switched model, set $\check{x}(0) = 0$ ($x(0) = 0$, $\bar{x}(0) = 0$), and the membership functions is selected as

$$\begin{cases} h_1^{[j]}(x_2^{[j]}(k)) \triangleq \frac{[1 - \sin(x_2(k))]}{2}, \\ h_2^{[j]}(x_2^{[j]}(k)) \triangleq \frac{[1 + \sin(x_2(k))]}{2}. \end{cases}$$

The input variable $\omega(\bullet)$ is chosen as

$$\omega(k) = \exp(-0.1k) \sin(0.9k), \quad k > 0.$$

Figure 1 depicts a hybrid stochastic switching signal, which is activated arbitrarily for different modes. The parameters of vertical axis, '1' and '2', represent the first and the second nonlinear stochastic submode, respectively. It

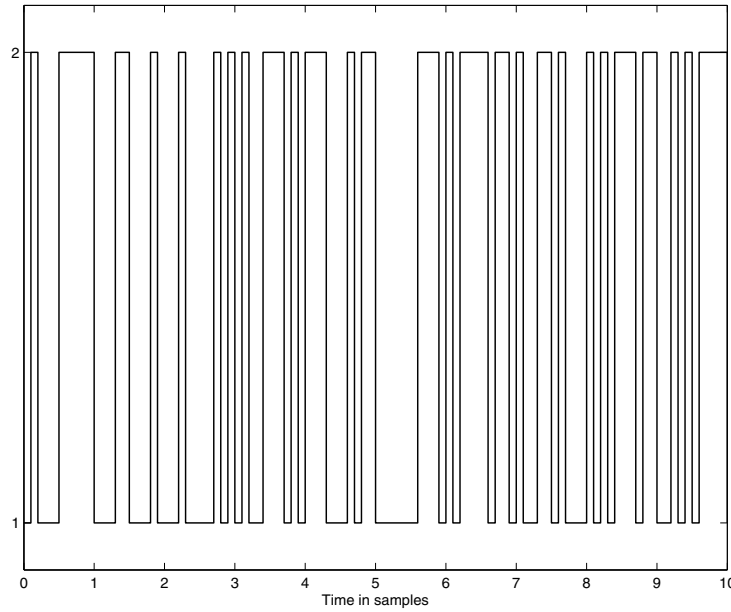


Figure 1. Stochastic switching signal with the average dwell time $T_a \geq 0.1$

can be easily seen from Figure 1 that the average dwell time $T_a \geq \frac{\ln \sigma}{\beta} = \frac{\ln 1.02}{0.8} = 0.0248$. Figure 2 shows that the measured outputs of the original hybrid switched system (30)–(31), the third-dimension hybrid switched model (34), the second-dimension hybrid switched model (33) and the first-dimension hybrid switched model (32), while the output errors between the original nonlinear hybrid switched system and the reduced-order hybrid switched models are described in Figure 3.

5. Conclusion

This paper investigates the reduced-order model approximation problem with a pre-specified system performance level for nonlinear hybrid stochastic switched systems in T-S fuzzy modelling. The steps of the proposed model reduction technique are: 1) given the high-order nonlinear hybrid switched system, construct a reduced-order hybrid switched model to approximate; 2) by applying the average dwell time analysis approach and the piecewisely blending quadratic Lyapunov function technique, the corresponding augmented error dynamic system is guaranteed to be mean-square exponentially stable with a given error performance level. Moreover, the establishment of the resulted feasibility condition for the reduced-order hybrid stochastic switched systems is shown by the projection technique. Combining with the cone complementary linearization algorithm, the parameters of the reduced-order hybrid switched models can be expressed as getting a feasible solution for a sequential minimization problem, which are represented in linear matrix inequality form. Finally, simulation results have illustrated the effectiveness of the presented model reduction technique.

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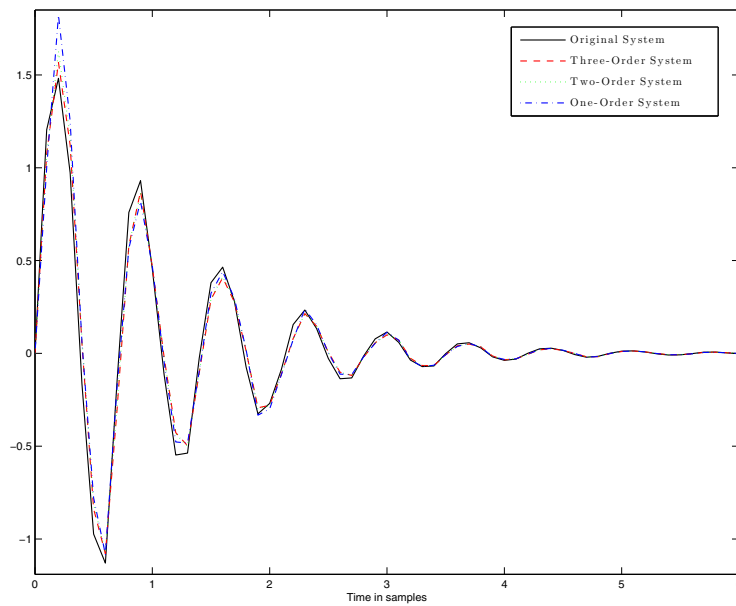


Figure 2. Outputs of the original nonlinear hybrid switched system and the reduced-order hybrid switched models

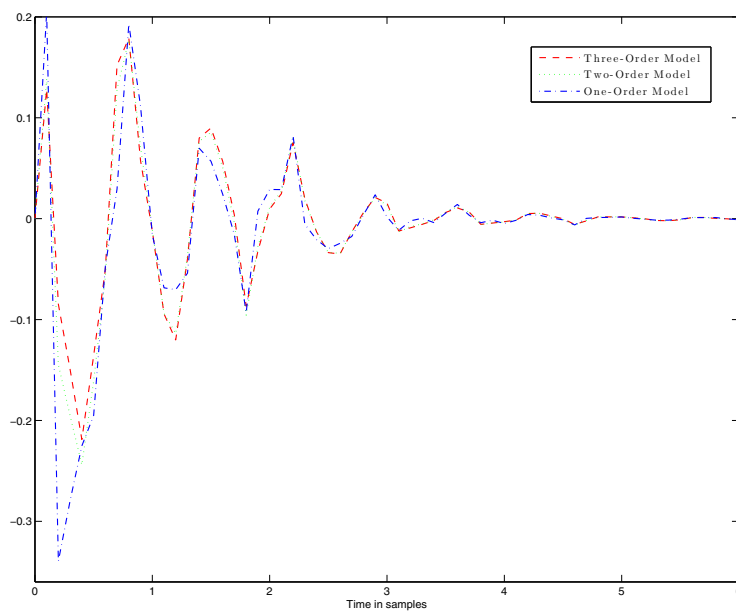


Figure 3. Output errors between the original nonlinear hybrid switched system and the reduced-order hybrid switched models

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