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# Delay-dependent stabilization condition for T-S fuzzy neutral systems

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***Abstract*—In this paper, the stabilization problems for a class of Takagi-Sugeno (T-S) fuzzy neutral systems are explored. Utilizing Pólya's theorem and some homogeneous polynomials techniques, the delay-dependent stabilization condition for T-S fuzzy neutral systems are proposed in terms of a linear matrix inequality (LMI) to guarantee the asymptotic stabilization of T-S fuzzy neutral systems. Lastly, an example is illustrated to demonstrate the effectiveness and applicability of the proposed method.**

***Keywords*—Fuzzy control, Takagi-Sugeno (T-S) fuzzy model, linear matrix inequality (LMI), Pólya's theorem.**

## I. Introduction

During the past decades, fuzzy logic control has been widely adopted to analyze nonlinear systems [1], [2]. In addition to fuzzy logic control, a great deal of effort has been devoted to describe a nonlinear model using Takagi-Sugeno (T-S) fuzzy model. Moreover, many fuzzy modelling approaches [3], [4] which are provided to represent nonlinear models as T-S fuzzy model. Through the modeling procedure, T-S fuzzy model can be described by fuzzy IF-THEN rules, which represents local linear input-output relations of a nonlinear model. In addition to fuzzy modeling scheme, the parallel distributed compensation (PDC) technique [5] is adopted to stabilize the overall T-S fuzzy model. For the stabilization condition analysis, Lyapunov direct method [6], [7] is mainly investigated to yield the stabilization conditions of T-S fuzzy model. Furthermore, many studies utilize linear matrix inequalities (LMIs) to find a feasible solution, if available. Furthermore, the solutions can be found via linear matrix inequalities (LMIs) techniques.

Time-delay phenomenon exists in many practical systems, such as chemical engineering systems, robotic arm systems and network systems [8], [9]. In general, the practical systems with time-delay are more complicated than those systems without time-delays. Besides, time-delay may cause instability and reduce the system performance under some situations; therefore, there has been an increasing interest in the stabilization problem for time-delay systems, and a lot of results on these topics have been explored in the literature [10], [11].

In addition, time-delay phenomenon exists in both the state and the derivative of the state in neutral systems. For this reason, stabilization problem for neutral systems has been explored in many studies [12], [13]. For example, in [14], a state matrix decomposition is adopted and a delay-dependent stability condition for fuzzy neutral systems was proposed. A descriptor system approach is adopted for uncertain fuzzy neutral system in [15]. Recently, a polynomial technique has been adopted to reduce the conservatism of the stabilization condition [16]–[18]. Inspired by these works and reference therein, we will explore the delay-dependent stabilization condition for T-S fuzzy neutral systems via polynomial technique.

Following the introduction, the paper is organized as follows. In Section II, a general description of T-S fuzzy neutral system is introduced and the state feedback fuzzy controller is also designed. In Section III, based on homogeneous polynomial technique and Pólya's theorem, a delay-dependent stabilization conditions for T-S fuzzy neutral system is formulated in terms of LMIs in this section. In Section IV, a numerical example is given to demonstrate the feasibility and effectiveness of the proposed approach. Finally, the conclusions are given in Section V.

**Notation**: The notations in this paper are quite standard. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the $n$-dimensional Euclidean space and the set of all $n\times m$ real matrices. $A^T$ denotes the transpose of matrix $A$. $X \leq Y$ or $X < Y$ , respectively, where $X$ and $Y$ are symmetric matrices, means that $X - Y$ is negative semi-definite or negative definite, respectively. $I$ is the identity matrix with a compatible dimension (without confusion). $diag[q_1, \cdots, q_n]$ represents a block-diagonal, $q_1 \cdots q_n$ as the diagonal elements. Denotes a symmetric matrix, where $\star$ represents the entries implied by symmetry. The matrices, if not explicitly stated, are assumed to have compatible dimensions. ! denotes factorial for combinatoric expression. Let $K(h)$ be the set of $r$-tuples defined as [19]:

$$K(h) = \{\, (\, k_1\, k_2 \cdots k_r) :\, k_1 + k_2 + \cdots + k_r = h, \quad \forall k_i \in I^+(\text{positive integers}),\, i = 1, 2, \cdots, r\}$$

where $h$ is the total polynomial degree. Since the number of fuzzy base is $r$, the number of elements in $K(h)$ is expressed by $J(h) = (r+h-1)!/(h!(r-1)!)$. For example, $r=2$, $h=3$

$$\begin{aligned} J(3) &= (2+3-1)!/(3!(2-1)!) = 4 \\ K(3) &= \{(30),\ (21),\ (12),\ (03)\} \\ &= \{t(1),\ t(2),\ t(3),\ t(4)\} \end{aligned}$$

For clarity, the following notations are adopted:

$$\begin{aligned} k &= k_1 k_2 \cdots k_r \\ \mu^k &= \mu_1^{k_1}\mu_2^{k_2}\cdots\mu_r^{k_r} \\ e_i &= 0\cdots\underbrace{1}_{i^{th}}\cdots 0 \\ k - e_i &= k_1k_2\cdots(k_i-1)\cdots k_r \\ \pi(k) &= (k_1!)\,(k_2!)\cdots(k_r!). \end{aligned}$$

## II. Preliminaries

To begin, consider the following T-S fuzzy neutral system:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r}\mu_i(t)[A_ix(t) + A_{ci}x(t-h(t)) + A_{di}\dot{x}(t-h(t)) \\ &\qquad + B_iu(t)] \\ &= A(t)x(t) + A_c(t)x(t-h(t)) + A_d(t)\dot{x}(t-h(t)) \\ &\quad + B(t)u(t) \\ x(t) &= \phi(t),\ \forall t\in[-max\{h_M\},0],\ i=1,...,r \end{aligned} \tag{1}$$

where $r$ is the number of fuzzy rules. The state of system $x(t)\in\mathbb{R}^{n\times 1}$ and the input $u(t)\in\mathbb{R}^{m\times 1}$. The matrices $A_i$, $A_{ci}$, $A_{di}\in\mathbb{R}^{n\times n}$, $B_i\in\mathbb{R}^{n\times m}$ are system matrices, and initial vector $\phi(t)$ belongs to the set of continuous functions. The time-varying delay $h(t)$ satisfies $0\le h(t)\le h_M$ and $0\le \dot{h}(t)\le h_D$. $\mu_i(t)=\omega_i(t)/\sum_{i=1}^{r}$, $\omega_i(t)=\prod_{i=1}^{p}M_{ij}(\xi(t))$. $M_{ij}$ is the membership degree of $\xi(t)$, and $\omega_i(t)\ge 0$ for all $t$, $i=1,\cdots,r$. It is clear that $\mu_i(t)\ge 0$, and $\sum_{i=1}^{r}\mu_i(t)=1$.

The state feedback fuzzy controller for T-S fuzzy neutral system (1) is represented as follows.

$$\begin{aligned} u(t) &= \sum_{k\in K(r-1),r\ge 2}\mu^kF_kx(t) \\ &= F(t)x(t) \end{aligned} \tag{2}$$

By substituting (2) into (1), the closed-loop system can be obtained as (3).

$$\begin{aligned} \dot{x}(t) =& (A(t)+B(t)F(t))x(t) + A_c(t)x(t-h(t)) \\ &+ A_d(t)\dot{x}(t-h(t)) \end{aligned} \tag{3}$$

## III. Main Results

Before discussing the proof of the theorems, here are some lemmas which are used in the proof.

*Lemma 1:* [20] For any positive symmetric constant matrix $R_1\in\mathbb{R}^{n\times n}$ and a scalar $h_M>0$, if there exists a vector function $\dot{x}(s):[0,h_M]\to\mathbb{R}^n$ such that the integrals $\int_{t-h(t)}^{t}\dot{x}(s)R_1\dot{x}(s)ds$ and $\int_{t-h(t)}^{t}\dot{x}(s)ds$ are well defined, then the following inequality holds:

$$\begin{aligned} -h_M\int_{t-h_M}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds &\le -h(t)\int_{t-h(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds \\ &\le -\left(\int_{t-h(t)}^{t}\dot{x}(s)ds\right)^T R_1\left(\int_{t-h(t)}^{t}\dot{x}(s)ds\right). \end{aligned}$$

*Lemma 2:* [21] (Pólya's theorem) For a positive integer $r$, $\{\triangle_r\colon(\mu_1,\ \cdots,\ \mu_r)\mid \mu_i\ge 0,\ \sum_{i=1}^{r}\mu_i=1\}$. If a real homogeneous polynomial $F(\mu_1,\ \cdots,\ \mu_r)$ is positive definite, then for a sufficiently large $d$, all the coefficients of

$$(\mu_1+\cdots+\mu_r)^d F(\mu_1,\cdots,\mu_r)$$

are positive.

*Lemma 3:* [16] Consider the T-S fuzzy system,

$$\dot{x}(t) = \sum_{i=1}^{r}\mu_i(t)[A_ix(t)+B_iu(t)] \tag{4}$$

where $x(t)$ is the state of system,$u(t)$ is the control input, $A(t)=\sum_{i=1}^{r}\mu_i(\xi(t))A_i$ $B(t)=\sum_{i=1}^{r}\mu_i(\xi(t))B_i$. The T-S fuzzy system (4) is asymptotically stabilizable via the state feedback controller $G_k=\overline{G}_kX^{-1}$ if and only if there exist a symmetric positive definite matrix $X>0$ and $d\in\mathbb{N}$ such that:

$$\begin{aligned} \sum_{k'\in K(d)}\sum_{i=1}^{r}\frac{d!}{\pi(k')}\Big(&\frac{(k_i-k_i')}{\pi(k-k')}(A_iX+\star) \\ &+(B_iG_{k-k'-e_i}+\star)\Big)<0 \end{aligned} \tag{5}$$

where $k\in K(d+r)$, $k\succ k'$, symbol $\succ$ is the componentwise.

Before discussing the following stability conditions, we first define the following function:

$$\begin{aligned} P_1(t) &= \sum_{i=1}^{r}\mu_i(t)P_{1i},\ \ P_2(t)=\sum_{i=1}^{r}\mu_i(t)P_{2i} \\ P_3(t) &= \sum_{i=1}^{r}\mu_i(t)P_{3i},\ \ S_1(t)=\sum_{i=1}^{r}\mu_i(t)S_{1i} \\ P_4(t) &= \sum_{j=1}^{r}\mu_i(t)P_{4j}. \end{aligned}$$

The main result on the asymptotic stability of the T-S fuzzy neutral system (1) is propounded in the following theorem.

*Theorem 1:* If there exist integers $d_a>0$, $d_b>0$, some positive matrices $\bar{P}_0$, $\bar{P}_{1i}$, $\bar{P}_{2i}$, $P_{3i}$, $P_{4j}\in\mathbb{R}^{n\times n}$, and matrix $\bar{S}_{1i}^T=[\bar{S}_{11i}^T\ \bar{S}_{12i}^T\ \bar{S}_{13i}^T\ \bar{S}_{14i}^T]$, $\bar{S}_{11i}^T,\cdots,\ \bar{S}_{14i}^T\in\mathbb{R}^{n\times n}$ such that the following inequalities (6) and (7) are satisfied for some positive scalar $g_1,\ g_2,\ g_3$ and $0\le h(t)\le h_M,\ 0\le\dot{h}(t)\le h_D$ then the fuzzy neutral system (1) is asymptotically stabilizable via the state feedback controller $F_k=\bar{F}_kX^{-1}$, $X\in\mathbb{R}^{n\times n}$.

$$\Omega<0 \tag{6}$$

$$P_{3i}<P_{4j} \tag{7}$$

where

$$\begin{aligned} \Omega &= diag\left[\Omega^{t_a(1),t_b(1)},...,\ \Omega^{t_a(J(d+2)),t_b(J(d+1))}\right] \\ \Omega^{k_a,k_b} &= \sum_{k_a'\in K(d_a)}\frac{d_a!}{\pi(k_a')}\sum_{k_b'\in K(d_b)}\frac{d_b!}{\pi(k_b')}\bar{\Omega} \\ k_a &\in K(d_a+2),\ \ k_b\in K(d_b+1), k_a\succ k_a', k_b\succ k_b' \end{aligned}$$

$$\bar{\Omega}=\begin{bmatrix}\bar{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13} & \bar{\Omega}_{14} & \bar{\Omega}_{15}\\ \star & \bar{\Omega}_{22} & \bar{\Omega}_{23} & \bar{\Omega}_{24} & \bar{\Omega}_{25}\\ \star & \star & \bar{\Omega}_{33} & \bar{\Omega}_{34} & \bar{\Omega}_{35}\\ \star & \star & \star & \bar{\Omega}_{44} & \bar{\Omega}_{45}\\ \star & \star & \star & \star & \bar{\Omega}_{55}\end{bmatrix}$$

$$\begin{aligned}
\bar{\Omega}_{11}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[(\bar{S}_{11i}+\star)+\bar{P}_{1i}\\
&+(A_iX+\star)]+\sum_{i=1}^{r}\frac{1!}{\pi(k_b-k_b^{'})}[(B_i\bar{F}_{k_a-k_a^{'}-e_i}+\star)]\\
\bar{\Omega}_{12}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[g_1X^TA_i^T\\
&+\bar{S}_{12i}^T+A_{ci}X-\bar{S}_{11i}^T]\\
&+\sum_{i=1}^{r}\frac{1!}{\pi(k_b-k_b^{'})}[g_1\bar{F}_{k_a-k_a^{'}-e_i}^TB_i^T]\\
\bar{\Omega}_{13}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[g_2X^TA_i^T\\
&+\bar{S}_{13i}^T]+\sum_{i=1}^{r}\frac{1!}{\pi(k_b-k_b^{'})}[g_2\bar{F}_{k_a-k_a^{'}-e_i}^TB_i^T]\\
&+\sum_{i=1}^{r}\frac{r!}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[\bar{P}_0^T-X]\\
\bar{\Omega}_{14}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[A_{di}X+\bar{S}_{14i}^T\\
&+g_3X^TA_i^T]+\sum_{i=1}^{r}\frac{1!}{\pi(k_b-k_b^{'})}[g_3\bar{F}_{k_a-k_a^{'}-e_i}^TB_i^T]\\
\bar{\Omega}_{15}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[\bar{S}_{11i}]\\
\bar{\Omega}_{22}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[(g_1A_{ci}X+\star)\\
&-(\bar{S}_{12i}+\star)]\\
&+\sum_{j=1}^{r}\frac{r!}{\pi(k_a-k_a^{'})}[-(1-h_D)\bar{P}_{1j})]\\
\bar{\Omega}_{23}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[g_2X^TA_{ci}^T\\
&-\bar{S}_{13i}^T]+\sum_{i=1}^{r}\frac{r!}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[-g_1X]\\
\bar{\Omega}_{24}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[g_3X^TA_{ci}^T\\
&+g_1A_{di}X-\bar{S}_{14i}^T]\\
\bar{\Omega}_{25}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[\bar{S}_{12i}]\\
\bar{\Omega}_{33}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[\bar{P}_{2i}+h_M\bar{P}_{3i}]\\
&+\sum_{i=1}^{r}\frac{r!}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[(-g_2X+\star)]\\
\bar{\Omega}_{34}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[g_2A_{di}X]\\
&+\sum_{i=1}^{r}\frac{r!}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[-g_3X]\\
\bar{\Omega}_{35}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[\bar{S}_{13i}]\\
\bar{\Omega}_{44}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[(g_3A_{di}X+\star)]\\
&+\sum_{j=1}^{r}\frac{r!}{\pi(k_a-k_a^{'})}[-(1-h_D)\bar{P}_{2j}]\\
\bar{\Omega}_{45}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[\bar{S}_{14i}]\\
\bar{\Omega}_{55}=&\sum_{i=1}^{r}\frac{(r-1)!(k_{ai}-k_{ai}^{'})}{\pi(k_a-k_a^{'})}\frac{1!}{\pi(k_b-k_b^{'})}[-h_M^{-1}\bar{P}_{3i}].
\end{aligned}$$

*Proof:* Firstly, let us consider the Lyapunov-Krasoviskii function

$$V(t)=V_1(t)+V_2(t)+V_3(t)+V_4(t) \tag{8}$$

where

$$\begin{aligned}
V_1(t)&=x^T(t)P_0x(t)\\
V_2(t)&=\int_{t-h(t)}^{t}x^T(s)P_1(s)x(s)ds\\
V_3(t)&=\int_{t-h(t)}^{t}\dot{x}^T(s)P_2(s)\dot{x}(s)ds\\
V_4(t)&=\int_{t-h_M}^{t}(s-(t-h_M))\dot{x}^T(s)P_3(s)\dot{x}(s)ds
\end{aligned}$$

and $P_0$, $P_1(s)$, $P_2(s)$, $P_3(s)$ are symmetric positive definite matrices.

The time derivative of Lyapunov-Krasoviskii function is

$$\dot{V}(t)=\dot{V}_1(t)+\dot{V}_2(t)+\dot{V}_3(t)+\dot{V}_4(t) \tag{9}$$

where

$$\begin{aligned}
\dot{V}_1(t)=&\dot{x}^T(t)P_0x(t)+x^T(t)P_0\dot{x}(t)\\
\dot{V}_2(t)=&x^T(t)P_1(t)x(t)\\
&-(1-\dot{h}(t))x^T(t-h(t))P_1(t-h(t))x(t-h(t))\\
\dot{V}_3(t)=&\dot{x}^T(t)P_2(t)\dot{x}(t)\\
&-(1-\dot{h}(t))\dot{x}^T(t-h(t))P_2(t-h(t))\dot{x}(t-h(t))\\
\dot{V}_4(t)=&h_M\dot{x}^T(t)P_3(t)\dot{x}(t)-\int_{t-h_M}^{t}\dot{x}^T(s)P_3(s)\dot{x}(s)ds.
\end{aligned}$$

According to $0\le\dot{h}(t)\le h_D$, we can obtain

$$\begin{aligned}
\dot{V}_2(t)=&x^T(t)P_1(t)x(t)-(1-h_D)\\
&\times x^T(t-h(t))P_1(t-h(t))x(t-h(t)) &(10)\\
\dot{V}_3(t)=&\dot{x}^T(t)P_2(t)\dot{x}(t)-(1-h_D)\\
&\times\dot{x}^T(t-h(t))P_2(t-h(t))\dot{x}(t-h(t)). &(11)
\end{aligned}$$

From Newton-Leibniz formula and the T-S fuzzy neutral system in (1), the following equalities are always hold:

$$\Pi_1 = 2\xi^T(t)S_1(t)[x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} \dot{x}(s)ds] = 0 \tag{12}$$

$$\begin{aligned}\Pi_2 =& 2\xi^T(t)S_2(t)[A(t)x(t) + A_c(t)x(t-h(t)) \\ &+ A_d(t)\dot{x}(t-h(t)) + B(t)K(t)x(t) - \dot{x}(t)] = 0\end{aligned} \tag{13}$$

$$\begin{aligned}\Pi_3 =& h_M\xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t) \\ &- \int_{t-h_M}^{t} \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)ds = 0\end{aligned} \tag{14}$$

where $\xi^T(t) = [x^T(t) \quad x^T(t-h(t)) \quad \dot{x}^T(t) \quad \dot{x}^T(t-h(t))]$.

From the condition $0 \leq h(t) \leq h_M$, we can obtain the following result

$$\begin{aligned}\Pi_3 =& h_M\xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t) \\ &- \int_{t-h_M}^{t} \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)ds \\ \leq& h_M\xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t) \\ &- \int_{t-h(t)}^{t} \xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)ds.\end{aligned} \tag{15}$$

By (9)-(15) with $P_3(s) \geq P_4(t)$ we can obtain

$$\begin{aligned}\dot{V}(t) \leq& \Lambda(t) + h_M\xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t) \\ &- \int_{t-h(t)}^{t} [(\dot{x}^T(s)P_3(s) + \xi^T(t)S_1(t)) \\ &\times P_3^{-1}(s)(P_3(s)\dot{x}(s) + S_1^T(t)\xi(t))]ds \\ \leq& \Lambda(t) + h_M\xi^T(t)S_1(t)P_4^{-1}(t)S_1^T(t)\xi(t)\end{aligned} \tag{16}$$

where

$$\begin{aligned}\Lambda(t) =& \dot{x}^T(t)P_0x(t) + x^T(t)P_0\dot{x}(t) + x^T(t)P_1(t)x(t) \\ &- (1-h_D)x^T(t-h(t))P_1(t-h(t))x(t-h(t)) \\ &- (1-h_D)\dot{x}^T(t-h(t))P_2(t-h(t))\dot{x}(t-h(t)) \\ &+ \dot{x}^T(t)P_2(t)\dot{x}(t) + h_M\dot{x}^T(t)P_3(t)\dot{x}(t) \\ &+ 2\xi^T(t)S_1(t)[x(t) - x(t-h(t))] \\ &+ 2\xi^T(t)S_2(t)[A(t)x(t) + A_c(t)x(t-h(t)) \\ &+ A_d(t)\dot{x}(t-h(t)) + B(t)K(t)x(t) - \dot{x}(t)].\end{aligned}$$

In order for $\dot{V}(x(t)) < 0$ for all $x(t) \neq 0$, (16) should be negative. By Schur complement and pre- and post-multiplying both sides with $diag[X, X, X, X, X]$ and define $X = S_{21}^{-1}$, $S_{22} = g_1S_{21}$, $S_{23} = g_2S_{21}$, $S_{24} = g_3S_{21}$, we can get the following result from (16)

$$\xi^T(t)\Xi(t)\xi(t) < 0 \tag{17}$$

where

$$\Xi(t) = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} \\ \star & \Xi_{22} & \Xi_{23} & \Xi_{24} & \Xi_{25} \\ \star & \star & \Xi_{33} & \Xi_{34} & \Xi_{35} \\ \star & \star & \star & \Xi_{44} & \Xi_{45} \\ \star & \star & \star & \star & \Xi_{55} \end{bmatrix}$$

$$\begin{aligned}
\Xi_{11} &= [(\bar{S}_{11}(t) + \star) + \bar{P}_1(t) + (A(t)X + \star) + (B(t)\bar{F}(t) + \star)] \\
\Xi_{12} &= [A_c(t)X + g_1X^TA(t)^T + \bar{S}_{12}^T(t) - \bar{S}_{11}^T(t) \\
&\quad + g_1\bar{F}^T(t)B^T(t)] \\
\Xi_{13} &= [g_2X^TA(t)^T + \bar{S}_{13}^T(t) + \bar{P}_0^T - X + g_2\bar{F}^T(t)B^T(t)] \\
\Xi_{14} &= [A_d(t)X + g_3X^TA(t)^T + \bar{S}_{14}^T(t) + g_3\bar{F}^T(t)B^T(t)] \\
\Xi_{15} &= [\bar{S}_{11}(t)] \\
\Xi_{22} &= [(g_1A_c(t)X + \star) - (\bar{S}_{12}(t) + \star) - (1-h_D)\bar{P}_1(t-h(t))] \\
\Xi_{23} &= [g_2X^TA_c(t)^T - \bar{S}_{13}^T(t) - g_1X] \\
\Xi_{24} &= [g_1A_d(t)X + g_3X^TA_c(t)^T - \bar{S}_{14}^T(t)] \\
\Xi_{25} &= [\bar{S}_{12}(t)] \\
\Xi_{33} &= [\bar{P}_2(t) + h_M\bar{P}_3(t) - (g_2X + \star)] \\
\Xi_{34} &= [g_2A_d(t)X - g_3X] \\
\Xi_{35} &= [\bar{S}_{13}(t)] \\
\Xi_{44} &= [(g_3A_d^T(t)X + \star) - (1-h_D)\bar{P}_2(t-h(t))] \\
\Xi_{45} &= [\bar{S}_{14}(t)] \\
\Xi_{55} &= [-h_M^{-1}\bar{P}_3(t)]
\end{aligned}$$

$\bar{S}_{11}(t) = XS_{11}(t)X,\ \bar{S}_{12}(t) = XS_{12}(t)X,\ \bar{P}_0(t) = XP_0(t)X,$
$\bar{S}_{13}(t) = XS_{13}(t)X,\ \bar{S}_{14}(t) = XS_{14}(t)X,\ \bar{P}_1(t) = XP_1(t)X,$
$\bar{P}_2(t) = XP_2(t)X,\ \bar{P}_1(t-h(t)) = XP_1(t-h(t))X$
$\bar{P}_3(t) = XP_3(t)X,\ \bar{P}_2(t-h(t)) = XP_2(t-h(t))X.$

Clearly, (17) is equivalent to (18)

$$\Phi(t) < 0 \tag{18}$$

where

$$\Phi(t) = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & \Phi_{15} \\ \star & \Phi_{22} & \Phi_{23} & \Phi_{24} & \Phi_{25} \\ \star & \star & \Phi_{33} & \Phi_{34} & \Phi_{35} \\ \star & \star & \star & \Phi_{44} & \Phi_{45} \\ \star & \star & \star & \star & \Phi_{55} \end{bmatrix}$$

$$\begin{aligned}
\Phi_{11} =& \sum_{j=1}^{r}\mu_j[(B(t)\bar{F}(t) + \star)] + \sum_{j=1}^{r}\mu_j(\sum_{i=1}^{r}\mu_i)^{r-1}[\bar{P}_1(t) \\
&+ (\bar{S}_{11}(t) + \star) + (A(t)X + \star)] \\
\Phi_{12} =& \sum_{j=1}^{r}\mu_j(\sum_{i=1}^{r}\mu_i)^{r-1}[A_c(t)X + g_1X^TA(t)^T \\
&+ \bar{S}_{12}^T(t) - \bar{S}_{11}^T(t)] + \sum_{j=1}^{r}\mu_j[g_1\bar{F}^T(t)B^T(t)] \\
\Phi_{13} =& \sum_{j=1}^{r}\mu_j(\sum_{i=1}^{r}\mu_i)^{r-1}[g_2X^TA(t)^T + \bar{S}_{13}^T(t)] \\
&+ \sum_{j=1}^{r}\mu_j[g_2\bar{F}^T(t)B^T(t)] + \sum_{j=1}^{r}\mu_j(\sum_{i=1}^{r}\mu_i)^{r}[\bar{P}_0^T - X] \\
\Phi_{14} =& \sum_{j=1}^{r}\mu_j(\sum_{i=1}^{r}\mu_i)^{r-1}[A_d(t)X + g_3X^TA(t)^T + \bar{S}_{14}^T(t)] \\
&+ \sum_{j=1}^{r}\mu_j[g_3\bar{F}^T(t)B^T(t)]
\end{aligned}$$

$$\Phi_{15} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [\bar{S}_{11}(t)]$$

$$\Phi_{22} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [(g_1 A_c(t) X + \star) - (\bar{S}_{12}(t) + \star)] - (\sum_{i=1}^{r} \mu_i)^{r} [(1 - h_D) \bar{P}_1(t - h(t))]$$

$$\Phi_{23} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [g_2 X^T A_c(t)^T - \bar{S}_{13}^T(t)] - \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r} [g_1 X]$$

$$\Phi_{24} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [g_1 A_d(t) X + g_3 X^T A_c(t)^T - \bar{S}_{14}^T(t)]$$

$$\Phi_{25} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [\bar{S}_{12}(t)]$$

$$\Phi_{33} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [\bar{P}_2(t) + h_M \bar{P}_3(t)] - \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r} [(g_2 X + \star)]$$

$$\Phi_{34} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [g_2 A_d(t) X] - \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r} [g_3 X]$$

$$\Phi_{35} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [\bar{S}_{13}(t)]$$

$$\Phi_{44} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [(g_3 A_d(t) X + \star)] + (\sum_{i=1}^{r} \mu_i)^{r} [(1 - h_D) \bar{P}_2(t - h(t))]$$

$$\Phi_{45} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [\bar{S}_{14}(t)]$$

$$\Phi_{55} = \sum_{j=1}^{r} \mu_j (\sum_{i=1}^{r} \mu_i)^{r-1} [-h_M^{-1} \bar{P}_3(t)].$$

By applying Lemma 2 and Lemma 3 to (18), yields

$$\begin{aligned}
&(\sum_{i=1}^{r} \mu_i(t))^{d_a} (\sum_{j=1}^{r} \mu_j(t - h(t)))^{d_b} \Phi(t) \\
&= \sum_{k' \in K(d_a)} \frac{d_a!}{\pi(k_a')} \sum_{k' \in K(d_b)} \frac{d_b!}{\pi(k_b')} \bar{\Omega} < 0 \\
&= \sum_{k_a' \in K(d_a)} \sum_{k_b' \in K(d_b)} \Omega^{ka,kb} < 0. \qquad (19)
\end{aligned}$$

Therefore, if (19) is satisfied which implies that $\Omega < 0$, and the closed-loop T-S fuzzy neutral system is asymptotically stable. This completes the proof of the theorem. ■

## IV. Numerical Example

In this section, a numerical example is provided to demonstrate the validity and feasibility of the proposed result.

*Example 1:* Consider the following T-S fuzzy neutral system:

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i(t) [A_i x(t) + A_{ci} x(t - h(t)) + A_{di} \dot{x}(t - h(t)) + B_i u(t)] \qquad (20)$$

where

$$A_1 = \begin{bmatrix} 0.3 & 0.6 \\ 0.8 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0.3 \\ 1 & 0.6 \end{bmatrix},$$
$$A_{c1} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix}, A_{c2} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} -0.5 & 1 \\ 0.4 & 0.3 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0.3 & 0 \\ 0.7 & -0.2 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \ h(t) = 0.3 + 0.2cos(t).$$

By applying the convex optimization problem in Theorem 1 with $g_1 = 0.32$, $g_2 = 2.55$, $g_3 = 0.1$, $h_M = 0.5$, and $h_D = 0.2$, the following matrices and controller gain can be obtained:

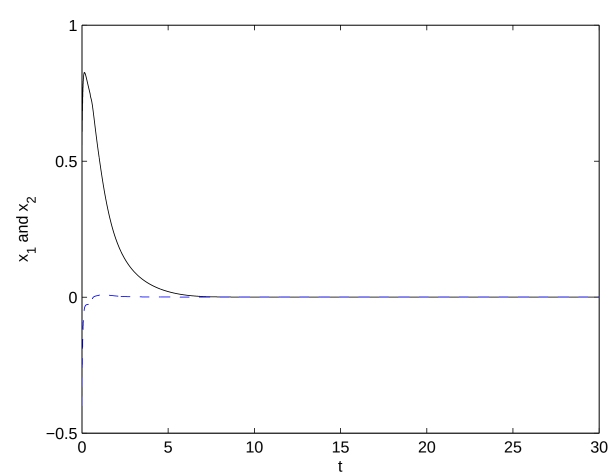


Fig. 1. The state response for closed-loop T-S fuzzy neutral systems with initial condition $x(0) = [0.5, -0.4]$.

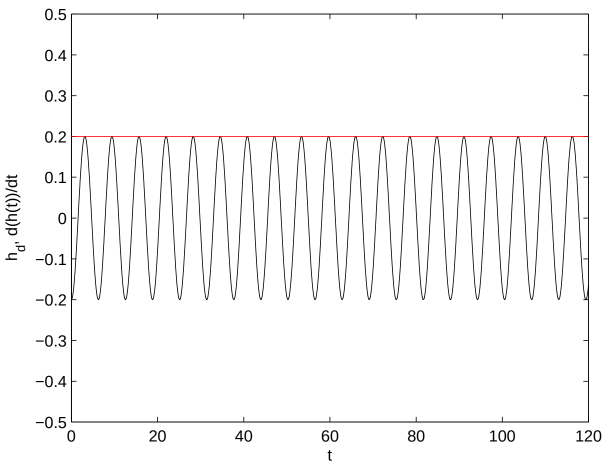


Fig. 2. The variation of $\dot{h}(t)$ and $h_D$.

$$P_0 = \begin{bmatrix} 14.1414 & 13.7123 \\ 13.7123 & 13.5008 \end{bmatrix},\ X = \begin{bmatrix} 0.4453 & 0.2166 \\ 0.2166 & 0.2209 \end{bmatrix},$$
$$P_{11} = \begin{bmatrix} 8.9149 & 8.3762 \\ 8.3762 & 8.1529 \end{bmatrix},\ P_{12} = \begin{bmatrix} 7.4549 & 7.0032 \\ 7.0032 & 6.9467 \end{bmatrix},$$
$$P_{21} = \begin{bmatrix} 0.8856 & 0.3300 \\ 0.3300 & 0.2413 \end{bmatrix},\ P_{22} = \begin{bmatrix} 1.4665 & 0.7449 \\ 0.7449 & 0.5225 \end{bmatrix},$$
$$P_{31} = \begin{bmatrix} 0.7179 & 0.1753 \\ 0.1753 & 0.1872 \end{bmatrix},\ P_{32} = \begin{bmatrix} 1.1204 & 0.1160 \\ 0.1160 & 0.0456 \end{bmatrix},$$
$$F_{10} = [-1.3569\ \ -23.2738],\ F_{01} = [-2.2770\ \ -22.3444].$$

The state responses for closed-loop T-S fuzzy neutral system with delay time $0.3+0.2cos(t)$ and $x(0) = [0.5, -0.4]$ is shown in Fig. 1. From the simulation results, it can be seen the designed fuzzy controller ensures the asymptotic stability of the closed-loop T-S fuzzy neutral system. One can observe that the states converge to the equilibrium states after some transient times. Fig. 2 shows the variation of $\dot{h}(t)$ and $h_D$.

## V. Conclusions

In this paper, a stabilization problem for T-S fuzzy neutral system is investigated. Based on the polynomial technique and some variable transformation, a delay-dependent stabilization condition is proposed for T-S fuzzy neutral system. Furthermore, the results can be formulated in terms of LMI forms. A numerical example is given to illustrate the effectiveness of the proposed methods.


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