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# Interval Type-2 Fuzzy-Model-Based Control Design for Time-Delay Systems under Imperfect Premise Matching

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**Abstract**—In this paper, the problems of stabilization for interval type-2 fuzzy systems with time-varying delay and parameter uncertainties are investigated. The objective is to design an interval type-2 fuzzy controller such that the closed-loop control system is asymptotically stable. The conditions for the existence of such a controller are delay dependent and membership function dependent in terms of linear matrix inequalities (LMIs). Based on a basic lemma, we formulate and solve the problem with more flexibility due to imperfect premise matching that the number of rules and premise membership functions are not necessary the same between the interval type-2 fuzzy model and interval type-2 fuzzy controller. A systematic approach making use of the information embedded in the lower and upper membership functions is employed to facilitate the stability analysis. A numerical example indicates the effectiveness of the derived results.

**Index Terms**—Interval type-2 fuzzy control, Time-varying delay, Imperfect premise matching

## I. INTRODUCTION

Type-2 fuzzy systems have drawn wide attention during the last decade, and many fruitful results have been accumulated on analysis and synthesis of these systems in both theory and practice (see, e.g. [1]–[7]). First introduced by Zadeh in 1975 [8], one motivation for studying such a class of systems is that type-2 fuzzy sets are better in representing and capturing uncertainties [9], [10], especially when the nonlinear plant suffers the parameter uncertainties while type-1 fuzzy sets do not contain uncertain information. Type-2 fuzzy systems like type-1 fuzzy systems are characterized by IF-THEN rules and are represented as weighted sum of local linear systems, and

type-2 fuzzy systems could be regarded as a bunch of type-1 fuzzy systems.

However, the type-2 fuzzy set was originally for general type-2 fuzzy systems rather than the fuzzy-model-based control framework. This stimulates the study on interval type-2 fuzzy model describing the nonlinear plant subject to parameter uncertainty captured by upper and lower membership functions. Interval type-2 fuzzy systems were proposed in [11] and then extended in [12] for a wider class of nonlinear systems. Since then, they have been supported by a wide range of applications such as image processing [13], face recognition [14], energy markets [15], supervisory adaptive tracking control [16], and linguistic summarization [17]. Preliminary stability results on interval type-2 fuzzy-model-based systems could be found in [12] under parallel distributed compensation scheme. [18] further increases design flexibility and reduces implementation complexity by considering the imperfect premise matching, which means the fuzzy controller and the fuzzy plant do not have to share the same number of fuzzy rules and/or same premise membership functions.

On the other hand, it is well known that most practical dynamic systems inherently involve time delays. Without taking the limitations into consideration, techniques developed may result in performance degradation or even instability of the closed-loop control system in practice. In recent years, fuzzy system with time delays has been probed widely. Just to name a few, in [19], the authors proposed delay partitioning approach to stabilize continuous time-delay Takagi-Sugeno fuzzy systems with time-varying parameter uncertainties. In [20], the authors investigated fault detection problem for Takagi-Sugeno fuzzy systems with time-varying delays via delta operator approach. In [21], the authors dealt with the network delay compensation problem for nonlinear networked

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control systems.

Although type-2 fuzzy sets demonstrate superiority compared to type-1 fuzzy sets, the formers will be complex in analysis and have heavy computational burden. In this paper we investigate interval type-2 fuzzy-model-based control design for systems with time-varying delays under imperfect premise matching. Unlike existing work under type-1 fuzzy logic frame, this paper formulates nonlinear systems with parameter uncertainties and time-varying delay under interval type-2 fuzzy logic frame. The uncertainties are presented by the upper and lower membership functions and the time-varying delay is restricted by its bound and derivative.

Motivated by the discussions above, by means of Lyapunov-Krasovskii functional, we have proposed sufficient conditions in terms of LMIs to assure the asymptotical stability of the closed-loop control system. The remainder of this paper is organized as follows. Section II is devoted to the mathematical model of the concerned system and some preliminaries. Synthesis of state-feedback type-2 fuzzy controller are presented in Section III. A numerical simulation in Section IV is to illustrate the feasibility and effectiveness of the proposed results and Section V concludes the paper.

## II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a nonlinear system with time-varying delays and parameter uncertainties represented by the following interval type-2 fuzzy model with lower and upper bound membership functions.

**Plant Rule  $i$ :**

IF  $\theta_1(x(t))$  is  $\tilde{M}_{i1}$ ,  $\theta_2(x(t))$  is  $\tilde{M}_{i2}$   $\cdots$  and  $\theta_\Psi(x(t))$  is  $\tilde{M}_{i\Psi}$ , THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di} x(t-d(t)) + B_i u(t) \\ x(t) = \varphi(t), \quad t = [-\bar{d}, 0) \end{cases} \quad (1)$$

where  $\tilde{M}_{i\alpha}$  is an interval type-2 fuzzy set of rule  $i$ ,  $\alpha = 1, 2, \dots, \Psi$  and  $i = 1, 2, \dots, p$ .  $x(t) \in \mathbb{R}^n$  is the state,  $u(t) \in \mathbb{R}^m$  is the control input,  $d(t)$  is the time-varying delay and satisfies  $d(t) \in (0, \bar{d}]$ ,  $\dot{d}(t) \leq m$ ,  $\bar{d}$  and  $m$  are known positive numbers,  $\varphi(t)$  is the initial sequence.  $A_i$ ,  $B_i$ ,  $A_{di}$  are known matrices as system matrices, input matrices and delayed-state matrices, respectively. The firing strength of rule  $i$  is the interval sets as follows:

$$W_i(x(t)) = [\underline{w}_i(x(t)), \bar{w}_i(x(t))], \quad i = 1, 2, \dots, p \quad (2)$$

where

$$\underline{w}_i(x(t)) = \prod_{\alpha=1}^{\Psi} \underline{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(x(t))) \geq 0 \quad (3)$$

$$\bar{w}_i(x(t)) = \prod_{\alpha=1}^{\Psi} \bar{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(x(t))) \geq 0 \quad (4)$$

in which  $\underline{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(x(t)))$  and  $\bar{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(x(t)))$  denote the lower and upper membership functions respectively satisfying the property  $\bar{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(x(t))) \geq \underline{\mu}_{\tilde{M}_{i\alpha}}(\theta_\alpha(x(t))) \geq 0$  and  $\underline{w}_i(x(t))$  and  $\bar{w}_i(x(t))$  denote the lower and upper grade of

membership respectively. The inferred internal type-2 fuzzy model is defined as follows:

$$\dot{x}(t) = \sum_{i=1}^p \tilde{w}_i(x(t))(A_i x(t) + A_{di} x(t-d(t)) + B_i u(t)) \quad (5)$$

where

$$\tilde{w}_i(x(t)) = \underline{w}_i(x(t))\underline{\alpha}_i(x(t)) + \bar{w}_i(x(t))\bar{\alpha}_i(x(t)) \geq 0 \quad \forall i \quad (6)$$

$$\sum_{i=1}^p \tilde{w}_i(x(t)) = 1 \quad (7)$$

in which  $\underline{\alpha}_i(x(t)) \in [0, 1]$ ,  $\bar{\alpha}_i(x(t)) \in [0, 1]$  are nonlinear functions with the property that  $\underline{\alpha}_i(x(t)) + \bar{\alpha}_i(x(t)) = 1$ .

**Controller Rule  $j$ :**

IF  $\sigma_1(x(t))$  is  $\tilde{N}_{j1}$ ,  $\sigma_2(x(t))$  is  $\tilde{N}_{j2}$   $\cdots$  and  $\sigma_\Omega(x(t))$  is  $\tilde{N}_{j\Omega}$ , THEN

$$u(t) = K_j x(t) \quad (8)$$

where  $\tilde{N}_{j\beta}$  is an interval type-2 fuzzy set of rule  $j$ ,  $\beta = 1, 2, \dots, \Omega$  and  $j = 1, 2, \dots, c$ .  $K_j$  are unknown feedback gains to be determined. The firing strength of rule  $j$  is the interval sets as follows:

$$M_j(x(t)) = [\underline{m}_j(x(t)), \bar{m}_j(x(t))], \quad j = 1, 2, \dots, c \quad (9)$$

where

$$\underline{m}_j(x(t)) = \prod_{\beta=1}^{\Omega} \underline{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(x(t))) \geq 0 \quad (10)$$

$$\bar{m}_j(x(t)) = \prod_{\beta=1}^{\Omega} \bar{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(x(t))) \geq 0 \quad (11)$$

in which  $\underline{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(x(t)))$  and  $\bar{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(x(t)))$  denote the lower and upper membership functions respectively satisfying the property  $\bar{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(x(t))) \geq \underline{\mu}_{\tilde{N}_{j\beta}}(\sigma_\beta(x(t))) \geq 0$  and  $\underline{m}_j(x(t))$  and  $\bar{m}_j(x(t))$  denote the lower and upper grade of membership respectively. The inferred internal type-2 fuzzy controller is defined as follows:

$$u(t) = \sum_{j=1}^c \tilde{m}_j(x(t)) K_j x(t) \quad (12)$$

where

$$\tilde{m}_j(x(t)) = \frac{\underline{m}_j(x(t))\underline{\beta}_j(x(t)) + \bar{m}_j(x(t))\bar{\beta}_j(x(t))}{\sum_{k=1}^c (\underline{m}_k(x(t))\underline{\beta}_k(x(t)) + \bar{m}_k(x(t))\bar{\beta}_k(x(t)))} \geq 0 \quad \forall j \quad (13)$$

$$\sum_{j=1}^c \tilde{m}_j(x(t)) = 1 \quad (14)$$

in which  $\underline{\beta}_j(x(t)) \in [0, 1]$ ,  $\bar{\beta}_j(x(t)) \in [0, 1]$  are predefined functions with the property that  $\underline{\beta}_j(x(t)) + \bar{\beta}_j(x(t)) = 1$ .

With the plant and controller expression and the property of  $\sum_{i=1}^p \tilde{w}_i(x(t)) = 1$ ,  $\sum_{j=1}^c \tilde{m}_j(x(t)) = 1$ ,

$\sum_{i=1}^p \sum_{j=1}^c \tilde{w}_i(x(t)) \tilde{m}_j(x(t)) = 1$ , we can have the closed-loop control system as

$$\dot{x}(t) = \sum_{i=1}^p \sum_{j=1}^c \tilde{h}_{ij}(x(t)) ((A_i + B_i K_j)x(t) + A_{di}x(t-d(t))) \quad (15)$$

where  $\tilde{h}_{ij}(x(t)) \triangleq \tilde{w}_i(x(t)) \tilde{m}_j(x(t))$ . In addition  $\tilde{h}_{ij}(x(t))$  could be reconstructed as  $\bar{\gamma}_{ij}(x(t)) \bar{h}_{ij}(x(t)) + \underline{\gamma}_{ij}(x(t)) \underline{h}_{ij}(x(t))$ , in which  $\underline{\gamma}_{ij}(x(t)) \in [0, 1]$ ,  $\bar{\gamma}_{ij}(x(t)) \in [0, 1]$  are functions with the property that  $\underline{\gamma}_{ij}(x(t)) + \bar{\gamma}_{ij}(x(t)) = 1$  and  $\bar{h}_{ij}(x(t))$  and  $\underline{h}_{ij}(x(t))$  are the upper and lower bound of  $\tilde{h}_{ij}(x(t))$  with definitions below from [18]

$$\bar{h}_{ij}(x(t)) = \sum_{k=1}^q \sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{r i_r k}(x_r(t)) \bar{\delta}_{ij i_1 i_2 \cdots i_n k} \quad (16)$$

$$\underline{h}_{ij}(x(t)) = \sum_{k=1}^q \sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 \prod_{r=1}^n v_{r i_r k}(x_r(t)) \underline{\delta}_{ij i_1 i_2 \cdots i_n k} \quad (17)$$

where  $0 \leq \underline{\delta}_{ij i_1 i_2 \cdots i_n k} \leq \bar{\delta}_{ij i_1 i_2 \cdots i_n k} \leq 1$  are scalars to be figured out,  $0 \leq \underline{h}_{ij}(x(t)) \leq \bar{h}_{ij}(x(t)) \leq 1$ ,  $v_{r i_r k}(x_r(t)) \in [0, 1]$  and  $v_{r i_1 k}(x_r(t)) + v_{r i_2 k}(x_r(t)) = 1$ , otherwise  $v_{r i_r k}(x_r(t)) = 0$ ,  $x(t) \in \Psi_k$ ,  $\cup_{k=1}^q \Psi_k = \Psi$  is the state space of interest.

*Remark 1:* With the above definitions, in the further stability analysis, we could use scalars  $\underline{\delta}_{ij i_1 i_2 \cdots i_n k}$  and  $\bar{\delta}_{ij i_1 i_2 \cdots i_n k}$  to deal with the term  $\underline{h}_{ij}(x(t))$  and  $\bar{h}_{ij}(x(t))$  through  $\prod_{r=1}^n v_{r i_r k}(x_r(t))$  which are independent of  $i$  and  $j$ . In a word, the stability conditions involving the membership function information ( $\bar{h}_{ij}(x(t))$  and  $\underline{h}_{ij}(x(t))$ ) as the upper and lower bound of  $\tilde{h}_{ij}(x(t))$  could be achieved by scalars  $\underline{\delta}_{ij i_1 i_2 \cdots i_n k}$  and  $\bar{\delta}_{ij i_1 i_2 \cdots i_n k}$ .

### III. MAIN RESULTS

For simplification reason, we denote  $\tilde{w}_i(x(t))$ ,  $\tilde{m}_j(x(t))$ ,  $\tilde{h}_{ij}(x(t))$ ,  $\bar{h}_{ij}(x(t))$  and  $\underline{h}_{ij}(x(t))$  as  $\tilde{w}_i$ ,  $\tilde{m}_j$ ,  $\tilde{h}_{ij}$ ,  $\bar{h}_{ij}$  and  $\underline{h}_{ij}$ , respectively.

We need to revisit a fundamental lemma to be used in the following proof.

*Lemma 1:* [22] For matrix  $N = \begin{bmatrix} -R & L \\ * & -R \end{bmatrix} \leq 0$ ,  $d(t) \in (0, \bar{d}]$ , and a vector function  $\dot{x} : [-\bar{d}, 0) \rightarrow \mathbb{R}^n$  such that the integration in the following inequality is well defined, then it holds that

$$-\bar{d} \int_{t-\bar{d}}^t \dot{x}^T(s) R \dot{x}(s) ds \leq v^T(t) W v(t) \quad (18)$$

where

$$W = \begin{bmatrix} -R & R+L & -L \\ * & -2R+L+L^T & R+L \\ * & * & -R \end{bmatrix} \quad (19)$$

$$v^T(t) = [x^T(t) \ x^T(t-d(t)) \ x^T(t-\bar{d})] \quad (20)$$

*Theorem 1:* Given constants  $m$ , positive scalar  $\bar{d}$ , predefined scalars  $\bar{\delta}_{ij i_1 i_2 \cdots i_n k}$  and  $\underline{\delta}_{ij i_1 i_2 \cdots i_n k}$  satisfying (16) and (17), if there exist positive matrices  $X$ ,  $\bar{Y}_{ij}$ ,  $\bar{Q}$ ,  $\bar{Z}$  and  $\bar{T}$  of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} -\bar{Z} & \bar{T} \\ * & -\bar{Z} \end{bmatrix} < 0 \quad (21)$$

$$\Xi_{ij} - \begin{bmatrix} \bar{Y}_{ij} & 0 \\ 0 & 0 \end{bmatrix} < 0 \quad \forall i, j \quad (22)$$

$$\sum_{i=1}^p \sum_{j=1}^c \left( \underline{\delta}_{ij i_1 i_2 \cdots i_n k} \Xi_{ij} + (\bar{\delta}_{ij i_1 i_2 \cdots i_n k} - \underline{\delta}_{ij i_1 i_2 \cdots i_n k}) \times \begin{bmatrix} \bar{Y}_{ij} & 0 \\ 0 & 0 \end{bmatrix} \right) < 0 \quad (23)$$

$\forall i, j, k, i_1, i_2, \dots, i_n$

where

$$\Xi_{ij} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{bmatrix} \quad (24)$$

with

$$\Xi_{11} = A_i X + B_i N_j + (A_i X + B_i N_j)^T + \bar{Q} - \bar{Z}/\bar{d} \quad (25)$$

$$\Xi_{12} = A_{di} X + (\bar{Z} + \bar{T})/\bar{d} \quad (26)$$

$$\Xi_{13} = -\bar{T}/\bar{d} \quad (27)$$

$$\Xi_{14} = \sqrt{\bar{d}}(X A_i^T + Y_j^T B_i^T) \quad (28)$$

$$\Xi_{22} = (m-1)\bar{Q} - (\bar{Z} + \bar{T} + \bar{T}^T)/\bar{d} \quad (29)$$

$$\Xi_{23} = (\bar{Z} + \bar{T})/\bar{d} \quad (30)$$

$$\Xi_{24} = \sqrt{\bar{d}} X A_i^T \quad (31)$$

$$\Xi_{33} = -\bar{Z}/\bar{d} \quad (32)$$

$$\Xi_{34} = 0 \quad (33)$$

$$\Xi_{44} = \bar{Z} - 2X \quad (34)$$

Then the closed-loop control system (15) is asymptotically stable. Moreover the interval type-2 fuzzy controller gains can be obtained by  $K_j = N_j X^{-1}$ .

**Proof.** Consider a candidate of Lyapunov-Krasovskii functional as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (35)$$

$$V_1(t) = x^T(t) P x(t) \quad (36)$$

$$V_2(t) = \int_{t-d(t)}^t x^T(s) Q x(s) ds \quad (37)$$

$$V_3(t) = \int_{-\bar{d}t+\theta}^0 \int_{-\bar{d}t+\theta}^t \dot{x}^T(s) Z \dot{x}(s) ds d\theta \quad (38)$$

Along the trajectories of the closed-loop control system, the corresponding time derivative of  $V(t)$  is given by

$$\begin{aligned}\dot{V}_1(t) &= 2x^T(t)P\dot{x}(t) \\ &= 2\sum_{i=1}^p\sum_{j=1}^c\tilde{w}_i\tilde{m}_jx^T(t) \\ &\quad \times P((A_i + B_iK_j)x(t) + A_{di}x(t-d(t))) \\ \dot{V}_2(t) &= x^T(t)Qx(t) - (1-\dot{d}(t))x^T(t-d(t))Qx(t-d(t)) \\ &\leq x^T(t)Qx(t) - (1-m)x^T(t-d(t))Qx(t-d(t)) \\ \dot{V}_3(t) &= \bar{d}\dot{x}^T(t)Z\dot{x}(t) - \int_{t-\bar{d}}^t \dot{x}^T(s)Z\dot{x}(s)ds\end{aligned}$$

By applying Lemma 1,  $\dot{V}_3(t)$  can be expressed as

$$\begin{aligned}\dot{V}_3(t) &\leq \bar{d}\sum_{i=1}^p\sum_{j=1}^c\tilde{w}_i\tilde{m}_j \\ &\quad \times ((A_i + B_iK_j)x(t) + A_{di}x(t-d(t)))^T Z \\ &\quad \times ((A_i + B_iK_j)x(t) + A_{di}x(t-d(t))) \\ &\quad + \frac{1}{\bar{d}}v^T(t) \begin{bmatrix} -Z & Z+T & -T \\ * & -2Z-T-T^T & Z+T \\ * & * & -Z \end{bmatrix} v(t)\end{aligned}$$

with  $v^T(t) = [x^T(t) \ x^T(t-d(t)) \ x^T(t-\bar{d})]$  and subject to  $\begin{bmatrix} -Z & T \\ * & -Z \end{bmatrix} \leq 0$ .

$$\text{Rewrite } \dot{V}(t) \text{ with } \Omega_{ij} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & -T/\bar{d} \\ * & \Lambda_{22} & (Z+T)/\bar{d} \\ * & * & -Z/\bar{d} \end{bmatrix},$$

where

$$\begin{aligned}\Lambda_{11} &= P(A_i + B_iK_j) + (A_i + B_iK_j)^T P + Q - Z/\bar{d} \\ &\quad + \bar{d}(A_i + B_iK_j)^T Z(A_i + B_iK_j) \\ \Lambda_{12} &= (Z+T)/\bar{d} + PA_{di} + \bar{d}(A_i + B_iK_j)^T ZA_{di} \\ \Lambda_{22} &= (m-1)Q - (2Z+T+T^T)/\bar{d} + \bar{d}A_{di}^T ZA_{di}\end{aligned}$$

Then we get

$$\begin{aligned}\dot{V}(t) &\leq \sum_{i=1}^p\sum_{j=1}^c\tilde{h}_{ij}v^T(t)\Omega_{ij}v(t) \\ &= \sum_{i=1}^p\sum_{j=1}^c(\underline{h}_{ij} + (\tilde{h}_{ij} - \underline{h}_{ij}))v^T(t)\Omega_{ij}v(t) \\ &\leq \sum_{i=1}^p\sum_{j=1}^c\underline{h}_{ij}v^T(t)\Omega_{ij}v(t) + (\bar{h}_{ij} - \underline{h}_{ij})v^T(t)Y_{ij}v(t) \\ &= \sum_{i=1}^p\sum_{j=1}^c v^T(t)(\underline{h}_{ij}\Omega_{ij} + (\bar{h}_{ij} - \underline{h}_{ij})Y_{ij})v(t)\end{aligned}$$

with

$$\begin{aligned}Y_{ij} &\geq 0 \\ Y_{ij} &\geq \Omega_{ij}\end{aligned}$$

The stability condition for the closed-loop control system would be

$$\sum_{i=1}^p\sum_{j=1}^c v^T(t)(\underline{h}_{ij}\Omega_{ij} + (\bar{h}_{ij} - \underline{h}_{ij})Y_{ij})v(t) < 0 \quad (39)$$

Recalling that  $\sum_{k=1}^q\sum_{i_1=1}^2\cdots\sum_{i_n=1}^2\prod_{r=1}^n v_{r,i_r,k}(x_r(t)) = 1$ , with (16) and (17), by using Schur Complement and congruence transformation, we can get the conditions as stated in the theorem with  $X = P^{-1}$ ,  $K_j X = N_j$ ,  $\bar{Q} = XQX$ ,  $\bar{Z} = XZX$ ,  $\bar{T} = XTX$ ,  $\bar{Y}_{ij} = \text{diag}\{X, X, X\}Y_{ij}\text{diag}\{X, X, X\}$ . ■

*Remark 2:* Theorem 1 introduces membership functions  $\tilde{h}_{ij}$  which are reconstructed by the upper bound  $\bar{h}_{ij}$  and lower bound  $\underline{h}_{ij}$ . Moreover  $\bar{h}_{ij}$  and  $\underline{h}_{ij}$  could be expressed by predefined scalars  $\bar{\delta}_{ij i_1 i_2 \dots i_n k}$  and  $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$  in the form of (16) and (17). This will allow us to just check conditions at certain points ( $\bar{\delta}_{ij i_1 i_2 \dots i_n k}$  and  $\underline{\delta}_{ij i_1 i_2 \dots i_n k}$ ) rather than every point of the upper bound  $\bar{h}_{ij}$  and lower bound  $\underline{h}_{ij}$ .

*Remark 3:* As Theorem 1 involves the information of the membership functions in control design, it is a membership function dependent method which is less conservative than the membership function independent method. While Theorem 1 could be reduced to the following corollary which is membership function independent for control design.

*Corollary 1:* Given constants  $m$ , positive scalar  $\bar{d}$ , if there exist positive matrices  $X$ ,  $\bar{Q}$ ,  $\bar{Z}$  and  $\bar{T}$  of appropriate dimensions such that the following LMIs hold:

$$\begin{bmatrix} -\bar{Z} & \bar{T} \\ * & -\bar{Z} \end{bmatrix} < 0 \quad (40)$$

$$\Xi_{ij} = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{22} & \Xi_{23} & \Xi_{24} \\ * & * & \Xi_{33} & \Xi_{34} \\ * & * & * & \Xi_{44} \end{bmatrix} < 0 \quad \forall i, j \quad (41)$$

the elements in  $\Xi_{ij}$  are the same as stated in Theorem 1. Then the closed-loop control system (15) is asymptotically stable. Moreover the interval type-2 fuzzy controller gains can be obtained by  $K_j = N_j X^{-1}$ .

*Remark 4:* In the derivation of Theorem 1, we introduce slack matrices  $Y_{ij}$  to bring more flexibility. We can include even more slack matrices based on some inequalities and equalities, but this will lead to high computational demand.

*Remark 5:* It could be noted that dividing the region of  $x$  into more partitions could further reduce the conservatism. The more upper and lower bounds of the membership functions involved in could lead to more relaxed results while the computation burden would be heavier.

*Remark 6:* Theorem 1 and Corollary 1 could be modified to tackle control systems without time-varying delay by removing  $V_2(t)$  and  $V_3(t)$  in  $V(t)$  following the similar derivation.

#### IV. NUMERICAL EXAMPLE

In this section, a numerical example will be presented to demonstrate the potential and validity of our developed theoretical results.

Consider a three-rule interval type-2 fuzzy model in the form of (1) with  $A_1 = \begin{bmatrix} 2.78 & -5.63 \\ 0.01 & 0.33 \end{bmatrix}$ ,  $A_2 = \begin{bmatrix} 0.2 & -3.22 \\ 0.35 & 0.12 \end{bmatrix}$ ,  $A_3 = \begin{bmatrix} -a & -4.33 \\ 0 & 0.05 \end{bmatrix}$ ,  $B_1 = \begin{bmatrix} 2 & -1 \end{bmatrix}^T$ ,  $B_2 = \begin{bmatrix} 8 & 0 \end{bmatrix}^T$ ,  $B_3 = \begin{bmatrix} -b+6 & -1 \end{bmatrix}^T$ ,  $A_{d1} = \begin{bmatrix} 0.5 & 0 \\ 0 & -0.5 \end{bmatrix}$ ,  $A_{d2} = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ ,  $A_{d3} = \begin{bmatrix} 0.4 & 0 \\ 0 & -0.6 \end{bmatrix}$ .  $a$  and  $b$  are constant parameters,  $x = [x_1 \ x_2]^T$ .  $d(t) = 0.1\bar{d}(1 + 9\sin^2 t)$ ,  $m = 0.9\bar{d}$ .  $\varphi(t) = 0$  when  $t \in [-\bar{d}, 0)$ .

The membership functions for the plant (1) are chosen as  $\tilde{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4+\eta(t))})$ ,  $\tilde{w}_2(x_1) = 1 - \tilde{w}_1(x_1) - \tilde{w}_3(x_1)$ ,  $\tilde{w}_3(x_1) = 1/(1 + e^{-(x_1-4+\eta(t))})$ . Due to parameter uncertainty  $\eta(t)$ , the membership functions are uncertain grades of membership. The lower and upper membership functions are chosen as  $\underline{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4+0.25)})$ ,  $\bar{w}_2(x_1) = 1 - \underline{w}_1(x_1) - \underline{w}_3(x_1)$ ,  $\underline{w}_3(x_1) = 1/(1 + e^{-(x_1-4-0.25)})$ ,  $\bar{w}_1(x_1) = 1 - 1/(1 + e^{-(x_1+4-0.25)})$ ,  $\bar{w}_3(x_1) = 1/(1 + e^{-(x_1-4+0.25)})$ ,  $\underline{w}_2(x_1) = 1 - \bar{w}_1(x_1) - \bar{w}_3(x_1)$ .

The lower and upper membership functions for the controller (12) are chosen as  $\underline{m}_1(x_1) = 1 - 1/e^{-(x_1+0.15)/2}$ ,  $\bar{m}_1(x_1) = 1 - 1/e^{-(x_1-0.15)/2}$ ,  $\underline{m}_2(x_1) = 1 - \bar{m}_1(x_1)$ ,  $\bar{m}_2(x_1) = 1 - \underline{m}_1(x_1)$ . From (13), we can get  $\tilde{m}_j(x_1)$ . Set  $\bar{\beta}_1 = \bar{\beta}_2 = 0.5$ , we can get stability regions by conditions in Theorem 1 subject to different values of  $a$  and  $b$ . We consider the grades of membership are capped and focus on the region  $x_1 \in [-10, 10]$ . We consider  $132 \leq a \leq 140$  at the interval of one and  $5 \leq b \leq 60$  at the interval of five. With  $\bar{d} = 0.19$ , we can see the stability region given by Theorem 1 (membership function dependent method) indicated by “o” is larger than that given by Corollary 1 (membership function independent method) indicated by “+”. The result is shown in Fig. 1. When the upper bound of the delay increases, for example  $\bar{d} = 1$ , membership function independent method would not give any feasible solution, and the stability region given by Theorem 1 (membership function dependent method) indicated by “o” is shown in Fig. 2 as  $132 \leq a \leq 140$  at the interval of one and  $5 \leq b \leq 60$  at the interval of five. These reveal the less of conservatism of the proposed membership function dependent method given in the paper.

With  $a = 136$ ,  $b = 30$ ,  $\bar{\beta}_1 = \bar{\beta}_2 = 0.5$ ,  $x(0) = [-5 \ 0]^T$ ,  $\bar{d} = 0.19$ , Fig. 3 gives the state response of the closed-loop control system which is asymptotically stable with the controller gains  $K_1 = \begin{bmatrix} -4.823987 & -0.042365 \end{bmatrix}$ ,  $K_2 = \begin{bmatrix} -5.283737 & -0.271163 \end{bmatrix}$  and  $P = \begin{bmatrix} 0.8758 & -0.4786 \\ -0.4786 & 3.5184 \end{bmatrix}$ . Fig. 4 gives the state response of the closed-loop control system with the same parameters used in Fig. 3 except for  $\bar{d} = 1$  with the controller gains  $K_1 = \begin{bmatrix} -0.577141 & 0.690671 \end{bmatrix}$ ,  $K_2 = \begin{bmatrix} -5.396162 & -0.340777 \end{bmatrix}$  and  $P = \begin{bmatrix} 0.0851 & -0.1199 \\ -0.1199 & 0.7701 \end{bmatrix}$ .

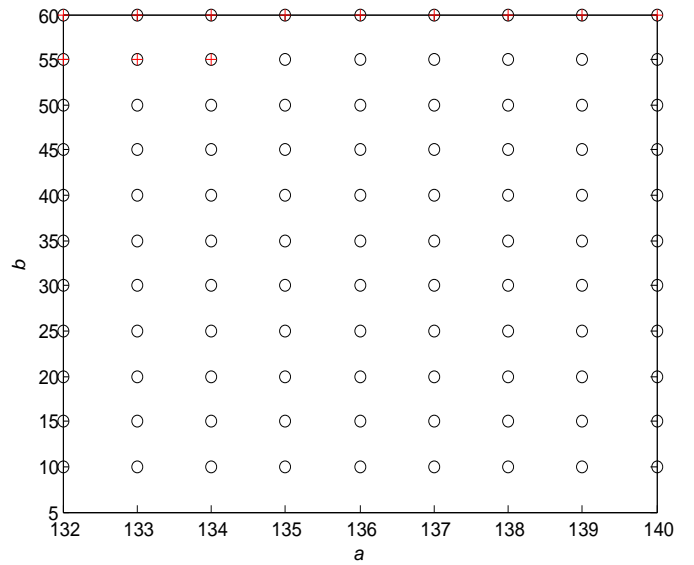


Fig. 1. Stability regions given by conditions in Theorem 1 (o, 99 points) and Corollary 1 (+, 12 points) with  $\bar{d} = 0.19$ .

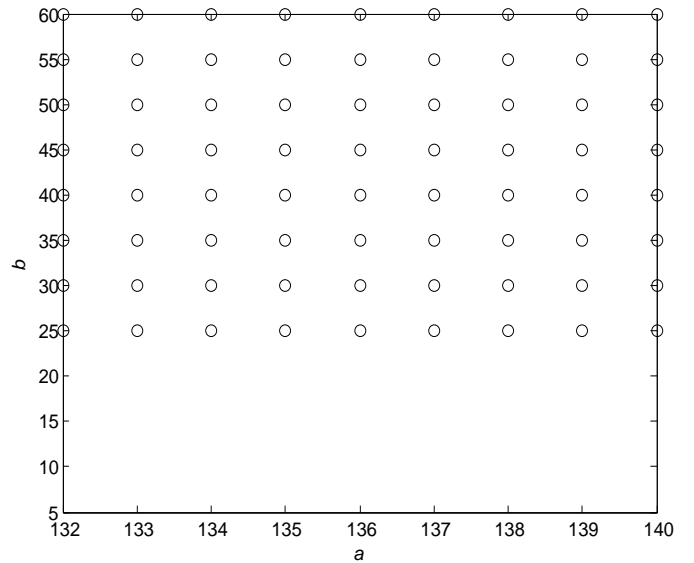


Fig. 2. Stability regions given by conditions in Theorem 1 (o, 72 points) and Corollary 1 (+, 0 point) with  $\bar{d} = 1$ .

## V. CONCLUSION

The stability of interval type-2 fuzzy-model-based control systems with time-varying delay and parameter uncertainties is investigated in this paper. We have proposed an interval type-2 fuzzy state feedback controller to ensure the asymptotic stability of the closed-loop control system under imperfect premise matching. This membership function dependent method shares more design flexibility, because it is not required that the interval type-2 fuzzy controller and interval type-2 fuzzy plant have the same premise membership function and/or number of fuzzy rules. The stability conditions come in LMI form

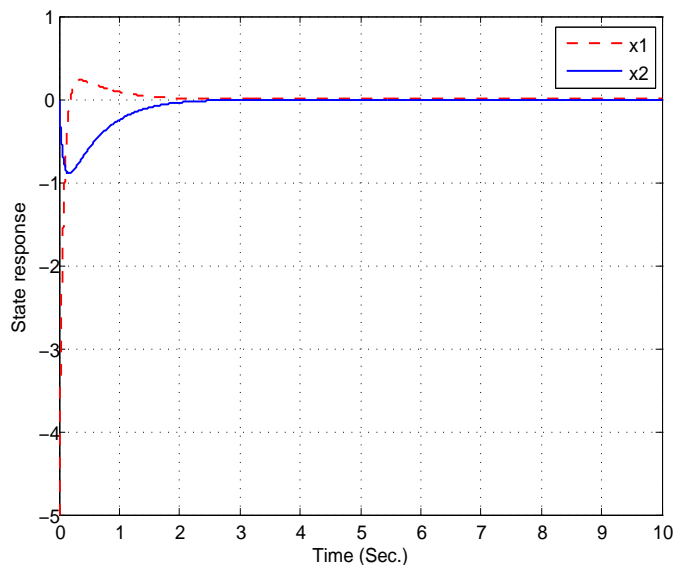


Fig. 3. State response of the closed-loop control system with  $a = 136$ ,  $b = 30$ ,  $\bar{\beta}_1 = \bar{\beta}_2 = 0.5$ ,  $x(0) = [-5 \ 0]^T$ ,  $\bar{d} = 0.19$ .

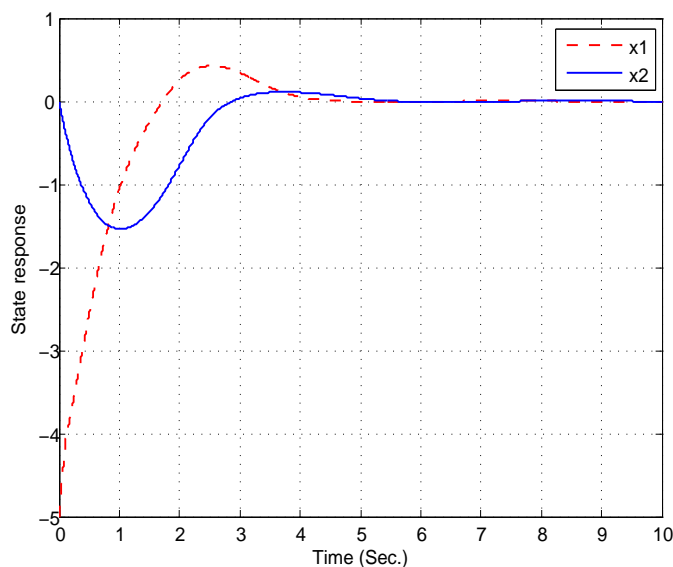


Fig. 4. State response of the closed-loop control system with  $a = 136$ ,  $b = 30$ ,  $\bar{\beta}_1 = \bar{\beta}_2 = 0.5$ ,  $x(0) = [-5 \ 0]^T$ ,  $\bar{d} = 1$ .

and include the information of the membership functions to be more relaxed than membership independent method. A numerical example is presented to show the effectiveness of the proposed approach.

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