1 Introduction

- analytic philosophy (AP) a complex tradition made up of various strands, some mutually reinforcing, some in creative tension with one another
  - AP often seen as originating in the rebellion by Bertrand Russell (1872–1970) and G. E. Moore (1873–1958) against British idealism around turn of 20th c.
  - but role of Gottlob Frege (1848–1925) even more important
- Frege created modern logic and used it in seeking to demonstrate logicism – that arithmetic is reducible to logic
- Russell, too, was a logicist and offered logical analyses of his own
  - most notably, the theory of descriptions
- Moore and Wittgenstein also co-founders of AP
  - but Frege’s and Russell’s work at heart of the ‘analytic revolution’

2a Frege’s logical revolution

- origins of analytic revolution in Frege’s logical revolution
  - Begriffsschrift of 1879: creation of quantificational logic
  - opened up semantic machinery of a host of complex sentences that had resisted effective analysis up to that point
- key: use of function–argument analysis
  - extended from maths (as in representing a line as $y = ax + b$) to logic
  - all sentences, not just equations, analysed in function–argument terms
- simple sentence such as ‘Gottlob is human’ analysed not as ‘$S$ is $P$’ but as ‘$S$’, where ‘$a$’ represents argument (‘Gottlob’) and ‘$F$’ the function (‘$x$ is human’)
- advantages start to become clear in case of relational sentences
  - analysed as functions of two or more arguments
  - e.g. ‘Gottlob is shorter than Bertrand’ analysed as ‘$R$’
  - enables a unified account of relational sentences to be provided

Gottlob Frege (1848-1925)

- German mathematician, logician and philosopher
- taught at Jena 1874-1917

1) Begriffsschrift (1879)
2) The Foundations of Arithmetic (1884)
3) ‘Function and Concept’ (1891)
4) ‘On Sense and Reference’ (1892)
5) ‘On Concept and Object’ (1892)
6) Basic Laws of Arithmetic (1893, 1903)
7) ‘Thought’ (1918)
2b Quantificational logic

- greater power of function-argument analysis only fully revealed in case of sentences involving quantifier terms such as 'all' and 'some'
  
  * \( l_1 \) : All logicians are human.
  
  * \( l_2 \) : \( (\forall x)(lx \rightarrow hx) \).
  
  * \( l_3 \) : Some logicians are human.
  
  * \( l_4 \) : \( (\exists x)(lx \land \neg hx) \).
    
    - both \( l_1 \) and \( l_3 \) have a quantificational logical form that is more complex than their surface (subject-predicate) grammatical form
    
    - subject terms 'analysed away'
  
- enables formalization of sentences with multiple quantifier terms
  
  * \( p_1 \) : Every philosopher respects some logician.
  
  * \( p_2 \) : \( (\forall x)(px \rightarrow (3y)(ly \land Ayx)) \).
  
  * \( p_3 \) : \( (3y)(ly \land (\forall x)(px \rightarrow Ayx)) \).
    
    - quantifier shift fallacy: mistakenly thinking \( (p_2) \) implies \( (p_3) \)

3a Frege’s use of logical analysis in logic

- 'Gottlob is human' and 'All logicians are human' have different logical forms
  
  - involve different logical relations: subsumption and subordination, resp.
  
- task of logical analysis to reveal the logical form of sentences, to solve philosophical problems
  
  Example: problem of negative existential statements
  
  * \( u_1 \) : Unicorns do not exist.
  
  * \( u_2 \) : Unicorns are non-existent.
    
    - S-P analysis is potentially misleading: must unicorns 'subsist'?
    
    - subject term 'analysed away'
    
    - \( u_2 \) : The concept unicorn is not instantiated.
  
- denying that something exists is saying that the relevant concept has no instances: there is no need to posit any mysterious object

3b Interpretive & decompositional analysis

- logical analysis can do genuine philosophical work: it can elucidate the logical structure of our thinking and help clear up confusions that may arise from misinterpreting statements we make
  
  - analysis does not just mean 'decomposition'
    
    - despite what S-P analysis of 'Gottlob is human' might suggest
  
- function-argument analysis yields constituents that are different from what a sentence’s grammatical form might indicate
  
    - as in e.g. 'All logicians are human'
  
- logical analysis proceeds in two steps:
  
  1) interpretive analysis: rephrases to reveal logical form
  
  2) decompositional analysis: identifies supposed (logically significant) constituents
    
    - in logical analysis there is no decomposition without interpretation

4a Frege’s logicist project

1879 \textit{Begriffsschrift}: develops logic; analyses math. induction

1884 \textit{The Foundations of Arithmetic}: formal account of his project

1893 \textit{Basic Laws of Arithmetic}: informal account of his project

- central claim of Foundations: number statements are assertions about concepts
  
  - just like existential statements

  * \( l_1 \) : Jupiter has four moons.
  
  * \( l_2 \) : The concept moon of Jupiter has four instances.
  
  * \( l_3 \) : \( (\exists v, w, x, y)(Mv \land Mw \land Mx \land My \land v = w \land x = y \land My(v = w \land x = y)) \).
    
    - \( l_3 \) has a more complex (quantificational) logical form than its surface (S-P) grammatical form suggests
4b Defining the natural numbers

- numbers defined as classes (extensions of concepts)
  - understood as abstract, logical objects
- taking logical concepts of identity and negation, we can form extension of the concept not identical with itself (i.e., class of things that are not self-identical)
  - nothing falls under this class, so this is the null class
  - 0 can then be defined in terms of the null class
- with our first object, we can then form extension of the concept is identical with 0, which has one member
  - 1 can then be defined, and so on
- starting with the null class, then, and using only logical concepts, we can define all the natural numbers

5a Russell’s paradox

- discovered in 1902, when Russell wrote to Frege, who recognized that it undermined his logicist project
  - threatens assumptions that for every concept, there is a class of things that fall under it, and that classes can be members of themselves

  Consider the class of horses. This class is not itself a horse, so the class is not a member of itself. Consider the class of non-horses. This class is not a horse, so the class is a member of itself. So classes divide into those that are members of themselves and those that are not members of themselves. Consider now the class of all classes that are not members of themselves. Is this a member of itself or not? If it is, then since it is the class of all classes that are not members of themselves, it is not. If it is not, then since this is the defining property of the classes it contains, it is. We have a contradiction.

  - defining condition for problematic class seems perfectly logical

5b Logical constructions

- Russell’s response to paradox: theory of types
  - a hierarchy: ‘genuine’ objects, classes, classes of classes, etc.
  - a class can only be a member of a higher class, hence not of itself
- classes are ‘logical fictions’ or ‘logical constructions’
  (A) The average British woman has 1.9 children.
  (A) The total number of children of British women divided by the total number of British women equals 1.9.
    - ‘the average British woman’ a logical fiction; convenient way of talking
  (C) The class of horses is a subclass of the class of animals.
  (C) Anything that falls under the concept horse falls under the concept animal.
  (C) (Wx (Hx → Ax).
    - talk of classes is ‘constructed’ out of our talk of concepts
6 Russell’s theory of descriptions

- problem: how definite descriptions (of form ‘the F’) can contribute to the meaning and truth-value of sentences even when they lack a referent

\[ (K_1) \text{ The present King of France is bald.} \]

\[ (K_2) \text{ There is at least one King of France. } (\exists x) Kx \]

\[ (K_3) \text{ There is at most one King of France. } (\forall x)(\forall y)(Kx \land Ky \rightarrow y = x) \]

\[ (K_4) \text{ Whatever is King of France is bald. } (\forall x) (Kx \rightarrow Bx) \]

\[ (K_5) \text{ There is a unique King of France and whatever is King of France is bald. } (\exists! x) (Kx \land (\forall y)(Ky \rightarrow y = x \land Bx)) \]

\[ (K_6) \text{ The concept King of France is uniquely instantiated and whatever instantiates this concept also instantiates the concept bald.} \]

\[ (K_7) \text{ The concept King of France is uniquely instantiated and subordinate to the concept bald.} \]

7 Interpretive analysis

- role played by interpretive analysis especially distinctive of AP
  - but Frege and Russell use it differently
  - Russell in an eliminativist and not just reductivist project

L_1. Jupiter has four moons.

\[ (L_1) \text{ There are } 4 \text{ moons. } (\exists x) \text{ Moon(x) } \land \text{ Moon(y) } \land \text{ Moon(z) } \land \text{ Moon(w) } \land \forall x \forall y \forall z \forall w (x \neq y \land y \neq z \land z \neq w \land w \neq x) \]

- ‘Four’ analysed away; but \( L_1 \) also equivalent to:

\[ (L_2) \text{ The number of Jupiter’s moons is (the number) four.} \]

- for Frege, shows that numbers are objects
- if sentences are true, constituent terms must have a ‘meaning’ (‘Bedeutung’)
- nevertheless, some of his analyses are implicitly eliminativist
- better to talk about numbers as logical constructions than logical fictions
- deciding whether or not numbers ‘exist’ distracts from real issue
- key point: interpretive analysis opens up a variety of projects

8 The paradox of analysis

- threatens to undermine the very possibility of interpretive analysis

\[ (L_1) \text{ All logicians are human. } (\forall x) (Lx \rightarrow Hx). \]

\[ (L_2) \text{ The concept logician is subordinate to the concept human.} \]

- does \( L_2 \) have the same meaning as \( L_1 \)?
  - minimum requirement: logically equivalent
  - key point: dynamic nature of process of analysis
  - an analysis is informative by being transformative
- an analysis offers richer conceptual tools to understand something
  - in thinking about analyses themselves, we invoke further concepts, such as those of meaning, reference, and so on
- all these logical and semantic concepts and relations might themselves be seen as logical constructions, which emerge in our activities of analysis

9 Conclusion

- analytic philosophy concerned with analysis
  - but ‘analysis’, in one form or another, has always been part of Western philosophy
- what is especially distinctive of (one central strand of) analytic philosophy is the role played by interpretive analysis
  - drawing on the powerful resources that the new quantifical logic provided
- brought with it a new set of concepts, which opened up a new set of questions concerning meaning, reference, and so on
  - these questions were especially addressed by the next generation of analytic philosophers, when the ‘linguistic turn’ was taken
- linguistic turn nevertheless rooted in Frege’s and Russell’s analytic revolution