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Distributed Event-Based Set-Membership Filtering for A Class of Nonlinear Systems with Sensor Saturations over Sensor Networks

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***Abstract*—In this paper, the distributed set-membership filtering problem is investigated for a class of discrete time-varying system with an event-based communication mechanism over sensor networks. The system under consideration is subject to sector-bounded nonlinearity, unknown but bounded noises and sensor saturations. Each intelligent sensing node transmits the data to its neighbors only when certain triggering condition is violated. By means of a set of recursive matrix inequalities, sufficient conditions are derived for the existence of the desired distributed event-based filter which is capable of confining the system state in certain ellipsoidal regions centered at the estimates. Within the established theoretical framework, two additional optimization problems are formulated: one is to seek the minimal ellipsoids (in the sense of matrix trace) for the best filtering performance, and the other is to maximize the triggering threshold so as to reduce the triggering frequency with satisfactory filtering performance. A numerically attractive chaos algorithm is employed to solve the optimization problems. Finally, an illustrative example is presented to demonstrate the effectiveness and applicability of the proposed algorithm.**

***Index Terms*—Nonlinear time-varying systems; Distributed set-membership filtering; Sensor networks; Event-based filtering; Sensor saturations; Unknown but bounded noise**

I. Introduction

The past decades have witnessed a rapid growth on the utilization of sensor networks consisting of a large number of sensing nodes geographically distributed in certain areas. Sensor networks have found extensive applications in various fields ranging from information collection, environmental monitoring, industrial automation, to intelligent buildings [6], [29], [31], [40], [44], [45]. The practical significance of sensor networks has recently led to considerable research interest on the distributed estimation or filtering problems whose aim is to extract the true signals based on the information measurements collected/transmitted via sensor networks. Compared with the traditional filtering algorithms in a single sensor system [21], [26], [33], [42], the key feature of the distributed filtering over sensor networks is that each sensor estimates the system state based not only on its own measurement but also on the neighboring sensors' measurements according to the topology [35]. So far, much effort has been made to the investigation on the distributed filtering problems and several effective strategies have been developed, see [7] for a survey. It is worth mentioning that, up to now, the resource efficiency issue has not been adequately addressed towards the distributed filtering problems especially for nonlinear time-varying systems, and this gives rise to the primary motivation of our current research.

With the nowadays revolution of microelectronics techniques, there is an incremental adoption of small-size micro-processors which are embedded in the sensing nodes responsible for information collecting, signal processing, data transmitting and sometimes instruction actuating within the sensor networks. In engineering practice, these micro-processors are apparently subject to limited resource such as battery storage. For energy-saving purposes, it is often favorable to exploit the event-based rules under which the information received by sensing nodes is transmitted to the controllers/filters only when some events occur. Compared to the traditional time-based communication mechanism, the event-based communication scheme has the advantage of improving the efficiency of resource utilization by reducing the unnecessary executions over the network, see [20] and the references therein for some earlier works. Due to its clear physical implication and promising application prospect, in the past decade or so, the event-based filtering problem has stirred remarkable interest and many research results have been reported in the literature, see e. g. [11], [27]. It is worth noting that, despite the recent progress in event-triggering filter/control, it remains an open problem to develop more generic triggering conditions that could play a reasonable tradeoff between the efficiency of the resource utilization and the specification of the system performance.

Apart from the aforementioned resource limitation issue, it is well known that the embedded micro-processors are typically of limited capacity within a sensor network due primarily to the physical and communication constraints. Consequently, some new phenomena (e.g. signal quantization, sensor saturation and actuator failures) have inevitably emerged that deserve particular attention in the system design.

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These phenomena are customarily referred to as the incomplete information that has attracted much research interest in developing filtering schemes [1], [8]–[10], [16]–[18], [22], [23], [25], [32], [37], [39], [41], [43], [46]. However, when it comes to the event-based distributed filtering problems with incomplete information, the corresponding results have been very few owing mainly to the lack of appropriate techniques for coping with 1) the complicated node coupling according to the topological information and 2) the demanding triggering mechanism accounting for the limited capability. As such, another motivation for our current investigation is to examine the impact of the incomplete information on the performance of the event-based distributed filtering over the sensor network with a given topology.

In real-world engineering, almost all practical systems are time-varying. For such time-varying systems, a filter that could provide better transient performance than those traditional methods developed to achieve specified steady-state performance is more effective and applicable. Therefore, the filtering problems for time-varying systems have stirred considerable research interests in the past few years. For example, the difference Riccati equation method has been proposed in [38] to solve the robust Kalman filtering problem for uncertain time-varying systems. Recently, the recursive linear matrix inequality (RLMI) method has become another effective approach to deal with the filtering and control problems for time-varying systems. Originally proposed in [12], the RLMI method has been so far widely recognized and extensively utilized in both theoretical research and engineering applications associated with time-varying systems, see e. g. [7], [32]. However, up to now, the distributed filtering problem has not been adequately investigated yet for systems subject to time-varying parameters, especially for the case where the event triggering mechanism and sensor saturation are also involved.

On another research frontier, the set-membership filtering problem originated in [36] aims to use the measurements to calculate recursively a bounding ellipsoid to the set of possible states, see [2], [30] and the references therein. Recently, there has been renewed interest in the set-membership filtering problems for various systems by developing computationally efficient algorithms. For instance, in [13], the convex optimization method has been utilized to handle the set-membership filtering with the guaranteed robustness against the system parameter uncertainties. In [14], the set-membership filtering issue has been discussed in frequency domain and an adaptive algorithm has been developed with applications in the frequency-domain equalization problem. It is worth mentioning that the set-membership filtering problem has been addressed in [37] for stochastic system in the presence of sensor saturations, where a recursive scheme has been provided for constructing an ellipsoidal state estimation set of all states consistent with the measured output and the given noise. Unfortunately, for large-scale distributed systems such as sensor networks, the set-membership filtering has not received adequate research attention, and this motivates us to investigate the set-membership filtering problem for nonlinear systems under an event-based distributed information processing mechanism.

Motivated by the above discussions, in this paper, it is our objective to design a distributed event-triggering set-membership filtering scheme for a class of discrete time-varying nonlinear systems subject to unknown but bounded noises and sensor saturations. A novel triggering condition with clear engineering insight is proposed to better reflect the reality of practical applications. The nonlinearity is assumed to satisfy the so-called sector condition, which is quite general and could cover several classes of nonlinearities as special cases. *We endeavor to answer the following questions: i) how to deal with the proposed triggering condition within the unified framework for filter analysis and synthesis? ii) how to quantify the influences on the filtering performance from the given topology, the sector-bounded nonlinearity, the unknown but bounded noises as well as the sensor saturations? iii) how to characterize the relationship between the triggering threshold and the filtering performance or, in other words, how to exploit the trade-offs between the size of the ellipsoids and the triggering threshold so as to make compromise between the filtering performance and the triggering frequency?* We shall respond to the three questions raised above by investigating the so-called distributed set-membership filtering problem.

The novelties of this paper lie in the following four aspects. *i) The system model under study is comprehensive that includes sector-bounded nonlinearity, unknown but bounded noises and sensor saturations. ii) A new ellipsoidal triggering condition is presented in which the threshold is adjustable to make the compromise between filtering performance and communication cost. iii) Two optimization problems are solved and the developed algorithms can be applied to seek the minimal ellipsoids ensuring the enhanced filtering performance and the maximal triggering threshold guaranteeing the reduced communication cost. iv) Within the established theoretical framework, we can easily handle the distributed event-based set-membership filtering problems for systems with heterogeneous structures and/or time-varying topology*.

The rest of this paper is organized as follows. Section II formulates the distributed event-based filter design problem for nonlinear discrete time-varying system with unknown but bounded noises as well as sensor saturations. Our main results are presented in Section III where sufficient conditions for the existence of the desired filter are given in terms of recursive linear matrix inequalities (RLMIs). Section IV gives a numerical example and Section V draws our conclusion.

Notation The notation used here is fairly standard except where otherwise stated. $\mathbb{R}^n$ denotes the *n*-dimensional Euclidean space, $\mathbf{1}_n$ denotes an n-dimensional column vector with all ones. I_n and 0_n denote the identity matrix and zero matrix of n dimensions, respectively. The notation $X \geq Y$ (respectively $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively positive definite). For matrices $A \in \mathbb{R}^{m\times n}$ and $B \in \mathbb{R}^{p\times q}$, their Kronecker product is a matrix in $\mathbb{R}^{mp\times nq}$ denoted as $A \otimes B$. The superscript "T" denotes the transpose. For a vector a, $\|a\| = a^{\mathrm{T}}a$. $\mathrm{tr}[A]$ means the trace of matrix A and $\mathrm{diag}\{F_1, F_2, \ldots, F_n\}$ denotes a block diagonal matrix whose diagonal blocks are given by $F_1, F_2, \ldots, F_n$. The notation $\mathrm{diag}_n\{A_i\}$ represents the block diagonal matrix

$\mathrm{diag}\{A_1, A_2, \ldots, A_n\}$ and $\mathrm{col}_n\{x_i\}$ denotes the column vector $[x_1^{\mathrm{T}} \;\; x_2^{\mathrm{T}} \;\; \ldots \;\; x_n^{\mathrm{T}}]^{\mathrm{T}}$.

II. Problem Formulation

In this paper, it is assumed that the sensor network has N sensor nodes which are distributed in the space according to a specific interconnection topology characterized by a directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{L})$, where $\mathscr{V} = \{1, 2, ..., N\}$ denotes the set of sensing nodes, $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is the set of edges, and $\mathscr{L} = [\theta_{ij}]_{N\times N}$ is the nonnegative adjacency matrix associated with the edges of the graph, that is, $\theta_{ij} > 0$ if and only if edge$(i, j) \in \mathscr{E}$ (i.e. there is information transmission from sensor j to sensor i). If $(i, j) \in \mathscr{E}$, then node j is called one of the neighbors of node i. Also, we assume that $\theta_{ii} = 1$ for all $i \in \mathscr{V}$ and, therefore, (i, i) can be regarded as an additional edge. The set of neighbors of node $i \in \mathscr{V}$ plus the node itself is denoted by $\mathscr{N}_i \triangleq \{j \in \mathscr{V} | (i, j) \in \mathscr{E}\}$.

Consider a time-varying nonlinear system with N sensors described by the following state-space model:

$$\begin{cases} x_{k+1} = A_k x_k + D_k w_k + f(x_k) \\ \;\; y_{i,k} = \sigma(C_{i,k} x_k) + E_{i,k} v_k \quad i = 1, 2, \ldots, N \end{cases} \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the system state and $y_{i,k} \in \mathbb{R}^m$ is the measurement output measured by sensor i. The parameters A_k, D_k, $C_{i,k}$ and $E_{i,k}$ are real-valued time-varying matrices of appropriate dimensions. $w_k \in \mathbb{R}^{\omega}$ and $v_k \in \mathbb{R}^{\nu}$ represent the process and measurement noises, respectively, which are deterministic and satisfy the following assumption.

Assumption 1: The noise sequences w_k and v_k are confined to the following ellipsoidal sets:

$$\begin{cases} \mathscr{S}_k \triangleq \{w_k : w_k^{\mathrm{T}} S_k^{-1} w_k \leq 1\} \\ \mathscr{R}_k \triangleq \{v_k : v_k^{\mathrm{T}} R_k^{-1} v_k \leq 1\} \end{cases} \tag{2}$$

where $S_k > 0$ and $R_k > 0$ are known matrices with compatible dimensions characterizing the sizes and orientations of the ellipsoids.

Remark 1: In practical engineering, due to the man-made electromagnetic interference as well as other natural sources, sometimes the noises are not really stochastic. Rather, they are deterministic, unknown but bounded (by energy or amplitude) [13]. As such, most statistics-based filtering algorithms (such as Kalman filtering scheme requiring exact information on the Gaussian noises) are no longer applicable. It is worth noting that the unknown but bounded noise serves as an important type of non-Gaussian noises that has received considerable research attention with respect to the filtering problems, see e.g. [13], [34]. In this paper, the process noises w_k and measurement noises v_k are assumed to be deterministic, unknown but bounded within certain ellipsoidal sets, and this gives rise to the set-membership filtering problem to be addressed in the sequel.

Definition 1: [19] Let K_1 and K_2 be some real matrices with $K \triangleq K_2 - K_1 > 0$. A nonlinearity $\kappa(\cdot)$ is said to satisfy the sector condition with respect to K_1 and K_2 if

$$\big(\kappa(y) - K_1 y\big)^{\mathrm{T}}\big(\kappa(y) - K_2 y\big) \leq 0. \tag{3}$$

In this case, the sector-bounded nonlinearity $\kappa(\cdot)$ is said to belong to the sector $[K_1, K_2]$.

Assumption 2: The nonlinear function $f(x_k)$ in the system (1) belongs to the sector $[U_1, \; U_2]$, where U_1 and U_2 are known real-valued matrices with appropriate dimensions.

The saturation function $\sigma(\cdot)$ is defined as

$$\sigma(\cdot) = \begin{bmatrix} \sigma_1(y^{(1)}) & \sigma_2(y^{(2)}) & \cdots & \sigma_m(y^{(m)}) \end{bmatrix} \tag{4}$$

where $\sigma_s(y^{(s)}) = \mathrm{sign}(y^{(s)})\min\{y_{\max}^{(s)}, |y^{(s)}|\}$ with $y^{(s)}$ representing the sth entry of the vector y. Note that, if there exist diagonal matrices G_{1i} and G_{2i} such that $0 \leq G_{1i} < I \leq G_{2i}$, then the saturation function $\sigma(C_{i,k} x_k)$ in (1) can be written as follows:

$$\sigma(C_{i,k} x_k) = G_{1i} C_{i,k} x_k + \varphi(C_{i,k} x_k) \tag{5}$$

where $\varphi(C_{i,k} x_k)$ is certain nonlinear vector-valued function satisfying the sector condition with $K_1 = 0$ and $K_2 = G_i \triangleq G_{2i} - G_{1i}$, that is, $\varphi(C_{i,k} x_k)$ satisfies the following inequality:

$$\varphi^{\mathrm{T}}(C_{i,k} x_k)\big(\varphi(C_{i,k} x_k) - G_i C_{i,k} x_k\big) \leq 0. \tag{6}$$

Before introducing the distributed *event-based* filter structure, we first recall traditional distributed time-based filter as follows:

$$\hat{x}_{i,k+1} = F_{i,k} \hat{x}_{i,k} + \sum_{j \in \mathscr{N}_i} \theta_{ij} H_{ij,k} r_{j,k} \quad i = 1, 2, \ldots, N \tag{7}$$

where $\hat{x}_{i,k} \in \mathbb{R}^n$ is the estimate of the system state based on the ith sensing node, $F_{i,k}$ and $H_{ij,k}$ are the filter parameters, and $r_{i,k}$ represents the innovation sequence defined by

$$r_{i,k} \triangleq y_{i,k} - C_{i,k} \hat{x}_{i,k}. \tag{8}$$

Remark 2: In traditional distributed filtering algorithms, it is usually assumed that the sensing nodes broadcast their local information at *every* periodic sampling instant, and this might result in unnecessary waste of communication resources especially when the energy saving becomes a concern. For the purpose of improving the efficiency of network utilization, as an alternative to the periodic control method, the *event-triggering* mechanism will be proposed here to reduce the network communication burden with guaranteed filtering performance, where the main idea is to broadcast *important* messages rather than *all* messages.

Let us now elaborate the event-triggering mechanism to be adopted. Suppose that the sequence of the triggering instants is $\{k_t^i\}$ $(t = 0, 1, 2, \ldots)$ satisfying $0 < k_0^i < k_1^i < k_2^i < \cdots < k_t^i < \cdots$, where k_t^i represents the time instant k at which the $(t+1)$th trigger occurs for agent i. Then, define

$$e_{i,k} \triangleq r_{i,k_t^i} - r_{i,k} \tag{9}$$

which indicates the difference of the broadcast innovations at the latest triggering time and the current time. With the notation of $e_{i,k}$, the sequence of event-triggering instants is defined iteratively by

$$k_{t+1}^i = \inf\{k \in \mathbb{Z}^+ | k > k_t^i, e_{i,k}^{\mathrm{T}} \Omega_{i,k}^{-1} e_{i,k} > 1\} \tag{10}$$

where $\Omega_{i,k} > 0$ $(i = 1, 2, \ldots, N)$ is referred to as the triggering threshold matrix of agent i at time instant k.

Remark 3: The ellipsoidal triggering condition defined in (10) is quite general that covers several well-studied triggering conditions as special cases. For example, it is observed from (10) that, when the matrix $\Omega_{i,k}$ is set to be a fixed positive scalar, then the ellipsoidal triggering condition specializes to the frequently used one as shown in [11] and the references therein. In particular, in the case that $\Omega_{i,k} \to 0$ (i.e. the size of ellipsoid approaches 0), the event-triggering mechanism will reduce to the traditional time-driven one. Moreover, another advantage of the proposed ellipsoidal triggering condition lies in the fact that the triggering threshold matrix $\Omega_{i,k}$ is actually a parameter that can be co-designed with the filter parameters, and this provides much flexibility in making trade-offs between the filtering performance and the triggering frequency, thereby achieving the balance between desired filtering accuracy and affordable resource consumption.

By incorporating (9)-(10) with (7), we come up with the following *event-based* filter structure to be adopted in this paper:

$$\hat{x}_{i,k+1} = F_{i,k}\hat{x}_{i,k} + \sum_{j\in\mathscr{N}_i} \theta_{ij} H_{ij,k} r_{j,k_t^j},\ k \in [k_t^i, k_{t+1}^i) \quad (11)$$

where $F_{i,k}$ and $H_{ij,k}$ $(i, j \in \mathscr{V})$ are the filter parameters to be designed.

By (9)–(11), we have established the structure of a distributed filter with an event-triggering mechanism, which invokes the transmission of information when the difference between the current value and its latest transmitted value exceeds certain threshold. Before proceeding further, we give the following assumption.

Assumption 3: The initial state x_0 and its estimate $\hat{x}_{i,0}$ satisfy

$$(x_0 - \hat{x}_{i,0})^{\mathrm{T}} P_0^{-1} (x_0 - \hat{x}_{i,0}) \le 1 \quad (12)$$

where $P_0 > 0$ is a given positive definite matrix.

The objective of this paper is twofold. ***Firstly***, for system (1) and filter (11), let the directed communication graph $\mathscr{G}$, the sequence of positive definite threshold matrices $\{\Omega_{i,k}\}_{k\ge0}$ and the sequence of positive definite matrices $\{P_k\}_{k\ge0}$ (constraints imposed on the filtering performance) be given. It is our first aim to design the sequences of filtering gains $\{F_{i,k}\}_{k\ge0}$ and $\{H_{ij,k}\}_{k\ge0}$ subject to the given triple $\big(\mathscr{G}, \{\Omega_{i,k}\}, \{P_k\}\big)$ such that the following inequality is satisfied:

$$\Delta_{i,k} \triangleq (x_k - \hat{x}_{i,k})^{\mathrm{T}} P_k^{-1} (x_k - \hat{x}_{i,k}) \le 1,\ i \in \mathscr{V},\ k \ge 0. \quad (13)$$

Secondly, two optimization problems will be investigated for minimizing P_k and maximizing $\Omega_{i,k}$ in the sense of matrix trace at each time instant, respectively. This problem is referred to as a distributed event-based set-membership filtering problem.

III. Distributed Event-Based Set-Membership Filter Design

In this section, we will design a distributed event-based filter of form (11) for system (1) subject to sector-bounded nonlinearity, unknown but bounded noises and sensor saturations. A sufficient condition for the existence of the desired filter will be formulated in terms of a set of recursive linear matrix inequalities (RLMIs). First of all, we recall two useful lemmas for our following development.

Lemma 1: (S-procedure [3]) Let $\psi_0(\cdot)$,$\psi_1(\cdot)$,. . .,$\psi_p(\cdot)$ be quadratic functions of the variable $\varsigma \in \mathbb{R}^n$: $\psi_j(\varsigma) \triangleq \varsigma^{\mathrm{T}} X_j \varsigma$ $(j = 0, \dots, p)$, where $X_j^{\mathrm{T}} = X_j$. If there exist $\epsilon_1 \ge 0, \dots, \epsilon_p \ge 0$ such that $X_0 - \sum_{j=1}^{p} \epsilon_j X_j \le 0$, then the following is true:

$$\psi_1(\varsigma) \le 0, \dots, \psi_p(\varsigma) \le 0 \to \psi_0(\varsigma) \le 0. \quad (14)$$

Lemma 2: (Schur Complement Equivalence) Given constant matrices $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$ where $\mathcal{S}_1 = \mathcal{S}_1^{\mathrm{T}}$ and $0 < \mathcal{S}_2 = \mathcal{S}_2^{\mathrm{T}}$, then $\mathcal{S}_1 + \mathcal{S}_3^{\mathrm{T}} \mathcal{S}_2^{-1} \mathcal{S}_3 < 0$ if and only if

$$\begin{bmatrix} \mathcal{S}_1 & \mathcal{S}_3^{\mathrm{T}} \\ \mathcal{S}_3 & -\mathcal{S}_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\mathcal{S}_2 & \mathcal{S}_3 \\ \mathcal{S}_3^{\mathrm{T}} & \mathcal{S}_1 \end{bmatrix} < 0. \quad (15)$$

A. *Filter Design Subject to Fixed Triple* $\big(\mathscr{G}, \{\Omega_{i,k}\}, \{P_k\}\big)$

For simplicity of notation, before giving the main results, we denote

$$\begin{aligned}
&\xi_k \triangleq \mathrm{col}_N\{x_k\}, \quad \hat{x}_k \triangleq \mathrm{col}_N\{\hat{x}_{i,k}\}, \quad e_k \triangleq \mathrm{col}_N\{e_{i,k}\},\\
&\tilde{f}_k \triangleq \mathrm{col}_N\{f(x_k)\}, \quad \tilde{\varphi}_k \triangleq \mathrm{col}_N\{\varphi(C_{i,k}x_k)\},\\
&\mathcal{G}_k \triangleq \mathrm{diag}_N\{G_{1i}C_{i,k}\}, \quad \mathcal{C}_k \triangleq \mathrm{diag}_N\{C_{i,k}\},\\
&\mathcal{E}_k \triangleq \mathrm{diag}_N\{E_{i,k}\}, \quad \mathcal{F}_k \triangleq \mathrm{diag}_N\{F_{i,k}\},\\
&\Phi_{\varrho,i} \triangleq \mathrm{diag}\{\underbrace{0_\varrho, \dots, 0_\varrho}_{i-1}, I_\varrho, \underbrace{0_\varrho, \dots, 0_\varrho}_{N-i}\}, \quad \varrho = \{n, q, m\},\\
&\mathcal{L}_{\varrho,i} \triangleq (\mathbf{1}_N^{\mathrm{T}} \otimes I_\varrho)\Phi_{\varrho,i}, \quad \varrho = \{n, q, m\}.
\end{aligned}$$

From system (1) and filter (11), the one-step-ahead estimation error is obtained as follows:

$$\begin{aligned}
&x_{k+1} - \hat{x}_{i,k+1}\\
=&A_k x_k + D_k w_k + f(x_k)\\
&- \Big(F_{i,k}\hat{x}_{i,k} + \sum_{j\in\mathscr{N}_i} \theta_{ij} H_{ij,k} r_{j,k_t^j}\Big)\\
=&A_k x_k + D_k w_k + f(x_k) - F_{i,k}\hat{x}_{i,k}\\
&- \Big(\sum_{j\in\mathscr{N}_i} \theta_{ij} H_{ij,k}\big(\sigma(C_{j,k}x_k) + E_{j,k}v_k\\
&- C_{j,k}\hat{x}_{j,k} + e_{j,k}\big)\Big). \qquad (16)
\end{aligned}$$

By denoting $\tilde{x}_{i,k} \triangleq x_k - \hat{x}_{i,k}$ and $\tilde{x}_k \triangleq \mathrm{col}_N\{\tilde{x}_{i,k}\}$, we rewrite the filtering error dynamics (16) into the following compact form:

$$\begin{aligned}
\tilde{x}_{k+1} =&(I_N \otimes A_k)\xi_k + (\mathbf{1}_N \otimes D_k)w_k + \tilde{f}_k\\
&- \mathcal{F}_k\hat{x}_k - \mathcal{H}_k\mathcal{G}_k\xi_k - \mathcal{H}_k\tilde{\varphi}_k\\
&- \mathcal{H}_k\mathcal{E}_k(\mathbf{1}_N \otimes I_\nu)v_k + \mathcal{H}_k\mathcal{C}_k\hat{x}_k - \mathcal{H}_k e_k \qquad (17)
\end{aligned}$$

where $\mathcal{H}_k \triangleq \big[\theta_{ij}H_{ij,k}\big]_{N\times N}$. Obviously, since $\theta_{ij} = 0$ when $j \notin \mathscr{N}_i$, $\mathcal{H}_k$ is a sparse matrix which can be expressed as

$$\mathcal{H}_k \in \mathscr{T}_{n\times m} \quad (18)$$

where $\mathscr{T}_{n\times m} \triangleq \big\{\mathcal{T} = [T_{ij}] \in \mathbb{R}^{nN\times mN} \big| T_{ij} \in \mathbb{R}^{n\times m}, T_{ij} = 0 \text{ if } j \notin \mathscr{N}_i\big\}$.

The following theorem gives a sufficient condition for the solvability of the addressed distributed event-based set-membership filtering problem.

Theorem 1: For system (1) and filter (11), let the triple $\big(\mathscr{G},\{\Omega_{i,k}\},\{P_k\}\big)$ be given. The design objective (13) is achieved if there exist sequences of real-valued matrices $\{\mathcal{F}_k\}_{k\geq 0}$ and $\{\mathcal{H}_k\}_{k\geq 0}$ $\big(\mathcal{H}_k\in\mathscr{T}_{n\times m}\big)$, sequences of non-negative scalars $\{\epsilon_{i,k}^{(1)}\}_{k\geq 0}$, $\{\epsilon_{i,k}^{(2)}\}_{k\geq 0}$, $\{\epsilon_{k}^{(3)}\}_{k\geq 0}$, $\{\epsilon_{k}^{(4)}\}_{k\geq 0}$, $\{\epsilon_{i,k}^{(5)}\}_{k\geq 0}$ and $\{\epsilon_{i,k}^{(6)}\}_{k\geq 0}$ $(i=1,2,\ldots,N)$ satisfying the following N recursive linear matrix inequalities:

$$\begin{bmatrix} -\Gamma_k & \Pi_k^{\mathrm{T}}\mathcal{L}_{n,i}^{\mathrm{T}} \\ \mathcal{L}_{n,i}\Pi_k & -P_{k+1}\end{bmatrix}\leq 0 \tag{19}$$

where

$$\begin{aligned}
\Gamma_k &= \sum_{i=1}^{N}\Big(\epsilon_{i,k}^{(5)}\Xi_{i,k}+\epsilon_{i,k}^{(6)}\Psi_{i,k}\Big)+\bar{\Gamma}_k, \qquad (20)\\
\bar{\Gamma}_k &= \mathrm{diag}\Big\{1-\sum_{i=1}^{N}\Big(\epsilon_{i,k}^{(1)}+\epsilon_{i,k}^{(2)}\Big)-\epsilon_k^{(3)}-\epsilon_k^{(4)},\\
&\qquad \sum_{i=1}^{N}\epsilon_{i,k}^{(1)}\mathcal{L}_{q,i}^{\mathrm{T}}\mathcal{L}_{q,i},\sum_{i=1}^{N}\epsilon_{i,k}^{(2)}\mathcal{L}_{m,i}^{\mathrm{T}}\Omega_{i,k}^{-1}\mathcal{L}_{m,i},\\
&\qquad \epsilon_k^{(3)}S_k^{-1},\epsilon_k^{(4)}R_k^{-1},0,0\Big\},
\end{aligned}$$

$$\Xi_{i,k}=\begin{bmatrix}
\hat{x}_{i,k}^{\mathrm{T}}U_1^{\mathrm{T}}U_2\hat{x}_{i,k} & \hat{x}_k^{\mathrm{T}}\big(I_N\otimes(\bar{U}Q_k)\big)\Phi_{q,i} & 0 & 0 & 0 & -\hat{x}_k^{\mathrm{T}}(I_N\otimes\tilde{U})\Phi_{n,i} & 0\\
* & I_N\otimes(Q_k^{\mathrm{T}}\bar{U}Q_k)\Phi_{q,i} & 0 & 0 & 0 & -I_N\otimes(Q_k^{\mathrm{T}}\tilde{U})\Phi_{n,i} & 0\\
* & * & 0 & 0 & 0 & 0 & 0\\
* & * & * & 0 & 0 & 0 & 0\\
* & * & * & * & 0 & 0 & 0\\
* & * & * & * & * & I_{nN}\Phi_{n,i} & 0\\
* & * & * & * & * & * & 0
\end{bmatrix}, \tag{21}$$

$$\begin{aligned}
\bar{U} &= \frac{U_1^{\mathrm{T}}U_2+U_2^{\mathrm{T}}U_1}{2}, \qquad \tilde{U}=\frac{U_1^{\mathrm{T}}+U_2^{\mathrm{T}}}{2},\\
\Psi_{i,k} &= \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \bar{\Psi}_{i,k}\end{bmatrix}, \qquad (22)\\
\bar{\Psi}_{i,k} &= \begin{bmatrix} -\hat{x}_k^{\mathrm{T}}\mathrm{diag}\{C_{i,k}^{\mathrm{T}}G_i^{\mathrm{T}}\}\Phi_{m,i}\\ -\mathrm{diag}\{Q_k^{\mathrm{T}}C_{i,k}^{\mathrm{T}}G_i^{\mathrm{T}}\}\Phi_{m,i}\\ 0\\ 0\\ 0\\ 0\\ 2I_{mN}\Phi_{m,i}\end{bmatrix},\\
\Pi_k &= \begin{bmatrix}\Pi_{11} & \Pi_{12} & -\mathcal{H}_k & \Pi_{14} & \Pi_{15} & I_{nN} & -\mathcal{H}_k\end{bmatrix}, \qquad (23)\\
\Pi_{11} &= \left(I_N\otimes A_k-\mathcal{F}_k-\mathcal{H}_k(\mathcal{G}_k-\mathcal{C}_k)\right)\hat{x}_k,\\
\Pi_{12} &= I_N\otimes(A_kQ_k)-\mathcal{H}_k\mathcal{G}_k(I_N\otimes Q_k),\\
\Pi_{14} &= \mathbf{1}_N\otimes D_k, \qquad \Pi_{15}=-\mathcal{H}_k\mathcal{E}_k(\mathbf{1}_N\otimes I_\nu),
\end{aligned}$$

with $Q_k\in\mathbb{R}^{n\times q}$ being a factorization of P_k $\big($i.e., $P_k=Q_kQ_k^{\mathrm{T}}\big)$.

Proof: The proof is performed by induction. First, it can be immediately known from Assumption 3 that $\Delta_{i,0}\leq 1$ holds. Then, suppose that $\Delta_{i,k}\leq 1$ is true at time instant $k>0$, we shall proceed to prove that $\Delta_{i,k+1}\leq 1$ holds.

According to [13], since $\Delta_{i,k}\leq 1$ and $P_k=Q_kQ_k^{\mathrm{T}}$, there exist $z_{i,k}\in\mathbb{R}^q$ $(i=1,2,\ldots,N)$ with $\|z_{i,k}\|\leq 1$ such that

$$x_k=\hat{x}_{i,k}+Q_kz_{i,k}. \tag{24}$$

Obviously, by denoting $z_k\triangleq\mathrm{col}_N\{z_{i,k}\}$, (24) is equivalent to

$$\xi_k=\hat{x}_k+(I_N\otimes Q_k)z_k. \tag{25}$$

From (24) and (25), the filtering error system (17) is rewritten as follows:

$$\begin{aligned}
\tilde{x}_{k+1}=&(I_N\otimes A_k)\big(\hat{x}_k+(I_N\otimes Q_k)z_k\big)\\
&+(\mathbf{1}_N\otimes D_k)w_k+\tilde{f}_k-\mathcal{F}_k\hat{x}_k\\
&-\mathcal{H}_k\mathcal{G}_k\big(\hat{x}_k+(I_N\otimes Q_k)z_k\big)\\
&-\mathcal{H}_k\tilde{\varphi}_k-\mathcal{H}_k\mathcal{E}_k(\mathbf{1}_N\otimes I_\nu)v_k\\
&+\mathcal{H}_k\mathcal{C}_k\hat{x}_k-\mathcal{H}_ke_k\\
=&\left(I_N\otimes A_k-\mathcal{F}_k-\mathcal{H}_k(\mathcal{G}_k-\mathcal{C}_k)\right)\hat{x}_k\\
&+\left(I_N\otimes(A_kQ_k)-\mathcal{H}_k\mathcal{G}_k(I_N\otimes Q_k)\right)z_k\\
&+(\mathbf{1}_N\otimes D_k)w_k+\tilde{f}_k-\mathcal{H}_k\tilde{\varphi}_k\\
&-\mathcal{H}_k\mathcal{E}_k(\mathbf{1}_N\otimes I_\nu)v_k-\mathcal{H}_ke_k.
\end{aligned} \tag{26}$$

Denoting

$$\eta_k\triangleq\begin{bmatrix}1 & z_k^{\mathrm{T}} & e_k^{\mathrm{T}} & w_k^{\mathrm{T}} & v_k^{\mathrm{T}} & \tilde{f}_k^{\mathrm{T}} & \tilde{\varphi}_k^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}, \tag{27}$$

the filtering error dynamics can be further expressed by

$$\tilde{x}_{k+1}=\Pi_k\eta_k \tag{28}$$

where Π_k is defined in (23).

It follows from (2), (10) and (24) that the vectors $z_{i,k}$, $e_{i,k}$, w_k and v_k are satisfying

$$\begin{cases}\|z_{i,k}\|\leq 1\\ e_{i,k}^{\mathrm{T}}\Omega_{i,k}^{-1}e_{i,k}\leq 1\\ w_k^{\mathrm{T}}S_k^{-1}w_k\leq 1\\ v_k^{\mathrm{T}}R_k^{-1}v_k\leq 1\end{cases} \tag{29}$$

which, by (27), can be rewritten in terms of η_k as follows:

$$\begin{cases}\eta_k^{\mathrm{T}}\mathrm{diag}\left\{-1,\mathcal{L}_{q,i}^{\mathrm{T}}\mathcal{L}_{q,i},0,0,0,0,0\right\}\eta_k\leq 0\\ \eta_k^{\mathrm{T}}\mathrm{diag}\left\{-1,0,\mathcal{L}_{m,i}^{\mathrm{T}}\Omega_{i,k}^{-1}\mathcal{L}_{m,i},0,0,0,0\right\}\eta_k\leq 0\\ \eta_k^{\mathrm{T}}\mathrm{diag}\left\{-1,0,0,S_k^{-1},0,0,0\right\}\eta_k\leq 0\\ \eta_k^{\mathrm{T}}\mathrm{diag}\left\{-1,0,0,0,R_k^{-1},0,0\right\}\eta_k\leq 0\end{cases} \tag{30}$$

We now proceed to investigate the sector-bounded nonlinearity $f(x_k)$ in system (1). From Assumption 2, $f(x_k)$ belongs to sector $[U_1,U_2]$, which can be formulated by

$$\big(f(x_k)-U_1x_k\big)^{\mathrm{T}}\big(f(x_k)-U_2x_k\big)\leq 0. \tag{31}$$

Substituting (24) into (31) results in

$$\begin{aligned}
&f^{\mathrm{T}}(x_k)f(x_k)+\hat{x}_{i,k}^{\mathrm{T}}U_1^{\mathrm{T}}U_2\hat{x}_{i,k}+z_{i,k}^{\mathrm{T}}Q_k^{\mathrm{T}}U_1^{\mathrm{T}}U_2Q_kz_{i,k}\\
&+\hat{x}_{i,k}^{\mathrm{T}}U_1^{\mathrm{T}}U_2Q_kz_{i,k}+z_{i,k}^{\mathrm{T}}Q_k^{\mathrm{T}}U_1^{\mathrm{T}}U_2\hat{x}_{i,k}-f^{\mathrm{T}}(x_k)U_2\hat{x}_{i,k}\\
&-f^{\mathrm{T}}(x_k)U_2Q_kz_{i,k}-\hat{x}_{i,k}^{\mathrm{T}}U_1^{\mathrm{T}}f(x_k)\\
&-z_{i,k}^{\mathrm{T}}Q_k^{\mathrm{T}}U_1^{\mathrm{T}}f(x_k)\leq 0
\end{aligned} \tag{32}$$

which can be expressed by η_k as

$$\eta_k^{\mathrm{T}}\Xi_{i,k}\eta_k \le 0 \tag{33}$$

with $\Xi_{i,k}$ being defined in (21).

By the same token, we have from the sensor saturation constraints in (6) that

$$\varphi^{\mathrm{T}}(C_{i,k}x_k)\big(\varphi(C_{i,k}x_k) - G_iC_{i,k}(\hat{x}_{i,k} + Q_kz_{i,k})\big) \le 0 \tag{34}$$

which can be formulated by η_k as

$$\eta_k^{\mathrm{T}}\Psi_{i,k}\eta_k \le 0 \tag{35}$$

with $\Psi_{i,k}$ being defined in (22).

In the following, according to the principle of induction, it remains to show that $\Delta_{i,k+1} \le 1$ is true if the condition of this theorem is satisfied at time instant k, i.e., there exist real-valued matrices $\mathcal{F}_k$ and $\mathcal{H}_k$ $\big(\mathcal{H}_k \in \mathscr{T}_{n\times m}\big)$, non-negative scalars $\epsilon_{i,k}^{(1)} \ge 0$, $\epsilon_{i,k}^{(2)} \ge 0$, $\epsilon_k^{(3)} \ge 0$, $\epsilon_k^{(4)} \ge 0$, $\epsilon_{i,k}^{(5)} \ge 0$ and $\epsilon_{i,k}^{(6)} \ge 0$ $(i = 1, 2, \ldots, N)$ satisfying RLMIs (19).

By resorting to the Schur Complement Equivalence (Lemma 2), it can be seen that the set of RLMIs (19) holds if and only if

$$\Pi_k^{\mathrm{T}}\mathcal{L}_{n,i}^{\mathrm{T}}P_{k+1}^{-1}\mathcal{L}_{n,i}\Pi_k - \Gamma_k \le 0 \tag{36}$$

which, by (20), is equivalent to

$$\begin{aligned}
&\Pi_k^{\mathrm{T}}\mathcal{L}_{n,i}^{\mathrm{T}}P_{k+1}^{-1}\mathcal{L}_{n,i}\Pi_k - \mathrm{diag}\{1,0,0,0,0,0,0\}\\
&-\sum_{i=1}^{N}\epsilon_{i,k}^{(1)}\mathrm{diag}\{-1,\mathcal{L}_{q,i}^{\mathrm{T}}\mathcal{L}_{q,i},0,0,0,0,0,0\}\\
&-\sum_{i=1}^{N}\epsilon_{i,k}^{(2)}\mathrm{diag}\left\{-1,0,\mathcal{L}_{m,i}^{\mathrm{T}}\Omega_{i,k}^{-1}\mathcal{L}_{m,i},0,0,0,0\right\}\\
&-\epsilon_k^{(3)}\mathrm{diag}\left\{-1,0,0,S_k^{-1},0,0,0\right\}\\
&-\epsilon_k^{(4)}\mathrm{diag}\left\{-1,0,0,0,R_k^{-1},0,0\right\}\\
&-\sum_{i=1}^{N}\epsilon_{i,k}^{(5)}\Xi_{i,k} - \sum_{i=1}^{N}\epsilon_{i,k}^{(6)}\Psi_{i,k} \le 0.
\end{aligned} \tag{37}$$

In view of (30), (33) and (35), it follows directly from the S-procedure (Lemma 1) that

$$\begin{aligned}
&\eta_k^{\mathrm{T}}\big(\Pi_k^{\mathrm{T}}\mathcal{L}_{n,i}^{\mathrm{T}}P_{k+1}^{-1}\mathcal{L}_{n,i}\Pi_k\\
&-\mathrm{diag}\{1,0,0,0,0,0,0\}\big)\eta_k \le 0.
\end{aligned} \tag{38}$$

It is evident that the following equivalences hold:

$$\begin{aligned}
&\text{Inequality (38)}\\
\Longleftrightarrow\ &\eta_k^{\mathrm{T}}\Pi_k^{\mathrm{T}}\mathcal{L}_{n,i}^{\mathrm{T}}P_{k+1}^{-1}\mathcal{L}_{n,i}\Pi_k\eta_k \le 1\\
\overset{(28)}{\Longleftrightarrow}\ &\tilde{x}_{k+1}^{\mathrm{T}}\mathcal{L}_{n,i}^{\mathrm{T}}P_{k+1}^{-1}\mathcal{L}_{n,i}\tilde{x}_{k+1} \le 1\\
\Longleftrightarrow\ &(x_{k+1} - \hat{x}_{i,k+1})^{\mathrm{T}}P_{k+1}^{-1}(x_{k+1} - \hat{x}_{i,k+1}) \le 1.
\end{aligned} \tag{39}$$

We can now conclude from (39) that $\Delta_{i,k+1} \le 1$ is achieved, and the induction is accomplished. Therefore, the design objective (13) is met with the obtained sequences of parameters $\{\mathcal{F}_k\}_{k\ge 0}$ and $\{\mathcal{H}_k\}_{k\ge 0}$ for fixed triple $\big(\mathscr{G}, \{\Omega_{i,k}\}, \{P_k\}\big)$. The proof is now complete. ■

In the following, an iterative algorithm is presented to compute the sequences of the filtering parameters $\{F_{i,k}\}_{k\ge 0}$ and $\{H_{ij,k}\}_{k\ge 0}$ recursively.

Algorithm 1: **Computational Algorithm for $\{F_{i,k}\}_{k\ge 0}$ and $\{H_{ij,k}\}_{k\ge 0}$**

1) Initialization: Set $k = 0$ and the maximum computation step $k_{\max}$. Set the triple $\big(\mathscr{G}, \{\Omega_{i,k}\}, \{P_k\}\big)$ for $0 \le k \le k_{\max}$. Then factorize $\{P_k\}$ appropriately to obtain the sequence of matrices $\{Q_k\}$. Select the initial values of x_0 and $\hat{x}_{i,0}$ satisfying (12). Then $\hat{x}_0 = \mathrm{col}_N\{\hat{x}_{i,0}\}$ is known.
2) With the obtained $\hat{x}_k$ and Q_k, solve the RLMIs (19) for $\mathcal{F}_k$ and $\mathcal{H}_k$. Then $F_{i,k}$ and $H_{ij,k}$ can be obtained.
3) With the obtained $\mathcal{F}_k$ and $\mathcal{H}_k$, compute $\hat{x}_{i,k+1}$ according to (11). Then $\hat{x}_{k+1} = \mathrm{col}_N\{\hat{x}_{i,k+1}\}$ is obtained.
4) Set $k = k + 1$. If $k > k_{\max}$, exit. Otherwise, go to *2)*.

B. Filter Design Subject to Constraint of Average Filtering Errors

In many cases, from a global point of view, we are more interested in the filtering performance in terms of an average of estimation errors among all the sensing nodes rather than the individual ones, see [32], [35] for references. As such, in this subsection, based on the results obtained so far, we will further discuss the distributed filtering problem subject to constraints imposed on the average of filtering errors. To begin with, we define the average filtering error ζ_k as follows:

$$\begin{aligned}
\zeta_k &\triangleq \sum_{i=1}^{N}\lambda_i(x_k - \hat{x}_{i,k})\\
&= (\mathbf{1}^{\mathrm{T}}\Lambda \otimes I_n)(\xi_k - \hat{x}_k)\\
&= (\mathbf{1}^{\mathrm{T}}\Lambda \otimes I_n)\tilde{x}_k
\end{aligned} \tag{40}$$

where the weighting parameters λ_i $(i = 1, 2, \ldots, N)$ represent the priorities with respect to the corresponding sensing nodes.

Assume $\zeta_0^{\mathrm{T}}Y_0^{-1}\zeta_0 \le 1$ where $Y_0 > 0$ is a given matrix. Let the triple $\big(\mathscr{G}, \{\Omega_{i,k}\}, \{Y_k\}\big)$ be given, where $\{Y_k\}_{k\ge 0}$ is a sequence of positive definite matrices describing the constraints imposed on the average filtering performance. It is our objective in this subsection to design the filter parameters in (11) such that the following requirement is met for $k \ge 0$:

$$\zeta_k^{\mathrm{T}}Y_k^{-1}\zeta_k \le 1. \tag{41}$$

Theorem 2: For system (1) and filter (11), let the triple $\big(\mathscr{G}, \{\Omega_{i,k}\}, \{Y_k\}\big)$ and the initial condition $\zeta_0^{\mathrm{T}}Y_0^{-1}\zeta_0 \le 1$ be given. The requirement (41) is achieved if there exist sequences of real-valued matrices $\{\mathcal{F}_k\}_{k\ge 0}$ and $\{\mathcal{H}_k\}_{k\ge 0}$ $\big(\mathcal{H}_k \in \mathscr{T}_{n\times m}\big)$, sequences of non-negative scalars $\{\epsilon_{i,k}^{(1)}\}_{k\ge 0}$, $\{\epsilon_{i,k}^{(2)}\}_{k\ge 0}$, $\{\epsilon_k^{(3)}\}_{k\ge 0}$, $\{\epsilon_k^{(4)}\}_{k\ge 0}$, $\{\epsilon_{i,k}^{(5)}\}_{k\ge 0}$ and $\{\epsilon_{i,k}^{(6)}\}_{k\ge 0}$ $(i = 1, 2, \ldots, N)$ satisfying the following RLMI:

$$\begin{bmatrix} -\Gamma_k & \Pi_k^{\mathrm{T}}(\mathbf{1}^{\mathrm{T}}\Lambda \otimes I_n)^{\mathrm{T}}\\ (\mathbf{1}^{\mathrm{T}}\Lambda \otimes I_n)\Pi_k & -Y_{k+1}\end{bmatrix} \le 0 \tag{42}$$

where Γ_k and Π_k are defined in (20) and (23), respectively.

Proof: With the fixed triple $\big(\mathscr{G}, \{\Omega_{i,k}\}, \{Y_k\}\big)$ and given the initial condition $\zeta_0^{\mathrm{T}}Y_0^{-1}\zeta_0 \le 1$, the proof of Theorem 2 can be accomplished by induction which is analogous to that of Theorem 1 and is therefore omitted here. ■

C. Optimization Algorithms

Theorems 1 and 2 in previous subsections outline the principles of designing the filtering parameters by solving the corresponding set of RLMIs. It should be pointed out that, however, neither of the proposed methodologies provides an optimal solution. As discussed previously, we now proceed to deal with the second part of our design objective, that is, minimizing $\{P_k\}_{k\geq 0}$ (for the locally best filtering performance) and maximizing $\{\Omega_{i,k}\}_{k\geq 0}$ (for the locally lowest triggering frequency) in the sense of matrix trace, respectively. In the following stage, two optimization problems based on Theorem 1 will be proposed to demonstrate the flexibility of our developed strategy. Such a kind of flexibility allows us making compromise between filtering performance and triggering frequency to achieve a balance between accuracy and cost.

Optimization Problem 1: *Minimization of $\{P_k\}_{k\geq 0}$ (in the sense of matrix trace) for the locally best filtering performance.*

Corollary 1: For the discrete time-varying nonlinear system (1) with filter (11), let the pair $\left(\mathscr{G},\{\Omega_{i,k}\}\right)$ be given. A sequence of minimized $\{P_k\}_{k\geq 0}$ (in the sense of matrix trace) is guaranteed if there exist sequences of real-valued matrices $\{\mathcal{F}_k\}_{k\geq 0}$ and $\{\mathcal{H}_k\}_{k\geq 0}$ $\left(\mathcal{H}_k\in\mathscr{T}_{n\times m}\right)$, sequences of non-negative scalars $\{\epsilon_{i,k}^{(1)}\}_{k\geq 0}$, $\{\epsilon_{i,k}^{(2)}\}_{k\geq 0}$, $\{\epsilon_{k}^{(3)}\}_{k\geq 0}$, $\{\epsilon_{k}^{(4)}\}_{k\geq 0}$, $\{\epsilon_{i,k}^{(5)}\}_{k\geq 0}$ and $\{\epsilon_{i,k}^{(6)}\}_{k\geq 0}$ $(i=1,2,\ldots,N)$ solving the following optimization problem:

$$\min_{P_{k+1},\mathcal{F}_k,\mathcal{H}_k,\epsilon_{i,k}^{(1)},\epsilon_{i,k}^{(2)},\epsilon_{k}^{(3)},\epsilon_{k}^{(4)},\epsilon_{i,k}^{(5)},\epsilon_{i,k}^{(6)}} \operatorname{trace}[P_{k+1}] \tag{43}$$

$$\text{subject to}$$

$$\begin{bmatrix} -\Gamma_k & \Pi_k^{\mathrm{T}}\mathcal{L}_{n,i}^{\mathrm{T}} \\ \mathcal{L}_{n,i}\Pi_k & -P_{k+1}\end{bmatrix}\leq 0. \tag{44}$$

Notice that the inequalities (44) are linear to the variables P_{k+1}, $\mathcal{F}_k$, $\mathcal{H}_k$, $\epsilon_{i,k}^{(1)}$, $\epsilon_{i,k}^{(2)}$, $\epsilon_{k}^{(3)}$, $\epsilon_{k}^{(4)}$, $\epsilon_{i,k}^{(5)}$ and $\epsilon_{i,k}^{(6)}$. Therefore, it follows directly from Corollary 1 that Optimization Problem 1 can be readily solved via the existing semi-definite programming methods [28].

Optimization Problem 2: *Maximization of triggering threshold matrices $\{\Omega_{i,k}\}_{k\geq 0}$ (in the sense of matrix trace) for the locally lowest triggering frequency*.

Corollary 2: For the discrete time-varying nonlinear system (1) with filter (11), let the pair $\left(\mathscr{G},\{P_k\}\right)$ be given. The locally lowest triggering frequency can be determined if there exist sequences of positive definite matrices $\{\Upsilon_{i,k}\}_{k\geq 0}$, sequences of real-valued matrices $\{\mathcal{F}_k\}_{k\geq 0}$ and $\{\mathcal{H}_k\}_{k\geq 0}$ $\left(\mathcal{H}_k\in\mathscr{T}_{n\times m}\right)$, non-negative scalars $\{\epsilon_{i,k}^{(1)}\}_{k\geq 0}$, $\{\epsilon_{i,k}^{(2)}\}_{k\geq 0}$, $\{\epsilon_{k}^{(3)}\}_{k\geq 0}$, $\{\epsilon_{k}^{(4)}\}_{k\geq 0}$, $\{\epsilon_{i,k}^{(5)}\}_{k\geq 0}$ and $\{\epsilon_{i,k}^{(6)}\}_{k\geq 0}$ $(i=1,2,\ldots,N)$ solving the following optimization problem:

$$\min_{\mathcal{F}_k,\mathcal{H}_k,\Upsilon_{i,k},\epsilon_{i,k}^{(1)},\epsilon_{i,k}^{(2)},\epsilon_{k}^{(3)},\epsilon_{k}^{(4)},\epsilon_{i,k}^{(5)},\epsilon_{i,k}^{(6)}} \operatorname{trace}\left[\sum_{i=1}^{N}\beta_i\Upsilon_{i,k}\right] \tag{45}$$

$$\text{subject to}$$

$$\begin{bmatrix} -\tilde{\Gamma}_k & \Pi_k^{\mathrm{T}}\mathcal{L}_{n,i}^{\mathrm{T}} \\ \mathcal{L}_{n,i}\Pi_k & -P_{k+1}\end{bmatrix}\leq 0 \tag{46}$$

where

$$\begin{aligned}\tilde{\Gamma}_k=&\sum_{i=1}^{N}\left(\epsilon_{i,k}^{(5)}\Xi_{i,k}+\epsilon_{i,k}^{(6)}\Psi_{i,k}\right)\\ &+\operatorname{diag}\Bigg\{1-\sum_{i=1}^{N}\left(\epsilon_{i,k}^{(1)}+\epsilon_{i,k}^{(2)}\right)-\epsilon_{k}^{(3)}-\epsilon_{k}^{(4)},\\ &\qquad\sum_{i=1}^{N}\epsilon_{i,k}^{(1)}\mathcal{L}_{q,i}^{\mathrm{T}}\mathcal{L}_{q,i},\sum_{i=1}^{N}\mathcal{L}_{m,i}^{\mathrm{T}}\epsilon_{i,k}^{(2)}\Upsilon_{i,k}\mathcal{L}_{m,i},\\ &\qquad\epsilon_{k}^{(3)}S_k^{-1},\epsilon_{k}^{(4)}R_k^{-1},0,0\Bigg\}\end{aligned}$$

and $\beta_i>0$ $(i=1,2,\ldots,N)$ are the weighting scalars satisfying $\sum_{i=1}^{N}\beta_i=1$. The threshold matrix $\Omega_{i,k}$ at each time instant can be determined by $\Omega_{i,k}=\Upsilon_{i,k}^{-1}$.

Remark 4: With the satisfaction of certain predetermined filtering performance (i.e., a prescribed sequence of $\{P_k\}_{k\geq 0}$), Corollary 2 presents a way to maximize certain combination of individual triggering threshold so as to reduce the triggering frequency. The values of weighting scalars β_i $(i=1,2,\ldots,N)$ are usually set to be equal but they can be adjusted according to the priorities of certain sensing nodes. Similarly, by borrowing idea from the proposed optimization problem (45)–(46), we are able to determine the "largest" noises that can be tolerated with the satisfaction of certain prespecified filtering performance.

Noting that (46) is actually a set of bilinear matrix inequalities (BMIs) because of the term $\epsilon_{i,k}^{(2)}\Upsilon_{i,k}$, in the following stage, we provide a numerical algorithm based on the chaos optimization [15] to solve the Optimization Problem 2. To start with, we introduce the following iterative chaotic mapping [15]:

$$\rho(\tau+1)=\sin\left(\frac{\alpha}{\rho(\tau)}\right) \tag{47}$$

where $\rho(\tau)\in[-1,\ 0)\cup(0,\ 1]$ is a chaotic variable and $\alpha>0$ is a properly selected parameter and τ $(\tau=0,1,2,\ldots)$ indicates the iterative counter.

Remark 5: Recently, the chaos optimization techniques have been employed to solve a variety of global optimization problems due to the properties of unique ergodicity and irregularity of the series generated by chaos [24]. Noticing that in the chaotic mapping (47), if the initial value is set as $\rho(0)\neq 0$, then the chaotic variable $\rho(\tau)$ is able to traverse every state in the interval $[-1,\ 0)\cup(0,\ 1]$ in an ergodic way, and each state is visited only once. As such, it makes $\rho(\tau)$ a potential candidate for solving the set of BMIs (46) iteratively.

We now present a detailed algorithm to solve Optimization Problem 2. Let $\epsilon_{i,k}^{(2)}$ in (46) be chaotic variable. We first need to determine the interval over which $\epsilon_{i,k}^{(2)}$ can traverse ergodically. It can be easily seen that if (46) holds, then it follows directly that

$$-\tilde{\Gamma}_k\leq 0 \tag{48}$$

which indicates by (21) that

$$\bar{f}_k+1-\sum_{i=1}^{N}\left(\epsilon_{i,k}^{(1)}+\epsilon_{i,k}^{(2)}\right)-\epsilon_{k}^{(3)}-\epsilon_{k}^{(4)}\geq 0 \tag{49}$$

where $\bar{f}_k \triangleq \sum_{i=1}^{N} \epsilon_{i,k}^{f} \hat{x}_{i,k}^{\mathrm{T}} U_1^{\mathrm{T}} U_2 \hat{x}_{i,k}$.

Taking into account that $\epsilon_{i,k}^{(1)} \geq 0$, $\epsilon_{i,k}^{(2)} \geq 0$, $\epsilon_{k}^{(3)} \geq 0$ and $\epsilon_{k}^{(4)} \geq 0$, we acquire $0 \leq \epsilon_{i,k}^{(2)} \leq \bar{f}_k + 1$. In order to determine the maximum of $\bar{f}_k$, we propose the following auxiliary optimization problem:

$$\max_{P_{k+1},\mathcal{F}_k,\mathcal{H}_k,\epsilon_{i,k}^{(1)},\epsilon_{i,k}^{(2)},\epsilon_{k}^{(3)},\epsilon_{k}^{(4)},\epsilon_{i,k}^{(5)},\epsilon_{i,k}^{(6)}} \bar{f}_k \quad \text{subject to} \quad (44) \tag{50}$$

We present the optimization problem (50) to seek the maximum of $\bar{f}_k$ thereby determining the interval in which $\epsilon_{i,k}^{(2)}$ is confined. If it is solvable, then we denote the optimal value as $\bar{f}_k^{*}$, and it follows directly that $\epsilon_{i,k}^{(2)} \in (0,\ \bar{f}_k^{*}+1]$. Denote the chaotic variable during the τth iteration as $\epsilon_{i,k}^{(2)}(\tau)$, and by taking (47) into consideration, it can be set as

$$\epsilon_{i,k}^{(2)}(\tau) = \frac{\bar{f}_k^{*}+1}{2}\Big(1+\rho_i(\tau)\Big), \qquad i = 1,2,\ldots,N \tag{51}$$

With $\epsilon_{i,k}^{(2)}(\tau)$ obtained during the τth iteration, the addressed optimization problem (45) subject to constraint (46) is converted into a semi-definite programming problem with LMI constraints which can be effectively solved by existing tools.

For time instant k, denote $\phi_k\big(\epsilon_{i,k}^{(2)}\big) \triangleq \mathrm{trace}\Big[\sum_{i=1}^{N}\beta_i \Upsilon_{i,k}\Big]$. Then, during the τth iteration, the intermediate index $\phi_k\big(\epsilon_{i,k}^{(2)}(\tau)\big)$ is given as follows:

$$\phi_k\big(\epsilon_{i,k}^{(2)}(\tau)\big) = \begin{cases} \mathrm{trace}\Big[\displaystyle\sum_{i=1}^{N}\beta_i \Upsilon_{i,k}(\tau)\Big], & \text{if (45) is solvable} \\ \vartheta, & \text{otherwise} \end{cases}$$

where $\vartheta > 0$ is a sufficiently large constant. In the following, we will formulate the detailed algorithm for solving Optimization Problem 2.

Algorithm 2: **Algorithm for Solving Optimization Problem 2**

1) Initialization: Set $k = 0$. Set the maximum step $k_{\max}$.
2) Set $\tau = 0$, $\phi_k^{*} = \vartheta$. Set the maximum iteration times $\tau_{\max}$ for the chaotic optimization and select the proper initial value of $\rho_i(0) \neq 0$.
3) Solve the optimization problem (50) and obtain $\bar{f}_k^{*}$.
4) Obtain $\epsilon_{i,k}^{(2)}(\tau)$ from equation (51) with known $\rho_i(\tau)$.
5) Solve the semi-definite programming problem (45) with constraint (46) by using the obtained $\epsilon_{i,k}^{(2)}(\tau)$. If $\phi_k\big(\epsilon_{i,k}^{(2)}(\tau)\big) < \phi_k^{*}$, then let $\phi_k^{*} = \phi_k\big(\epsilon_{i,k}^{(2)}(\tau)\big)$ and $\epsilon_{i,k}^{*} = \epsilon_{i,k}^{(2)}(\tau)$. Otherwise, go to *6)*.
6) Set $\tau = \tau + 1$. Calculate $\rho_i(\tau)$ according to (47).
7) If $\tau > \tau_{\max}$, or ϕ_k^{*} does not change after certain iteration times, output the optimal values and go to *8)*. Otherwise, go to *4)*.
8) Set $k = k + 1$. If $k > k_{\max}$, exit. Otherwise, go to *3)*.

Up to now, the addressed distributed event-based set-membership filtering problem has been discussed for the nonlinear systems subject to unknown but bounded noises and sensor saturations. In terms of the feasibility of certain RLMIs, the sufficient conditions for the solvability of the addressed problem have been presented. Moreover, in order to show the advantage of our developed algorithm, two optimization problems have been investigated to demonstrate the flexibility of filter design technique which is capable of making compromise between filtering accuracy and communication cost. It will be further verified later in Section IV by an illustrative example.

Remark 6: The advantages of the developed method can be summarized as follows: i) within the established generic framework, the sector-bounded nonlinearity, unknown but bounded noises, sensor saturations can be tackled simultaneously with the proposed ellipsoidal triggering condition; ii) it allows much flexibility in making trade-offs between the filtering accuracy and communication cost, while both of the essential objectives can be met at the same time; iii) the proposed method has the potential to deal with the distributed filtering problem over sensor networks with time-varying topology. It is worth pointing out that one of our possible research topics in future is to consider the distributed filtering problems with other performance requirements such as H_∞ specifications investigated in [4], [5].

IV. An Illustrative Example

In this section, an illustrative example is presented to show the validity of the proposed filter design strategy. Consider a nonlinear discrete time-varying nonlinear system with 3 sensing nodes and have the following parameters:

$$\begin{aligned}
A_k &= \begin{bmatrix} 0.55+0.11\sin(0.5k) & 0.01+0.01\sin(2k) \\ 0.01 & 0.55+0.11\sin(0.5k) \end{bmatrix}, \\
D_k &= \begin{bmatrix} -0.1+0.05\cos(3k) \\ 0.2+0.04\mathbf{e}^{-k} \end{bmatrix}, \\
C_{1,k} &= \begin{bmatrix} 0.73+0.2\sin(k) & 0.1 \end{bmatrix}, \\
E_{1,k} &= 0.2+0.05\cos(3k), \\
C_{2,k} &= \begin{bmatrix} 0.1 & 0.75+0.4\sin(2k) \end{bmatrix}, \\
E_{2,k} &= 0.2+0.15\sin(2k), \\
C_{3,k} &= \begin{bmatrix} 0.75 & 0.1 \end{bmatrix}, \\
E_{3,k} &= 0.15+0.05\sin(2k).
\end{aligned}$$

Suppose that the sensor network is represented by a directed graph $\mathscr{G} = (\mathscr{V},\mathscr{E},\mathscr{L})$ where the set of nodes $\mathscr{V} = \{1,2,3\}$, the set of edges $\mathscr{E} = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}$, and the adjacency elements associated with the edges of the graph are $\theta_{ij} = 1$.

Let the disturbances be $w_k = 1.2\sin(2k)$ and $v_k = 1.5\cos(5k)$ and set $S = 2$ and $R = 3$. It then can be easily checked that w_k and v_k belong to the ellipsoidal sets defined in (2). Let the initial values be given as follows:

$$x_0 = \begin{bmatrix} 5 \\ 3 \end{bmatrix}, \quad \hat{x}_{1,0} = \hat{x}_{2,0} = \hat{x}_{3,0} = \begin{bmatrix} 5.02 \\ 3.01 \end{bmatrix},$$
$$P_0 = \begin{bmatrix} 0.1 & 0.01 \\ 0.01 & 0.1 \end{bmatrix}.$$

Then it can be easily verified that Assumption 3 is satisfied.

The nonlinear function $f(x_k)$ is chosen as

$$f(x_k)=\left[\begin{array}{c}-0.1x_k^{(1)}+0.15x_k^{(2)}+\frac{0.1x_k^{(2)}\sin(x_k^{(1)})}{\sqrt{(x_k^{(1)})^2+(x_k^{(2)})^2+10}}\\ -0.05x_k^{(1)}+0.05x_k^{(2)}\end{array}\right]$$

where $x_k^{(1)}\triangleq[1\ \ 0]x_k$ and $x_k^{(2)}\triangleq[0\ \ 1]x_k$ represent the first and second entries of the system state, respectively. It can be verified that $f(x_k)$ belongs to the sector $[U_1,U_2]$ with

$$U_1=\left[\begin{array}{cc}-0.4 & 0\\ -0.2 & -0.3\end{array}\right],\qquad U_2=\left[\begin{array}{cc}0.2 & 0.3\\ 0.1 & 0.4\end{array}\right].$$

Denote the first and second entries of $C_{i,k}x_k$ as $\tilde{y}_{i,k}^{(1)}$ and $\tilde{y}_{i,k}^{(2)}$, respectively. With the purpose of showing the influence from the sensor saturation on the filtering performance, we consider the following two cases:

Case 1:

$$\begin{cases}\sigma\big(\tilde{y}_{1,k}^{(1)}\big)_{\max}=\sigma\big(\tilde{y}_{1,k}^{(2)}\big)_{\max}=20,\\ \sigma\big(\tilde{y}_{2,k}^{(1)}\big)_{\max}=\sigma\big(\tilde{y}_{2,k}^{(2)}\big)_{\max}=30,\\ \sigma\big(\tilde{y}_{3,k}^{(1)}\big)_{\max}=\sigma\big(\tilde{y}_{3,k}^{(2)}\big)_{\max}=20.\end{cases}\tag{52}$$

Case 2:

$$\begin{cases}\sigma\big(\tilde{y}_{1,k}^{(1)}\big)_{\max}=\sigma\big(\tilde{y}_{1,k}^{(2)}\big)_{\max}=10,\\ \sigma\big(\tilde{y}_{2,k}^{(1)}\big)_{\max}=\sigma\big(\tilde{y}_{2,k}^{(2)}\big)_{\max}=15,\\ \sigma\big(\tilde{y}_{3,k}^{(1)}\big)_{\max}=\sigma\big(\tilde{y}_{3,k}^{(2)}\big)_{\max}=10.\end{cases}\tag{53}$$

In this section, we proceed to utilize the algorithms proposed in Corollary 1 and Corollary 2 to solve Optimization Problem 1 (*OP1*) and Optimization Problem 2 (*OP2*), respectively. The simulations are performed by means of Matlab software (YALMIP 3.0), and the results are shown in Figs. 1–8.

Fig. 1 and Fig. 2 depict the time instants of each agent when the trigger occurs in *OP1* and *OP2*, respectively. By comparison between triggering times shown in the figures, we can obviously see that 1) the proposed event-triggering mechanism can effectively reduce the frequency of the innovation broadcasting; 2) the total triggering times in *OP2* is much less than that in *OP1*, indicating that the triggering frequency can be further reduced if we implement the strategy provided in *OP2*, exactly as anticipated. For the system subject to the saturation in *Case 1*, the trajectories of the estimation errors of the system state entries $x_k^{(1)}$, $x_k^{(2)}$ are shown in Figs. 3–6. Generally, the adoption of the scheme developed in Corollary 1 results in a better filtering accuracy as expected, since it can be seen that the values of estimation errors shown in Fig. 3 and Fig. 4 are smaller than those shown in Fig. 5 and Fig. 6, respectively.

In order to figure out the impact from the saturation bound on the filtering performance, we now consider the proposed *Case 2* expressed by (53). The simulation results are shown in Fig. 7 and Fig. 8, where the figures of the estimation errors are presented. We can see that the filtering performance can be improved if the saturation bound becomes larger.

In summary, the proposed design technique for the desired distributed event-based filter offers much flexibility in making trade-offs between the two essential requirements (i.e., filtering performance and triggering frequency) and therefore provides the engineers with an effective methodology of achieving the balance between accuracy and cost in practical applications.

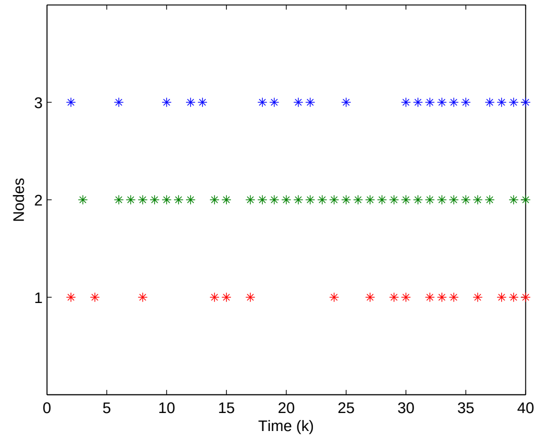

Fig. 1. The triggering sequences for ***OP1*** (*Case 1*).

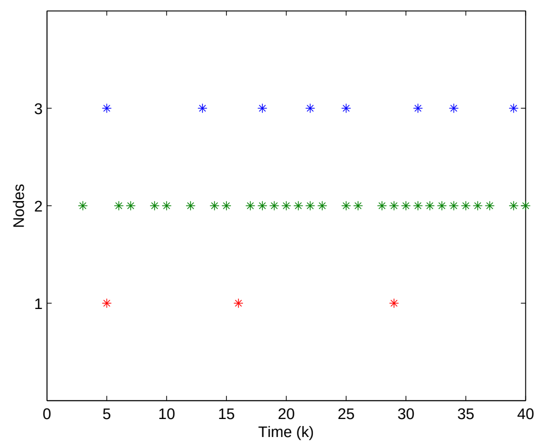

Fig. 2. The triggering sequences for ***OP2*** (*Case 1*).

V. Conclusion

In this paper, the distributed event-based set-membership filtering problem has been addressed for a class of discrete nonlinear time-varying systems subject to unknown but bounded noises and sensor saturations over sensor networks. A novel event-triggering communication mechanism has been proposed for the sake of reducing the sensor data transmission rate and the energy consumption. By means of recursive linear matrix inequalities approach, the sufficient conditions have been established for the existence of the desired distributed event-triggering filter. With the established framework, two optimization problems have been discussed to demonstrate the flexibility of the proposed methodology in making trade-offs between accuracy and cost. Finally, a numerical simulation example has been exploited to verify the effectiveness of the distributed event-triggering filtering strategy.

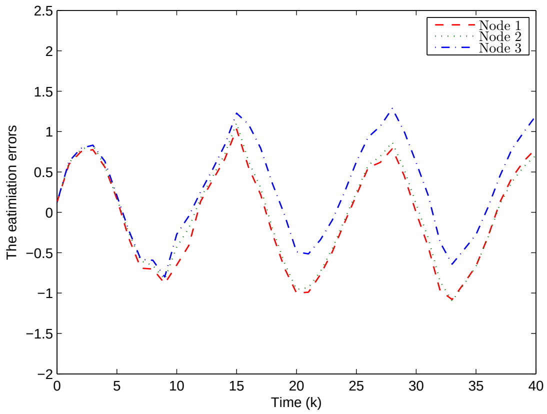

Fig. 3. The estimation errors of $x_k^{(1)}$ for ***OP1*** (*Case 1*).

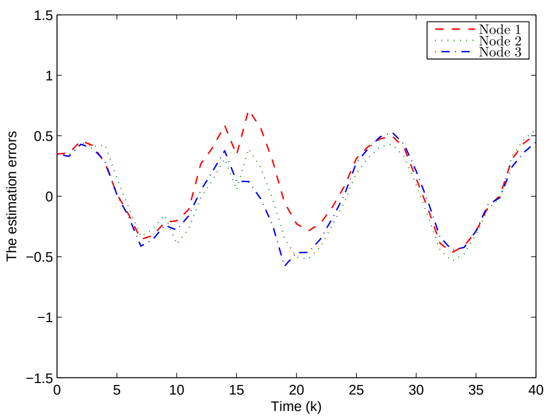

Fig. 4. The estimation errors of $x_k^{(2)}$ for ***OP1*** (*Case 1*).

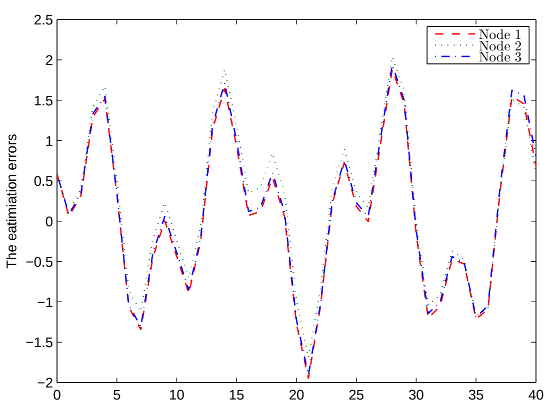

Fig. 5. The estimation errors of $x_k^{(1)}$ for ***OP2*** (*Case 1*).

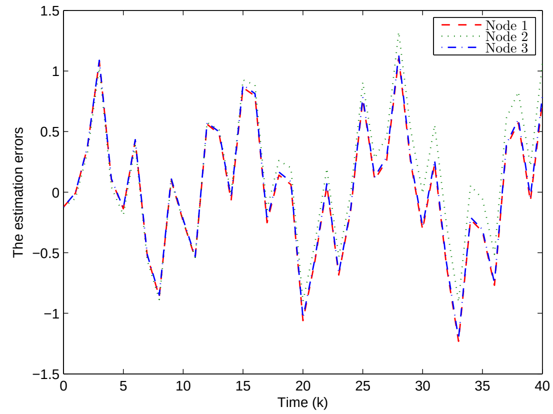

Fig. 6. The estimation errors of $x_k^{(2)}$ for ***OP2*** (*Case 1*).

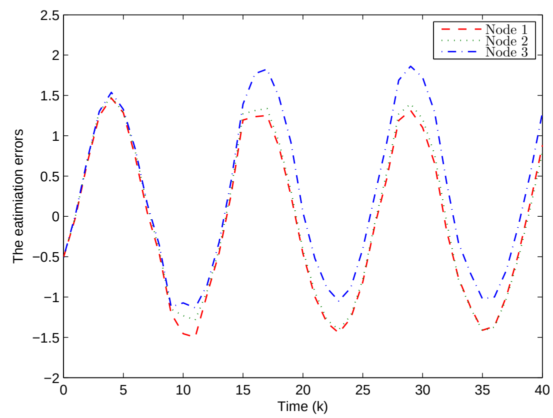

Fig. 7. The estimation errors of $x_k^{(1)}$ for ***OP1*** (*Case 2*).

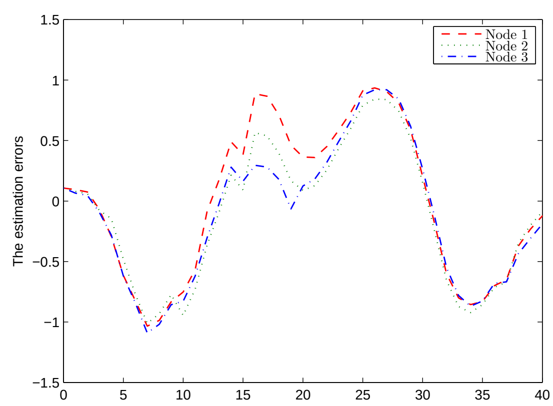

Fig. 8. The estimation errors of $x_k^{(2)}$ for ***OP1*** (*Case 2*).

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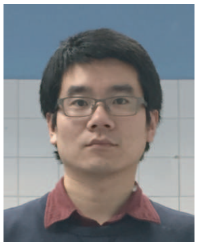

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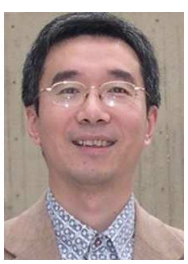

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