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DOI:

[10.1109/TFUZZ.2017.2648855](https://doi.org/10.1109/TFUZZ.2017.2648855)

*Document Version*

Peer reviewed version

[Link to publication record in King's Research Portal](#)

*Citation for published version (APA):*

Li, H., Wang, J., Wu, L., Lam, H. K., & Gao, Y. (2018). Optimal Guaranteed Cost Sliding-Mode Control of Interval Type-2 Fuzzy Time-Delay Systems. *IEEE Transactions on Fuzzy Systems*, 26(1), 246-257. Article 7807230. <https://doi.org/10.1109/TFUZZ.2017.2648855>

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# Optimal Guaranteed Cost Sliding Mode Control of Interval Type-2 Fuzzy Time-Delay Systems

Hongyi Li, Jiahui Wang, Ligang Wu, Hak-Keung Lam and Yabin Gao

## Abstract

The paper is concerned with the optimal guaranteed cost sliding mode control problem for interval type-2 (IT2) Takagi-Sugeno (T-S) fuzzy systems with time-varying delays and exogenous disturbances. **The time-varying weight coefficients reflecting the uncertain parameters hidden in membership functions are handled via adaptive method.** A new integral sliding surface is presented based on system output. By designing a novel adaptive sliding mode controller, system perturbation or model error can be compensated, and the reachability of the sliding surface can be guaranteed with the ultimate uniform boundedness of the closed-loop system. Optimal conditions of an  $\mathcal{H}_2$  guaranteed cost function and an  $\mathcal{H}_\infty$  performance index are established for the resulting time-delay control system. Finally, simulation results are provided to illustrate the advantages and effectiveness of the proposed control scheme.

**Keywords:** Interval type-2 fuzzy systems; Sliding mode control; Guaranteed cost control; Time-delay systems.

## I. INTRODUCTION

Takagi-Sugeno (T-S) fuzzy model [1], [2] is acknowledged to be effective in representing a complex nonlinear plant. T-S fuzzy model uses several local linear systems connected by membership functions to describe the nonlinear systems. Based on T-S fuzzy systems, stability analysis and synthesis results

This work was partially supported by the National Natural Science Foundation of China (61525303, 61573070, 61622302, 6167307), the Heilongjiang Outstanding Youth Science Fund (JC201406) and the Fok Ying Tung Education Foundation (141059).

H. Li is with the College of Engineering, Bohai University, Jinzhou 121013, China. Email: lihongyi2009@gmail.com

J. Wang is with the College of Information Science and Technology, Bohai University, Jinzhou 121013, Liaoning, China. Email: wangmaodou2014@gmail.com

L. Wu and Y. Gao are with the Research Institute of Intelligent Control and Systems, Harbin Institute of Technology, Harbin 150001, China. E-mail: ligangwu@hit.edu.cn, gaoyb2012@gmail.com

H.K. Lam is with the Department of Informatics, King's College London, London, WC2R 2LS, U.K. Email: hak-keung.lam@kcl.ac.uk

have been reported for classes of nonlinear systems, such as uncertain systems [3]–[6], jumping systems [7], time-delay systems [8], stochastic systems [9] and networked control systems [10]. However, the control approaches of the T-S fuzzy systems based on type-1 fuzzy sets (called type-1 fuzzy systems) are not well in dealing uncertainties existing in membership functions. Recently, interval type-2 (IT2) T-S fuzzy models [11], [12] were proposed based on type-2 fuzzy sets. IT2 fuzzy systems use the idea of well-defined lower and upper membership functions (LUMFs) to deal with uncertain parameters existing in membership functions. It has been demonstrated that IT2 T-S fuzzy systems are more universal, and the control synthesis results based on IT2 fuzzy models possess less conservativeness comparing to type-1 T-S fuzzy systems [12]. Some typical control approaches on the basis of IT2 T-S fuzzy systems have been reported [13]–[15]. For example, a sampled-data controller for IT2 fuzzy systems with actuator fault was designed in [16]. The work in [17] dealt with the model reduction problem for uncertain nonlinear systems via IT2 fuzzy model approach. Trajectory stabilization of a computer simulated model car with uncertain velocity was dealt with in [18] via type-2 fuzzy control systems.

In terms of the uncertain parameters of membership function, there still exist additional uncertainties including external disturbance, modeling error and complex system perturbation. In order to compensate the uncertainties and disturbances, several control methods [19]–[26] have been proposed. Among of these results, sliding mode control (SMC) has been recognized as a forceful robust control approach. It has attracted significant attention and successfully applied in a wide of practical systems [27]–[29]. Using SMC strategy to a control system, strong robustness for uncertainties and fast response of system stability can be achieved. To point a few, a second order sliding mode observer in [30] was designed for multi-cell converters formulated as a type of hybrid system. The work in [31] utilized the SMC approach to resolve the stabilization problem for a class of uncertain nonlinear systems described by type-1 T-S fuzzy models. By using the SMC strategy, an IT2 fuzzy SMC was incorporated in [32] in order to compensate for undesired mechanical couplings and to match the resonant frequencies for application in single axis micro-electromechanical systems vibratory gyroscopes. The adaptive SMC problem for IT2 T-S fuzzy systems has been solved in [33]. However, there are few results about the IT2 T-S fuzzy time-delay systems controlled by SMC strategy when considering the guaranteed cost performance index. The topic of SMC with guaranteed cost performance for the IT2 T-S fuzzy time-delay systems is interesting and challenging, which motivates this study.

Based on the aforementioned discussion, the optimal guaranteed cost SMC problem for a class of IT2 fuzzy time-delay systems with  $\mathcal{H}_\infty$  performance is studied in this paper. Firstly, IT2 T-S fuzzy model is used to represent the nonlinear system with uncertainties.  $\mathcal{H}_2$  guaranteed cost function and

$\mathcal{H}_\infty$  performance index are established for the resulting time-delay system. Secondly, an integral sliding surface is designed, and the desired  $\mathcal{H}_2$  guaranteed cost constraints with  $\mathcal{H}_\infty$  constraints are analyzed using Lyapunov stability theory. Finally, a novel sliding mode controller is proposed to guarantee that the closed-loop system is uniformly ultimately bounded (UUB). The advantages of the proposed results in this paper can be summarized as: 1) parameter uncertainties, time-delay in states and system perturbation or model error are considered in the plant represented by IT2 T-S fuzzy model, 2) optimization algorithms of SMC in terms of guaranteed cost and  $\mathcal{H}_\infty$  performance indices for the IT2 fuzzy time-delay systems are provided, and 3) a new sliding mode controller via output-feedback control approach is designed to guarantee that the closed-loop system is UUB.

The rest of the paper is organized as follows. Section II describes the IT2 T-S fuzzy plant with time-delay and uncertainties, and presents the  $\mathcal{H}_2$  guaranteed cost function and  $\mathcal{H}_\infty$  performance index. In Section III, the new sliding surface and sliding mode controller via output-feedback control method are designed, and the optimal schemes of guaranteed-cost-based SMC and  $\mathcal{H}_\infty$ -based SMC are detailed. In Section IV, a simulation example is presented. Section V concludes the paper.

*Notation.* The notion  $\mathcal{L}_2[0, \infty)$  is used for vector-valued functions and its  $\mathcal{L}_2$  norm is defined as  $\|w(t)\| = \sqrt{\int_0^\infty w^T(t)w(t)dt}$ . The superscripts “ $T$ ” and “ $-1$ ” stand for the matrix transpose and inverse, respectively. The symbols “ $*$ ” and “ $I$ ” denote the transposed elements in the symmetric positions of a matrix, and an identity matrix with appropriate dimensions, respectively.  $X > 0$  ( $X < 0$ ) means matrix  $X$  is positive (negative) definite.  $\lambda_{\min}(X)$  denotes the smallest eigenvalue of the matrix  $X$ , and  $\text{trace}(X)$  is the trace of matrix  $X$ .  $\|\cdot\|$  and  $\|\cdot\|_1$  represent the Euclidean norm and 1-norm (sum of absolute values), respectively.  $\text{sgn}(\cdot)$  is the symbolic function.

## II. PROBLEM FORMULATION

The following IT2 fuzzy systems are considered to represent a nonlinear plant with uncertainties. Fuzzy Rule  $i$ : IF  $f_1(\theta(t))$  is  $M_1^i$  and  $\dots$  and  $f_j(\theta(t))$  is  $M_j^i$  and  $\dots$  and  $f_s(\theta(t))$  is  $M_s^i$ , THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di} x(t-d(t)) + B_i u(t) + D_i v(t), \\ y(t) = C_i x(t), \\ z_2(t) = E_i x(t) + E_{di} x(t-d(t)) + F_i w(t), \\ z_\infty(t) = \bar{E}_i x(t) + \bar{E}_{di} x(t-d(t)) + \bar{F}_i w(t), \\ x(t) = \varphi(t), \quad -\bar{d} \leq t \leq 0, \end{cases} \quad (1)$$

where  $M_j^i$  is the IT2 fuzzy set of  $i$ -th rule,  $f_j(\theta(t))$  is the  $j$ -th measurable premise variable,  $i = 1, 2, \dots, p$ ,  $j = 1, 2, \dots, s$ ,  $p$  is the number of the fuzzy rules, and  $s$  is the number of the fuzzy sets.

$x(t) \in \mathbb{R}^n$  and  $u(k) \in \mathbb{R}^m$  stand for the system state vector and the control input to be designed, respectively.  $v(t) \in \mathbb{R}^g$  stands for a family of uncertainties, such as system perturbation or model error, which satisfies the following structure

$$\|v(t)\| \leq \delta + \eta \|g(x(t), y(t))\|,$$

where  $\delta$  and  $\eta$  are determined by the complex structure of the uncertainties  $v(t)$ , and  $g(x(t), y(t))$  is a known function.  $z_2(t) \in \mathbb{R}^l$  is the desired output with  $\mathcal{H}_2$  guaranteed cost performance (introduced in (9)), and  $z_\infty(t) \in \mathbb{R}^l$  is the desired output with  $\mathcal{H}_\infty$  disturbance attenuation performance (introduced in (10)).  $y(t) \in \mathbb{R}^n$  is the desired controlled output.  $w(t) \in \mathbb{R}^h$  denotes external disturbances. The time delay  $d(t)$  is a time-varying continuous function that satisfies

$$0 \leq d(t) \leq \bar{d}, \quad \dot{d}(t) \leq \tau < 1, \quad (2)$$

where  $\bar{d}$  and  $\tau$  are constants.  $\varphi(t)$  denotes the initial condition for  $-\bar{d} \leq t \leq 0$ , which is a constant scalar or differentiable function [34].  $A_i \in \mathbb{R}^{n \times n}$ ,  $A_{di} \in \mathbb{R}^{n \times n}$ ,  $B_i \in \mathbb{R}^{n \times m}$ ,  $D_i \in \mathbb{R}^{n \times g}$ ,  $C_i \in \mathbb{R}^{n \times n}$ ,  $E_i \in \mathbb{R}^{l \times n}$ ,  $E_{di} \in \mathbb{R}^{l \times n}$ ,  $F_i \in \mathbb{R}^{l \times h}$ ,  $\bar{E}_i \in \mathbb{R}^{l \times n}$ ,  $\bar{E}_{di} \in \mathbb{R}^{l \times n}$  and  $\bar{F}_i \in \mathbb{R}^{l \times h}$  are known system matrices. It is assumed that each local linear system in (1) is completely controllable and observable. In this paper, we use  $C_1 = C_2 = \dots = C_p = C$  for analysis, where  $C$  is a known constant matrix. The firing interval of the  $i$ -th rule is as follows:

$$\Psi_i(\theta(t)) \in \left[ \underline{\psi}_i(\theta(t)), \bar{\psi}_i(\theta(t)) \right], \quad (3)$$

where

$$\underline{\psi}_i(\theta(t)) = \prod_{j=1}^s \underline{\mu}_{M_j^i}(f_j(\theta(t))) \geq 0, \quad \bar{\psi}_i(\theta(t)) = \prod_{j=1}^s \bar{\mu}_{M_j^i}(f_j(\theta(t)))$$

in which  $\underline{\mu}_{M_j^i}(f_j(\theta(t))) \in [0, 1]$  and  $\bar{\mu}_{M_j^i}(f_j(\theta(t))) \in [0, 1]$  stand for the lower and upper membership functions, respectively. It indicates the property that  $\bar{\mu}_{M_j^i}(f_j(\theta(t))) \geq \underline{\mu}_{M_j^i}(f_j(\theta(t)))$ , which causes  $\bar{\psi}_i(\theta(t)) \geq \underline{\psi}_i(\theta(t))$  for all  $i$  and  $t \geq 0$ . Then, the established IT2 T-S fuzzy model in (1) can be rewritten as follows:

$$\mathcal{G} : \begin{cases} \dot{x}(t) = \sum_{i=1}^p \psi_i(\theta(t)) [A_i x(t) + A_{di} x(t-d(t)) + B_i u(t) + D_i v(t)], \\ z_2(t) = \sum_{i=1}^p \psi_i(\theta(t)) [E_i x(t) + E_{di} x(t-d(t)) + F_i w(t)], \\ z_\infty(t) = \sum_{i=1}^p \psi_i(\theta(t)) [\bar{E}_i x(t) + \bar{E}_{di} x(t-d(t)) + \bar{F}_i w(t)], \\ x(t) = \varphi(t), \quad -\bar{d} \leq t \leq 0, \end{cases} \quad (4)$$

with a linear measurable output  $y(t) = Cx(t)$ , where  $\psi_i(\theta(t))$  is the grade of membership of the  $i$ -th local system, which is defined as

$$\psi_i(\theta(t)) = \underline{v}_i(\theta(t)) \underline{\psi}_i(\theta(t)) + \bar{v}_i(\theta(t)) \bar{\psi}_i(\theta(t)) \geq 0, \quad (5)$$

with

$$0 \leq \underline{v}_i(\theta(t)) \leq 1, \quad 0 \leq \bar{v}_i(\theta(t)) \leq 1, \quad (6)$$

$$\sum_{i=1}^p \psi_i(\theta(t)) = 1, \quad \underline{v}_i(\theta(t)) + \bar{v}_i(\theta(t)) = 1, \quad (7)$$

where  $\psi_i(\theta(t))$  in (5) represents the membership function of  $i$ -th subsystem, and  $\underline{v}_i(\theta(t))$  and  $\bar{v}_i(\theta(t))$  are weighting coefficient functions which can represent the change of the uncertain parameters (unknown but time-varying or time-invariant). To design an adaptive sliding mode controller in this paper, the weight  $\bar{v}_i(\theta(t))$  of the  $i$ -th membership grade function in (5) satisfies the following assumption:

$$0 \leq \bar{v}_i(\theta(t)) \leq \alpha_i \leq 1, \quad (8)$$

where  $\alpha_i$  is the upper bound of  $\bar{v}_i(\theta(t))$ , which are unknown. Moreover, from (7), it follows that  $0 \leq 1 - \alpha_i \leq \underline{v}_i(\theta(t)) \leq 1$ .

Based on the system (4), we will design an adaptive SMC law to compensate all nonlinearities, time-varying delays and uncertainties, and the following requirements are simultaneously achieved:

(R1) The state trajectories of system (4) (i.e. (1)) are globally driven onto the pre-designed sliding surface, and subsequently, the sliding motion is asymptotically stable.

(R2) In the case when  $w(t) = 0$ , the cost function [34] for the stabilized time-delay system (4) is

$$\mathcal{J}_c = \int_{t_0}^{\infty} \|z_2(t)\|^2 dt < \beta. \quad (9)$$

where  $\beta$  is a positive scalar denoting the upper bound of the cost function.

(R3) For a determined scalar  $\gamma > 0$ , under zero-initial condition with  $w(t) \neq 0$ , the controlled output  $z_{\infty}(t)$  satisfies

$$\int_{t_0}^{\infty} \|z_{\infty}(t)\|^2 dt < \gamma^2 \int_{t_0}^{\infty} \|w(t)\|^2 dt. \quad (10)$$

The problem addressed above is referred to as the  $\mathcal{H}_{\infty}$ -based guaranteed cost SMC for IT2 fuzzy systems (4) with time-varying delays and uncertainties. In general, the design of SMC law consists of two steps: (i) designing a sliding surface such that, in the sliding mode, the system response acts like the desired dynamics performances, and (ii) synthesizing the SMC law ensuring that the sliding mode can be reached and the system states maintain the sliding mode thereafter, and simultaneously, the desired

$\mathcal{H}_2$  guaranteed cost performance in (9) and  $\mathcal{H}_\infty$  disturbance attenuation performance in (10) can be guaranteed.

### III. MAIN RESULTS

In this section, we will present the SMC design procedure for system (4). At first, an integral-type sliding surface is designed based on **measurable** system output. The sliding surface parameter  $K$  is designed subject to different optimal objective, which involves  $\mathcal{H}_2$  guaranteed cost constraints and  $\mathcal{H}_\infty$  disturbance attenuation performance. Then, based on the designed sliding surface, an adaptive sliding mode controller is designed to guarantee that the closed-loop system is UUB.

#### A. Sliding surface design

We introduce an auxiliary variable  $\bar{x}(t) = Uy(t) = UCx(t)$  with a given constant matrix  $U \in \mathbb{R}^{m \times n}$  and  $UC$  is nonsingular. Denote

$$\begin{aligned} \mathbf{A}(t) &\triangleq \sum_{i=1}^p \psi_i(\theta(t)) UCA_i, \quad \mathbf{A}_d(t) \triangleq \sum_{i=1}^p \psi_i(\theta(t)) UCA_{di}, \\ \mathbf{B}(t) &\triangleq \sum_{i=1}^p \psi_i(\theta(t)) UCB_i, \quad \mathbf{D}(t) \triangleq \sum_{i=1}^p \psi_i(\theta(t)) UCD_i, \\ \bar{\mathbf{A}}(t) &\triangleq \sum_{i=1}^p \bar{\psi}_i(\theta(t)) A_iUC, \quad \bar{\mathbf{A}}_d(t) \triangleq \sum_{i=1}^p \bar{\psi}_i(\theta(t)) A_{di}UC, \\ \bar{\mathbf{B}}(t) &\triangleq \sum_{i=1}^p \bar{\psi}_i(\theta(t)) B_iKUC, \quad \hat{\mathbf{B}}(t) \triangleq \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \|GUCB_i\|, \end{aligned}$$

for saving space in the following context. Based on the measurable output of the plant, we choose the following integral function as sliding surface.

$$s(t) = GUy(t) - \int_0^t \sum_{i=1}^p \bar{\psi}_i(\theta(\tau)) G[(A_i + B_iK)Uy(\tau) + A_{di}Uy(\tau - d(\tau))] d\tau, \quad (11)$$

where  $G \in \mathbb{R}^{m \times n}$  is a known constant matrix satisfying  $\text{rank}(GUC) = m$  and  $GUCB_i$  ( $i = 1, 2, \dots, p$ ) is nonsingular with  $GUCB_i > 0$  or  $GUCB_i < 0$  for all  $i = 1, 2, \dots, p$ , and  $K \in \mathbb{R}^{m \times n}$  is the sliding surface parameter to be determined. Based on (4) and (11), we have

$$\begin{aligned} \dot{s}(t) &= G[(\mathbf{A}(t) - \bar{\mathbf{A}}(t) - \bar{\mathbf{B}}(t))x(t) + (\mathbf{A}_d(t) - \bar{\mathbf{A}}_d(t))x(t - d(t)) \\ &\quad + \mathbf{B}(t)u(t) + \mathbf{D}(t)v(t)]. \end{aligned} \quad (12)$$

When the state trajectories of the system enter the sliding motion, we know  $s(t) = 0$  and  $\dot{s}(t) = 0$ . Consequently, according to (12) and  $\dot{s}(t) = 0$ , we can get the equivalent control law:

$$u_{eq}(t) = (GB(t))^{-1} G [(\bar{\mathbf{A}}(t) + \bar{\mathbf{B}}(t) - \mathbf{A}(t)) x(t) + (\bar{\mathbf{A}}_d(t) - \mathbf{A}_d(t)) x(t - d(t)) - \mathbf{D}(t) v(t)].$$

Then, by substituting (13) into (4), the following sliding motion dynamics can be obtained.

$$\mathcal{G}_s : \begin{cases} \dot{x}(t) = \mathbf{A}_s(t) x(t) + \mathbf{A}_{ds}(t) x(t - d(t)), \\ z_2(t) = \mathbf{E}_s(t) x(t) + \mathbf{E}_{ds}(t) x(t - d(t)) + \mathbf{F}_s(t) w(t), \\ z_\infty(t) = \bar{\mathbf{E}}_s(t) x(t) + \bar{\mathbf{E}}_{ds}(t) x(t - d(t)) + \bar{\mathbf{F}}_s(t) w(t), \\ x(t) = \varphi(t), \quad -\bar{d} \leq t \leq 0, \end{cases} \quad (13)$$

where

$$\begin{aligned} \mathbf{A}_s(t) &\triangleq \sum_{i=1}^p \psi_i(\theta(t)) \left[ B_i (GB(t))^{-1} G (\bar{\mathbf{A}}(t) + \bar{\mathbf{B}}(t) - \mathbf{A}(t)) + A_i \right], \\ \mathbf{A}_{ds}(t) &\triangleq \sum_{i=1}^p \psi_i(\theta(t)) \left[ B_i (GB(t))^{-1} G (\bar{\mathbf{A}}_d(t) - \mathbf{A}_d(t)) + A_{di} \right], \\ \mathbf{E}_s(t) &\triangleq \sum_{i=1}^p \psi_i(\theta(t)) E_i, \quad \mathbf{E}_{ds}(t) \triangleq \sum_{i=1}^p \psi_i(\theta(t)) E_{di}, \quad \mathbf{F}_s(t) \triangleq \sum_{i=1}^p \psi_i(\theta(t)) F_i, \\ \bar{\mathbf{E}}_s(t) &\triangleq \sum_{i=1}^p \psi_i(\theta(t)) \bar{E}_i, \quad \bar{\mathbf{E}}_{ds}(t) \triangleq \sum_{i=1}^p \psi_i(\theta(t)) \bar{E}_{di}, \quad \bar{\mathbf{F}}_s(t) \triangleq \sum_{i=1}^p \psi_i(\theta(t)) \bar{F}_i. \end{aligned}$$

System  $\mathcal{G}_s$  shows a complexly fuzzified system dynamics, which also can be served as the sliding motion.

Based on system (13), we will analyze the stability of the sliding motion, and determine the sliding mode parameter  $K$ . The desired guaranteed cost performance and  $\mathcal{H}_\infty$  performance are analyzed in the following part.

*Remark 1:* The stability of the fuzzy time-delay system (4) is prerequisite for analyzing the desired performances. On the other hand, the sliding mode controller is designed to force the trajectories of the plant (4) onto the predesigned sliding surface, whose stability is determined by the sliding motion (13). Therefore, the stability of the sliding motion (13) is important for the overall system stability and performances once the reachability of the sliding surface holds with a sliding mode controller (presented in (41)).

1) *Stability analysis of the sliding motion:* For the dynamics of the sliding motion in the first expression of system  $\mathcal{G}_s$  in (13), Theorem 1 gives an asymptotic stability criterion based on Lyapunov stability theory.

*Theorem 1:* The sliding motion in (13) is asymptotically stable if there exist matrices  $0 < P = P^T \in \mathbb{R}^{m \times m}$ ,  $0 < Q = Q^T \in \mathbb{R}^{m \times m}$ ,  $0 < Z = Z^T \in \mathbb{R}^{m \times m}$ ,  $0 < X_{11} = X_{11}^T \in \mathbb{R}^{n \times n}$ ,  $0 < X_{22} = X_{22}^T \in$



$\mathbb{R}^{n \times n}$ ,  $X_{12} \in \mathbb{R}^{n \times n}$ ,  $Y \in \mathbb{R}^{n \times n}$ ,  $T \in \mathbb{R}^{n \times n}$ ,  $U \in \mathbb{R}^{n \times n}$  and  $K \in \mathbb{R}^{m \times n}$  such that the following matrix inequalities ( $i = 1, 2, \dots, p$ ) hold:

$$\Phi_i \triangleq \begin{bmatrix} -\bar{d}Z & \bar{d}ZG(A_iUC + B_iKUC) & \bar{d}ZGA_{di}UC \\ * & \Phi_{1i} & \Phi_{2i} \\ * & * & \Phi_3 \end{bmatrix} < 0, \quad (14)$$

where

$$\begin{aligned} \Phi_{1i} &\triangleq \left[ (A_iUC + B_iKUC)^T G^T PGUC + Y \right]_s + (GUC)^T QGUC + \bar{d}X_{11}, \\ \Phi_{2i} &\triangleq (GA_{di}UC)^T PGUC - Y + T^T + \bar{d}X_{12}, \quad \Phi_3 \triangleq (\tau - 1)(GUC)^T QGUC - [T]_s + \bar{d}X_{22}. \end{aligned}$$

*Proof:* For the dynamics of system (13), consider the Lyapunov functional candidate  $V_s(t)$  as follows:

$$\begin{aligned} V_s(t) &= x^T(t) (GUC)^T PGUC x(t) + \int_{t-d(t)}^t x^T(s) (GUC)^T QGUC x(s) d(s) \\ &\quad + \int_{-\bar{d}}^0 \int_{t+\theta}^t \dot{x}^T(s) (GUC)^T ZGUC \dot{x}(s) d(s) d(\theta). \end{aligned} \quad (15)$$

Based on the above statement, we have

$$\begin{aligned} \dot{V}_s(t) &= 2\dot{x}^T(t) (GUC)^T PGUC x(t) + x^T(t) (GUC)^T QGUC x(t) \\ &\quad - \left(1 - \dot{d}(t)\right) x^T(t-d(t)) (GUC)^T QGUC x(t-d(t)) \\ &\quad + \bar{d}\dot{x}^T(t) (GUC)^T ZGUC \dot{x}(t) - \int_{t-\bar{d}}^t \dot{x}^T(s) (GUC)^T ZGUC \dot{x}(s) d(s) \\ &\leq 2 \left[ (\bar{\mathbf{A}}(t) + \bar{\mathbf{B}}(t)) x(t) + G\bar{\mathbf{A}}_d(t) x(t-d(t)) \right]^T G^T PGUC x(t) \\ &\quad + x^T(t) (GUC)^T QGUC x(t) - (1-\tau) x^T(t-d(t)) (GUC)^T QGUC x(t-d(t)) \\ &\quad + \bar{d}\dot{x}^T(t) (GUC)^T ZGUC \dot{x}(t) - \int_{t-\bar{d}}^t \dot{x}^T(s) (GUC)^T ZGUC \dot{x}(s) d(s). \end{aligned} \quad (16)$$

We introduce free-weighting matrices  $Y \in \mathbb{R}^{n \times n}$  and  $T \in \mathbb{R}^{n \times n}$  with the following equation based on Leibniz–Newton formula holding:

$$2 \left( x^T(t) Y + x^T(t-d(t)) T \right) \left( x(t) - x(t-d(t)) - \int_{t-d(t)}^t \dot{x}(s) ds \right) = 0. \quad (17)$$

For matrices  $X = \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0$  and  $\Psi > 0$  with  $Z > 0$ , we know that

$$\bar{d}\zeta^T(t) X \zeta(t) - \int_{t-d(t)}^t \zeta^T(s) X \zeta(s) d(s) \geq 0, \quad (18)$$

$$\int_{t-d(t)}^t \zeta^T(s, t) \Psi \zeta(s, t) ds \geq 0 \quad (19)$$

always hold, where  $\varsigma(t) \triangleq \begin{bmatrix} x^T(t) & x^T(t-d(t)) \end{bmatrix}^T$ ,  $\zeta(s,t) \triangleq \begin{bmatrix} \dot{x}^T(s) & x^T(t) & x^T(t-d(t)) \end{bmatrix}^T$ , and

$$\Psi \triangleq \begin{bmatrix} (GUC)^T ZGUC & Y & T \\ * & X_{11} & X_{12} \\ * & * & X_{22} \end{bmatrix}.$$

Then adding (17)–(19) to  $\dot{V}_s(t)$  in (16) yields:

$$\begin{aligned} \dot{V}_s(t) &\leq 2 \left[ (\bar{\mathbf{A}}(t) + \bar{\mathbf{B}}(t))x(t) + \bar{\mathbf{A}}_d(t)x(t-d(t)) \right]^T G^T P GUC x(t) \\ &\quad + x^T(t) (GUC)^T QGUC x(t) - (1-\tau)x^T(t-d(t)) (GUC)^T QGUC x(t-d(t)) \\ &\quad + x^T(t) \Omega^T (\bar{d}Z)^{-1} \Omega x(t) - \int_{t-\bar{d}}^t \dot{x}^T(s) (GUC)^T ZGUC \dot{x}(s) ds \\ &\quad + 2x^T(t) Y x(t) - 2x^T(t) Y x(t-d(t)) - \int_{t-d(t)}^t 2x^T(t) Y \dot{x}(s) ds \\ &\quad + 2x^T(t-d(t)) T x(t) - 2x^T(t-d(t)) T x(t-d(t)) \\ &\quad - \int_{t-d(t)}^t 2x^T(t-d(t)) T \dot{x}(s) ds + \bar{d}\zeta^T(t) X \zeta(t) - \int_{t-d(t)}^t \zeta^T(s) X \zeta(s) ds \\ &\quad + \int_{t-d(t)}^t \zeta^T(s,t) \Psi \zeta(s,t) ds \\ &\leq \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \varsigma^T(t) \left[ \Theta_i + \Omega_i^T (\bar{d}Z)^{-1} \Omega_i \right] \varsigma(t), \end{aligned} \quad (20)$$

where

$$\Theta_i \triangleq \begin{bmatrix} \Phi_{1i} & \Phi_{2i} \\ * & \Phi_3 \end{bmatrix}, \quad \Omega_i \triangleq \begin{bmatrix} \bar{d}ZG(A_iUC + B_iKUC) & \bar{d}ZGA_{di}UC \end{bmatrix}. \quad (21)$$

By Schur complement, we know condition  $\Phi_i < 0$  in (14) is equivalent to  $\Theta_i + \Omega_i^T (\bar{d}Z)^{-1} \Omega_i < 0$ , which implies  $\dot{V}_s(t) < 0$  for  $\varsigma(t) \neq 0$ . Therefore, the sliding motion (with the output  $z_2(t)$  when  $w(t) = 0$ ) is asymptotically stable if the conditions (14) have a feasible solution.  $\blacksquare$

*Remark 2:* In terms the form of inequality (14), there exists complex couplings with the matrix  $K$ . For a solvable solution via convex approach, we can give the matrices  $P$  and  $Z$  to receive a feasible solution in the form of linear convex. Similar problems in Theorems 2–4 below can be coped with by this concise approach.

2) *Analysis of  $\mathcal{H}_2$  guaranteed cost performance:* When the state trajectories of the plant in (4) are forced to be driven onto the designed sliding surface by the sliding mode controller presented in (41), the desired  $\mathcal{H}_2$  performance index (respectively, the optimal  $\mathcal{H}_\infty$  disturbance attenuation performance index) can be designed based on the stabilized sliding motion in (13). According to the required  $\mathcal{H}_2$  guaranteed

cost performance in (9) for system (4), Theorem 2 provides a condition of optimizing  $\mathcal{H}_2$  guaranteed cost performance.

*Theorem 2:* According to the fuzzy system (13) with  $w(t) = 0$ , if there exist matrices  $P, Q, Z, X_{11}, X_{12}, X_{22}, Y, T, U, K$  as illustrated in Theorem 1 and positive definite matrices  $\Upsilon_1 \in \mathbb{R}^{n \times n}, \Upsilon_2 \in \mathbb{R}^{n \times n}, \Upsilon_3 \in \mathbb{R}^{n \times n}$  such that:

$$\begin{bmatrix} -I_l & 0_{l \times m} & E_i & E_{di} \\ * & -\bar{d}Z & \bar{d}ZG(A_iUC + B_iKUC) & \bar{d}ZGA_{di}UC \\ * & * & \Phi_{1i} & \Phi_{2i} \\ * & * & * & \Phi_3 \end{bmatrix} < 0, \quad (22)$$

$$\begin{bmatrix} -\Upsilon_1 & a^{\frac{1}{2}}(GUC)^T P \\ * & -P \end{bmatrix} \leq 0, \quad (23)$$

$$\begin{bmatrix} -\Upsilon_2 & b^{\frac{1}{2}}(GUC)^T Q \\ * & -Q \end{bmatrix} \leq 0, \quad (24)$$

$$\begin{bmatrix} -\Upsilon_3 & c^{\frac{1}{2}}(GUC)^T Z \\ * & -Z \end{bmatrix} \leq 0, \quad (25)$$

then the system (13) is asymptotically stable, and the optimal  $\mathcal{H}_2$  guaranteed cost bound defined in (9) is

$$\mathcal{J}_c = \int_{t_0}^{\infty} z_2^T(t) z_2(t) dt < \text{trace}(\Upsilon_1) + \text{trace}(\Upsilon_2) + \text{trace}(\Upsilon_3) = \beta. \quad (26)$$

Furthermore, the optimal guaranteed cost bound  $\beta_{\min}$  can be **obtained** from the minimization problem

$$\min \text{trace}(\Upsilon_1) + \text{trace}(\Upsilon_2) + \text{trace}(\Upsilon_3), \quad \text{s.t. (22)–(25)}. \quad (27)$$

*Proof:* Recalling  $\sum_{i=1}^p \psi_i(\theta(t)) = 1$ , we introduce the following inequality based on Lemma 2 in [34].

$$\sum_{i=1}^p \psi_i(\theta(t)) X_i^T \sum_{j=1}^p \psi_j(\theta(t)) X_j \leq \sum_{i=1}^p \psi_i(\theta(t)) X_i^T X_i. \quad (28)$$

Considering Lyapunov function  $V_s(t)$  in (15) for system (13), and the following performance index

$$\mathcal{J}_2 = \dot{V}_s(t) + z_2^T(t) z_2(t).$$

Based on the inequality in (28), we have

$$\begin{aligned} \mathcal{J}_2 &\leq \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \varsigma^T(t) \left[ \Theta_i + \Omega_i^T (\bar{d}Z)^{-1} \Omega_i \right] \varsigma(t) \\ &\quad + \sum_{i=1}^p \psi_i(\theta(t)) (E_i x(t) + E_{di} x(t-d(t)))^T \sum_{j=1}^p \psi_j(\theta(t)) (E_j x(t) + E_{dj} x(t-d(t))) \end{aligned}$$

$$\begin{aligned}
&\leq \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \varsigma^T(t) \left[ \Theta_i + \Omega_i^T (\bar{d}Z)^{-1} \Omega_i \right] \varsigma(t) \\
&\quad + \sum_{i=1}^p \bar{\psi}_i(\theta(t)) (E_i x(t) + E_{di} x(t-d(t)))^T (E_i x(t) + E_{di} x(t-d(t))) \\
&= \varsigma^T(t) \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \tilde{\Phi}_i \varsigma(t). \tag{29}
\end{aligned}$$

According to Schur complement, we know  $\tilde{\Phi}_i < 0$  in (29) is equivalent to (22), which means  $\mathcal{J}_2 = \dot{V}_s(t) + z_\infty^T(t) z_2(t) < 0$  for all  $\varsigma(t) \neq 0$ . Furthermore, the condition (22) is equivalent to (14) in Theorem 1, the asymptotically stability of the sliding motion in (13) can be guaranteed with  $V_s(t) \rightarrow 0$  when  $t \rightarrow \infty$ . Hence, we have

$$\int_{t_0}^{\infty} \left( \dot{V}_s(t) + z_2^T(t) z_2(t) \right) dt = V_s(\infty) - V_s(t_0) + \int_{t_0}^{\infty} z_2^T(t) z_2(t) dt < 0,$$

i.e.

$$\mathcal{J}_c = \int_{t_0}^{\infty} \|z_2(t)\|_2^2 dt < V_s(t_0). \tag{30}$$

Additionally, when the initial condition  $\varphi(t)$  is a constant, it implies that  $\dot{\varphi}(t) \equiv 0$  for all  $t \in [-\bar{d}, 0]$ . When  $\varphi(t)$  is a differentiable function,  $\dot{\varphi}(t)$  is not relative to the differential (13) because it is invalid before zero interval. Overall,  $V_s(t_0) < \beta$  holds, where  $\beta$  is defined as (26). Therefore, the cost function is bounded by  $\beta$  given in (26). Let

$$\begin{aligned}
a &= \varphi^T(t_0) \varphi(t_0), \\
b &= \int_{-d(t_0)}^{t_0} \varphi^T(\tau) \varphi(\tau) d(\tau), \\
c &= \int_{-\bar{d}}^{t_0} \int_t^{t_0} \dot{\varphi}^T(\tau) \dot{\varphi}(\tau) d(\tau) d(t).
\end{aligned}$$

Recalling the conditions in (23)–(25), according to Schur complement, it follows

$$a^{\frac{1}{2}} (GUC)^T PGUC a^{\frac{1}{2}} \leq \Upsilon_1, \tag{31}$$

$$b^{\frac{1}{2}} (GUC)^T QGUC b^{\frac{1}{2}} \leq \Upsilon_2, \tag{32}$$

$$c^{\frac{1}{2}} (GUC)^T ZGUC c^{\frac{1}{2}} \leq \Upsilon_3. \tag{33}$$

Then using the property  $\text{trace}(MN) = \text{trace}(NM)$  with (31)–(33), we have

$$\begin{aligned}
\varphi^T(t_0) [(GUC)^T PGUC]^{-1} \varphi(t_0) &= \text{trace}(a[(GUC)^T PGUC]^{-1}) \\
&= \text{trace}(a^{\frac{1}{2}} [(GUC)^T PGUC]^{-1} a^{\frac{1}{2}}) \\
&\leq \text{trace}(\Upsilon_1), \tag{34}
\end{aligned}$$

$$\begin{aligned}
\int_{-d(t_0)}^{t_0} \varphi^T(\tau) [(GUC)^T QGUC]^{-1} \varphi(\tau) d(\tau) &= \text{trace}(b[(GUC)^T QGUC]^{-1}) \\
&= \text{trace}(b^{\frac{1}{2}} [(GUC)^T QGUC]^{-1} b^{\frac{1}{2}}) \\
&\leq \text{trace}(\Upsilon_2), \tag{35}
\end{aligned}$$

$$\begin{aligned}
\int_{-\bar{d}}^{t_0} \int_t^{t_0} \dot{\varphi}^T(\tau) [(GUC)^T ZGUC]^{-1} \dot{\varphi}(\tau) d(\tau) d(t) &= \text{trace}(c[(GUC)^T ZGUC]^{-1}) \\
&= \text{trace}(c^{\frac{1}{2}} [(GUC)^T ZGUC]^{-1} c^{\frac{1}{2}}) \\
&\leq \text{trace}(\Upsilon_3). \tag{36}
\end{aligned}$$

Consequently, the  $\mathcal{H}_2$  guaranteed cost function is bounded by

$$\mathcal{J}_c = \int_{t_0}^{\infty} z_2^T(t) z_2(t) dt < V_s(t_0) \leq \text{trace}(\Upsilon_1) + \text{trace}(\Upsilon_2) + \text{trace}(\Upsilon_3). \tag{37}$$

Meanwhile, when an optimizer is applied to solve the minimization problem in (27), an optimal guaranteed cost bound  $\beta_{\min}$  can be obtained with  $\beta_{\min} = \text{trace}(\Upsilon_1) + \text{trace}(\Upsilon_2) + \text{trace}(\Upsilon_3)$ . It completes the proof.  $\blacksquare$

*Remark 3:* Theorem 2 directly gives the minimum  $\mathcal{H}_2$  guaranteed cost bound. Actually, from the proof of Theorem 2, the desired guaranteed cost bound can be  $V_s(t_0)$  in (30) without optimization design. [The utilization of guaranteed cost bound design has been developed in many systems, detailed presentations can be found in \[35\]–\[37\], etc.](#)

3) *Analysis of  $\mathcal{H}_\infty$  disturbance attenuation performance:* Considering the required  $\mathcal{H}_\infty$  performance in (10) for system (4), we give the condition of optimizing  $\mathcal{H}_\infty$  disturbance attenuation level in the following theorem.

*Theorem 3:* The plant (4) can achieve the  $\mathcal{H}_\infty$  performance in (10) with an optimal disturbance attenuation level  $\gamma > 0$ , if there exist matrices  $P, Q, Z, X_{11}, X_{12}, X_{22}, Y, T, K$  as illustrated in Theorem 1 such that the following optimization problem has a feasible solution.

$$\min \quad \gamma^* \quad \text{s.t. } \Gamma_i < 0 \text{ with } \gamma^* = \gamma^2, i = 1, 2, \dots, p, \tag{38}$$

where

$$\Gamma_i \triangleq \begin{bmatrix} -I_l & 0_{l \times m} & \bar{E}_i & \bar{E}_{di} & \bar{F}_i \\ * & -\bar{d}Z & \bar{d}ZG(A_iUC + B_iKUC) & \bar{d}ZGA_{di}UC & 0_{r \times h} \\ * & * & \Phi_{1i} & \Phi_{2i} & 0_{n \times h} \\ * & * & * & \Phi_3 & 0_{n \times h} \\ * & * & * & * & -\gamma^2 I_h \end{bmatrix},$$

and  $\Phi_{1i}, \Phi_{2i}$  and  $\Phi_3$  are defined in (14).

*Proof:* **First**, it is easily found that  $\Gamma_i < 0$  is equivalent to the condition (14), which is an asymptotic stability criterion of the sliding motion. Then considering the system in (13) under the zero initial condition, we establish the following index for  $\mathcal{H}_\infty$  performance analysis:

$$\mathcal{J}_\infty = \dot{V}_s(t) + z_\infty^T(t) z_\infty(t) - \gamma^2 w^T(t) w(t).$$

Following the Lyapunov function  $V_s(t)$  expressed in (15) for system  $\mathcal{G}_s$  when  $w(t) \neq 0$ , we have

$$\begin{aligned} \mathcal{J}_\infty &= \dot{V}_s(t) + z_\infty^T(t) z_\infty(t) - \gamma^2 w^T(t) w(t) \\ &\leq \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \varsigma^T(t) \left[ \Theta_i + \Omega_i^T (\bar{d}Z)^{-1} \Omega_i \right] \varsigma(t) \\ &\quad + \sum_{i=1}^p \psi_i(\theta(t)) \left( \bar{E}_i x(t) + \bar{E}_{di} x(t-d(t)) + \bar{F}_i w(t) \right)^T \\ &\quad \times \sum_{j=1}^p \psi_j(\theta(t)) \left( \bar{E}_j x(t) + \bar{E}_{dj} x(t-d(t)) + \bar{F}_j w(t) \right) - \gamma^2 w^T(t) w(t) \\ &\leq -\gamma^2 w^T(t) w(t) + \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \left( \bar{E}_i x(t) + \bar{E}_{di} x(t-d(t)) + \bar{F}_i w(t) \right)^T \\ &\quad \times \left( \bar{E}_i x(t) + \bar{E}_{di} x(t-d(t)) + \bar{F}_i w(t) \right) + \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \varsigma^T(t) \left[ \Theta_i + \Omega_i^T (\bar{d}Z)^{-1} \Omega_i \right] \varsigma(t) \\ &= \bar{\varsigma}^T(t) \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \bar{\Phi}_i \bar{\varsigma}(t), \end{aligned}$$

where

$$\begin{aligned} \bar{\Phi}_i &\triangleq \begin{bmatrix} \Phi_{1i} & \Phi_{2i} & 0_{n \times h} \\ * & \Phi_3 & 0_{n \times h} \\ * & * & -\gamma^2 I_h \end{bmatrix} + \bar{\Omega}_i^T (\bar{d}Z)^{-1} \bar{\Omega}_i + \Xi_i^T \Xi_i, \quad \Xi_i \triangleq \begin{bmatrix} \bar{E}_i & \bar{E}_{di} & \bar{F}_i \end{bmatrix}, \\ \bar{\Omega}_i &\triangleq \begin{bmatrix} \bar{d}ZG(A_i UC + B_i KUC) & \bar{d}ZGA_{di} UC & 0_{m \times l} \end{bmatrix}, \quad \bar{\varsigma}(t) \triangleq \begin{bmatrix} \varsigma^T(t) & w^T(t) \end{bmatrix}^T. \end{aligned}$$

According to Schur complement, we have  $\Gamma_i < 0$  in (38) is equivalent to  $\bar{\Phi}_i < 0$ , which means  $\mathcal{J}_\infty = \dot{V}_s(t) + z_\infty^T(t) z_\infty(t) - \gamma^2 w^T(t) w(t) < 0$ . Considering the  $\mathcal{H}_\infty$  performance in (10) under the zero initial condition with  $V_s(\infty) \geq 0$ , we have

$$\begin{aligned} &\int_{t_0}^{\infty} \left( \dot{V}_s(t) + z_\infty^T(t) z_\infty(t) - \gamma^2 w^T(t) w(t) \right) dt \\ &= V_s(\infty) - V_s(0) + \int_{t_0}^{\infty} \left( z_\infty^T(t) z_\infty(t) - \gamma^2 w^T(t) w(t) \right) dt \\ &= V_s(\infty) + \int_{t_0}^{\infty} \left( z_\infty^T(t) z_\infty(t) - \gamma^2 w^T(t) w(t) \right) dt < 0, \end{aligned}$$

which means

$$\int_{t_0}^{\infty} (z_{\infty}^T(t) z_{\infty}(t) - \gamma^2 w^T(t) w(t)) dt < -V_s(\infty) \leq 0. \quad (39)$$

From (39), we know that the requirement (R3) can be achieved. This completes the proof.  $\blacksquare$

4) *Analysis of optimal  $\mathcal{H}_{\infty}$ -based guaranteed cost performance:* When the requirements (R2) and (R3) are both desired, the following theorem gives a sufficient criterion based on the results above.

*Theorem 4:* The plant (4) can achieve the optimal  $\mathcal{H}_{\infty}$  performance in (10) with guaranteed cost performance in (9), if there exist matrices  $P, Q, Z, X_{11}, X_{12}, X_{22}, Y, T, K$  such that the optimization problem

$$\begin{aligned} \min \quad & \text{trace}(\Upsilon_1) + \text{trace}(\Upsilon_2) + \text{trace}(\Upsilon_3) + \gamma^*, \\ \text{s.t.} \quad & (22)\text{--}(23) \text{ and } (38) \text{ with } \gamma^* = \gamma^2 \end{aligned} \quad (40)$$

has feasible solutions for all  $i = 1, 2, \dots, p$ . Moreover, the optimal guaranteed cost is bounded by  $\beta_{\min} = \text{trace}(\Upsilon_1) + \text{trace}(\Upsilon_2) + \text{trace}(\Upsilon_3)$  and the optimal  $\mathcal{H}_{\infty}$  disturbance attenuation level is  $\gamma_{\min} = \sqrt{\gamma^*}$ .

*Proof:* The proof is similar to the proofs of Theorem 2 and Theorem 3, and is omitted because of space limitation.  $\blacksquare$

### B. Adaptive sliding mode controller design

This section gives the desired adaptive sliding mode controller to stabilize the IT2 fuzzy systems (4). The reachability of the sliding mode for the IT2 fuzzy system (4) is analyzed. The following theorem gives a sliding mode controller for system (4). Define  $\tilde{\delta}(t) = \hat{\delta}(t) - \delta$ ,  $\tilde{\eta}(t) = \hat{\eta}(t) - \eta$  and  $\tilde{\alpha}_i(t) = \hat{\alpha}_i(t) - \alpha_i$ , where  $\hat{\delta}(t)$ ,  $\hat{\eta}(t)$  and  $\hat{\alpha}_i(t)$  are adaptive parameters that estimate unknown parameters  $\delta$ ,  $\eta$  and  $\alpha_i$ , respectively.

*Theorem 5:* The feasible sliding surface has been obtained by Theorem 1, then the following controller of the closed-loop system (4) is designed that the sliding surface in (11) and all signals are UUB.

$$u(t) = \begin{cases} -u_s(t) - \frac{1}{\|s^T(t)GUC\|} \hat{\mathbf{B}}^{-1}(t) GUC (GUC)^T s(t) u_a(t), & \text{if } \|s^T(t)GUC\| \neq 0; \\ -u_s(t), & \text{if } \|s^T(t)GUC\| = 0, \end{cases} \quad (41)$$

with

$$\begin{aligned} u_s(t) & \triangleq \mu \hat{\mathbf{B}}^{-1}(t) \text{sgn}(s(t)), \\ u_a(t) & \triangleq \frac{1}{\|s^T(t)GUC\|} \|s^T(t)G \sum_{i=1}^p \bar{\psi}_i(\theta(t)) (A_i + B_i K) U y(t)\| \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\|s^T(t)GUC\|} \|s^T(t)G \sum_{i=1}^p \bar{\psi}_i(\theta(t)) A_{di} U y(t-d(t))\| \\
& + \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \|A_i(UC)^{-1} U y(t)\| + \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \|A_{di}(UC)^{-1} U y(t-d(t))\| \\
& + \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \|D_i\| (\hat{\delta}(t) + \hat{\eta}(t) \|g(x(t), y(t))\|) + \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \hat{\alpha}_i(t),
\end{aligned}$$

and the adaptive laws

$$\dot{\hat{\delta}}(t) = -\lambda_1 \varepsilon_1 \hat{\delta}(t) + \lambda_1 \|s^T(t)GUC\| \left\| \sum_{i=1}^p \bar{\psi}_i(\theta(t)) D_i \right\|, \quad (42)$$

$$\dot{\hat{\eta}}(t) = -\lambda_2 \varepsilon_2 \hat{\eta}(t) + \lambda_2 \|s^T(t)GUC\| \left\| \sum_{i=1}^p \bar{\psi}_i(\theta(t)) D_i \right\| \|g(x(t), y(t))\|, \quad (43)$$

$$\begin{aligned}
\dot{\hat{\alpha}}_i(t) &= \text{Proj}_{[0,1]} \{\phi_i(t)\} \\
&= \begin{cases} 0, & \text{if } \hat{\alpha}_i(t) = 0 \text{ and } \phi_i(t) < 0, \text{ or if } \hat{\alpha}_i(t) = 1 \text{ and } \phi_i(t) > 0, \\ \phi_i(t), & \text{otherwise,} \end{cases}
\end{aligned} \quad (44)$$

where  $\text{Proj}\{\cdot\}$  is the projection operator [38], whose role is to project the estimates to the interval  $[0,1]$ ,  $\phi_i(t) \triangleq q_i \bar{\psi}_i(\theta(t)) \hat{\alpha}_i(t) \|s^T(t)GUC\|$ ,  $\mu, \lambda_1, \lambda_2, \varepsilon_1, \varepsilon_2$  and  $q_i$  are positive scalars with  $0 \leq \hat{\alpha}_i(0) \leq 1$  ( $i = 1, 2, \dots, p$ ).

*Proof:* Choose the following Lyapunov function:

$$V(t) = \frac{1}{2} \left( s^T(t)s(t) + \frac{1}{\lambda_1} \tilde{\delta}^2(t) + \frac{1}{\lambda_2} \tilde{\eta}^2(t) + \sum_{i=1}^p \frac{1}{q_i} \tilde{\alpha}_i^2(t) \right). \quad (45)$$

As  $GUCB_i > 0$  or  $GUCB_i < 0$  for all  $i = 1, 2, \dots, p$ , then it follows:

$$-\|\psi_i(\theta(t))GUCB_i\| \leq -\|\underline{\psi}_i(\theta(t))GUCB_i\|. \quad (46)$$

When  $\|s^T(t)GUC\| \neq 0$ , with the controller in (41) and the adaptive laws (42)–(44) for system (4) based on (46), it follows

$$\begin{aligned}
\dot{V}(t) &= s^T(t)\dot{s}(t) + \frac{1}{\lambda_1} \tilde{\delta}(t) \dot{\tilde{\delta}}(t) + \frac{1}{\lambda_2} \tilde{\eta}(t) \dot{\tilde{\eta}}(t) + \sum_{i=1}^p \frac{1}{q_i} \tilde{\alpha}_i(t) \dot{\tilde{\alpha}}_i(t) \\
&= s^T(t) \left\{ GUC \sum_{i=1}^p \psi_i(\theta(t)) (A_i x(t) + A_{di} x(t-d(t)) + B_i u(t) + D_i v(t)) \right. \\
&\quad \left. - G \sum_{i=1}^p \bar{\psi}_i(\theta(t)) [(A_i + B_i K) UC x(t) + A_{di} UC x(t-d(t))] \right\} \\
&\quad - \varepsilon_1 \tilde{\delta}(t) \hat{\delta}(t) - \varepsilon_2 \tilde{\eta}(t) \hat{\eta}(t) + \|s^T(t)GUC\| \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \tilde{\alpha}_i(t) \hat{\alpha}_i(t)
\end{aligned}$$



$$\begin{aligned}
&\leq s^T(t) GUC \sum_{i=1}^p \psi_i(\theta(t)) (A_i x(t) + A_{di} x(t-d(t))) \\
&\quad - \|s^T(t) GUC\| \left[ \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \hat{\alpha}_i(t) + \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \|A_i x(t)\| \right. \\
&\quad + \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \|A_{di} x(t-d(t))\| + \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \|D_i\| (\hat{\delta}(t) + \hat{\eta}(t) \|g(x(t), y(t))\|) \\
&\quad + \frac{1}{\|s^T(t) GUC\|} \|s^T(t) G \sum_{i=1}^p \bar{\psi}_i(\theta(t)) (A_i + B_i K) UCx(t)\| \\
&\quad \left. + \frac{1}{\|s^T(t) GUC\|} \|s^T(t) G \sum_{i=1}^p \bar{\psi}_i(\theta(t)) A_{di} UCx(t-d(t))\| \right] \\
&\quad + \|s^T(t) GUC\| \left\| \sum_{i=1}^p \psi_i(\theta(t)) D_i (\delta + \eta \|g(x(t), y(t))\|) \right\| \\
&\quad + \|s^T(t) G \sum_{i=1}^p \bar{\psi}_i(\theta(t)) (A_i + B_i K) UCx(t)\| \\
&\quad + \|s^T(t) G \sum_{i=1}^p \bar{\psi}_i(\theta(t)) A_{di} UCx(t-d(t))\| - \varepsilon_1 \tilde{\delta}(t) \hat{\delta}(t) - \varepsilon_2 \tilde{\eta}(t) \hat{\eta}(t) \\
&\quad + \|s^T(t) GUC\| \left\| \sum_{i=1}^p \bar{\psi}_i(\theta(t)) D_i (\tilde{\delta}(t) + \tilde{\eta}(t) \|g(x(t), y(t))\|) \right\| \\
&\quad + \|s^T(t) GUC\| \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \tilde{\alpha}_i(t) \hat{\alpha}_i(t) - \mu \|s(t)\|_1. \tag{47}
\end{aligned}$$

Substituting (5) and (7) into (47) and considering  $1 - \tilde{\alpha}_i(t) = 1 - \hat{\alpha}_i(t) + \alpha_i > 0$  and

$$\left\| \sum_{i=1}^p \psi_i(\theta(t)) D_i \right\| \leq \sum_{i=1}^p \psi_i(\theta(t)) \|D_i\| \leq \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \|D_i\|,$$

we obtain

$$\begin{aligned}
\dot{V}(t) &\leq -\|s^T(t) GUC\| \sum_{i=1}^p \bar{\psi}_i(\theta(t)) \hat{\alpha}_i(t) (1 - \tilde{\alpha}_i(t)) - \varepsilon_1 \left( \hat{\delta}(t) - \frac{1}{2} \delta \right)^2 \\
&\quad - \varepsilon_2 \left( \hat{\eta}(t) - \frac{1}{2} d \right)^2 + \frac{\varepsilon_1}{4} \delta^2 + \frac{\varepsilon_2}{4} \eta^2 - \mu \|s(t)\|_1 \\
&\leq -\mu \|s(t)\|_1 + \frac{\varepsilon_1}{4} \delta^2 + \frac{\varepsilon_2}{4} \eta^2. \tag{48}
\end{aligned}$$

Therefore, from (48), applying the terminology and result in [39], we know that the ultimate uniform boundedness of system (4) can be achieved under the controller (41). The proof is completed.  $\blacksquare$

*Remark 4:* Theorem 5 gives the control input of plant (4). When the plant is stabilized by the controller (41), the desired  $\mathcal{H}_2$  guaranteed cost performance, the  $\mathcal{H}_\infty$  disturbance attenuation performance and even

the  $\mathcal{H}_\infty$ -based guaranteed cost performance can be achieved once there exist feasible solutions of the conditions in Theorems 2–4. Meanwhile, the optimal problems presented in Theorems 2–4 indicate the optimal level of the desired performances, respectively. The presented optimal SMC schemes are alternative according to the practical situation for engineering applications.

#### IV. SIMULATION RESULTS

In this section, we will provide the inverted pendulum model shown in Fig. 1 to demonstrate the effectiveness and superiority of the proposed results.

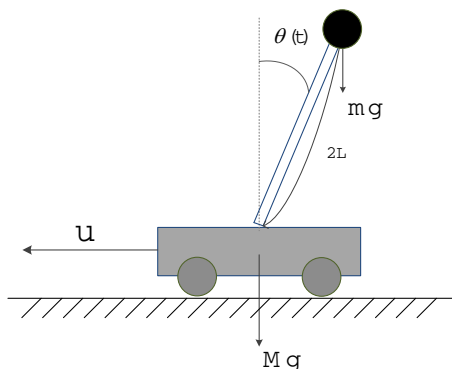


Fig. 1. Inverted pendulum system.

To test the superiority of the designed sliding mode control law subject to the desired guaranteed cost performance, a type-2 fuzzy controller [12] is compared. The dynamics of the inverted pendulum [12] is described as

$$\ddot{\theta}(t) = \frac{3g \sin(\theta(t)) - 3am_p L (\dot{\theta}(t))^2 \sin(2\theta(t))/2 - 3a \cos(\theta(t)) u(t)}{4L - 3am_p L \cos^2(\theta(t))}, \quad (49)$$

where  $\theta(t)$  denotes the angular displacement of the pendulum,  $2L = 1$  m is the length of the pendulum, the gravity acceleration is  $g = 9.8$  m/s<sup>2</sup>,  $m_p$  denotes the mass of the pendulum,  $m_c$  denotes the mass of the cart,  $a = 1/(m_p + m_c)$ , and  $u(t)$  denotes the force applied to the cart. We define  $x(t) = [x_1(t) \ x_2(t)]^T = [\theta(t) \ \dot{\theta}(t)]^T$ . Then the state space equation of the dynamics can be expressed by

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ f_1(t) & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ f_2(t) \end{bmatrix} u(t), \quad (50)$$

where

$$f_1(t) = \frac{\sin(x_1(t)) (g - am_p L x_2^2(t) \cos(x_1(t)))}{x_1(t) (4L/3 - am_p L x_2^2(t) \cos^2(x_1(t)))},$$

$$f_2(t) = \frac{-a \cos(x_1(t))}{4L/3 - am_p L \cos^2(x_1(t))}.$$

Assume that the uncertain parameters  $m_p$  and  $m_c$  satisfy  $m_{p\min} = 1 \text{ kg} \leq m_p \leq m_{p\max} = 2 \text{ kg}$  and  $m_{c\min} = 2 \text{ kg} \leq m_c \leq m_{c\max} = 3 \text{ kg}$ , respectively. The following four-rule IT2 fuzzy model is used to describe the inverted pendulum with a desired output  $z_2(t)$  and  $z_\infty(t)$  considering state delay  $x(t - d(t))$ , model uncertainties  $v(t)$  and external disturbances  $w(t)$ .

Fuzzy Rule  $i$ : IF  $f_1(t)$  is  $M_1^i$  and  $f_2(t)$  is  $M_2^i$ , THEN

$$\begin{cases} \dot{x}(t) = (1 - \sigma) A_i x(t) + \sigma A_i x(t - d(t)) + B_i u(t) + D_i v(t), \\ z_2(t) = (1 - \sigma) E_i x(t) + \sigma E_i x(t - d(t)) + F_i w(t), \\ z_\infty(t) = (1 - \sigma) \bar{E}_i x(t) + \sigma \bar{E}_i x(t - d(t)) + \bar{F}_i w(t), \end{cases} \quad (51)$$

where

$$A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ f_{1\min} & 0 \end{bmatrix}, A_3 = A_4 = \begin{bmatrix} 0 & 1 \\ f_{1\max} & 0 \end{bmatrix}, B_1 = B_3 = \begin{bmatrix} 0 \\ f_{2\min} \end{bmatrix},$$

$$B_2 = B_4 = \begin{bmatrix} 0 \\ f_{2\max} \end{bmatrix}, E_1 = \bar{E}_1 = \begin{bmatrix} 15 & 24 \\ -300 & 0.6 \end{bmatrix}, E_2 = \bar{E}_2 = \begin{bmatrix} 15 & 24 \\ -300 & 0.6 \end{bmatrix},$$

$$E_3 = \bar{E}_3 = \begin{bmatrix} 15 & 24 \\ -300 & 0.6 \end{bmatrix}, E_4 = \bar{E}_4 = \begin{bmatrix} 15 & 24 \\ -300 & 0.6 \end{bmatrix}, D_1 = \begin{bmatrix} 0 \\ 0.02 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0 \\ 0.04 \end{bmatrix}, D_3 = \begin{bmatrix} 0 \\ 0.02 \end{bmatrix}, D_4 = \begin{bmatrix} 0 \\ 0.04 \end{bmatrix}, F_1 = \bar{F}_1 = \begin{bmatrix} 0.5 & 2.5 \\ 0.05 & -7.5 \end{bmatrix},$$

$$F_2 = \bar{F}_2 = \begin{bmatrix} 0.5 & 2.5 \\ 0.1 & -7.5 \end{bmatrix}, F_3 = \bar{F}_3 = \begin{bmatrix} 0.5 & 2.5 \\ 0.05 & -7.5 \end{bmatrix}, F_4 = \bar{F}_4 = \begin{bmatrix} 0.5 & 2.5 \\ 0.1 & -7.5 \end{bmatrix}.$$

$\sigma = 0.1$  denotes the delay coefficient,  $d(t) = 0.1 + 0.1 \sin(t)$ ,  $v(t) \leq 0.02 + 0.01 \|y_2(t)\|$ .  $f_{1\min} = 9.2902$ ,  $f_{1\max} = 23.5200$ ,  $f_{2\min} = -0.6667$ , and  $f_{2\max} = -0.0792$  by considering a proper workplace with  $x_1 \in [-3\pi/8, 3\pi/8]$  and  $x_2 \in [-3, 3]$  for the inverted pendulum. The lower and upper membership functions are chosen in Table I. Besides, we set  $\underline{v}_i(\theta(t)) = q_1 \sin^2(t)$  ( $\bar{v}_i(\theta(t)) = 1 - \underline{v}_i(\theta(t))$ ) for  $i = 1, 2, 3, 4$ , which satisfy  $\sum_{i=1}^4 \psi_i(t) = 1$  to describe the parametric uncertainties.

Assume an external disturbance  $w(t) = [0.1 \sin(t) \ e^{-2t}]^T$  and a measurable output  $y(t) = Cx(t)$

TABLE I  
LOWER AND UPPER MEMBERSHIP FUNCTIONS

Lower membership functions	Upper membership functions
$\underline{\mu}_{M_1^1}(x_1) = 1 - e^{-(x_1^2/1.5)}$	$\bar{\mu}_{M_1^1}(x_1) = 0.25e^{-(x_1^2/0.3)}$
$\underline{\mu}_{M_1^2}(x_1) = \underline{\mu}_{M_1^1}(x_1)$	$\bar{\mu}_{M_1^2}(x_1) = \bar{\mu}_{M_1^1}(x_1)$
$\underline{\mu}_{M_1^3}(x_1) = 1 - \bar{\mu}_{M_1^1}(x_1)$	$\bar{\mu}_{M_1^3}(x_1) = 1 - \underline{\mu}_{M_1^1}(x_1)$
$\underline{\mu}_{M_1^4}(x_1) = \underline{\mu}_{M_1^3}(x_1)$	$\bar{\mu}_{M_1^4}(x_1) = \bar{\mu}_{M_1^3}(x_1)$
$\underline{\mu}_{M_2^1}(x_1) = 0.4e^{-(x_1^2/0.2)}$	$\bar{\mu}_{M_2^1}(x_1) = e^{-(x_1^2/2.5)}$
$\underline{\mu}_{M_2^2}(x_1) = 1 - \bar{\mu}_{M_2^1}(x_1)$	$\bar{\mu}_{M_2^2}(x_1) = 1 - \underline{\mu}_{M_2^1}(x_1)$
$\underline{\mu}_{M_2^3}(x_1) = \underline{\mu}_{M_2^2}(x_1)$	$\bar{\mu}_{M_2^3}(x_1) = \bar{\mu}_{M_2^2}(x_1)$
$\underline{\mu}_{M_2^4}(x_1) = \underline{\mu}_{M_2^3}(x_1)$	$\bar{\mu}_{M_2^4}(x_1) = \bar{\mu}_{M_2^3}(x_1)$

with  $C = \begin{bmatrix} 0.1 & 0 \\ 0 & 1 \end{bmatrix}$ . In the following part, we compare the proposed SMC approach with a type-2 fuzzy control by considering plant under two cases: the  $\mathcal{H}_\infty$  disturbance attenuation performance without guaranteed cost constraint and the  $\mathcal{H}_\infty$  disturbance attenuation performance with guaranteed cost constraint.

*Case I:  $\mathcal{H}_\infty$  disturbance attenuation performance without guaranteed cost constraint*

i. Interval type-2 fuzzy control approach

Applying the method in [14], we use the following fuzzy controller to control the system (51) with time-delay

$$u(t) = \sum_{i=1}^2 \varphi_i(\theta(t)) K_i x(t), \quad (52)$$

where  $\theta(t)$  and  $\varphi_i(\theta(t))$  are defined in [14]. In terms of the  $\mathcal{H}_\infty$  performance, we obtain a feasible solution  $K_1 = [ 277.7 \quad 6.8 ]$ ,  $K_2 = [ 270.3 \quad 6.6 ]$  and  $\gamma_{\min} = 7.9086$ . The simulation results are shown in Figs. 2–4 in black dashdot line. Fig. 2 and Fig. 3 show the state responses of the close-loop system (51), respectively. Fig. 4 illustrates the response of the control force.

ii. SMC approach

Based on the given system matrices, we choose

$$G = [ 0.1 \quad 0.1 ], \quad U = \begin{bmatrix} 0.3 & 0.04 \\ 5.0 & 0.045 \end{bmatrix}.$$

According to Theorem 3 with  $P = 2$  and  $Z = 1$ , we obtain the matrix  $K = [ 94.2 \quad -27.8 ]$ , and  $\gamma_{\min} = 7.9101$ . Under the designed adaptive sliding mode controller in (41), where  $\mu = 0.01$ ,  $\lambda_1 = 2$ ,  $\lambda_2 = 1$ ,  $\varepsilon_1 = 1.4$ ,  $\varepsilon_2 = 1.2$ ,  $q_1 = q_2 = q_3 = q_4 = 100.0$ ,  $\hat{\delta}(0) = 0.01$ ,  $\hat{\eta}(0) = 0.01$ ,  $\hat{\alpha}_1(0) = 0.2$ ,  $\hat{\alpha}_2(0) = 0.4$ ,  $\hat{\alpha}_3(0) = 0.6$  and  $\hat{\alpha}_4(0) = 0.8$ , the responses of the closed-loop system (51) are obtained, which are shown in Figs. 2–4 (pink solid lines). Fig. 2 and Fig. 3 describe the state responses of the closed-loop system, respectively. Obviously, the stability of the inverted pendulum system is better under the presented SMC approach. The control force is depicted in Fig. 4.

*Case II:  $\mathcal{H}_\infty$  disturbance attenuation performance with guaranteed cost constraint*

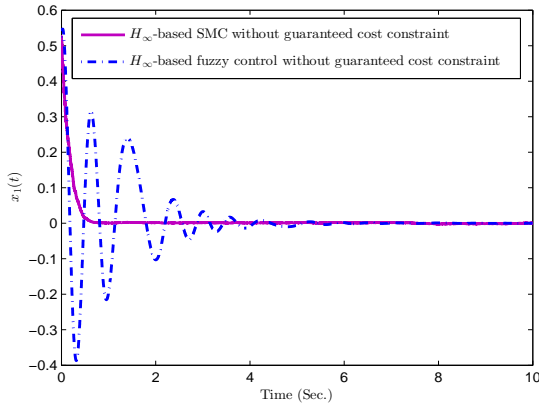
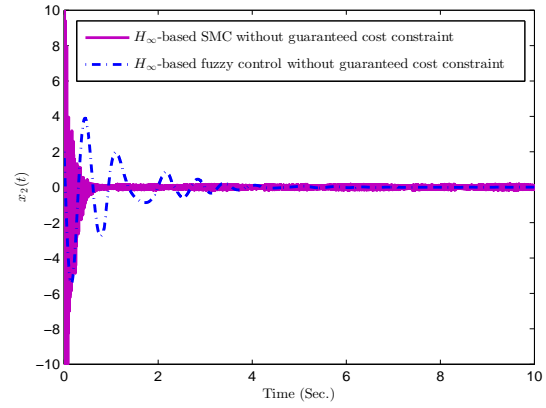
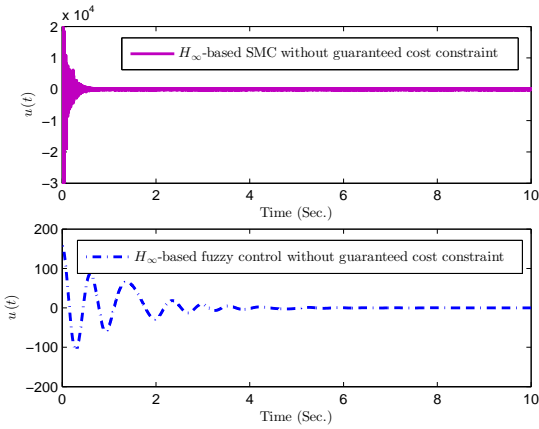
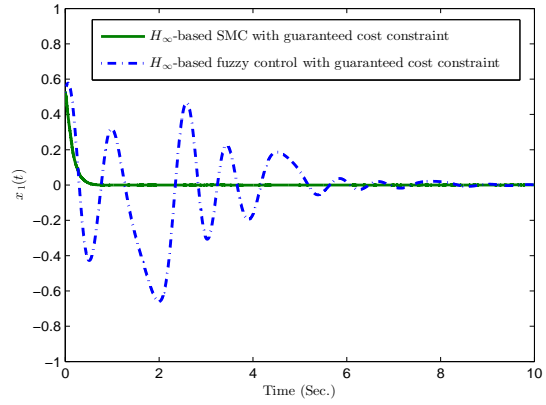
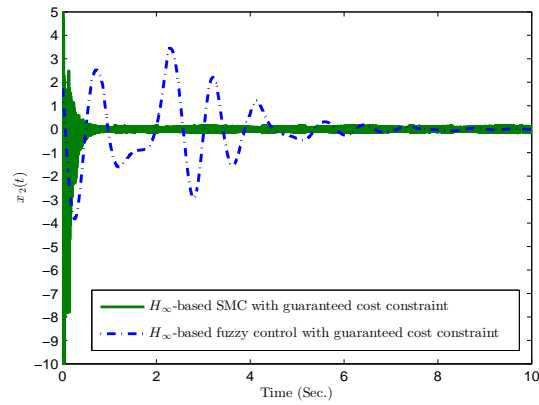
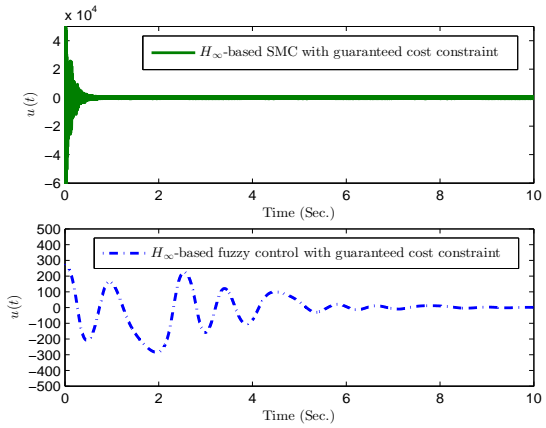
i. Interval type-2 fuzzy control approach

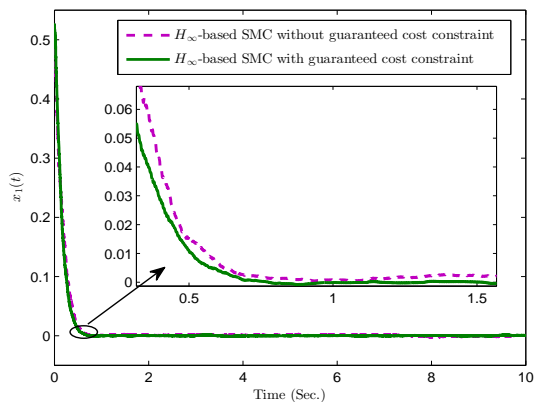
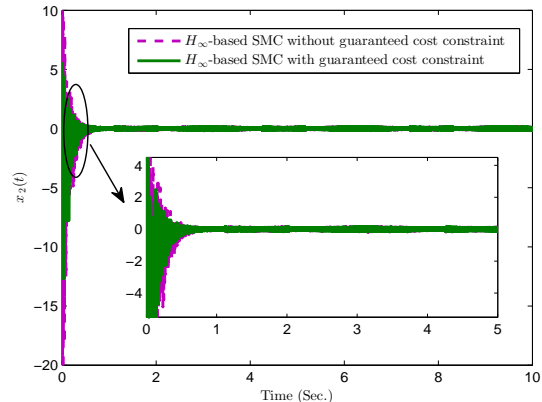
Considering the guaranteed cost function constraint, according to the method in [14], we obtain the controller gains  $K_1 = [ 549.5 \quad 16.4 ]$ ,  $K_2 = [ 343.9 \quad 9.0 ]$  and  $\gamma_{\min} = 7.9093$ ,  $\beta_{\min} = 0.1764$ . For the system (51) considering desired  $\mathcal{H}_\infty$  disturbance attenuation performance level  $\gamma_{\min}$  with guaranteed cost constraint index  $\beta_{\min}$ , we use the type-2 fuzzy controller (52) to control the inverted pendulum, the simulation results are shown in Figs. 5–7. The state responses of the closed-loop system (51) are shown with black dashdot line in Fig. 5 and Fig. 6. Fig. 7 describes the controller force.

ii. SMC approach

Considering desired  $\mathcal{H}_\infty$  disturbance attenuation performance with guaranteed cost constraint, according to Theorem 4 with the same given parameters, we obtain  $K = [ 129.5 \quad 69.6 ]$  and  $\gamma_{\min} = 7.9086$ ,  $\beta_{\min} = 0.1081$ . Using the designed adaptive sliding mode controller (41), the responses characteristics of the closed-loop system (51) are shown with pink solid lines in Fig. 5 and Fig. 6. Fig. 7 describes the controller force. Therefore, the SMC approach to the IT2 fuzzy time-delay systems with external disturbances is better than type-2 fuzzy control when considering  $\mathcal{H}_\infty$  disturbance attenuation performance and  $\mathcal{H}_2$  guaranteed cost performance in simulations.

*Remark 5:* Additionally, in order to compare the presented SMC approach with guaranteed cost constraint with the presented SMC approach without guaranteed cost constraint, we take the state response  $x_1$  from Fig. 2 and Fig. 5 into a frame, and similarly  $x_2$  from Fig. 3 and Fig. 6 into a frame. The compared results are shown in Fig. 8 and Fig. 9, from which we can find the presented SMC approach with guaranteed cost constraint is performed better than the presented SMC approach without guaranteed cost constraint.

Fig. 2. State response  $x_1(t)$  of the closed-loop system.Fig. 3. State response  $x_2(t)$  of the closed-loop system.Fig. 4. Control force of  $u(t)$ .Fig. 5. State response  $x_1(t)$  of the closed-loop system.Fig. 6. State response  $x_2(t)$  of the closed-loop system.Fig. 7. Control force of  $u(t)$ .

Fig. 8. State response  $x_2(t)$  of the closed-loop system.Fig. 9. Control force of  $u(t)$ .

## V. CONCLUSION

In this paper, the optimal  $\mathcal{H}_\infty$  guaranteed cost SMC problem has been solved for a class of IT2 fuzzy systems with time-varying delays and uncertainties. A new sliding surface has been designed in terms of different constraints. Then, by the designed sliding surface, the optimal  $\mathcal{H}_2$  guaranteed cost performance and  $\mathcal{H}_\infty$  performance of the plant have been analyzed. A novel adaptive sliding mode controller via output-feedback strategy has been presented to guarantee the reachability of the pre-specified sliding surface and ultimate uniform boundedness of the closed-loop system. Optimal schemes of guaranteed-cost-based SMC and  $\mathcal{H}_\infty$ -based SMC have been provided. Finally, the utilization of the inverted pendulum system has demonstrated the effectiveness of the control schemes proposed in this paper. However, the phenomenon of chattering can not be completely eliminated, which still be a research topic about how to reduce the chattering in the future studies. The recent techniques to reducing chattering in sliding mode control can be found in [40], [41].

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