DFT-based Channel Estimation Techniques for Massive MIMO Systems

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***Abstract*—In massive MIMO systems, efficient and highly accurate channel state information (CSI) at the base station are essential requirements for tackling the effect of pilot contamination to achieve the potential benefits of the systems. In this paper, we propose two discrete Fourier transform (DFT)-based channel estimation techniques for massive MIMO systems. The proposed methods mitigate the pilot contamination significantly via modifying the DFT-based estimation through iterations and most significant (MST) approaches. The simulation results obtained through these approaches demonstrated the effectiveness of the DFT-based channel estimation techniques in alleviate/eliminate the pilot contamination when compared to conventional channel estimators, in terms of channel estimation accuracy and achievable uplink rate.**

***Index Terms*—Massive MIMO; channel estimation; discrete Fourier transform (DFT).**

I. Introduction

Massive multiple input multiple output (MIMO) is currently considered a promising technology to achieve the next generation of mobile communication requirements. Since it can achieve dramatic improvements in the spectral and energy efficiencies, data rate and cell throughput. By employing a large number of antennae at the base station, significant beamforming gain can be achieved and the system capacity can serve an enormous number of users. In addition, a huge amount of mutual interference, thermal noise and small-scaled fading will be vanished [1]-[2]. The base station requires an accurate knowledge of the channel state information (CSI) to achieve the massive MIMO potential features. The CSI is obtained by transmitting predefined pilot sequences and estimating the channel coefficients from the received signals by applying an appropriate estimation algorithm [1]-[2]. To attain the optimum channel estimation accuracy, perfect orthogonal pilots allocations to the users are required, unfortunately, this requirement is not practical since the pilot overhead has to be proportional to the number of users in the system [1]-[3]. The size of the channel coherence block limits the number of orthogonal pilots. Thus, the pilot signal need to be reused in the neighbouring cells, creating spatially correlated inter-cell interference known as pilot contamination that degrades the massive MIMO performance [1]-[3] .

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Addressing the pilot contamination problem has motivated several approaches to address the pilot contamination problem i.e. by reducing the number of the employed pilots such as compressed sensing (CS) techniques in [4] and [5], and the blind estimation scheme in [6] to eliminate the pilot contamination problem completely. However, up to date, most of the proposed CS schemes and blind methods have major disadvantages, such as high complexity and low quality performance when compared with pilot assisted methods. The authors of [3] proposed a superimposed channel estimation approach by adding a pilot signal with low power to the data signal at the transmitter. The superimposed signal is then utilised at the receiver for channel estimation. However, a proportion of the power allocated to the training signal is wasted, which has a negative impact on energy efficiency. In addition, this scheme causes inter-cell interference, which known as cross contamination.

Meanwhile, orthogonal frequency division multiplexing (OFDM) can also improve system performance and increase spectral efficiency under frequency selective channels. Therefore, integration of both massive MIMO and OFDM technologies, which is usually denoted as massive MIMO-OFDM, is essential to enhance system capacity. Massive MIMO-OFDM is becoming a promising alternative among the upcoming 5G technologies [7]. Since OFDM has been design in way that the symbol time is larger than the duration of the channel impulse response (CIR) significantly so that the channel coefficient components are concentrated at the lower frequency region while the noise and interference are spread over the whole time domain region.

Given the aforementioned background and motivation, in this paper, the pilot contamination problem of massive MIMO systems is addressed using two modified DFT-based channel estimation approaches, i.e., iterative DFT and DFT-based Most Significant Taps Approach (DMSTA).

In DFT-based approach, the channel frequency response (CFR) is first estimated by applying the conventional channel estimation, i.e., linear minimum mean-square error (LMMSE) algorithm in the frequency domain. Then, the IDFT is applied to obtain the (CIR) in time domain [8], [9]. Then a truncation of the CIR is performed to improve the estimation accuracy approaches in the time domain. Finally, DFT is applied to the truncated CIR to obtain the estimated CFR [8], [9]. In iterative

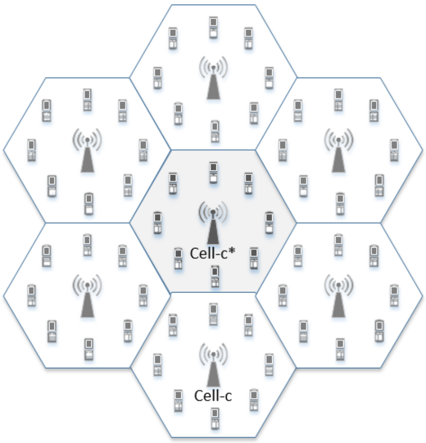

Fig. 1: Illustration of the system model of a multi-cell multi-user massive MIMO.

DFT, the previously described procedure in DFT estimation is executed several times to address the pilot contamination problem. While in DMSTA, more CIR truncation will be applied based on a threshold value to chose the taps with the most significant power.

The remainder of this paper is organised as follows. The massive MIMO system model is described in Section II. The conventional-DFT based estimator is presented in Section III. The iterative DFT-based channel estimation is introduced in Section IV. The achievable uplink rate is analysed in Section V. Complexity computation is illustrated in section VI. Numerical illustrations are provided in Section VII. Finally, conclusions are drawn in Section VIII.

The following notations are adopted throughout the paper. The superscripts $(.)^H$, $(.)^*$ and $(.)^{-1}$ denote the conjugate transpose operator, conjugate operator and the inverse operation operator, respectively. $tr(.)$ denotes the trace operator, and I denotes the identity matrix (of appropriate dimensions). The operation $\circledast$ denotes convolution process . Underlined symbols denotes time domain, while the non-underline symbols represents the frequency domain. The Frobenius norm of a matrix X are denoted by $\|X\|_F$. The $E\{.\}$ denotes expectation with regard to all random variables within the brackets. A Gaussian stochastic variable o is the denoted by $o \sim CN(r, q)$, where r is the mean and q is the variance. A circularly symmetric complex Gaussian stochastic vector $\mathbf{x}$ is denoted by $x \sim CN(x, Q)$, where x is the mean and $\mathbf{Q}$ is the covariance matrix.

II. System Model

We consider a time division duplexing (TDD) multi-cell massive MIMO system with C cells. Each cell comprises of M antennas at the BS and N single antenna users, i.e., $M >> N$. To improve the spectral efficiency, orthogonal frequency division multiplexing (OFDM) is adopted [10], [11].

At the beginning of the transmission, all mobile stations in all cells synchronously transmit OFDM pilot symbols to their serving base stations. Let the OFDM pilot symbol of user n in the c-th cell be denoted by $x_c^n[K]$, where K is the number of subcarriers. The OFDM transmission partition the multipath channel between the user and each antenna of the BS into parallel independent additive white Gaussian noise (AWGN) sub-channels in the frequency domain. Each sub-channel is associated with a subcarrier. Let $h^n_{c^*,c,i}[k]$ denote the k-th sub-channel coefficient between the n-th user in the c-th cell and the i-th antenna of the BS of cell c^* in the uplink. The received signal $y_{c^*,i}[k]$ by the i-th antenna element of the cell c^* at the k-th subcarrier can be expressed as

$$y_{c^*,i}[k] = \sum_{n=1}^{N} h^n_{c^*,c^*,i}[k] x^n_{c^*}[k] + \sum_{c=1,c\neq c^*}^{C} \sum_{n=1}^{N} h^n_{c^*,c,i}[k] x^n_c[k] + v_{c^*,i}[k], \tag{1}$$

for all $1 \leq i \leq M$ and $1 \leq c \leq C$, where $v_{c^*,i}[k] \sim CN(0, \sigma_v^2)$ is the AWGN at the i-th antenna of the BS in cell c^* at the k-th subcarrier.

The channel coefficient is modelled as $h^n_{c^*,c^*,i}[k] = \sqrt{\phi_{c^*,c,i}[k]}\psi_{c^*,c,i}[k]$, where $\phi_{c^*,c,i}[k]$ model the path-loss and shadowing (large-scale fading), while the term $\psi_{c^*,c,i}[k]$ is assumed to be independent identical distribution (i.i.d) of unknown random variable with $CN(0,1)$ (small-scale fading) [4].

The estimated channel using the conventional (LMMSE) estimator that completed entirely in the frequency domain based upon the observation of $y_{c^*,i}[k]$ can be expressed as

$$\hat{h}^{n,LMMSE}_{c^*,c^*,i}[k] = E[h^n_{c^*,c^*,i}[k](y_{c^*,i}[k])^H](E[y_{c^*,i}[k] (y_{c^*,i}[k])^H])^{-1} y_{c^*,i}[k], \tag{2}$$

$$\hat{h}^{n,LMMSE}_{c^*,c^*,i}[k] = E[h^n_{c^*,c^*,i}[k](\sum_{n=1}^{N} h^n_{c^*,c^*,i}[k] x^n_{c^*}[k] + \sum_{c=1,c\neq c^*}^{C} \sum_{n=1}^{N} h^n_{c^*,c,i}[k] x^n_c[k] + v_{c^*,i}[k])^H] E[(\sum_{n=1}^{N} h^n_{c^*,c^*,i}[k] x^n_{c^*}[k] + \sum_{c=1,c\neq c^*}^{C} \sum_{n=1}^{N} h^n_{c^*,c,i}[k] x^n_c[k] + v_{c^*,i}[k])(\sum_{n=1}^{N} h^n_{c^*,c^*,i}[k] x^n_{c^*}[k] + \sum_{c=1,c\neq c^*}^{C} \sum_{n=1}^{N} h^n_{c^*,c,i}[k] x^n_c[k] + v_{c^*,i}[k])^H])^{-1} y_{c^*,i}[k]. \tag{3}$$

Assuming that the channel components are uncorrelated, then $E\{h^n_{c^*,c^*,i}[k](h^n_{c^*,c^*,i}[k])^H\} = 1$. Hence (3) can expressed as (4) at the bottom of this page.

$$\hat{h}^{n,LMMSE}_{c^*,c^*,i}[k] = \sum_{n=1}^{N} (x^n_{c^*}[k])^H \underbrace{[\sum_{n=1}^{N} x^n_{c^*}[k](x^n_{c^*}[k])^H + \sum_{c=1}^{C}\sum_{n=1}^{N} x^n_c[k](x^n_c[k])^H + \sigma_v^2]^{-1}}_{=\xi} y_{c^*,i}[k]. \tag{4}$$

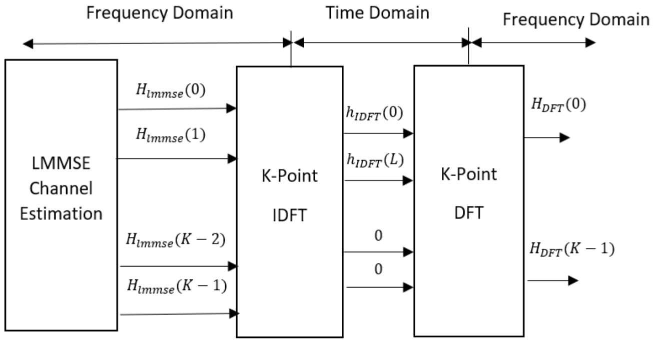

Fig. 2: DFT-based channel estimation.

III. DFT-Based Channel Estimator

To improve the LMMSE estimation accuracy, the DFT-based channel estimation algorithm has been proposed to reduce the noise and interference components in the time domain. The LMMSE estimated sample of CIR can be expressed

$$\underline{h}_{c^*,c^*,i}^{n,LMMSE}[\omega] = IDFT_k\{\hat{h}_{c^*,c^*,i}^{n,LMMSE}[k]\}, \tag{5}$$

for $1 \leq k \leq K$, where $IDFT_k\{\}$ indicates K-point IDFT and ω is the time domain sample index. As the estimated CIR from the LMMSE has most of its power concentrated in a few initial samples, the CIR is typically limited to the number of the channel taps L, which is much smaller than the number of subcarriers. Hence, (5) can be expressed as:

$$\hat{\underline{h}}_{c^*,c^*,i}^{n,IDFT}[\omega] = \begin{cases} \underline{h}_{c^*,c^*,i}^{n,LMMSE}[\omega], \ 0 \leq l \leq L \\ 0, \ Others \end{cases}. \tag{6}$$

In doing so, more noise and interference are cancelled, and the intended channel information is retained. Next, the DFT operation is conducted to recover the channel responses into the frequency domain, as follow:

$$\hat{h}_{c^*,c^*,i}^{n,DFT}[k] = DFT_k\{\hat{\underline{h}}_{c^*,c^*,i}^{n,IDFT}[\omega]\}. \tag{7}$$

From what has been described earlier, it is clear that compared with LMMSE, the DFT based channel estimation method makes use of IDFT/DFT to suppress noise and interference in the time domain [12], as shown in the basic block diagram of the DFT-based estimation in Fig. 2 (for notational simplicity in Fig.2, we refer to $\hat{h}_{c^*,c^*,i}^{n,LMMSE}[k], \hat{h}_{c^*,c^*,i}^{n,IDFT}[w]$ and $\hat{h}_{c^*,c^*,i}^{n,DFT}[w]$ as $H_{lmmse}(k), h_{IDFT}(L)$ and $H_{DFT}(k)$, respectively).

We now derive an expression for the approximated $\hat{h}_{c^*,c^*,i}^{n,DFT}[k]$, by taking the IFFT and FFT to the CFR of the LMMSE estimated channel multiplied by the rectangular function representing the limitation of the CIR of the DFT-based estimator definition that can be written as follow

$$\hat{h}_{c^*,c^*,i}^{n,DFT}[k] = \mathcal{F}\{Rect[\omega]\mathcal{F}^{-1}\{\hat{h}_{c^*,c^*,i}^{n,LMMSE}[k]\}\}, \tag{8}$$

where $\mathcal{F}/\mathcal{F}^{-1}$ stands for the IDFT/DFT operations, respectively, where $Rect[\omega]$ can be given as:

Algorithm 1 Iterative-based DFT Channel Estimation

1: **Inputs:** Number of the channel taps L, Threshold value δ.
2: Perform the LMMSE estimation to obtain the $\hat{h}_{c^*,c^*,i}^{n,LMMSE}[k]$.
3: $p = 0$
4: Perform the IFFT to transform the CFR to time domain, as described in section III.
5: Truncate the CIR for the time delay $L-1$.
6: Perform the DFT-based estimator to obtain $\hat{h}_{c^*,c^*,i}^{n,DFT,p}[k]$.
7: **Iterative Procedure:**
8: If $\delta \leq max|\hat{h}_{c^*,c^*,i}^{n,DFT,p}[k] - \hat{h}_{c^*,c^*,i}^{n,LMMSE}[k]|$, then $p = p+1$ and **Repeat:** 4 to 6.
10: **Else if:**
9: **Output:** Return the estimated channel $\hat{h}_{c^*,c^*,i}^{n,DFT,p+1}[k]$

$$Rect[\omega] = \begin{cases} 1, \ \omega < L \\ 0, \ \omega \geq L \end{cases},$$

$$\hat{h}_{c^*,c^*,i}^{n,DFT}[k] = \{\{Rect[\omega]\} \circledast \hat{h}_{c^*,c^*,i}^{n,LMMSE}[k], \tag{9}$$

$$\hat{h}_{c^*,c^*,i}^{n,DFT}[k] = \psi(k) \circledast \hat{h}_{c^*,c^*,i}^{n,LMMSE}[k], \tag{10}$$

where $\boldsymbol{\psi}(k)$ can be given as $\boldsymbol{\psi}(k) = \frac{1}{\sqrt{LK}} e^{\frac{-j\pi(L-1)}{K}} Sinc(\frac{\pi k L}{K})$.

IV. Iterative based-DFT Channel Estimation

To address the pilot contamination problem, we proposed an iterative algorithm for a DFT-based estimator for a massive MIMO-OFDM system. Accurate channel estimation can be obtained by applying algorithm 1 [12] and [13].

V. DMSTA-Based Approach

Practically, the CIR contains many taps with no significant energy, by neglecting those taps, the noise and interference through pilot contamination will be eliminated and that can improve the channel estimation performance significantly. In this paper we will use thresholding approach by retaining the channel taps whose energy is above a threshold value η and set the other taps to zero, while the suitable value of η Based on the facts mentioned in the section I, that most channel taps will concentrated on the lower region in time domain while the noise and pilot contamination components will be spread over the CIR. So, by estimating the power of the noise and interference by averaging the samples on the noise and interference region [14]-[16]. Thus, the threshold value can be given as follows

$$\eta = \zeta\alpha. \tag{11}$$

where ζ can be given as

$$\zeta = \frac{1}{L}\sum_{k=L}^{K} |\hat{H}_{c^*,c^*}^{n,DFT}|^2. \tag{12}$$

and α is a scaling factor that can be adjusted as noise margin.

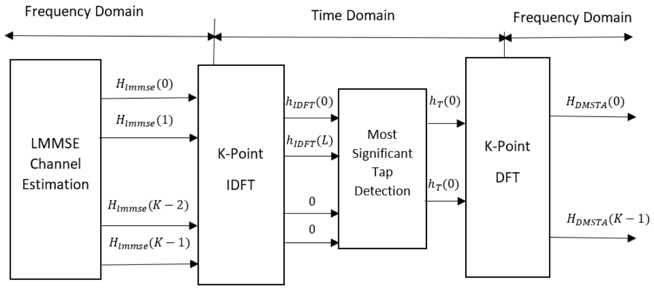

Fig. 3: DMSTA-based channel estimation.

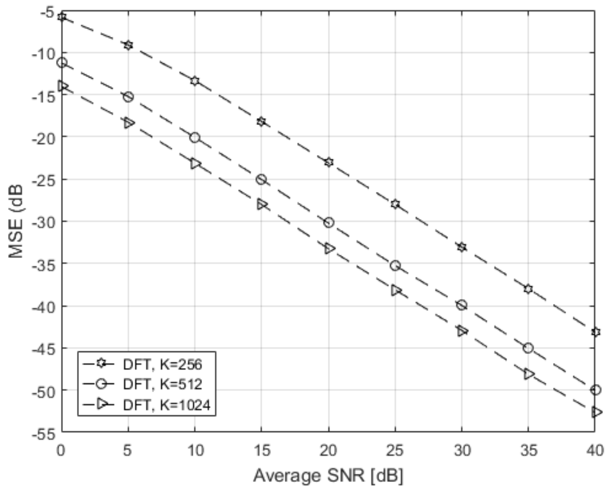

Fig. 4: The MSE of the DFT-based channel estimation versus the SNR for the number of path is 128 and for different values of $\{K = 256, 512$ and $1024\}$ so the compression ratio (CR) (i.e. L/K) is to be $CR = \{0.5, 0.25$ and $0.125\}$.

So, the most significant taps block will detect the significant channel taps [13], as follows:

$$\hat{\mathbf{h}}_{c^*,c^*}^{n,T} = \begin{cases} \hat{h}_{c^*,c^*}^{n,T}, |\hat{h}_{c^*,c^*}^{n,DFT}| > \eta \\ 0, \; otherwise \end{cases} . \tag{13}$$

Therefore, the DMSTA-based channel estimation algorithm can be expressed in the frequency domain as

$$\hat{H}_{c^*,c^*}^{n,DMSTA} = DFT\{\hat{h}_{c^*,c^*}^{n,T}\} \tag{14}$$

More elaborations on DMSTA approach can be found in Fig. 3.

VI. Numerical Results

In this section, various computer simulations are carried out to evaluate the performance of the LMMSE, DFT-based and iterative-DFT estimators. We assumed that $N = 20$ users, $M = 100$ antennas and the system under the influence of strong pilot contamination i.e. $\phi_{c^*,c^*,i} = 1$ and $\phi_{c^*,c^*,i} = 0.7$. The relative MSE can be written as

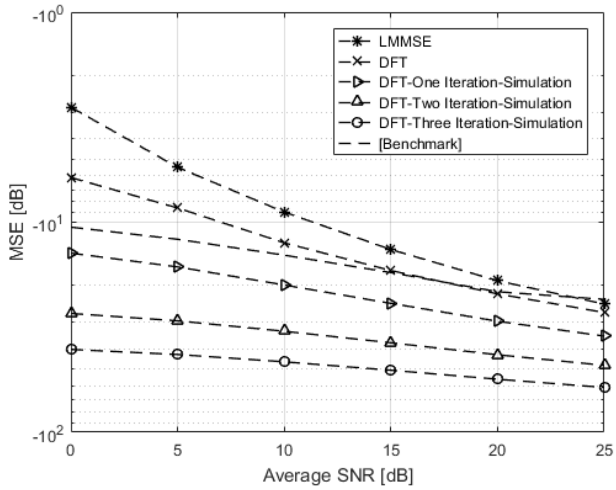

Fig. 5: The MSE comparison of LMMSE, DFT and iterative-DFT-based estimators as a function of SNR.

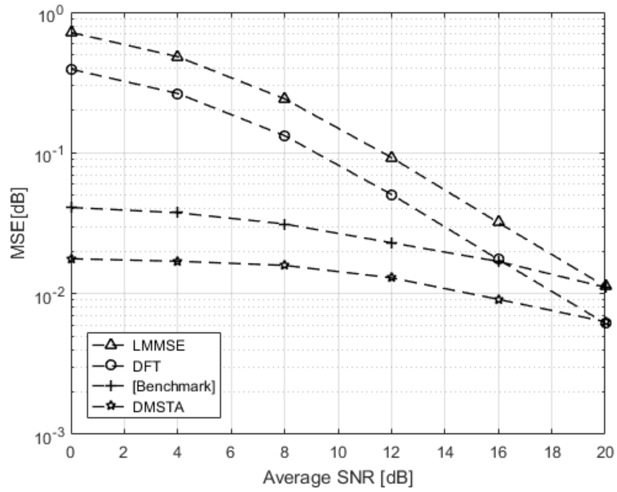

Fig. 6: MSE of the conventional LMMSE, DFT and DMSTA and the modified DFT-based in [9] versus SNR.

$$MSE = \frac{E\{||h_{c^*,c^*,i}^{n,DFT} - \hat{h}_{c^*,c^*,i}^{n,DFT}||_F^2\}}{E\{||h_{c^*,c^*,i}^{n,DFT}||_F^2\}}. \tag{15}$$

Fig. 4 shows the MSE performance of the DFT-based channel estimation with different values of $K = \{256, 512$ and $1024\}$, $L = 128$ taps, so the compression ratio (CR), i.e. L/K is to be $CR = \{0.5, 0.25$ and $0.125\}$. The results indicate that the performance of the conventional DFT-based estimator is enhanced when decreasing the number of the CR, as a result of eliminating more components of noise and interference.

Fig. 5 demonstrates the MSE performance of the conventional LMMSE, DFT, the modified DFT-based estimator in [9], and the iterative DFT-based channel estimation with one, two and three iterations. The results obviously show the iterative DFT estimators outperforms the other estimators.

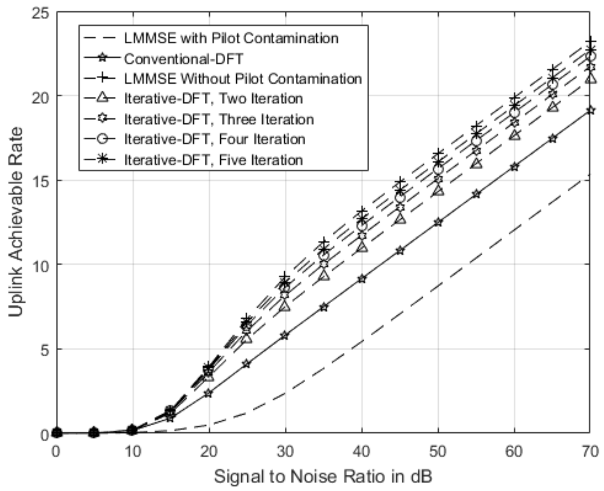

Fig. 7: Uplink achievable rate for LMMSE with no pilot contamination, conventional DFT, Iterative DFT and LMMSE under the effect of strong pilot contamination versus SNR.

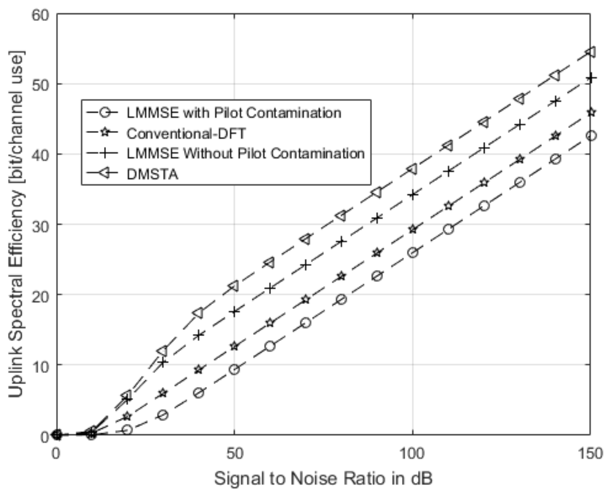

Fig. 8: Uplink achievable rate for LMMSE with no pilot contamination, conventional DFT, DMSTA and LMMSE under the effect of strong pilot contamination versus SNR.

Fig. 6 demonstrates the MSE performance of the conventional LMMSE, DFT, DMSTA, the modified DFT-based estimator in [9]. Obviously, it can be seen that the DMSTA provide better estimation performance compared to other estimators.

Fig. 7 and Fig. 8 shows the uplink achievable rate of the matched receiver combining (MRC) for the iterative DFT-based (2-5 iterations) and DMSTA, respectively. The performance of the proposed techniques is compared with the LMMSE with pilot contamination, conventional DFT-based,and the exact LMMSE (LMMSE with no noise and pilot contamination). It can be seen that the iterative DFT-based estimators and DMSTA performed closed to the exact LMMSE estimator with increasing the iterations. This demonstrates a significant improvement in addressing the pilot contamination problem.

VII. Conclusion

In this paper, we propose two modified DFT-based channel estimation techniques. The proposed estimators tackled the pilot contamination problem by applying the iterative DFT and the MST approaches. The simulation results showed that the proposed technique provides much better performance compared to the conventional methods in terms of addressing the pilot contamination problem. Furthermore, the estimation performance of the DFT-based based estimator can be enhanced by increasing the number of subcarriers.

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